ANALYSIS OF VIBRATION FREQUENCY IN TRANSVERSELY-ISOTROPIC SEMILINEAR ELASTIC THIN PLATE

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ABSTRACT

This study examined the vibration and frequency of transversely-isotropic semilinear elastic thin plate in comparison to that of an isotropic linear elastic thin plate. This was with a view to obtaining the effect of finite deformation and consequent nonlinearity on the transversely isotropic body. The method of double series solution was used to obtain the equation for the frequency of an elastic thin plate in vibration. Solving the vibration problem, the frequencies evaluated were compared for linear and semilinear materials. It was observed that semilinear elastic thin plate has higher frequency during vibration compared to that of linear elastic thin plate. The study concluded that this phenomenon of increased frequency during vibration in semilinear elastic material constitutes a nonlinear effect due to the finite deformation approach adopted.

Keywords: Semilinear, Elastic Thin Plate, Transversely-Isotropy, Finite Deformation

INTRODUCTION

In continuum mechanics, plate theories are mathematical descriptions of the mechanics of flat plates which is an extension of the theory of beams. Plates are defined as plane structural elements with a small thickness compared to the planar dimensions [1]. The typical ratio of length a or breadth b to thickness h of a plate structure is between 1 and 80. Thus, it is classified into the following [2]:

$$1 \leq \frac{min(a,b)}{h} < 8 \Rightarrow \text{Thick plate theory}$$

$$8 \leq \frac{min(a,b)}{h} < 80 \Rightarrow \text{Thin plate theory}$$

$$\frac{min(a,b)}{h} \geq 80 \Rightarrow \text{Membrane theory}$$

A plate theory takes advantage of this disparity in length scale to reduce the full three-dimensional solid mechanics problem to a two-dimensional problem. Plate finds application in engineering; as typical examples we may mention concrete and reinforced concrete plates used in structures, for ship hulls [3], airport tarmac and panels used in various constructions and surface designs. Vibration is the motion of a particle or a body or system of connected bodies displaced from a position of equilibrium. One of the most important measure of the implication of vibratory motion on structures or components design is the oscillation frequency, defined as the number of cycles completed in a certain period of time. Consequently, design and manufacture of machine elements are carefully controlled to eliminate unbalanced rotating forces and the concomitant inherent vibrations which induce increased stresses, energy losses, added wear, increased bearing loads, fatigue and passenger discomfort in vehicles [4], among others.

In modern designs, increasing use is made of materials which are transversely-isotropic. These are materials having an axis of symmetry and are in general characterized by 5 elastic constants [5]. A good representative of such a material is a layered plates [6].

Recently, several researchers have made tremendous efforts in the study of dynamics and vibration of structures. Among others, Igor, Jan and Vladimir [7] analysed the natural in-plane vibration of rectangular plates using the homotopy perturbation method. Using a boundary layer function, Jomehzadeh and Saidi [8] discussed the analytical solution for free vibration of transversely isotropic sector plates. Jovanovic [9] solved the transverse vibration response of a beam with a viscous boundary using Fourier series. Omolofe and Ogunyebi [10] considered the transverse vibrations of elastic thin beam resting on variable elastic foundations and subjected to travelling distributed forces. Oni and Awodola [11] considered the dynamic response under a moving load of an elastically supported non-prismatic Bernoulli-Euler beam on variable elastic foundation. Xing and Liu [12,13] solved the free vibrations of rectangular thin plates and orthotropic rectangular thin plates respectively by symplectic dual method. Ghugal and Shimpi [14] discussed the refined shear deformation theories of isotropic and anisotropic laminated plates.

In this work, the effect of finite deformation on vibration of transversely isotropic semilinear elastic thin plate is studied. The theory of finite elasticity is inherently nonlinear and is mathematically quite complex. The main difficulty with problems in finite deformations is encountered in how to obtain a workable energy function or a constitutive law [15] in developing relevant mathematical models. From both the practical and theoretical points of view the consideration of finite deformation is of interest, since amongst other things, it enables some phenomena, usually referred to as effects and often supressed through the small (or infinitesimal) deformation approach, to be detected. Here, the finite deformation approach is exploited to examine the influence of flexural rigidity on frequency of vibration of the plate.

MATHEMATICAL DERIVATION

The pertinent energy function on the basis of the semilinear material (John material) had been developed by John [16]. Therefore, the hypothesis of hyperelasticity by taking the Frechet derivative of the energy with respect to the tensor of deformation to obtain the constitutive relation (Piola stress tensor) is invoked. Using the double series solution, the vibration problem is solved to obtain the expression for the frequency of a transversely isotropic semilinear elastic thin plate. Also, the nonlinear effect which may be due to the finite deformation consideration is discussed.

Vibration Equation For Small Deformation

We recall the classical theory for infinitesimal deformation. The dynamic equation of an homogeneous isotropic elastic thin plate is given [4] by:

$$D\nabla^4 w = -\rho h \partial^2 w / \partial t^2 , \qquad (1)$$

Where, $D = Eh^3 / [12(1-\nu^2)]$ is the stiffness of the plate with linear material, h is the thickness of the plate, ν is the Poisson ratio of the linear material, ρ is the mass density of the plate and E is the Young modulus of the plate.

Double Series Solution for Linear Material

We assume solution of the form:

$$w = \sum\sum (B_{mn}\cos\lambda_{mn}t + B^*_{mn}\sin\lambda_{mn}t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} , \qquad (2)$$

Substitute (2) into (1), we obtain:

$$\left[\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4\right][B_{mn}\cos\lambda_{mn}t + B^*_{mn}\sin\lambda_{mn}t]\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$= \lambda^2_{mn}\frac{\rho h}{D}[B_{mn}\cos\lambda_{mn}t + B^*_{mn}\sin\lambda_{mn}t]\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \quad (3)$$

$$\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4 = \lambda^2_{mn}\frac{\rho h}{D}, \quad (4)$$

$$\lambda_{mn} = \pi^2\sqrt{\frac{D}{\rho h}}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right), \quad (5)$$

where, λ_{mn} is the frequency of the linear material.

Vibration Equation For Finite Deformation

Problem setting

Let Ω be a transversely isotropic elastic thin plate, a subset of three dimensional Euclidean space E^3 (i.e. $\Omega \in E^3$). We seek the equation involving the motion of the plate. But we need to know the bending equation of the static problem. This shows that the plate is deflected from an initial configuration Ω_o with position vector $\boldsymbol{r}$ into the current configuration Ω with the position vector R. We have the displacement of the particle as $\boldsymbol{u}=\boldsymbol{R}-\boldsymbol{r}$ where, (x_1, x_2, x_3) and (X_1, X_2, X_3) are the local coordinates in the initial and the current configurations, respectively.

The position vectors of particles in the initial configuration and current configuration is given respectively as:

$\boldsymbol{r} = \boldsymbol{r}(x_1, x_2, x_3)$ and $\boldsymbol{R} = \boldsymbol{R}(X_1, X_2, X_3)$.

Let the displacement in component form be

$$\boldsymbol{u} = \left(-x_3\frac{\partial w}{\partial x_1}, -x_3\frac{\partial w}{\partial x_2}, w\right).$$

Then,

$$\boldsymbol{R} = \boldsymbol{r} + \boldsymbol{u} = \left(x_1 - x_3\frac{\partial w}{\partial x_1}, x_2 - x_3\frac{\partial w}{\partial x_2}, x_3 + w\right). \quad (6)$$

Geometry of Deformation

Let the geometry of deformation of Ω from initial configuration Ω_o denoted by position vector $r=x_1\boldsymbol{k}_1+x_2\boldsymbol{k}_2+x_3\boldsymbol{k}_3$ onto the current configuration Ω denoted by position vector $R=X_1K_1+X_2K_2+X_3K_3$ be the gradient tensor $\dot{\nabla}\boldsymbol{R}$, where $\dot{\nabla}$ is the gradient operator in the initial configuration Ω_o .

From (6), we write the gradient tensor as:

$$\dot{\nabla}\boldsymbol{R} = \dot{\nabla}\left(x_1 - x_3\frac{\partial w}{\partial x_1}, x_2 - x_3\frac{\partial w}{\partial x_2}, x_3 + w\right).$$

This implies that the gradient tensor of the position vector R in Ω taken in the initial configuration $\Omega_o(r)$ is

$$\dot{\nabla}\boldsymbol{R} = \begin{pmatrix} 1 - x_3\frac{\partial^2 w}{\partial x_1^2} & -x_3\frac{\partial^2 w}{\partial x_1 \partial x_2} & \frac{\partial w}{\partial x_1} \\ -x_3\frac{\partial^2 w}{\partial x_1 \partial x_2} & 1 - x_3\frac{\partial^2 w}{\partial x_2^2} & \frac{\partial w}{\partial x_2} \\ -\frac{\partial w}{\partial x_1} & -\frac{\partial w}{\partial x_2} & 1 \end{pmatrix}, \quad (7)$$

and the transpose of the gradient tensor $\dot{\nabla}\boldsymbol{R}^T$ is given as:

$$\dot{\nabla}\boldsymbol{R}^T = \begin{pmatrix} 1 - x_3\frac{\partial^2 w}{\partial x_1^2} & -x_3\frac{\partial^2 w}{\partial x_1 \partial x_2} & -\frac{\partial w}{\partial x_1} \\ -x_3\frac{\partial^2 w}{\partial x_1 \partial x_2} & 1 - x_3\frac{\partial^2 w}{\partial x_2^2} & -\frac{\partial w}{\partial x_2} \\ \frac{\partial w}{\partial x_1} & \frac{\partial w}{\partial x_2} & 1 \end{pmatrix}. \quad (8)$$

We proceed to obtain the symmetric stretch tensor $\widetilde{U}$ such that

$\widetilde{U}^2 = \dot{\nabla}\boldsymbol{R} \cdot \dot{\nabla}\boldsymbol{R}^T = \boldsymbol{r}^m \boldsymbol{R}_m \cdot \boldsymbol{R}_n \boldsymbol{r}^n = G_{mn}\boldsymbol{r}^m \boldsymbol{r}^n = \widetilde{G}$ (9)

$$\widetilde{U} = \begin{pmatrix} 1 - x_3 \frac{\partial^2 w}{\partial x_1^2} & -x_3 \frac{\partial^2 w}{\partial x_1 \partial x_2} & -\frac{\partial w}{\partial x_1} \\ -x_3 \frac{\partial^2 w}{\partial x_1 \partial x_2} & 1 - x_3 \frac{\partial^2 w}{\partial x_2^2} & -\frac{\partial w}{\partial x_2} \\ -\frac{\partial w}{\partial x_1} & -\frac{\partial w}{\partial x_2} & -1 \end{pmatrix}. \quad (10)$$

Recall that, any square matrix A can be decomposed into its symmetric part A^S and orthogonal part A^O, i.e $A = A^S \cdot A^O$.

Therefore, we decompose the gradient tensor $\dot{\nabla}\boldsymbol{R}$ into stretch symmetric tensor $\widetilde{U}$ and orthogonal rotation tensor $\widetilde{\boldsymbol{O}}^{\boldsymbol{D}}$ such that

$\dot{\nabla}\boldsymbol{R} = \widetilde{\boldsymbol{U}} \cdot \widetilde{\boldsymbol{O}}^{\boldsymbol{D}}$. (11)

The orthogonal rotation tensor $\widetilde{O}^D$ is expressed as

$\widetilde{\boldsymbol{O}}^{\boldsymbol{D}} = \widetilde{\boldsymbol{U}}^{-1} \cdot \dot{\nabla}\boldsymbol{R}$, (12)

$$\widetilde{\boldsymbol{O}}^{\boldsymbol{D}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (13)$$

Constitutive Law for Transversely-Isotropic Plane Body

We recall the stress potential for the generalized Hooke's body

$W = \frac{1}{2}\tilde{C} \cdot\cdot \tilde{\varepsilon}\tilde{\varepsilon}$, (14)

where $\tilde{C}$ is the rank-4 elasticity tensor and $\tilde{\varepsilon}$ is the rank-2 small strain tensor [5].

When the body becomes isotropic medium, the above energy function reduces to

$W = \frac{1}{2}\lambda I_1^2(\tilde{\varepsilon}) + \mu I_1(\tilde{\varepsilon}^2)$, (15)

where λ, μ are the Lame constants and $I_1(\tilde{\varepsilon})$ is the first invariant of the rank-2 small strain tensor [5].

On the basis of Equation (15), John [5,16] constructed energy function for isotropic semilinear material in finite deformation as

$W = \frac{1}{2}\lambda S_1^2 + \mu S_2$, (16)

where $S_1 = I_1(\widetilde{U} - \widetilde{E})$ and $S_2 = I_1(\widetilde{U} - \widetilde{E})^2$ are the invariants of deformation [5,16].

On the basis of equation (16) we take the energy function for a transversely isotropic semilinear material in the case of plane deformation as [5]:

$W = \lambda_2 S_2 + \frac{1}{2}\lambda_1 S_1^2 + \lambda_0 S_0$ (17)

$S_0 = \bar{c} \cdot \widetilde{U}^2 \cdot \bar{c}$,

$S_1 = \widetilde{E} \cdot\cdot (\widetilde{U} - \widetilde{E}) \equiv I_1(\widetilde{U} - \widetilde{E})$,

$S_2 = I_1(\widetilde{U} - \widetilde{E})^2$, (18)

where, S_0 is an additional invariant of deformation, due to anisotropy. $\bar{c}$ is the unit vector that characterises the direction of anisotropy. $\lambda_0, \lambda_1, \lambda_2$ are the material constants. In the case of randomly unidirectional fibre reinforced composite the material constants are the effective moduli [17]:

$$\lambda_2 = \langle \mu \rangle ,$$

$$\lambda_1 = \langle \lambda \rangle + \frac{\langle \lambda/(\lambda + 2\mu) \rangle}{1/(\lambda + 2\mu)} - \langle \frac{\lambda^2}{\lambda + 2\mu} \rangle ,$$

$$\lambda_3 = \frac{1}{\langle 1/\mu \rangle}, \qquad \lambda_0 = \lambda_0(\lambda_2, \lambda_3) ,$$

and we note that in the case of degeneracy into isotropic, the energy function (17) automatically reduces to energy function (16) and according to the effective modulli $\lambda_3, \lambda_2, \lambda_1$, while $\lambda_0 = 2(\lambda_3 - \lambda_2)$ vanishes, i.e.

$\lambda_3 = \lambda_2 = \mu, \quad \lambda_1 = 0, \quad \lambda_0 = 0$. (19)

For any finite function $\varphi(\xi, t) \in \Omega \times [0,T)$, $\langle \varphi \rangle$ denotes its geometric average over Ω with volume $|\Omega|$: $\langle \varphi \rangle \left(1/_{|\Omega|} \right) \int_{\Omega} \varphi d\Omega$..

Now invoking the hypothesis of hyperelasticity of Cauchy-Truesdell [18], we take the frechet derivative [15] of the energy with respect to the geometry of the deformation (the deformation gradient) $\dot{\nabla}\boldsymbol{R}$ and obtain Piola stress tensor $\widetilde{\boldsymbol{P}}$ to which it is energy conjugate:

$$\widetilde{\boldsymbol{P}} = \frac{\partial W}{\partial \dot{\nabla}\boldsymbol{R}} ,$$

$$\widetilde{\boldsymbol{P}} = 2\lambda_2 \dot{\nabla}\boldsymbol{R} + (\lambda_1 S_1 - 2\lambda_2)\widetilde{\boldsymbol{O}}^D + 2\lambda_0 \boldsymbol{cc} \cdot \dot{\nabla}\boldsymbol{R}\,. \quad (20)$$

We observe that depending on the direction of anisotropy the vector $\boldsymbol{c}$ could be in any direction, including $\boldsymbol{k}_1$, $\boldsymbol{k}_2$ and $\boldsymbol{k}_3$. For our own medium, the transverse isotropy is in the transverse direction, hence we choose $\boldsymbol{c}$ to coincide with $\boldsymbol{k}_3$.

Dynamic Equation For A Transversely Isotropic Thin Plate

The equation of vibration for a transversely isotropic semilinear elastic thin plate is

$$D_o \nabla^4 \mathrm{w} + D_h \nabla^2 \mathrm{w} = -\rho h \, {\partial^2 w}/{\partial t^2}\,. \quad (21)$$

Double Series Solution for Semilinear Material

We assume solution of the form:

$$w = \sum\sum (B_{mn} \cos \lambda_{mn} t + B^*_{mn} \sin \lambda_{mn} t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\,. \quad (22)$$

By Substituting (22) into (21), we obtain:

$$\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4 - \frac{D_h}{D_o}\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right) = \lambda^2_{mn}\frac{\rho h}{D_o}\,.$$

Note: $-D_h = D = h(2\lambda_2 + \lambda_1)$, we then obtain;

$$\lambda^*_{mn} = \sqrt{\pi^4 \frac{D_o}{\rho h}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2 + \pi^2 \frac{D}{\rho h}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)}\,, \quad (23)$$

where λ^*_{mn} is the frequency of the semilinear material.

RESULTS

The tabular comparison of the frequency of linear and semilinear materials is shown for a given square plate with sides $a=b=100$units, such that min $(a,b)= 100$. According to the theory of thin plate, we have $8 \leq \dfrac{min(a,b)}{h} < 80$, implying that $1.25 < h \leq 12.5$.

The tables below show the comparison of λ_{mn} and λ^*_{mn} for fixed value of Young modulus E which is set to be $1000000000 N/m^2$ while we vary the Poisson ratio (ν) from 0.1 to 0.49 and the thickness (h) from 2 to 12.

Table 3.1: Frequency (λ_{mn} and λ^*_{mn}) Values at Different Modes (m,n) for a Linear and Semilinear Elastic Thin Plate, Given $E = 1000000000N/m^2$, $\nu = 0.1$ and $h = 2$.

(m,n)	0		20		40		60		80	
	$(\times 10^3)$ λ_{mn}	$(\times 10^3)$ λ^*_{mn}	$(\times 10^3)$ λ_{mn}	$(\times 10^3)$ λ^*_{mn}	$(\times 10^3)$ λ_{mn}	$(\times 10^3)$ λ^*_{mn}	$(\times 10^3)$ λ_{mn}	$(\times 10^3)$ λ^*_{mn}	$(\times 10^3)$ λ_{mn}	$(\times 10^3)$ λ^*_{mn}
0	0	0	0.0836	0.2468	0.3346	0.5733	0.7528	1.0288	1.3384	1.6355
20	0.0836	0.2468	0.1673	0.3688	0.4182	0.6680	0.8365	1.1166	1.4220	1.7212
40	0.3346	0.5733	0.4182	0.6680	0.6692	0.9402	1.0874	1.3773	1.6729	1.9775
60	0.7528	1.0288	0.8365	1.1166	1.0874	1.3773	1.5056	1.8067	2.0912	2.4028
80	1.3384	1.6355	1.4220	1.7212	1.6729	1.9775	2.0912	2.4028	2.6767	2.9962
100	2.0912	2.4028	2.1748	2.4877	2.4258	2.7421	2.8440	3.1654	3.4295	3.7571

λ_{mn} is the frequency values for a linear elastic thin plate at different modes (m,n). λ^*_{mn} is the frequency values for a semilinear elastic thin plate at different modes (m, n).

Table 3.2: Frequency (λ_{mn} and λ^*_{mn}) Values at Different Modes (m, n) for a Linear and Semilinear Elastic Thin Plate, Given $E = 1000000000N/m^2$, $v = 0.4$ and $h = 2$.

(m,n)	0		20		40		60		80	
	$(\times 10^3)$ λ_{mn}	$(\times 10^3)$ λ^*_{mn}	$(\times 10^3)$ λ_{mn}	$(\times 10^3)$ λ^*_{mn}	$(\times 10^3)$ λ_{mn}	$(\times 10^3)$ λ^*_{mn}	$(\times 10^3)$ λ_{mn}	$(\times 10^3)$ λ^*_{mn}	$(\times 10^3)$ λ_{mn}	$(\times 10^3)$ λ^*_{mn}
0	0	0	0.0908	0.3573	0.3632	0.8299	0.8173	1.4891	1.4529	2.3674
20	0.0908	0.3573	0.1816	0.5338	0.4540	0.9670	0.9081	1.6163	1.5438	2.4914
40	0.3632	0.8299	0.4540	0.9670	0.7265	1.3610	1.1805	1.9937	1.8162	2.8624
60	0.8173	1.4891	0.9081	1.6163	1.1805	1.9937	1.6346	2.6153	2.2702	3.4781
80	1.4529	2.3674	1.5438	2.4914	1.8162	2.8624	2.2702	3.4781	2.9059	4.3369
100	2.2702	3.4781	2.3610	3.6009	2.6335	3.9692	3.0875	4.5819	3.7232	5.4384

λ_{mn} is the frequency values for a linear elastic thin plate at different modes (m, n). λ^*_{mn} is the frequency values for a semilinear elastic thin plate at different modes (m, n).

Table 3.3: Frequency (λ_{mn} and λ^*_{mn}) Values at Different Modes (m, n) for a Linear and Semilinear Elastic Thin Plate, Given $E = 1000000000N/m^2$, $v = 0.1$ and $h = 7$.

(m,n)	0		20		40		60		80	
	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}
0	0	0	0.0293	0.0375	0.1171	0.1266	0.2635	0.2741	0.4684	0.4804
20	0.0293	0.0375	0.0586	0.0674	0.1464	0.1562	0.2928	0.3036	0.4977	0.5099
40	0.1171	0.1266	0.1464	0.1562	0.2342	0.2446	0.3806	0.3920	0.5855	0.5982
60	0.2635	0.2741	0.2928	0.3036	0.3806	0.3920	0.5270	0.5393	0.7319	0.7456
80	0.4684	0.4804	0.4977	0.5099	0.5855	0.5982	0.7319	0.7456	0.9368	0.9518
100	0.7319	0.7456	0.7612	0.7750	0.8490	0.8634	0.9954	1.0107	1.2003	1.2169

λ_{mn} is the frequency values for a linear elastic thin plate at different modes (m, n). λ^*_{mn} is the frequency values for a semilinear elastic thin plate at different modes (m, n).

Table 3.4: Frequency (λ_{mn} and λ^*_{mn}) Values at Different Modes (m, n) for a Linear and Semilinear Elastic Thin Plate, Given $E = 1000000000N/m^2$, $v = 0.4$ and $h = 7$.

(m,n)	0		20		40		60		80	
	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}
0	0	0	0.0318	0.0543	0.1271	0.1833	0.2860	0.3968	0.5085	0.6954
20	0.0318	0.0543	0.0636	0.0976	0.1589	0.2260	0.3178	0.4394	0.5403	0.7380
40	0.1271	0.1833	0.1589	0.2260	0.2543	0.3541	0.4132	0.5674	0.6357	0.8660
60	0.2860	0.3968	0.3178	0.4394	0.4132	0.5674	0.5721	0.7807	0.7946	1.0792
80	0.5085	0.6954	0.5403	0.7380	0.6357	0.8660	0.7946	1.0792	1.0171	1.3777
100	0.7946	1.0792	0.8264	1.1218	0.9217	1.2498	1.0806	1.4630	1.3031	1.7615

λ_{mn} is the frequency values for a linear elastic thin plate at different modes (m, n). λ^*_{mn} is the frequency values for a semilinear elastic thin plate at different modes (m, n).

Table 3.5: Frequency (λ_{mn} and λ^*_{mn}) Values at Different Modes (m, n) for a Linear and Semilinear Elastic Thin Plate, Given $E = 1000000000N/m^2$, $v = 0.1$ and $h = 12$.

(m,n)	0		20		40		60		80	
	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}
0	0	0	0.0502	0.5560	0.2008	0.2073	0.4517	0.4598	0.8030	0.8133
20	0.0502	0.0556	0.1004	0.1062	0.2509	0.2578	0.5019	0.5103	0.8532	0.8638
40	0.2008	0.2073	0.2509	0.2509	0.4015	0.4093	0.6524	0.6618	1.0038	1.0153
60	0.4517	0.4598	0.5019	0.5019	0.6524	0.6618	0.9034	0.9143	1.2547	1.2678
80	0.8030	0.8133	0.8532	0.8532	1.0038	1.0153	1.2547	1.2678	1.6060	1.6214
100	1.2547	1.2678	1.3049	1.3049	1.4555	1.4698	1.7064	1.7224	2.0577	2.0759

λ_{mn} is the frequency values for a linear elastic thin plate at different modes (m, n). λ^*_{mn} is the frequency values for a semilinear elastic thin plate at different modes (m, n) .

Table 3.6: Frequency (λ_{mn} and λ^*_{mn}) Values at Different Modes (m, n) for a Linear and Semilinear Elastic Thin Plate, Given $E = 1000000000N/m^2$, $v = 0.4$ and $h = 12$.

(m,n)	0		20		40		60		80	
	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}	$(\times 10^4)$ λ_{mn}	$(\times 10^4)$ λ^*_{mn}
0	0	0	0.0545	0.0804	0.2179	0.3000	0.4904	0.6656	0.8718	1.1773
20	0.0545	0.0804	0.1090	0.1537	0.2724	0.3731	0.5449	0.7387	0.9263	1.2504
40	0.2179	0.3000	0.2724	0.3731	0.4359	0.5925	0.7083	0.9580	1.0897	1.4697
60	0.4904	0.6656	0.5449	0.7387	0.7083	0.9580	0.9807	1.3235	1.3621	1.8352
80	0.8718	1.1773	0.9263	1.2504	1.0897	1.4697	1.3621	1.8352	1.7435	2.3469
100	1.3621	1.8352	1.4166	1.9083	1.5801	2.1276	1.8525	2.4931	2.2339	3.0048

λ_{mn} is the frequency values for a linear elastic thin plate at different modes (m, n). λ^*_{mn} is the frequency values for a semilinear elastic thin plate at different modes (m, n).

DISCUSSION OF RESULTS

Comparing Equations (5) and (23), we observed from Tables 3.1 to 3.6 that λ^*_{mn} which is the frequency of the semilinear material is always greater than λ_{mn}, the frequency of the linear material. This was observed at different modes (m, n) of the elastic plate. Furthermore, the frequency of both linear material and semilinear material at different modes (m, n) of the elastic plate increases as the value of the Poisson ratio(ν) increased from 0.1 to 0.49. Also, the frequency at different modes (m, n) of the elastic plate increases as the value of the thickness (h) increased from 2 to 12. It was also observed from Tables 3.1 to 3.6 that the frequency at mode (m, n) is the same as that of the mode (n, m).

CONCLUSIONS

Here, we obtained mathematical expression, equation (23), for the vibration frequency of thin plate in finite deformation on the basis of the semilinear material. The effect of finite deformation obtained is that the frequency which is of significant importance in civil exploitation is higher for a semilinear material (John's material) compared to that of a linear material (Hooke's material).

Specifically, the vibration frequency of a semilinear material is greater than that of a linear material for a given value of the plate thickness and the Poisson's ratio. In addition, the frequency increases for both the linear and semilinear materials for an increased value of thickness and

Poisson ratio. The results of this work will find practical applications in design and modelling of thin walled structures with respect to the issues of structure failure and noise control.

REFERENCES

Timoshenko, S. and Woinowsky-Krieger, S., 1959. *Theory of Plates and Shells*, McGraw-Hill, New York.

Ding, H., Chen, W. and Zhang, L., 2006. *Elasticity of Transversely Isotropic Materials*, Springer, Netherlands.

Amenzade, Y., 1979. *Theory of Elasticity*, MIR Publisher, Moscow.

Haym, B., 1998. *Mechanical Vibration: Analysis, Uncertainties and Control*. Prentice Hall, New Jersey.

Akinola, A., 1999. *An* Energy Function for Transversely-Isotropic Elastic Material and the Ponyting Effect, *Korean J. comput. and Appl. Math.* 6(3), 639-649.

Ventsel, E. and Krauthanmer, T., 2001. *Thin Plates and Shells: theory, analysis and applications.* Marcel Dekker, Inc., New York and Basel.

Igor, V.A., Jan, A. and Vladimir, C., 2006. *Analysis of Natural In-Plane vibration of Rectangular Plates Using Homotopy Perturbation Approach* Hindawi Publishing Corporation, Mathematical Problems in Engineering, 2006, 1-8.

Jomehzadeh, E. and Saidi, A.R., 2009. Analytical Solution for Free Vibration of Transversely Isotropic Sector Plates Using a Boundary Layer Function, *Thin-walled Structures* 47(1), 82-88.

Jovanovic V., 2010. A Fourier Series Solution for the Transverse Vibration Response of a Beam with a Viscous Boundary, *Journal of Sound and Vibration* 330(7), 1504-1515.

Omolofe, B. and Ogunyebi, S.N., 2009. Transverse Vibrations of Elastic Thin Beam Resting on Variable Elastic Foundations and Subjected to Travelling Distributed Forces, *Pacific Journal of Science and Technology* 10(2), 112-119.

Oni, S.T. and Awodola, T.O., 2010. Dynamic Response Under a Moving Load of an Elastically Supported Non-Prismatic Bernoulli-Euler Beam on Variable Elastic Foundation, *Latin American Journal of Solids and Structures* 7, 3-20.

Xing, Y.F. and Liu, B., 2009. *New Exact Solutions for Free Vibrations of Rectangular Thin Plates by Symplectic Dual Method*, Acta *Mechanica Sinica* 25, 265-270.

Xing, Y.F. and Liu, B., 2009. New Exact Solutions for Free Vibrations of Thin Orthotropic Rectangular Plates, *Composite Structures,* 89(4), 567-574.

Ghugal, Y.M. and Shimpi, R.P., 2002. A Review of Refined Shear Deformation Theories of Isotropic and Anisotropic Laminated Plates, *Journal of Reinforced Plastics and Composites* 21(9), 775-813.

Green, A.E. and Adkins, J., 1960. *Large Elastic Deformations*, Oxford University Press, Oxford, United kingdom.

John, F., 1960. Plain Strain Problem for a Perfectly Elastic Material of Harmonic Type *Commun. Pure and Appl. Math.* 13(2), 239-296.

Akinola, A., 2001. An Indifference Constitutive Law in Finite Elasticity, *Korean J. Comput. And Appl. Math.* 8(3), 695-710.

Akinola, A.P., Layeni, O.P., and Olagunju, M.A., 2004. A Solution by Anisotropic expansion for a Composite Parallelepiped Deformed Into Cylinder, *Applied Mathematics and Computation* 149, 599-611.