SOLUTIONS OF SELECTED PSEUDO LOOP EQUATIONS IN WATER DISTRIBUTION NETWORK USING MICROSOFT EXCEL SOLVER

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(Received: 23rd Oct., 2015; Accepted: 13th April, 2016)

ABSTRACT

Adequate potable water supply (quality and quantity) is relevant to the community development while water consumption per capita per day and sanitation practices of the people in a community are functions of adequate potable water supply. This paper demonstrated the use of Microsoft Excel Solver (a computer package) in solving selected pseudo loop equations in pipe network analysis problems. Two pipe networks with pumps and overhead tanks were used to demonstrate the use of Microsoft Excel Solver in solving pseudo loops (open loops; networks with pumps or reservoirs in series) equations in water distribution network from groundwater sources. Continuity and headloss equations (non linear and linear, transform into linear equations using linear theory method) were developed. These equations were solved using Microsoft Excel Solver. Pump characteristics parameters and headloss were determined from the solutions. These flows and headloss in the pseudo loops calculated were evaluated statistically using Analysis of variance (ANOVA); total error, root squared error, sum of squared error, mean squared error, model of selection criterion (MSC) and average error. The study revealed that the flows calculated were similar for the two equations and there is no significant difference between the flows expected and flows calculated as well as headloss in pipe network using the two equations ($F = 5.37 \times 10^{-5}$; $p = 0.99995$) at 95 % confidence level. Total errors for linear and non-linear equations were 0.002 and 0.008 with root squared errors of 0.0459 and 0.0875, while MSC values were 10.220 and 8.926 respectively. It was concluded that solving pseudo loops using these two equations provided information on pumps characteristics and would aid in pumps selection as well as provide necessary information for water, hydraulics, mechanical and environmental engineers.

Keywords: Microsoft Excel Solver, Pipe Network Analysis, Pseudo loops, Statistical Analysis, Headloss, Flow

INTRODUCTION

Water distribution system is a hydraulic infrastructure consisting of various fixed elements, which include pipes, tanks (reservoirs), pumps and valves. It is crucial in the provision of potable water to the consumers. Effective potable water supply is of paramount importance in community and country. Adequate water supply (quality and quantity) to the community is a function of sanitation practice, health, economics and development of the community (JMP, 2013). Figures 1 and 2 present global trend of access to piped water and global trend of sanitation practices respectively. In designing a new water distribution network or in expanding the existing one, it is essential to investigate and establish a reliable network to ensure adequate head loss and flows at the node. With the high cost of surface water treatment and distribution coupled with infrastructure decay (Otun *et al.,* 2011), irregular government water policy and economic meltdown, groundwater sources as water supply schemes are the common trend in developing countries (Figure 3). Figures 3d and 3e show relationship between potable water supply, sanitation practices and wealth in Sub- Sahara.

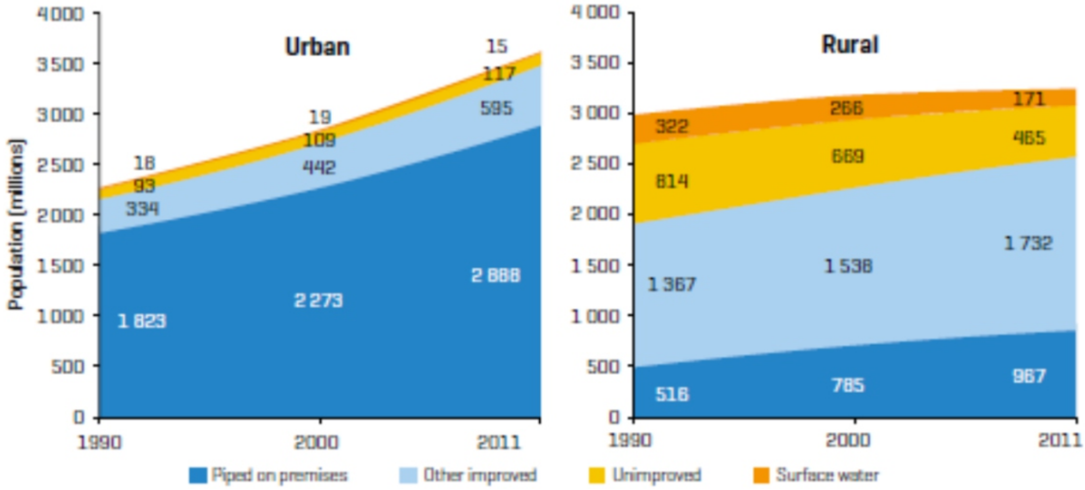

Figure 1a: Global Drinking water Coverage Trends in Urban and Rural Areas, 1990 – 2011 (JMP, 2013; Oke *et al.,* 2015a; b reprinted from WHO and UNICEF 2013 with permission number 199273)

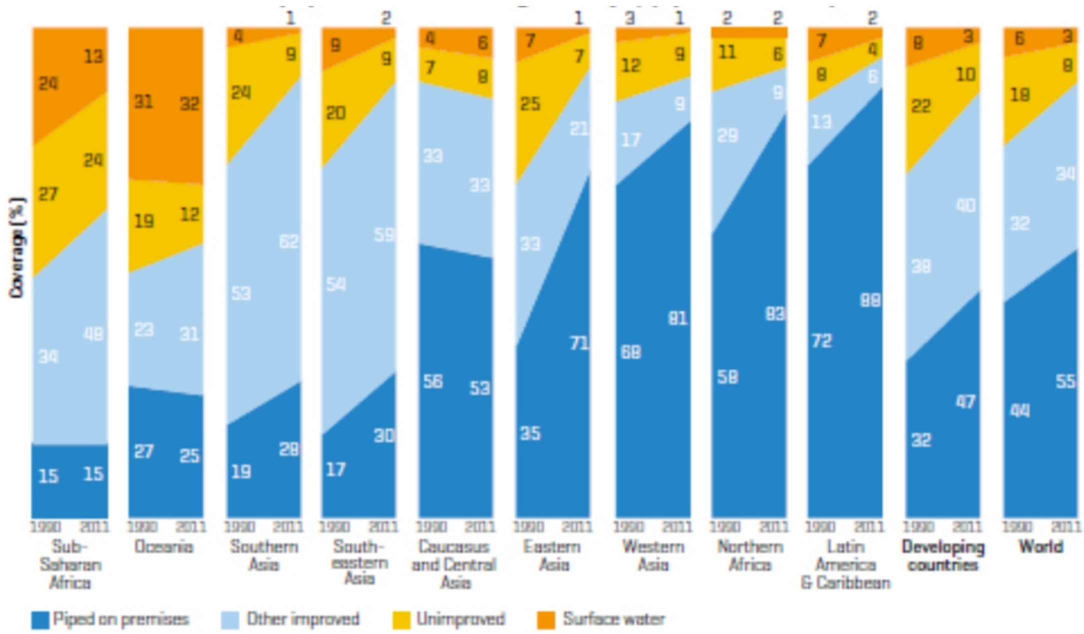

Figure 1b: Drinking water Coverage Trends by Developing Regions and the World, 1990 – 2011 (JMP, 2013; Oke *et al.,* 2015; b reprinted from WHO and UNICEF 2013 with permission number 199273)

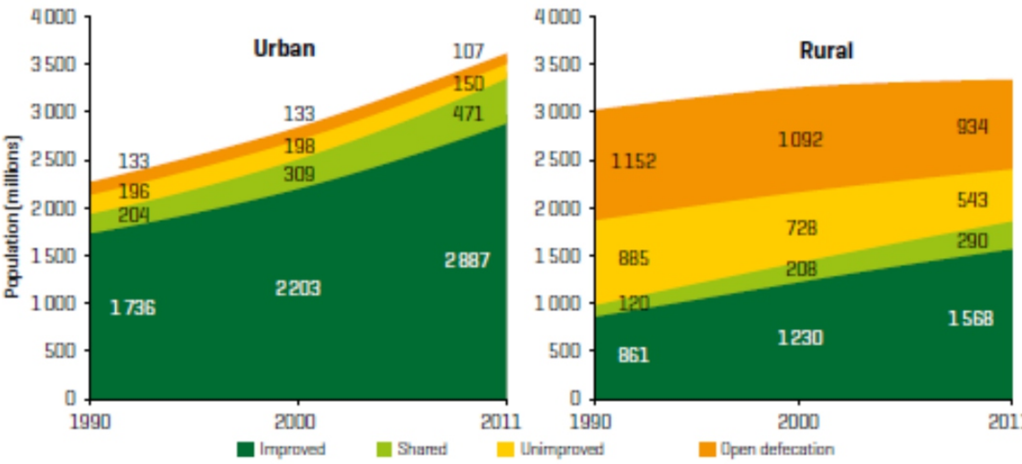

Figure 2a: Global Sanitation Coverage Trends in urban and Rural Areas, 1990 – 2011 (JMP, 2013; Oke *et al.,* 2015a; b with permission number 199273)

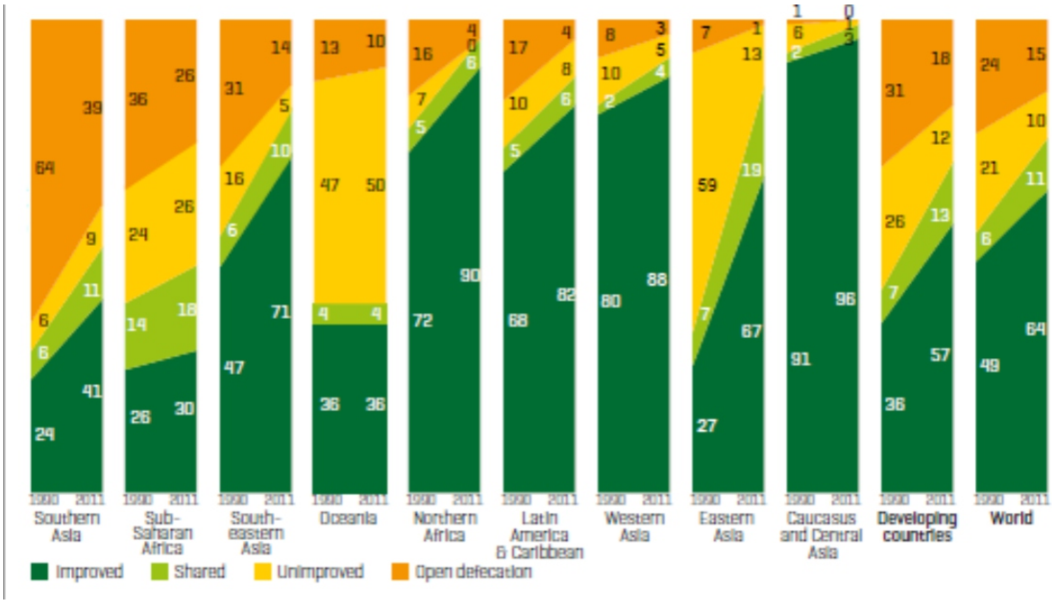

Figure 2b: Sanitation Coverage Trends by Developing Regions and the World, 1990 – 2011 (JMP, 2013; Oke *et al.,* 2015a; b with permission number 199273)

Figure 3a: Solar Powered Bore hole water Supply scheme in Katsina State

Figure 3b: Withdrawal Point Solar Powered Bore hole water Supply scheme in Katsina State

Figure 3c: Solar Powered Bore hole water Supply scheme in Osun State

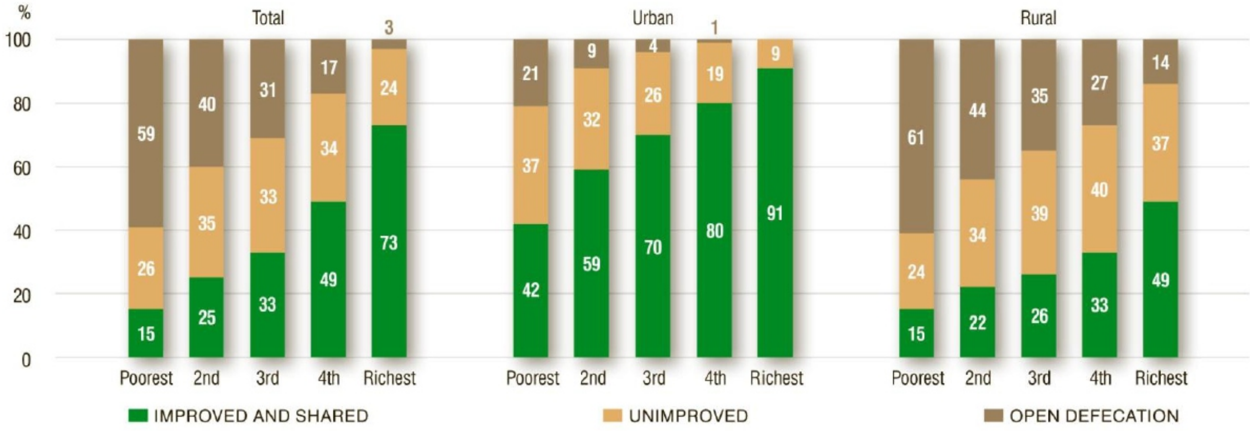

Figure 3d: Relationship between Wealth and Sanitation practices in Sub- Sahara (Source : JMP for Africa, 2012 with permission number 199273)

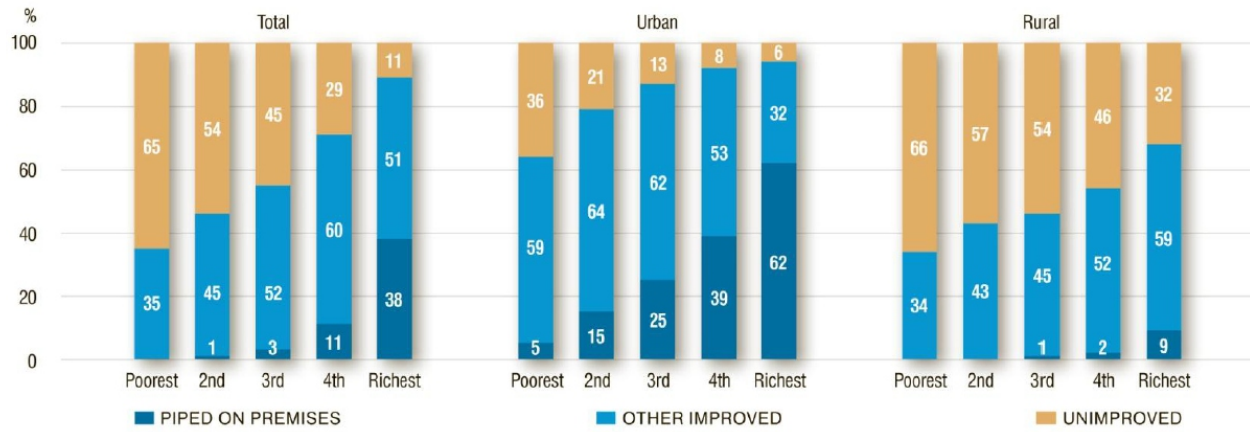

Figure 3e : Relationship between Wealth and Access to Piped Water in Sub- Sahara (Source : JMP for Africa, 2012 with permission number 199273)

Figures 3d and 3e show relationship between piped water (safe water), wealth and sanitation practices in sub sahara. This indicates that source of water such as piped groundwater as an alternative to public piped water is of a greater importance in safe water access and supply. Computation of flows and pressures in water distribution network (pipeline system) from groundwater sources to the community has been of great value and interest to water, hydraulics and environmental engineers and those involved with designs, construction and maintenance of public water distribution systems (Jeppson (1974; 1979), Jeppson and Davis (1976)). Analysis and design of pipes in water distribution networks generally create a relatively complex problem, particularly when the network consists of large number of pipes as frequently occurs in water distribution systems of large metropolitan areas and cities (Saminu *et al.,* 2013). There are various types of loops in pipeline systems or water distribution networks. The loops can be open (pseudo; which is path of pipes between two points of known energy level such as tank and pumps or two tanks; Ali 2000); closed loops (loop networks with no pumps); horizontal loop; vertical and pond loops (Chad , 2016). In pseudo loops, energy loops that begin at one supply source and end at another point (the water level between the two tanks should balance the summation of head lost through the pseudo loop).

In closed loops, many methods have been used in the pipe network analysis in the past to compute flows in pipe network. The methods range from graphical methods to the use of physical analogies and finally to the use of mathematical models. More methods or software of network analysis have been developed and implemented on the computer over the last fifty years. More on pipe network analysis can be found in literature such as Cross (1936); Mc corale and Deliany (1960);

Shamir and Howard (1968); Mcllroy (1949); Mcpherson *et al.* (1974); Elhay and Simpson (2011); Naser and Mohammad (2014); Todini and Pilati (1989). Literature such as Wood and Charles (1972); Wood (1981); Steel and McGhee (1979); Isaacs and Mills (1980); Dake (1982); Featherstone and Nalluri (1982); Viessman and Hammer (1993); Jeppson (1974; 1979), Jeppson and Davis (1976); Martins and Peters (1963); Mays (2000), Wood (1980), Nielsen (1989); Ellis and Simpson (1996); Chin (2000); Oke (2010) and Nelson *et al.* (2013) present techniques for pipe network analysis as Linear theory, Hardy Cross, equivalent pipe, circle theory and Newton Raphson techniques. Tukur (2006); Roossman (1993), Adeniran and Oyelowo (2013), and Dini and Tabesh (2014) highlighted different software that can be used for pipe network analysis. Advancement in computer has led to development of various programs for pipe network analysis among which EPANET. The limitation of EPANET is that there are no options for pressure-driven demand and leakage simulation in the EPANET program. It is well known that in the absence of significant fluid acceleration, the behavior of a network can be determined by a sequence of steady state conditions, which form a small but vital component for assessing the adequacy of a network (Saminu *et al.,* 2013). Such analysis is needed each time changing pattern of consumption occur at a significant level or when features such as water supply from groundwater sources (wells and boreholes), booster pumps, pressure regulating valves or storage tanks are added. The analysis of hydraulic and design problems of these situations have received little or no considerable attention in the past.

Pipe network for groundwater supply involves pumps and overhead tanks, which make it an application of pseudo loop. In pseudo loops, each open–loop (pseudo loop) makes the connection between a node with a known piezometric head (reservoir) or with a determined relation discharge – piezometric head (pump station), and another node with a known piezometric head or a determined relation discharge – piezometric head (Sarbu and Valea, 2011). Literature on applications and solutions of pseudo loops are limited. Jeppson (1974; 1979), Jeppson and Davis (1976) utilized Newton Raphson methods in solving selected pseudo loops equations. Hence, in this work there is a need to apply the Microsoft Excel Solver in solving pseudo loop equations in water distribution network with a view to shed more lights on this alternate source of water. Previous studies on Microsoft Excel Solver or similar package include Barati (2013) and Bhattacharjya (2010) used solver for groundwater flow; Gay and Middleton (1971) developed solutions for closed loop pipe network, Jewell (2001) and Huddleston *et al.* (2004) used excel sheet for closed loop pipe network analysis; Canakci (2007) used solver for pile foundation design; Tay *et al.* (2014) used the Microsoft Excel Solver for solving non-linear equation of closed loop pipeline and Oke *et al.* (2014; 2015a; b) use the Microsoft Excel Solver in solving linear theory and Hardy Cross methods in closed loop pipe network analysis.

MATERIALS AND METHOD

Pipe network of communities were adopted from literature Oke (2007) and Jeppson (1974; 1979). These pipe networks are as presented in Figures 4 and 5. Figure 4 presents a community without an overhead tank (case study A) with three boreholes (with three pumps) as sources of water supply and 4 pipes distribution network. Figure 5 shows a community with two overhead tanks and a borehole (with a pump) as a source of water supply and 6 pipes as distribution network (Case study B). Table 1 presents basic information of the pipe systems in the examples (Case studies A and B). These two case studies are applications of pseudo loops in pipe systems. In figure 4, it was assumed that the community has 1800 people and water demand is 80l/p.d. There are three pumps in the loop which supply water to the community. The pumps operate for two hours daily. In figure 5, the community has 500 people and water demand is 80l/p.d. There is a pump in the loop which supplies water to the community for two hours daily. Continuity equations were developed for the networks (equations 1 and 2) and the continuity equations were transformed into linear forms using linear theory method (equation 3). Pump characteristics curve equation was written (equations 4 to 6). The pump characteristics curves were transformed in to pseudo loops. Microsoft Excel Solver technique was used to solve developed continuity linear and non – linear equations developed from these two pipe networks (Case studies A and B). Microsoft Excel Solver technique was selected based on its advantages over other methods (Jeppson, 1974; 1979; Oke *et al.,* 2015), which include: it does not require an initialization and it always converges in a relatively little iterations. More on Microsoft Excel Solver can be found in literature such as Barati (2013); Bhattacharjya (2010); Gay and Middleton (1971); Jewell (2001); Briti *et al.* (2013); Tay *et al.* (2014) and Oke *et al.* (2014; 2015a ; b). Detailed procedures for Microsoft Excel Solver for solving these equations are presented in another paper.

The continuity and headloss equations used are as follows:

Sum of flow at any node is equal to zero;

$$\sum_{i=1}^{n} Q_i = 0 \tag{1}$$

where, Q_i is the flow at any node.

Sum of headloss in a closed loop is equal to zero;

$$\sum_{i=1}^{n} h_{li} = 0 \tag{2}$$

where, h_{li} is the headloss in a closed loop.

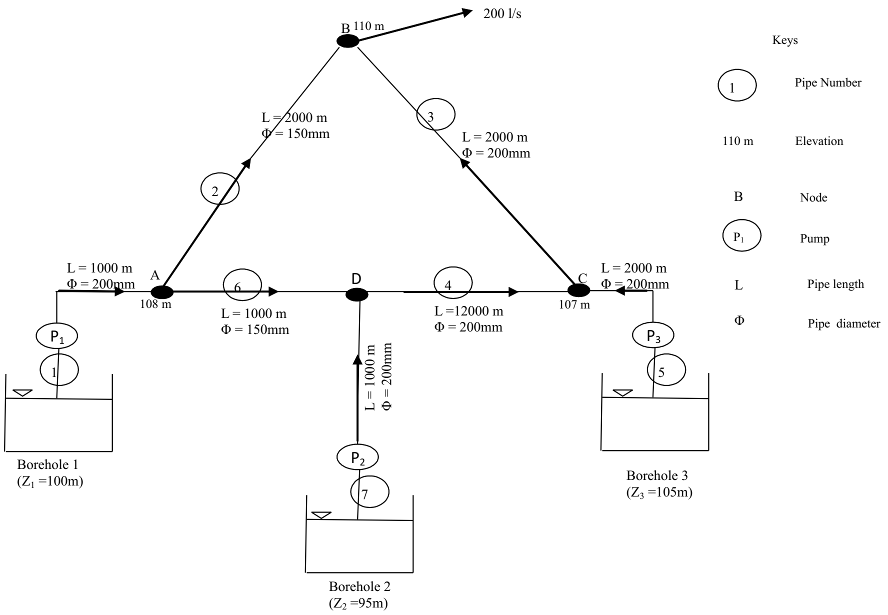

Figure 4: Pipe network of a community with 1800 people with 80l/p.d (Adopted from Jeppson, 1974; 1979; Case Study A)

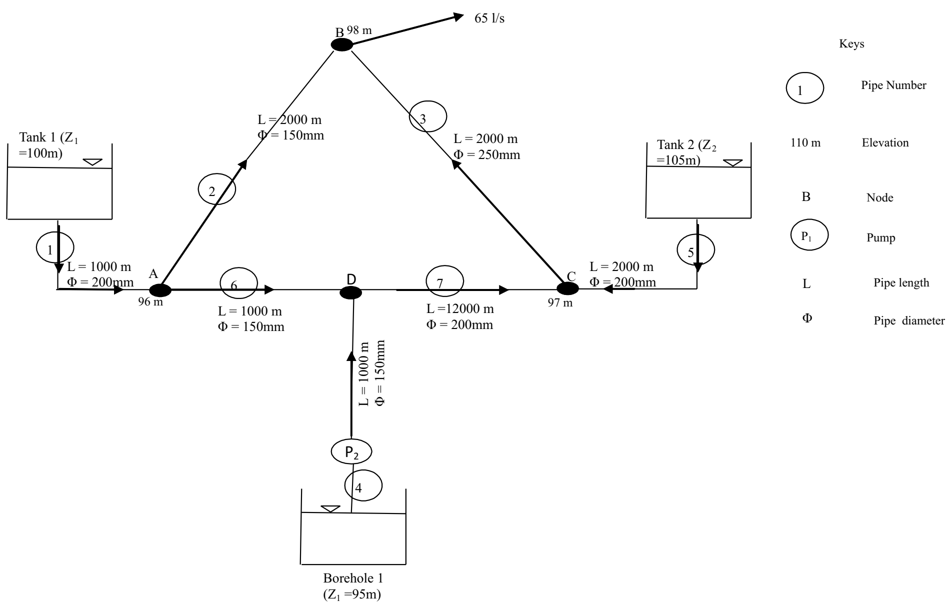

Figure 5: Pipe network of a community with 500 people with 80l/p.d (Case Study B; Adopted from Jeppson, 1974; 1979)

Table 1: Properties the Pipes in the Network

Description	Pipe Number	Diameter of the pipe (D, mm)	Length of the pipe (L, m)	Darcy fiction factor (f)	K $K=\frac{8\,fL}{\pi^2 gD^5}$
Example 1 (Case Study A; Figure 4)	1	200	1000	0.02	5159.883
	2	150	2000	0.02	43487.41
	3	200	2000	0.02	10319.77
	4	200	1200	0.02	6191.86
	5	200	2000	0.02	10319.77
	6	150	1000	0.02	21743.71
	7	200	1000	0.02	5159.883
Example 2 (Case Study B; Figure 4)	1	200	1000	0.02	5159.883
	2	150	2000	0.02	43487.41
	3	200	2000	0.02	10319.77
	4	200	2000	0.02	10319.77
	5	200	1000	0.02	5159.883
	6	150	1000	0.02	21743.71
	7	200	1200	0.02	6191.86

Linear Theory transforms the non-linear headloss equations into linear equations by approximating the headloss (H_l) in each pipe as:

$$H_l = \left(K_i Q_i^{n-1}\right) Q_i = K' Q_i \qquad (3)$$

Where; K' is the product of K (K $=\frac{8fL}{\pi^2 gD^5}$) and assumed flow (Q).

In pseudo loops, the sum of head loss is not equal to zero but must account for the difference in the reservoirs elevations and head produced by the pump or pumps as follows (Sarbu, 1997; Sarbu and Valea, 2011):

$$H_l = Z_a - Z_b = KQ_i^2 \qquad (3a)$$

Where: Z_a and Z_b are piezometric heads at pressure devices at the entrance or exit from the loop; H_{pij} – the pumping head of the booster pump integrated on the pipe ij, for the discharge Q_{ij}, approximated by parabolic interpolation on the pump curve given by points (Sarbu, 1997; Sarbu and Valea, 2011):

$$H_{pij} = AQ_{ij}^2 + B\left|Q_{ij}\right| + C \qquad (4a)$$

The coefficients A, B, C can be determined from three points of operating data. General equation for the pump characteristic curve is as indicated (Fairbanks, 1965; Jeppson, 1974; 1979; Oke, 2010):

$$h_p = AQ^2 + BQ + H_0 \qquad (4)$$

Where; h_p is the headloss; Q is the flow in the pipe, H_0 is the suction head; A and B are coefficients for the pump curve. Transformation of pump characteristics curve parameters can be expressed (Jeppson, 1974; 1979):

$$G_{pi} = Q_i + \frac{B_i}{2A_i} \qquad (5)$$

$$h_p = A_i G_{pi}^2 + H_0 \qquad (6)$$

Where, G_{pi} is the characteristic curve parameter. The flows calculated using the two developed equations (linear and non-linear) were evaluated statistically using total error, root squared error, sum of error, mean squared error, model of selection criterion (MSC) and average error.

Total error (Err^2) can be computed using equation (7) as follows:

$$Err^2 = \sum_{i=1}^{n}\left(Y_{obsi} - Y_{cali}\right)^2 \qquad (7)$$

where, Y_{obsi} is the observed flow and Y_{cali} is the calculated flow. MSC can be computed using equation (8) as follows:

$$MSC = \ln \frac{\sum_{i=1}^{n}\left(Y_{obsi} - \bar{Y}_{obsi}\right)^2}{\sum_{i=1}^{n}\left(Y_{obsi} - Y_{cali}\right)^2} - \frac{2p}{n} \qquad (8)$$

where, p is the number of parameters and n is the number of data points. Root squared error , sum of error, mean squared error and average error are as follows:

$$Err = \sqrt{\sum_{i=1}^{n}\left(Y_{obsi} - Y_{cali}\right)^{2}} \qquad (9)$$

$$SErr^{2} = \sum_{i=1}^{n}\left(Y_{obsi} - Y_{cali}\right) \qquad (10)$$

$$MSErr = \frac{\sum_{i=1}^{n}\left(Y_{obsi} - Y_{cali}\right)^{2}}{N} \qquad (11)$$

$$MErr = \frac{\sum_{i=1}^{n}\left(Y_{obsi} - Y_{cali}\right)}{N} \qquad (12)$$

RESULTS AND DISCUSSION

Results from this study are presented as follows: equations and solutions of the problems; pump characteristics curve and statistical evaluation of the equations.

Equations and Solutions of the Problems

The equations developed were square matrices. Equations 13 to 21 are for non-linear equation method (example 1; Case Study A), while equations 22 to 25 are for linear equation method and for example 1 (Case Study A). Equations 26 and 27 are the matrices for non- linear and linear equations for the same example. Equations 28 to 35 are for non-linear equation method (example 2; Case Study B), while equations 36 to 37 are for linear equation method and for example 2. Equations 40 and 49 are the matrices for non-linear and linear equations for the same example. They were in matrix forms of the number of pipes (in rows and in columns). Continuity equations (non-linear equations, flows into the nodes are positive, flow out of the node or withdraw are negative) for Example 1 (Case Study A; Figure 4) are as follows:

At Node A $\quad 0 = Q_1 - Q_2 - Q_6 \qquad (13)$

At node B $\quad 0 = Q_3 + Q_2 - 200 \qquad (14)$

At node C $\quad 0 = Q_4 + Q_5 - Q_3 \qquad (15)$

At node D $\quad 0 = Q_6 + Q_7 - Q_4 \qquad (16)$

where, Q_1, Q_2 and Q_3 are flows in pipes 1, 2 and 3 respectively.

Non linear headloss equation in a closed loop (headloss in clockwise direction is positive and headloss in anticlockwise direction is negative):

$$0 = K_2Q_2^2 - K_3Q_3^2 - K_4Q_4^2 - K_6Q_6^2 \qquad (17)$$

Transformed Pseudo loop equations for example 1 are as follows:

Pseudo loop from pump 1 to pump 3:

$$-5 = K_1Q_1^2 + K_4Q_4^2 + K_6Q_6^2 - K_5Q_5^2 + AG_{p1} - AG_{p3} \qquad (18)$$

Pseudo loop from pump 1 to pump 2:

$$5 = K_1Q_1^2 + K_6Q_6^2 - K_7Q_7^2 + AG_{p1} - AG_{p2} \qquad (19)$$

$$Q_4 = G_{p3} - \frac{B}{2A} \qquad (20)$$

$$Q_1 = G_{p1} - \frac{B}{2A} \text{ and } Q_5 = G_{p2} - \frac{B}{2A} \qquad (21)$$

Linear transformed continuity equations (linear equation, flows into the node are positive, flow out of the node or withdraw are negative) for Example 1 (Case Study A; Figure 4) are as follows:

At Node A $\quad 0 = Q_1 - Q_2 - Q_6 \qquad (13)$

At node B $\quad 0 = Q_3 + Q_2 - 200 \qquad (14)$

At node C $\quad 0 = Q_4 + Q_5 - Q_3 \qquad (15)$

At node D $\quad 0 = Q_6 + Q_7 - Q_4 \qquad (16)$

Headloss equation in a closed loop (headloss in clockwise direction is positive and headloss in anticlockwise direction is negative):

$$0 = K_2^{'}Q_2 - K_3^{'}Q_3 - K_4^{'}Q_4 - K_6^{'}Q_6 \qquad (22)$$

where $H_1 - H_2 = K_1Q_1^{2} = \Delta H_{12}$ and

$$Q_1 = \sqrt{\frac{\Delta H_{12}}{K_1}} \text{ and } K_1^{'} = K_1Q_1 \qquad (23)$$

Linear transformed Pseudo loop equations for example 1 are as follows:

Pseudo loop from pump 1 to pump 3:

$$-5 = K_1^{'}Q_1 + K_4^{'}Q_4 + K_6^{'}Q_6 - K_5^{'}Q_5 + AG_{p1} - AG_{p3} \qquad (24)$$

Pseudo loop from pump 1 to pump 2:

$$5 = K_1^{'}Q_1 + K_6^{'}Q_6 - K_7^{'}Q_7 + AG_{p1} - AG_{p2} \qquad (25)$$

The 7 x 7 matrices (non-linear and linear) of these equations on example 1 are as follows in equations (26) and (27). Continuity equations (non-linear equations, flows into the node are positive, flow out of the node or withdraw are negative) for Example 2 (Case Study B; Figure 5) are as follows:

At Node A $$0 = Q_1 - Q_2 - Q_6 \quad (28)$$

At node B $$0 = Q_3 + Q_2 - 65 \quad (29)$$

At node C $$0 = Q_7 + Q_5 - Q_3 \quad (30)$$

At node D $$0 = Q_6 - Q_7 + Q_4 \quad (31)$$

Non linear headloss equation in a closed loop (headloss in clockwise direction is positive and headloss in anticlockwise direction is negative):

$$0 = K_2 Q_2^2 - K_3 Q_3^2 - K_7 Q_7^2 - K_6 Q_6^2 \quad (32)$$

Transformed Pseudo loop equations for example 2 (Case Study B; Figure 5) are as follows:
Pseudo loop from pump 1 to pump 3:

$$-5 = K_1 Q_1^2 + K_7 Q_7^2 + K_6 Q_6^2 - K_5 Q_5^2 + AG_{p1} - AG_{p3} \quad (33)$$

Pseudo loop from pump 1 to pump 2:

$$5 = K_1 Q_1^2 + K_6 Q_6^2 - K_4 Q_4^2 + AG_{p1} - AG_{p2} \quad (34)$$

Transformation

$$Q_4 = G_{p3} - \frac{B}{2A} ; \; Q_1 = G_{p1} - \frac{B}{2A} \text{ and}$$

$$Q_5 = G_{p2} - \frac{B}{2A} \quad (35)$$

Linear transformed continuity equations (linear equation, flows into the node are positive, flow out of the node or withdraw are negative) for Example 2 (Case Study B; Figure 5) are as follows:

At Node A $$0 = Q_1 - Q_2 - Q_6 \quad (28)$$

At node B $$0 = Q_3 + Q_2 - 65 \quad (29)$$

At node C $$0 = Q_7 + Q_5 - Q_3 \quad (30)$$

At node D $$0 = Q_6 - Q_7 + Q_4 \quad (31)$$

Linear headloss equation in a closed loop (headloss in clockwise direction is positive and headloss in anticlockwise direction is negative):

$$0 = K_2^{'} Q_2 - K_3^{'} Q_3 - K_7^{'} Q_7 - K_6^{'} Q_6 \quad (36)$$

Transformed Pseudo loop equations for example 2 are as follows:
Pseudo loop from pump 1 to tank 2 (pump 3):

$$-5 = K_1^{'} Q_1 + K_7^{'} Q_7 + K_6^{'} Q_6 - K_5^{'} Q_5 + AG_{p1} - AG_{p3} \quad (37)$$

Pseudo loop from pump 1 to tank 1 (pump 2):

$$5 = K_1^{'} Q_1 + K_6^{'} Q_6 - K_4^{'} Q_4 + AG_{p1} - AG_{p2} \quad (38)$$

Transformation expressions are as follows:

$$Q_4 = G_{p3} - \frac{B}{2A} ; \; Q_1 = G_{p1} - \frac{B}{2A} \text{ and}$$

$$Q_5 = G_{p2} - \frac{B}{2A} \quad (39)$$

The 7 x 7 matrices (non-linear and linear) of these equations on example 2 are as follows in equations (40) and (41).

The values of flows and headloss calculated for each of the method based on case studies A and B were as presented in Figures 6 and 7, respectively. Detailed solutions of these equations by the Microsoft Excel Solver and Linear theory method were presented at the Appendices (Appendices A, B, C and D). The flows can be grouped into two; positive (in the same direction as indicated) and negative (flow in opposite direction). The flows range from 0.0054 to 0.124 m^3/s. The headloss is the range of 0.90 and 250.75 m. The lowest headloss was from linear equations and highest from non- linear equation. Analysis of variance of these calculated flows revealed that there is no significant difference between the flows at 95 % confidence level. Table 2 presents the results of analysis of variance (ANOVA). From the Figures the values of flows varied with the method as well as the problem. These indicated that flows in pipe network analysis are functions of the network, pipe's properties and the method used. This is in agreement with literature such as Dillingham (1967); De NeufVille and James (1969); Jeppson (1979); Steel and McGhee (1979); Featherstone and Nalluri (1982); Chin (2000); Oke (2007), which stated that flow in pipes in any network depends on the method and the network in place.

$$\begin{pmatrix} \sqrt{\frac{\Delta h_1}{K_1}} & -\sqrt{\frac{\Delta h_2}{K_2}} & 0 & 0 & 0 & -\sqrt{\frac{\Delta h_1}{K_1}} & 0 \\ 0 & -\sqrt{\frac{\Delta h_2}{K_2}} & \sqrt{\frac{\Delta h_3}{K_3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{\frac{\Delta h_3}{K_3}} & \sqrt{\frac{\Delta h_4}{K4}} & \sqrt{\frac{\Delta h_5}{K_5}} & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{\frac{\Delta h_4}{K_4}} & 0 & \sqrt{\frac{\Delta h_6}{K_6}} & \sqrt{\frac{\Delta h_7}{K_7}} \\ 0 & K_2 & -K_3 & -K_4 & 0 & -K_6 & 0 \\ K_1 & 0 & 0 & K_4 & -K_5 & K_6 & 0 \\ K_1 & 0 & 0 & 0 & 0 & -K_6 & K \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \end{pmatrix} = \begin{pmatrix} 0 \\ 200 \\ 0 \\ 0 \\ 0 \\ -5 \\ 5 \end{pmatrix} \tag{26}$$

Linear Transformed

$$\begin{pmatrix} \sqrt{\frac{\Delta h_1}{K_1}} & -\sqrt{\frac{\Delta h_2}{K_2}} & 0 & 0 & 0 & -\sqrt{\frac{\Delta h_1}{K_1}} & 0 \\ 0 & -\sqrt{\frac{\Delta h_2}{K_2}} & \sqrt{\frac{\Delta h_3}{K_3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{\frac{\Delta h_3}{K_3}} & \sqrt{\frac{\Delta h_4}{K4}} & \sqrt{\frac{\Delta h_5}{K_5}} & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{\frac{\Delta h_4}{K_4}} & 0 & \sqrt{\frac{\Delta h_6}{K_6}} & \sqrt{\frac{\Delta h_7}{K_7}} \\ 0 & K_2^{/} & -K_3^{/} & -K_4^{/} & 0 & -K_6^{/} & 0 \\ K_1^{/} & 0 & 0 & K_4^{/} & -K_5^{/} & K_6^{/} & 0 \\ K_1^{/} & 0 & 0 & 0 & 0 & -K_6^{/} & K_7^{/} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \end{pmatrix} = \begin{pmatrix} 0 \\ 200 \\ 0 \\ 0 \\ 0 \\ -5 \\ 5 \end{pmatrix} \tag{27}$$

$$\begin{pmatrix} \sqrt{\frac{\Delta h_1}{K_1}} & -\sqrt{\frac{\Delta h_2}{K_2}} & 0 & 0 & 0 & -\sqrt{\frac{\Delta h_1}{K_1}} & 0 \\ 0 & -\sqrt{\frac{\Delta h_2}{K_2}} & \sqrt{\frac{\Delta h_3}{K_3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{\frac{\Delta h_3}{K_3}} & \sqrt{\frac{\Delta h_4}{K4}} & \sqrt{\frac{\Delta h_5}{K_5}} & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{\frac{\Delta h_4}{K_4}} & 0 & \sqrt{\frac{\Delta h_6}{K_6}} & \sqrt{\frac{\Delta h_7}{K_7}} \\ 0 & K_2 & -K_3 & -K_4 & 0 & -K_6 & 0 \\ K_1 & 0 & 0 & K_4 & -K_5 & K_6 & 0 \\ K_1 & 0 & 0 & 0 & 0 & -K_6 & K \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \end{pmatrix} = \begin{pmatrix} 0 \\ 65 \\ 0 \\ 0 \\ 0 \\ -5 \\ 5 \end{pmatrix} \tag{40}$$

Linear Transformed

$$\begin{pmatrix} \sqrt{\frac{\Delta h_1}{K_1}} & -\sqrt{\frac{\Delta h_2}{K_2}} & 0 & 0 & 0 & -\sqrt{\frac{\Delta h_1}{K_1}} & 0 \\ 0 & -\sqrt{\frac{\Delta h_2}{K_2}} & \sqrt{\frac{\Delta h_3}{K_3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{\frac{\Delta h_3}{K_3}} & \sqrt{\frac{\Delta h_4}{K4}} & \sqrt{\frac{\Delta h_5}{K_5}} & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{\frac{\Delta h_4}{K_4}} & 0 & \sqrt{\frac{\Delta h_6}{K_6}} & \sqrt{\frac{\Delta h_7}{K_7}} \\ 0 & K_2^{/} & -K_3^{/} & -K_4^{/} & 0 & -K_6^{/} & 0 \\ K_1^{/} & 0 & 0 & K_4^{/} & -K_5^{/} & K_6^{/} & 0 \\ K_1^{/} & 0 & 0 & 0 & 0 & -K_6^{/} & K_7^{/} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \end{pmatrix} = \begin{pmatrix} 0 \\ 65 \\ 0 \\ 0 \\ 0 \\ -5 \\ 5 \end{pmatrix} \tag{41}$$

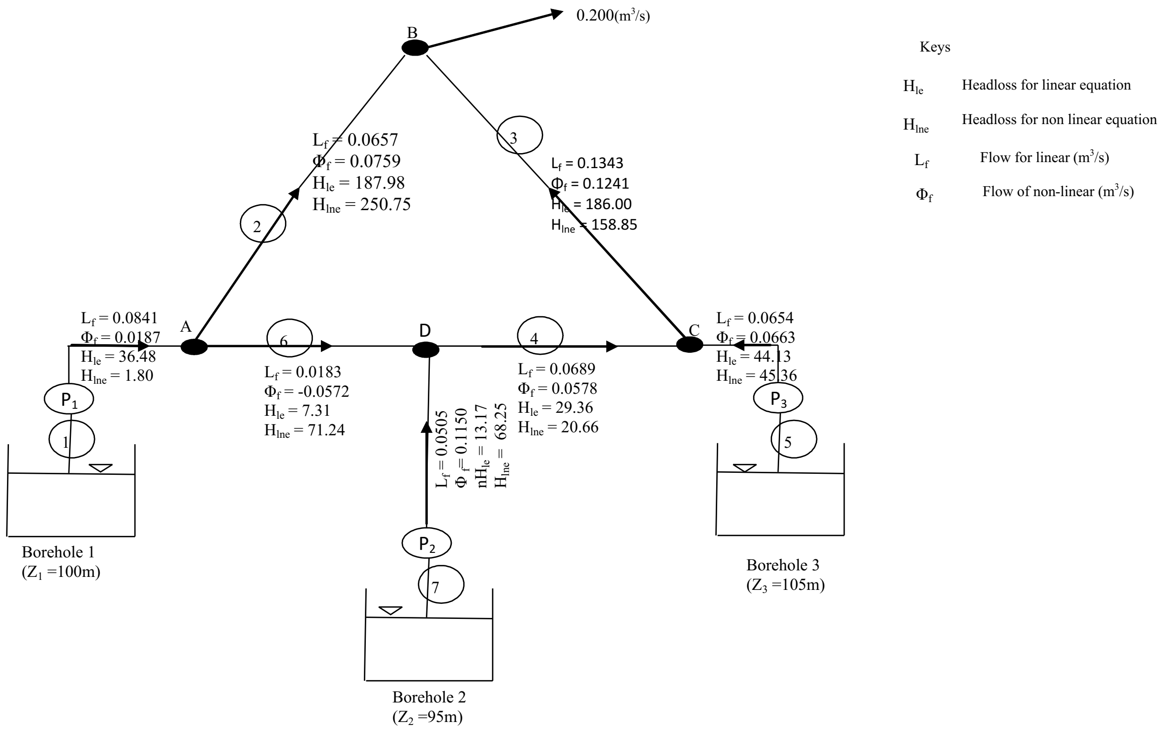

Figure 6: Parameter Solutions Obtained Using Case Study A; Figure 4

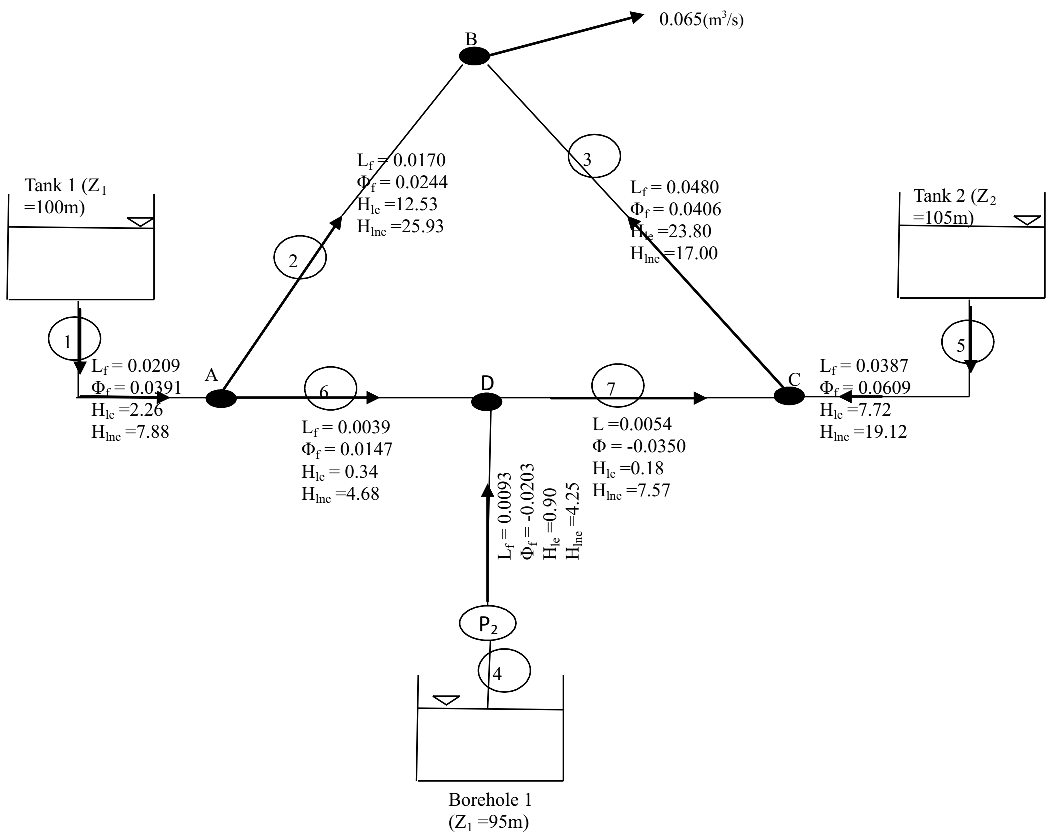

Figure 7: Parameter Solutions Obtained Using Case Study B; Figure 5

Table 2: Statistical Analysis of the calculated flows

Source of Variation	Sum Squared	Degree Freedom	Mean of Squared	F- value	P-value	F- critical at 95 % confidence level
Between Flows Calculated	0.000516	2	0.000258	5.37 x 10^{-05}	0.999946	3.259446
Within Flows Calculated	173.1267	36	4.809074			
Total	173.1272	38				

The network can be any of the followings, network with:

a) overhead tank as a source of water (gravity supply);
b) pump from clear well as sources of water supply (direct line);
c) supply from booster station (this involves pseudo loops in the network analysis) with overhead tanks; and
d) overhead tanks with valves and meters

Pump Characteristic Curve

Table 3 presents pump characteristic curve parameters for the pumps. From the table, it can be seen that the power of the required pump is a function of the calculated flows and suction head. Fairbank (1965) reported that when the pumping requirements are variable, it may be more desirable to install several small pumps in parallel rather than use a single large one. When the demand drops, one or more smaller pumps may be shut down, thus allowing the remainder to operate at or near peak efficiency. If a single pump is used with lowered demand, the discharge must be throttled, and it will operate at reduced efficiency. Moreover, when smaller units are used opportunity is provided during slack demand periods for repairing and maintaining each pump in turn, thus avoiding plant shut-downs which would be necessary with single units. Similarly, multiple pumps in series may be used when liquid must be delivered at high heads. In planning such installations a head-capacity curve for the system must first be drawn. The head required by the system is the sum of the static head (difference in elevation and/or its pressure equivalent) plus the variable head (friction and shock losses in the pipes, heaters, etc.). The former is usually constant for a .given system whereas the latter increases approximately with the square of the flow. The resulting curve is represented as line AB in Figure 8. Connecting two pumps in parallel to be driven by one motor is not a very common practice and to have such an arrangement may appear more expensive than a single pump. However, it should be remembered that in most cases it is possible to operate such a unit at about 40 per cent higher speed, which may reduce the cost of the motor materially. Thus, the cost of two high-speed pumps may not be much greater than that of a single slow-speed pump. For. units to operate satisfactorily in parallel, they must be working on the portion of the characteristic curve which drops off with increased capacity in order to secure an even flow distribution. Consider the action of two pumps operating in parallel. The system -head-capacity curve AB shown in figure 8a starts at H static when the flow is zero and rises parabolically with increased flow. Curve CD represents the characteristic curve of pump A operating alone; the similar curve for pump' B is represented by EF. Pump B will not start delivery until the discharge pressure of pump A falls below that of the shut-off head of B (point E). The combined delivery for a given head is equal to the sum of the individual capacities of the two pumps at that head.

Table 3 : Pump Characteristics Parameters

Description	Pump	Linear Transform				Non-Linear Transform			
		A	B	G	H_p	A	B	G	H_p
Example 1	1	-2.1458	2.8210	-0.5733	$-2.1458Q^2+2.8210Q+H_0$	-2.1251	2.8365	-0.6487	$-2.1251Q^2+2.8365Q+H_0$
	2	-2.1458	2.8210	-0.5919	$-2.1458Q^2+2.8210Q+H_0$	-2.1251	2.8365	-0.6011	$-2.1251Q^2+2.8365Q+H_0$
	3	-2.1458	2.8210	-0.6068	$-2.1458Q^2+2.8210Q+H_0$	-2.1251	2.8365	-0.5524	$-2.1251Q^2+2.8365Q+H_0$
Example 2	1	-1.6367	2.2981	-0.6811	$-1.6367Q^2+2.29810Q+H_0$	-1.6206	2.3094	-0.6734	$-1.6206Q^2+2.3094Q+H_0$
	2	-1.6367	2.2981	-0.6634	$-1.6367Q^2+2.29810Q+H_0$	-1.6206	2.3094	-0.6516	$-1.6206Q^2+2.3094Q+H_0$
	3	-1.6367	2.2981	-0.6967	$-1.6367Q^2+2.29810Q+H_0$	-1.6206	2.3094	-0.7474	$-1.6206Q^2+2.3094Q+H_0$

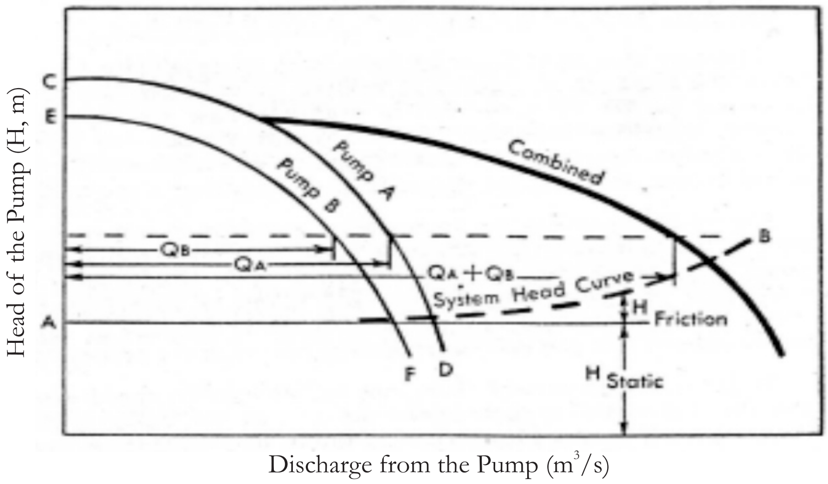

Figure 8a: Head Capacity Curves of Pumps Operating in Parallel (Source: Fairbank, 1965)

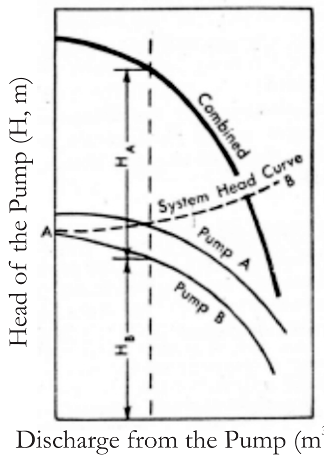

Figure 8b: Head Capacity Curves of Pumps Operating in Series (Source: Fairbank, 1965)

Table 4: Statistical Evaluation of the Equations

Description	Total error	Root squared error	Sum of error	Mean Squared Error	Model Selection Criterion	Average Error
Linear Equation	0.002	0.0459	0.057	0.0002	10.220	0.0044
Non- Linear Equation	0.008	0.0875	0.125	0.0006	8.926	0.0096

Table 5: Analysis of Variance of the Statistical Evaluation

Source of Variation	Sum Squared	Degree of freedom	Mean of Squared	F- value	P-value	F – value at 95 % confidence level
Between Errors	151.6676	5	30.33352	216.5579	1.09×10^{-06}	4.387374
Within Errors	0.840427	6	0.140071			
Total	152.508	11				

For a given combined delivery head, the capacity is divided between the pumps as 'noted' on the figures Q_A and Q_B. The combined characteristic curve shown on the figure is found by plotting these summations. More on pump selection can be found in literature such as

Statistical Evaluation of the Methods

Table 4 presents the detailed computation of the statistical evaluation. Four different statistical expressions were used to evaluate the performance of the flow estimations or to compare the method of estimating the flow in the pipe. Table 4 shows the values of that total error, root squared error, sum of error, mean squared error; model selection criterion and average error were 0.002 and 0.008; 0.0459 and 0.0875; 0.057 and 0.125; 0.0002 and 0.0006; 10.220 and 8.926; and 0.0044 and 0.096 for linear and non- linear equation respectively It is well known that the lower the value of errors the better the accuracy of the equation and the higher the dependability of the equation. This indicated that out of these two equations linear equations was the most dependable and more accurate than the other equation. This can be attributed to exponential nature of the non linear equation, which contribute to multiplication of the error.. Table 5 presents results of analysis of variance (ANOVA), which indicates that there is significant difference ($F = 216.5579$; $p = 1.09 \times 10^{-06}$) between the errors values and the equations at 95 % confidence level.

CONCLUSIONS

The study was on evaluation of linear and non-linear equations in pseudo loop of pipe network analysis. Two practical networks were used. Flows, headloss and pump characteristics parameters were calculated from these equations using Microsoft Excel Solver (a computer package) and these parameters were evaluated statistically. It can be concluded based on the study that:

a) flow in pipes is a function of equation used and withdrawal in the pipe network analysis at the node;
b) Linear equation is better than non- Linear equation in the estimation of flows and headloss at a node based on the value of errors; and
c) There is the need to conduct economics evaluation of these equations based on the headloss across the pipes in the loops to ascertain the reliability of the equations.
d) Comparison of the results obtained by the Microsoft Excel Solver with results from existing methods in literature such as Newton Raphson method may be desirable as a further extension of the study.

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APPENDICES

APPENDIX A

(Solver sheet for Linear Equation Example 1)

Pipe No	D (mm)	L(m)	f		K	Q'	K'	
1		200	1000	0.02	5159.883201		0.023	118.6773136
2		150	2000	0.02	43487.41068		0.06	2609.244641
3		200	2000	0.02	10319.7664		0.1	1031.97664
4		200	1200	0.02	6191.859841		0.04	247.6743936
5		200	2000	0.02	10319.7664		0.06	619.1859841
6		150	1000	0.02	21743.70534		0.04	869.7482136
7		200	1000	0.02	5159.883201		0.08	412.790656
Target	-6.30607×10^{-13}							
Sum of Headloss in a loop								
Variables (Flows)				G1	G2	G3	A	B
Q1 0.0841				-0.5733	-0.5919	-0.6068	-2.1458	2.8210
Q2 0.065746436								
Q3 0.134253564					$hp\text{-} H_0$			
Q4 0.06885706				Pump 1		0.222030311		
Q5 0.0654				Pump 2		0.137044838		
Q6 0.018336184				Pump 3		0.175309557		
Q7 0.0505								
Constraints								
Transformation At A		0.0841						
Transformation At B		0.0654						
Transformation At C		0.0505						
Node At A	0.0000							
Node At C	0.0000							
Node At B	0							
Node At D	0.0000							
Pseudo Loop 1		-5.00000028						
Pseudo Loop 2		5.000000117						

APPENDIX B

(Solver sheet for Linear Equation Example 2)

Pipe No	D (mm)	L(m)	f		K	Q'	K'	
1		200	1000	0.02	5159.883201		0.03	154.796496
2		150	2000	0.02	43487.41068		0.03	1304.62232
3		200	2000	0.02	10319.7664		0.03	309.592992
4		200	2000	0.02	10319.7664		0.04	412.790656
5		200	1000	0.02	5159.883201		0.07	361.191824
6		150	1000	0.02	21743.70534		0.04	869.7482136
7		200	1200	0.02	6191.859841		0.05	309.592992

Target 1.02585 x 10^{-13}

Sum of Headloss in a loop

Variables (Flows)								
				G1	G2	G3	A	B
Q1	0.0209	-0.6811	-0.6634	-0.6967	-1.6367	2.2981		
Q2	0.016977642							
Q3	0.048022358			$hp- H_0$				
Q4	0.009332154	Pump 1		0.047362547				
Q5	0.0387	Pump 2		0.01233633				
Q6	0.003943439	Pump 3		0.086464245				
Q7	0.0054							

Constraints

Transformation At A		0.0209
Transformation At B		0.0387
Transformation At C		0.0054
Node At A	0.0000	
Node At C	0.0000	
Node At B	0	
Node At D	0.0000	
Pseudo Loop 1		-4.999999871
Pseudo Loop 2		5

APPENDIX C

(Solver sheet for Non- Linear Equation Example 1)

Pipe No	D (mm)	L(m)	f	K
1	200	1000	0.02	5159.883201
2	150	2000	0.02	43487.41068
3	200	2000	0.02	10319.7664
4	200	1200	0.02	6191.859841
5	200	2000	0.02	10319.7664
6	150	1000	0.02	21743.70534
7	200	1000	0.02	5159.883201

Target 5.70803 $\times 10^{-07}$

Sum of Headloss in a loop

Variables (Flows)

			G1	G2	G3	A	B
Q1	0.0187	-0.6487	-0.6011	-0.5524	-2.1251	2.8365	
Q2	0.075934078						
Q3	0.124065922			$hp- H_0$			
Q4	0.057767571		Pump 1	0.052285778			
Q5	0.0663	Pump 2	0.298111728				
Q6	-0.057239183		Pump 3	0.178716032			
Q7	0.1150						

Constraints

Transformation At A	0.0187
Transformation At B	0.0663
Transformation At C	0.1150
Node At A	0.0000
Node At C	0.0000
Node At B	0
Node At D	0.0000
Pseudo Loop 1	-5.000000187
Pseudo Loop 2	4.999999601

APPENDIX D

(Solver sheet for Non- Linear Equation Example 2)

Pipe No	D (mm)	L(m)	f	K	
1	200		1000	0.02	5159.883201
2	150		2000	0.02	43487.41068
3	200		2000	0.02	10319.7664
4	200		2000	0.02	10319.7664
5	200		1000	0.02	5159.883201
6	150		1000	0.02	21743.70534
7	200		1200	0.02	6191.859841

Target 3.0092×10^{-07}

Sum of Headloss in a loop

Variables (Flows)

			G1	G2	G3	A	B
Q1	0.0391		-0.6734	-0.6516	-0.7474	-1.6206	2.3094
Q2	0.024416387						
Q3	0.040583613				$hp- H_0$		
Q4	-0.020284316			Pump 1		0.087799405	
Q5	0.0609			Pump 2		-0.082714029	
Q6	0.014674622			Pump 3		0.13456274	
Q7	-0.0350						

Constraints

Transformation At A	0.0391
Transformation At B	0.0609
Transformation At C	-0.0350
Node At A	0.0000
Node At C	0.0000
Node At B	0
Node At D	0.0000