

## MODIFIED FUNCTION PROJECTIVE SYNCHRONIZATION OF MULTISTABLE SYSTEMS

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### ABSTRACT

The design of generalized form of control functions capable of engineering desired form of synchronization such as complete synchronization, antisynchronization, projective synchronization and function projective has very important applications in real life situations. Inspired by practical application of generalized form of synchronization, controllers which enable Modified Function Projective Synchronization (MFPS) between two multistable chaotic systems are derived based on the Routh-Hurwitz criterion via Open Plus Closed Loop (OPCL) technique. The controllers are derived in such a way that the transformation matrix can be chosen arbitrarily to achieve a desired synchronization scheme. Numerical simulation results presented show the effectiveness of the proposed MFPS for transformation matrix with different constant values and transformation matrix with time dependent functions. The result shows that the proposed MFPS could be used to securely transmit different information signals at the same time.

**Keywords:** Modified Function Projective Synchronization (MFPS); OPCL; Routh-Hurwitz Criterion; Multistable Chaotic System; Open Plus Closed Loop (OPCL)

### INTRODUCTION

The fundamental principle of synchronization in dynamical systems has been shown to be useful in gaining new insights into collective behaviour of dynamical systems. Synchronization of chaotic systems is an important feature of nonlinear dynamical systems which has been intensively investigated due to its potential practical application in secure communication, neuron systems, laser dynamics, image processing, chemical, biological systems and information science (Chen and Dong, 1998; Sprott, 2003; Aguirre *et al.*, 2006; Gross *et al.*, 2006; Miliou *et al.*, 2007; Ghosh and Bhattacharya, 2010). Practical implementation of application of synchronization to secure communication via public domain fiber-optic links is presented in (Argyris *et al.*, 2005). Based on the importance of synchronization, different types of synchronization such as complete synchronization (Yao *et al.*, 2013; Singh *et al.*, 2014), antisynchronization (Ojo *et al.*, 2011; Yang, 2012), projective synchronization (Wang and Chen, 2010; Nian and Wang, 2013), phase synchronization (Wang *et al.*, 2010; Gholizade *et*

*al.*, 2013), generalized synchronization (Feng *et al.*, 2010; Koronovskii *et al.*, 2013), modified projection synchronization (Li, 2007; Farivar *et al.*, 2012), function projective synchronization (Min, 2013; Kareem *et al.*, 2012), modified function projective synchronization (Yu *et al.*, 2013; Lv *et al.*, 2012) and others (Runzi *et al.*, 2011; Ojo *et al.*, 2014) have been developed. Sequel to the discovery of different types synchronization, several linear and nonlinear methods such as active control (Njah, 2011; Ojo *et al.*, 2013), linear feedback (Ma *et al.*, 2012; Baogui Xin and Zhiheng Wu, 2015), backstepping (Njah *et al.*, 2010; Njah and Ojo 2010), sliding mode control (Jawaada *et al.*, 2012; Zribi *et al.*, 2010), impulsive control (Lu *et al.*, 2013; Li *et al.*, 2015), adaptive control (Liu *et al.*, 2008; Yang, 2011) and open-plus-closed-loop (Roy *et al.*, 2011; Grosu *et al.*, 2009; Sudheer and Sabirn, 2010; Padmanaban *et al.*, 2011) have been discovered in search of improved and effective methods for achieving stable synchronization. Among the nonlinear methods of synchronization, the OPCL is outstanding because it has been applied to synchronize both identical and non- identical systems loop (Roy *et*

al., 2011; Grosu *et al.*, 2009; Sudheer and Sabirn, 2010; Padmanaban *et al.*, 2011). Also, it has been discovered that it can be practically implemented (Padmanaban *et al.*, 2012).

Meanwhile, multistability is a nonlinear dynamical behaviour in which more than one attractor coexist in a dynamical system for a given set of parameters and transition from one form of attractor to another form of attractor in this dynamical system can be achieved by switching the initial conditions. This phenomenon of multistability has attracted a lot attention due to its existence and occurrence in various fields of life such as neuroscience, laser optics, biology, physics and chemistry (Brambilla *et al.*, 1991; Prengel *et al.*, 1994; Schiff *et al.*, 1994; Marmillot *et al.*, 1991). Lyapunov exponents of multistable systems vary with respect to the choice of initial conditions. The basins of attraction of different attractors of multistable systems are interwoven in a complex manner and separated by one or several chaotic saddles with the dimension of the basin boundaries very close to the dimension of the state space (Saha *et al.*, 2014). In a nutshell multistable systems are characterized by a high degree of complex dynamical behaviour due to the interaction among the coexisting attractors (Chudzik *et al.*, 2011). To the best of our knowledge, Modified function projective synchronization of the multistable system has not been considered via the OPCL method despite the practical applications of the method and the system to real life situation.

Motivated by the above discussion, this paper considers a generalized form MFPS for a multistable system that is capable of engineering a desired synchronization scheme such as complete synchronization, antisynchronization, projective

synchronization and function projective synchronization and which has very important applications in real life situations. The rest of this paper is organized as follows. In section 2, detail description of the material and method are presented. Section 3 gives the result and discussion while section 4 concludes the paper.

## MATERIALS AND METHOD

The multistable system in (Saha *et al.*, 2014) is described by the set of first order differential equations below

$$\begin{aligned}\dot{x} &= yz + a \\ \dot{y} &= x^2 - y \\ \dot{z} &= 1 - bx\end{aligned}\quad (1)$$

For the given differential equations (1), the matrix  $J$  is

$$J = \begin{pmatrix} 0 & z & y \\ 2x & -1 & 0 \\ -b & 0 & 0 \end{pmatrix}\quad (2a)$$

The equilibrium point for the system is given as  $x_0 = 1/b, y_0 = 1/b^2, z_0 = -ab^2$  and its Jacobian is

$$J_0 = \begin{pmatrix} 0 & -ab^2 & \frac{1}{b^2} \\ \frac{2}{b} & -1 & 0 \\ -b & 0 & 0 \end{pmatrix}\quad (2b)$$

and the corresponding characteristic equation is

$$\lambda^3 + \lambda^2 + (b^{-1} + 2ab)\lambda + b^{-1} = 0\quad (3)$$

It is worthy of note that though the system has one fixed point it exhibit complex dynamical nonlinear behaviour such as chaos, shrimp and multistability (Saha *et al.*, 2014). Of particular interest to us among these nonlinear behaviour is multistability where the bifurcation figures varies with the initial conditions and some of them are shown in Figure 1. Two-dimension attractors for the initial conditions used in this research work is shown in Figure 2

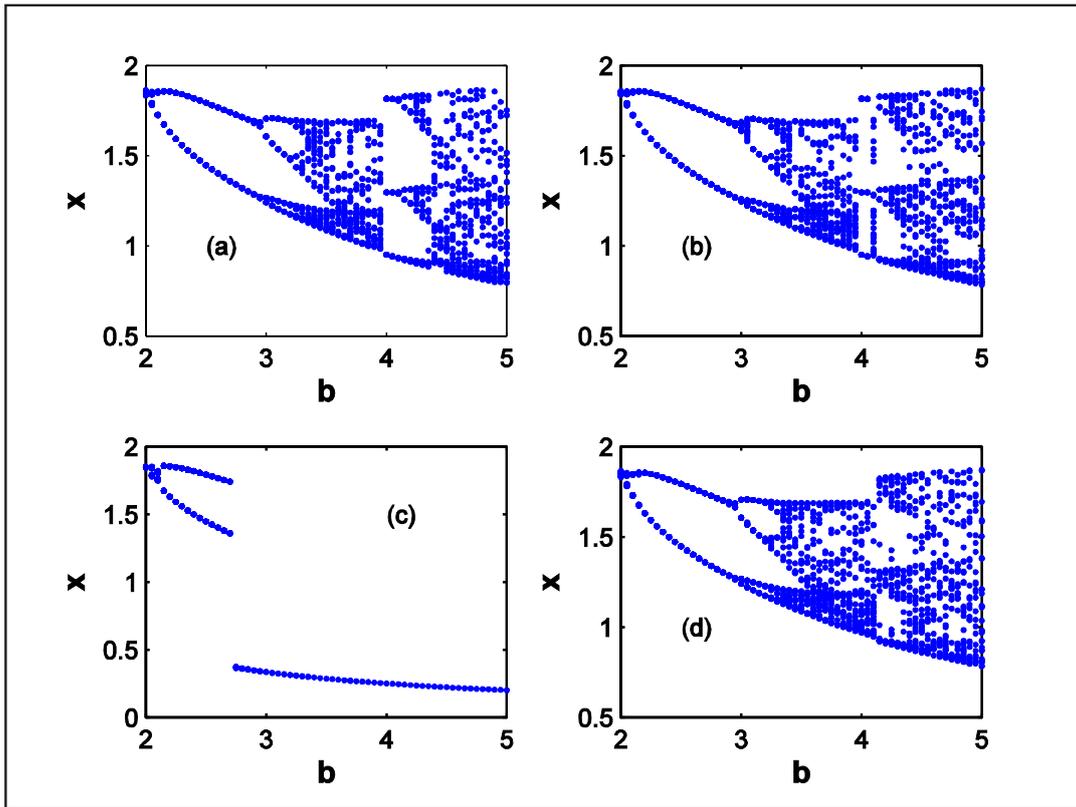


Figure 1: Bifurcation diagram for  $a = 0.01$  and  $b=4.0$  with initial condition (a) 2.2, 3.1, 1.1 (b)  $-0.2, -0.1, -0.1$  (c) 0.2, 0.1, 0.1 (d)  $-0.2, -0.1, 0.1$

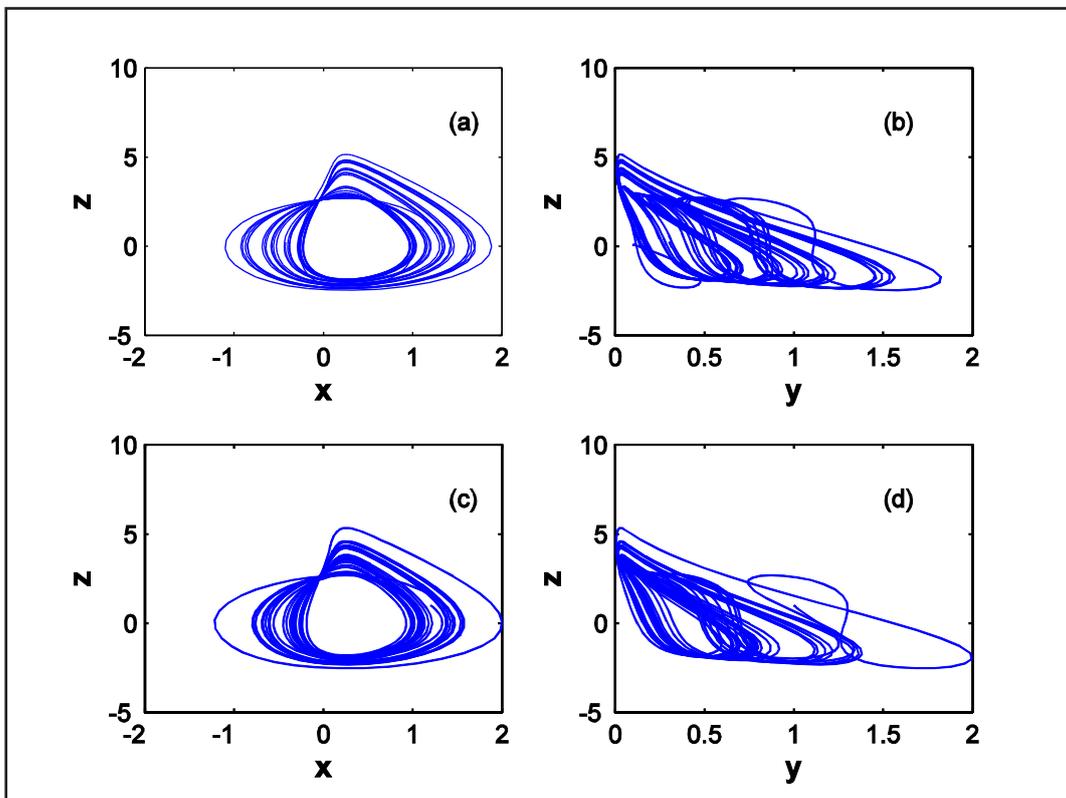


Figure 2: Phase portrait of chaotic attractor for initial conditions 1.0, 0.1, 0.1 ((a) and (b)) and for initial conditions 1.2, 1, 1 ((c) and (d))

Meanwhile, using the OPCL technique discussed by (Roy *et al.*, 2011; Grosu *et al.*, 2009), the technique is applied to realize modified function projective synchronization in two nonlinear systems. Now, to achieve this, consider a drive nonlinear system:

$$\dot{x} = Ax + f(x) \tag{4}$$

and the response nonlinear system:

$$\dot{y} = By + f(y) \tag{5}$$

where  $x = (x^1, x^2, \dots, x^n)^T \in \mathbb{R}^n$  and  $y = (y_1, y_2, \dots, y_n)^T \in \mathbb{R}^n$  are the state vectors of system (4) and (5) respectively.  $A, B \in \mathbb{R}^{n \times n}$ , are the constant matrices.  $n$  is the order of the systems and  $f(x), f(y)$  and are nonlinear functions of the systems which are continuous and differentiable. The response system dynamics after the control function has been applied is

$$\dot{y} = Cy + f(y) + U \tag{6}$$

where  $U$ , the coupling function is defined as

$$U = \dot{g} - f(g) + (H - J(g)) e \tag{7}$$

where  $J(g)$  is the Jacobian of  $f(g)$  dynamical system,  $H$  is an arbitrary constant Hurwitz matrix ( $n \times n$ ) whose elements are such that all its eigenvalues have negative real parts and goal dynamics  $g = \alpha(t)x$  where  $g = (g_1, g_2, g_3, \dots, g_n)^T$  and  $\alpha = (\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n)^T$  are arbitrary scaling constants or functions. Also the error state variable  $e = (y - g)$  is the error between the response and the goal dynamics at any time. Expanding  $f(y) = f(g + e)$  using Taylor's yields

$$f(y) = f(g) + \frac{\partial f(g)}{\partial g} e + \dots \tag{8}$$

Using (6) and (8), the error dynamics of the drive and response systems can be written as

$$\dot{e} = He \tag{9}$$

where  $H$  is Hurwitz matrix. Then, if  $e \rightarrow 0$  as  $t \rightarrow \infty$  asymptotic stable synchronization is achieved.

### RESULTS AND DISCUSSION

In this section, detail derivation of generalized form of control functions that is capable of realizing MFPS between two multistable systems is presented. To achieve this goal the drive

multistable system is given as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} x_2 x_3 + a \\ x_1^2 - x_2 \\ 1 - b x_1 \end{pmatrix} \tag{10}$$

while the response is given as:

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{pmatrix} = \begin{pmatrix} y_2 y_3 + a \\ y_1^2 - y_2 \\ 1 - b y_1 \end{pmatrix} \tag{11}$$

The Jacobian of the response system (11) is

$$J(y) = \begin{pmatrix} 0 & y_3 & y_2 \\ 2y_1 & -1 & 0 \\ -b & 0 & 0 \end{pmatrix} \tag{12}$$

For simplicity the Hurwitz matrix is chosen as

$$H = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{13}$$

The choice of the transformation matrix is

$$\alpha = \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{pmatrix} \tag{14}$$

To achieve the desired dynamical goal,

$$\begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix} = \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \alpha_1 x_1 \\ \alpha_2 x_2 \\ \alpha_3 x_3 \end{pmatrix} \tag{15}$$

Using the expression in (7) the expression of control functions that achieve modified function projective synchronization is given as

$$U = \begin{pmatrix} \alpha_1 x_1 + \alpha_1 \dot{x}_1 - g_2 g_3 - a \\ \alpha_2 x_2 + \alpha_2 \dot{x}_2 - g_1^2 + g_2 \\ \alpha_3 x_3 + \alpha_3 \dot{x}_3 + b g_1 - 1 \end{pmatrix} + \begin{pmatrix} -e_1 - g_3 e_2 - g_2 e_3 \\ -2g_1 e_1 \\ b e_1 - e_3 \end{pmatrix} \tag{16}$$

In order to ascertain the effectiveness of the analytical result, we solved equations (10), (11) and (16) using ode45 fourth order Runge-Kutta algorithm run on MATLAB. Furthermore, the system parameter values and initial conditions shown in Figure 2 were used to ensure chaotic dynamics of the state variables of the multistable system. While, the choice of  $\alpha$  determines by a desired synchronization scheme such as complete synchronization, antisynchronization, projective synchronization, hybrid synchronization to be achieved. However, in this numerical simulation only two cases shall be considered

#### I. Modified Projective Synchronization:

Choosing the scaling parameter value as  $\alpha_1 = 1/3$ ,

$a_2 = 1/2$ ,  $a_3 = 2.0$ , modified projective synchronization of the drive system (10) and the response system (11) is achieved when the controllers are activated for  $t \geq 70$ . This is indicated by the convergence to zero of the error

state variables as shown in Figure 3 and the projection of the state variables of the drive multistable system on the response system as shown in Figure 4. The recovered scaling parameters are shown in Figure 5.

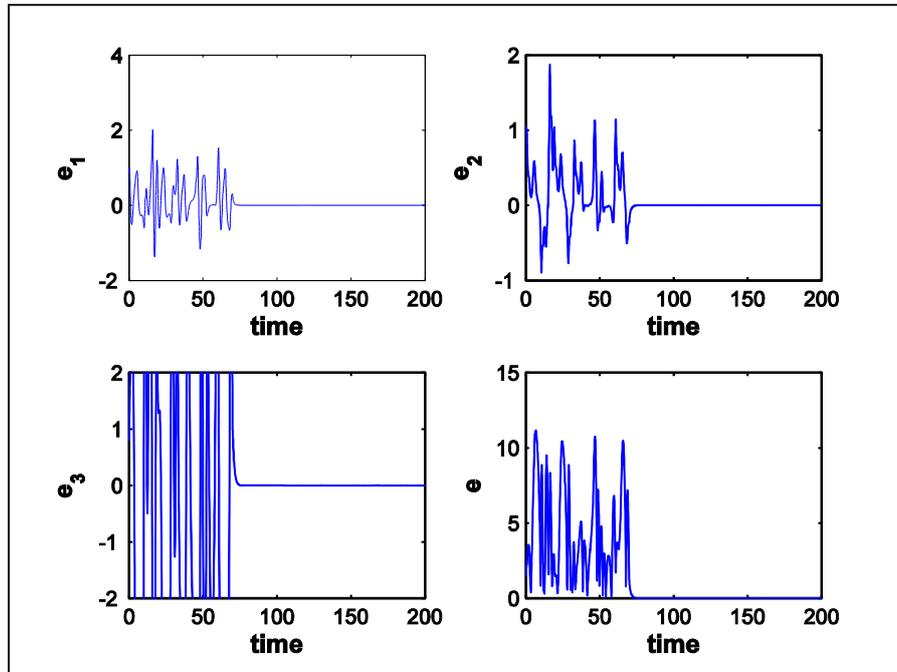


Figure 3: The dynamics of the error variables between the drive and the response systems with controllers deactivated for  $0 < t < 70$  and activated for  $t \geq 70$  where  $e_1 = x_1 - g_1$ ,  $e_2 = x_2 - g_2$ ,  $e_3 = x_3 - g_3$  and  $e = \sqrt{e_1^2 + e_2^2 + e_3^2}$

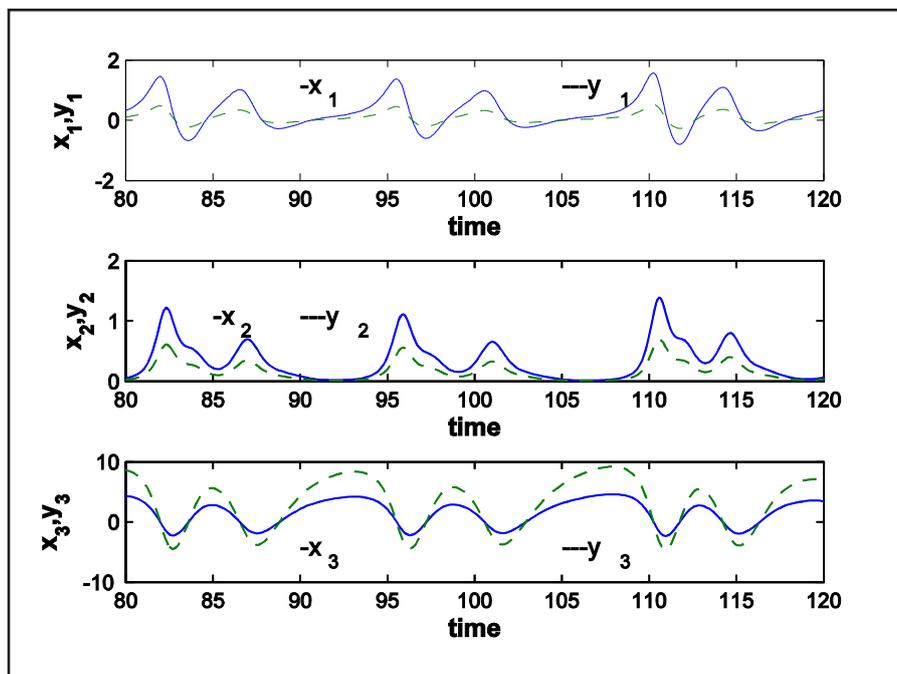


Figure 4: Time evolution of the state variables of the drive (solid line) and the response (dashed line) variables when the controllers have been activated.

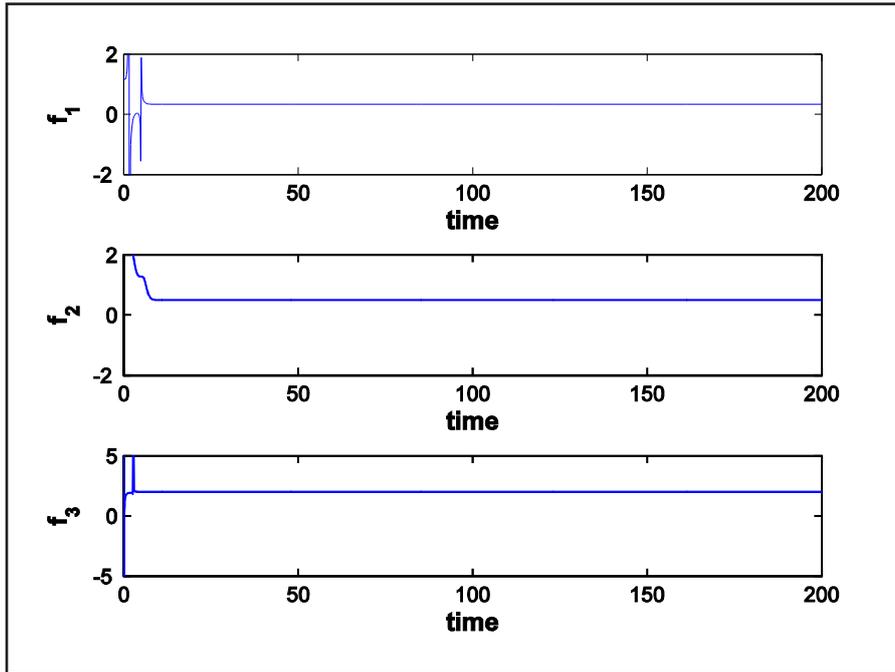


Figure 5: Time evolution of the recovered scaling constants where  $f_1 = \frac{\|x_1\|}{\|y_1\|}$ ,  $f_2 = \frac{\|x_2\|}{\|y_2\|}$ ,  $f_3 = \frac{\|x_3\|}{\|y_3\|}$

**ii. Modified Function Projective Synchronization:**

Choosing the scaling functions as  $a_1 = 0.01 + 0.05t$ ,  $a_2 = 2 + 0.01t^2$ ,  $a_3 = 1 + 2 \sin 0.2\pi t$ , modified function projective synchronization of the drive system (10) and the response system (11) is

achieved when the controllers are activated for  $t \geq 70$ . This is indicated by the convergence of the error state variables to zero as shown in Figure 6 and the projection of the state variables of the drive multistable system on the response system as shown in Figure 7. The recovered scaling functions are shown in Figure 8

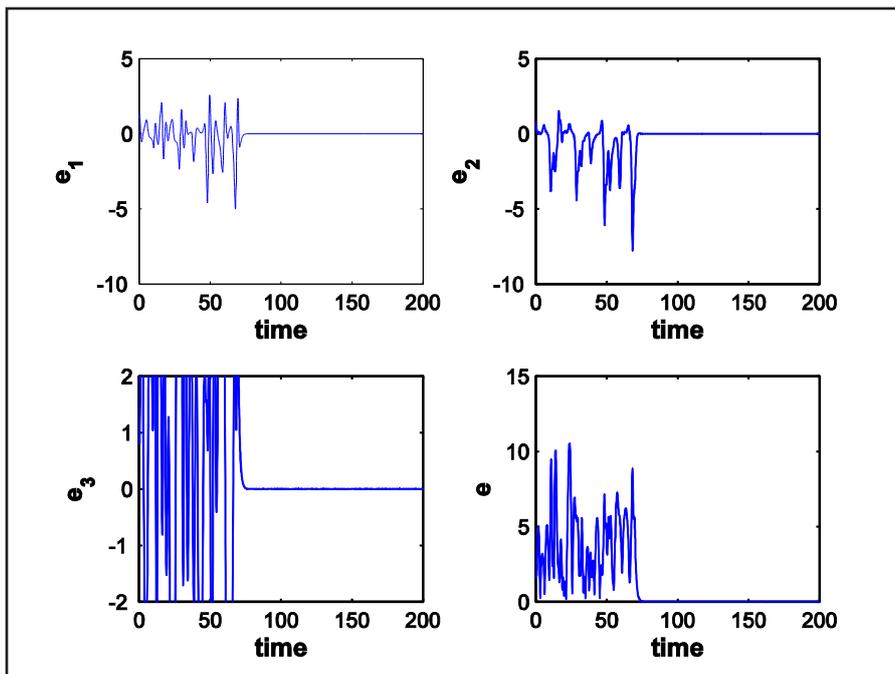


Figure 6: The dynamics of the error variables between the drive and the response systems with controllers deactivated for  $0 < t < 70$  and activated for  $t \geq 70$  where  $e_1 = x_1 - g_1$ ,  $e_2 = x_2 - g_2$ ,  $e_3 = x_3 - g_3$  and  $e = \sqrt{e_1^2 + e_2^2 + e_3^2}$

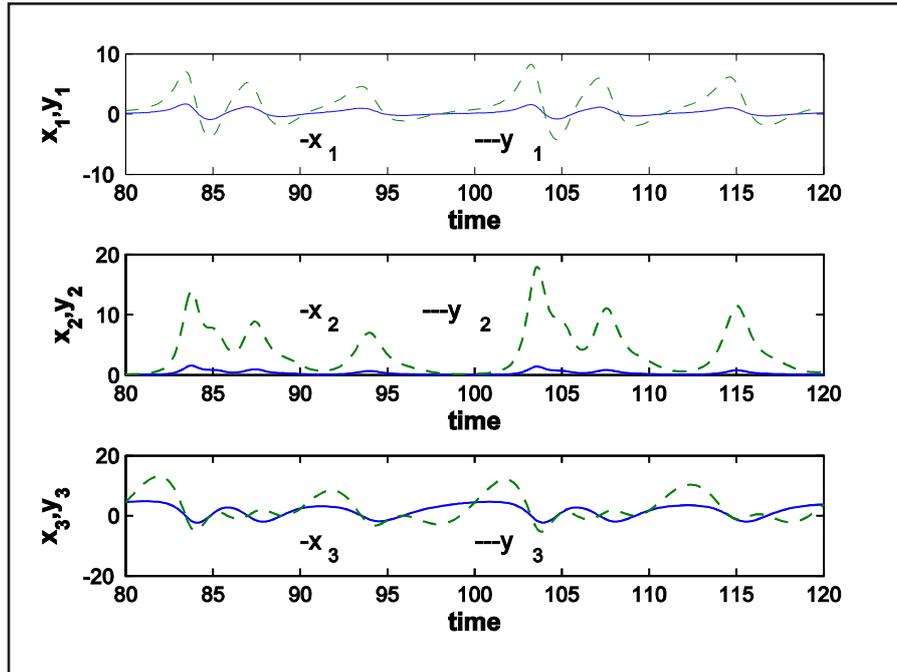


Figure 7: Time evolution of the state variables of the drive (solid line) and the response (dashed line) variables when the controllers have been activated

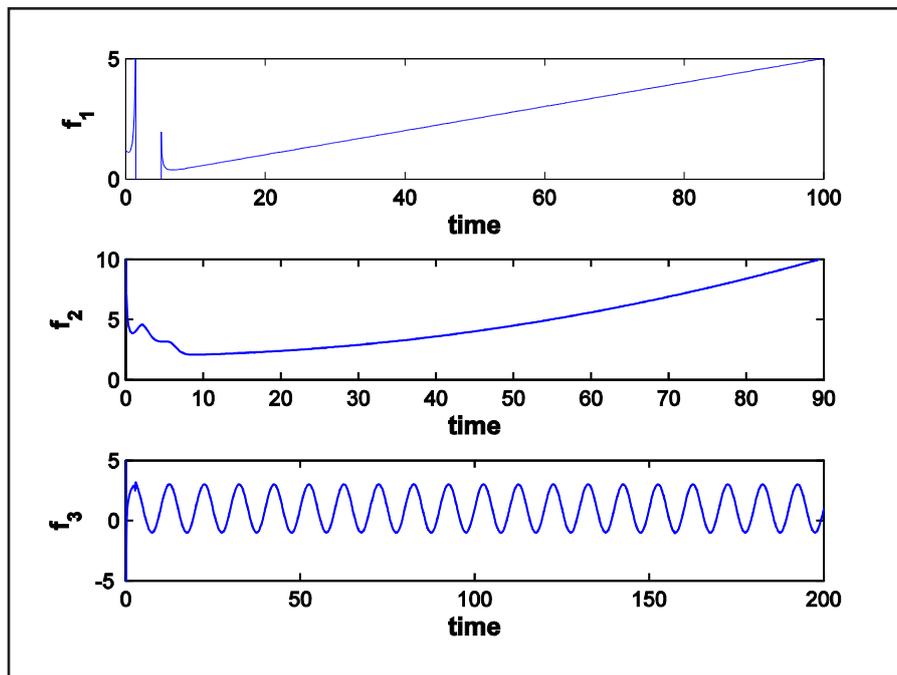


Figure 8: Time evolution of the recovered scaling functions where  $f_1 = \frac{\|x_1\|}{\|y_1\|}$ ,  $f_2 = \frac{\|x_2\|}{\|y_2\|}$ ,  $f_3 = \frac{\|x_3\|}{\|y_3\|}$

**CONCLUSION**

Modified function projective synchronization of multistable chaotic system have been achieved based on the Routh Hurwitz criteria via the Open-plus-closed-Loop method. It has been shown from the analytical and numerical results that

different controllers which are suitable for different types of synchronization scheme can be achieved from the general results. The modified function projective of multistable system provides higher and better security of information transmission as result of different

scaling function involved and the multistable nature of the system. The recovery of the scaling functions is an indication that more than one message can be securely transmitted and recovered after synchronization process has been completed.

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