ON THE THEORY OF ONE DIMENSIONAL INTEGRATED AUTOREGRESSIVE BILINEAR TIME SERIES MODELLING

Ojo J.F.

Department of Statistics, University of Ibadan, Ibadan, Nigeria.

(Received: September, 2010; Accepted: May, 2011)

ABSTRACT

The theory of one-dimensional integrated autoregressive bilinear time series models which are capable of achieving stationarity for all nonlinear series are proposed in this paper. These models are denoted by BL (p, d, 0, r, 1). The sufficient conditions for stationarity of this bilinear time series models are derived. The conditions for the invertibility of the model are also included. The parameters of the proposed models are estimated using proposed algorithm and robust nonlinear least squares method and statistical properties of the derived estimates are investigated. The bilinear models are fitted to Wolfer sunspot numbers and stationarity conditions are satisfied.

Keywords Non-linear Least Squares, Parameters, Wolfer Sunspot Numbers, Invertibility and Stationarity

INTRODUCTION

Building probability models for time series data is an important activity that enables a statistician to understand the underlying random mechanisms generating the series. Better still, it provides invaluable assistance in forecasting the future. Linear time series, such as the autoregressive (AR) models, have been widely and successfully used in many fields. This is mainly because these models can be easily analyzed and provide fairly good approximations of the underlying random mechanisms of numerous real-life time series.

Nevertheless, in some situations linear time series models may be insufficient in explaining the underlying random mechanisms. This is, for instance, the case with sunspot data and the Canadian lynx data set. Linear time series models cannot adequately describe them, and the test proposed by Subba Rao and Gabr (1980) does confirm that linear Gaussian models fail to describe the above series. Thus a natural alternative that suggests itself is nonlinear models. Undoubtedly, the nonlinear time series models are more complex than linear ones for several reasons. These are difficult parameter estimation of these models; intricate studying of statistical properties of most nonlinear models and sampling distribution of the estimates; and lastly, difficult evaluation of optimal forecasts for several steps in the future from these models. Yet despite these problems it seems reasonable to expect that in many situations nonlinear time series model should work better than a linear time series one.

Special nonlinear models considered by Granger and Andersen (1978) and Subba Rao (1981) are known as bilinear (BL) time series models. Providing a good fit, this class of time series has been found useful in many areas of biological sciences, ecology and engineering (e.g., Bruni et al. 1974). Thus many researchers have studied various bilinear models (e.g., Pham and Tran 1981, Gabr and Subba Rao 1981, Rao et al. 1983, Liu 1992, Cathy 1997, Gonclaves et al. 2000, Shangodoyin and Ojo 2003, Wang and Wei 2004, Boonchai and Eivind 2005, Bibi 2006, Doukhan et al. 2006, Drost et al. 2007, Usoro and Omekara 2008). This wide use and usefulness notwithstanding, the above models could not achieve stationarity for all nonlinear series. As a

result we proposed bilinear models that could achieve stationarity for all non linear series.

PROPOSED ONE-DIMENTIONAL INTEGRATED AUTOREGRESSIVE BILINEAR TIME SERIES MODELS

The models majorly considered by the above authors have its specification as BL (p, 0, r; 1) and can be defined as follows: $x_t + \sum_{j=1}^{p} \phi_j x_{t-j} = e_t + \left(\sum_{j=1}^{r} \beta_{1i} x_{t-r} \right) e_{t-1}$ and these models could not achieve stationarity for all nonlinear series. Therefore the model proposed that could achieve stationarity for all nonlinear series which is an important improvement over other bilinear time series models is called one-dimensional integrated autoregressive bilinear (BL) time series models and this is defined as follows:

$$\psi(B)X_t = \phi(B)\nabla^d X_t + \left(\sum_{k=1}^{r} b_{k1} X_{t-k} \right) e_{t-1} + e_t, \qquad (1)$$

denoted as BL (p, d, 0, r, 1) where $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 ..... - \phi_p B^p$

In the model, $\psi(B)X_t = \phi(B)\nabla^d$ is called the generalized autoregressive operator; it is a non stationary operator; p is the order of the autoregressive component; d is the degree of consecutive differencing required to achieve stationarity and b's are the component of nonlinear term Also e_t are independently and identically distributed as $N(0, \sigma_e^2)$. We assume that the models are invertible and have a realization $(X_1, X_2, ..., X_n)$ on the time series $\{X_t\}$ (Ojo (2009)).

The Vector Form of BL (p, d, 0, r, 1)

It is convenient to study the properties of a process when the model is in the state space form because of the Markovian nature of the model (Akaike (1974)).

Let

$$\underset{p\times p}{\Psi} = \begin{pmatrix} -\psi_1 & -\psi_2 & -\psi_3 & ..... & -\psi_{p+d-1} & -\psi_{p+d} \\ 1 & 0 & 0 & ..... & 0 & 0 \\ 0 & 1 & 0 & .... & 0 & 0 \\ 0 & 0 & 0 & ..... & 1 & 0 \end{pmatrix}$$

$$\underset{r\times r}{B_j} = \begin{pmatrix} b_{11} & b_{21} & b_{31} & .... & b_{r1} \\ 0 & 0 & 0 & .... & 0 \\ 0 & 0 & 0 & .... & 0 \end{pmatrix} \quad j=1$$

and vectors $\underset{1\times p}{C^T} = (1,0,0,.....,0)$ and let $\underset{1\times p}{X^T} = (X_t, X_{t-1}, ..., X_{t-p+d})$, (Here T stands for the transpose of a matrix) t = ...-1, 0, 1,..... With this notation, we can write the model (1) in the vector form as:

$$\mathbf{X}_t = \Psi \mathbf{X}_{t-1} + \mathbf{B}\mathbf{X}_{t-1} e_{t-1} + \mathbf{C} e_t \qquad (2)$$

STATIONARITY AND CONVERGENCE OF BL (p, d, 0, r, 1)

In this section, we give a sufficient condition for the existence of strictly stationary process and convergence conforming to the bilinear model (1). This we do through the following theorem.

Theorem 1

Let $\{e_t, t \in Z\}$ be a sequence of i.i.d. random variables defined on a probability space (Ω, F, P) such that $Ee_t = 0$ and $Ee_t^2 = \sigma^2 < \infty$. Let $\mathbf{C}$ be any column vector with components $c_1, c_2, \ldots, c_p$. Ψ and $\mathbf{B}$ be two matrices of order $p \times p$ such that $\rho(\Psi \otimes \Psi + \sigma^2 \mathbf{B} \otimes \mathbf{B}) = \lambda < 1$. The series of random vectors

$$\sum_{r \geq 1} \prod_{j=1}^{r} (\Psi + \mathbf{B}e_{t-j}) \mathbf{C}e_{t-r}$$

converges absolutely almost surely as well as in the mean for every fixed t in Z. Further, if

$$X_t = \mathbf{C}e_t + \sum_{r \geq 1} \prod_{j=1}^{r} (\Psi + \mathbf{B}e_{t-j}) \mathbf{C}e_{t-r}, \quad t \in Z,$$

then for every t in Z, $\{X_t, t \in Z\}$ is a strictly stationary process conforming to the bilinear model

$$\mathbf{X}_t = \Psi \mathbf{X}_{t-1} + \mathbf{B}\mathbf{X}_{t-1}e_{t-1} + \mathbf{C}e_t.$$

Conversely, if is a strictly stationary process satisfying

$$X_t = \mathbf{C}e_t + \Psi X_{t-1} + \mathbf{B}X_{t-1}e_{t-1} \qquad (3)$$

for every t in Z for some sequence $\{e_t, t \in Z\}$ of i.i.d. random variables with $Ee_t = 0$ and $Ee_t^2 = \sigma^2 < \infty$ and some matrices Ψ, B, $\mathbf{C}$ of orders $p \times p$, $r \times r$, $p \times 1$ with $\rho(\Psi \otimes \Psi + \sigma^2 \mathbf{B} \otimes \mathbf{B}) = \lambda < 1$, then for every t in Z.

Proof of theorem 1 is given in Appendix A.

INVERTIBILITY CONDITION OF BL(p, d, 0, r, 1)

For a time series model to be useful for forecasting purposes, it is necessary that it should be invertible. The invertibility of linear time series models has been discussed by Box and Jenkins (1970). Granger and Andersen (1978c) have provided another definition of invertibility which can be applied to both linear and nonlinear time series models. Their definition is as follows:

Let be a discrete parameter time series satisfying the model

$$X_t = f\{X_{t-j}, e_{t-j}, j = 1,2,\ldots,P\} + e_t \qquad (4)$$

where the $\{e_t\}$ are independent random variables. The random variables $\{e_t\}$ are not observable. Let $\{\hat{e}_t\}$ be an "estimate" of $\{e_t\}$, and let the initial values of $\{\hat{e}_t\}$ be set equal to zero. The model (4) is said to be invertible if $\lim_{t \to \infty} E\{e_t - \hat{e}_t\}^2 \to 0$ when the model and the parameters are known completely. The above definition holds for the bilinear models under study. Using this definition, we obtain a sufficient condition for the invertibility of the VBL(p, d, 0, r, 1) model.

Consider the VBL(p) model given by (2) and let $\mathbf{X}_t = \mathbf{H}^T\mathbf{X}_t$. Let $\{\hat{e}_t\}$ be an estimate of $\{e_t\}$ satisfying the difference equation

$$X_t = H^T \Psi X_{t-1} + H^T B X_{t-1} \hat{e}_{t-1} + H^T C \hat{e}_t \qquad (5)$$

$$H^T C \xi_t = -H^T B X_{t-1} \xi_{t-1} \qquad (6)$$

where $\xi_t = e_t - \hat{e}_t$. Assuming the process X_t to be ergodic

$$E\{\xi_t^2\} \leq [E\{\zeta_s^2\}]^t, \qquad (7)$$

where $\zeta_s = \{H^T B X_s\} / H^T C$. Taking limits on both sides of (7)

$$\lim_{t \to \infty} E\{\xi_t^2\} \leq \lim_{t \to \infty} [E\{\zeta_s^2\}]^t.$$

The right-hand term of the inequality tends to zero as t→∞ if $E\{\zeta_s^2\} < 1$ which implies that

$$H^T B E\{X_s X_s^T\} B^T H < (H^T C)^2 \qquad (8)$$

The condition (8) is a sufficient condition for the invertibility of the VBL (p, d, 0, r, 1) model.

Algorithm for fitting One-dimensional integrated autoregressive bilinear time series models

We will fit full autoregressive models with difference operator of various orders and choose that model for which the Akaike Information (AIC) is minimum. Let the order of this full linear autoregressive model with difference operator be $p+d$ and let the model be

$$X_t = \psi_1 X_{t-1} + ,\ldots\ldots + \psi_{p+d} X_{t-p-d} + e_t$$

Now, let the mean sum of squares of the residuals be $\hat{\sigma}_e^{2(1)}$ and its AIC be equal to AIC(1). The estimation of the full autoregressive models with difference operator can be done by the least squares method. Having fitted the full autoregressive with difference operator and with the minimum AIC (1), we can now fit the best subset autoregressive model with difference operator by considering the 2^k–1subsets.

The subsets can be fitted using the approach of Hagan and Oyetunji (1980).The model for which AIC is minimum among the fitted subset models is chosen. This is the best subset autoregressive model with difference operator. Let the best subset autoregressive model with difference operator be

$$X_t = \sum_{i=1}^{k} \psi_{m_i+d} X_{t-m_i-d} + e_t$$

where $m_1, m_2, \ldots\ldots, m_{k+d}$ are subsets of the integers (1, 2, …., $p+d$). Let the mean sum of squares of the residuals be and the AIC value be AIC (2), AIC(2) d" AIC(1). The estimation of the subsets autoregressive model with difference operator is done using the least squares method. Having fitted the subset autoregressive model, we now define the bilinear model as follows.

$$X_t = \psi_1 X_{t-1} + \ldots\ldots + \psi_{p+d} X_{t-p-d} + e_t + \left(\sum_{k=1}^{r} b_{k1} X_{t-k} \right) e_{t-1}$$

Using the estimation method described in the following subsection, we fit models of various orders and choose the model for which AIC is minimum. Let the minimum AIC be AIC (3), AIC (3) d" AIC (2). This is the fitted one-dimensional integrated autoregressive bilinear models.

Estimation of parameters of BL (p, d,0, r, 1)

The joint density function of $(e_m, e_{m+1}, \ldots, e_n)$, where $m = \max(r, 1)$, is given by

$$\frac{1}{(2\pi\sigma_e^2)^{(n-m+1)/2}} \exp\left(\frac{1}{-2\sigma_e^2} \sum_{m}^{n} e_t^2\right) \qquad (9)$$

Since the Jacobian of the transformation from $(e_m, e_{m+1}, \ldots, e_n)$ to $(X_m, X_{m+1}, \ldots, X_n)$ is unity, the likelihood function of $(X_m, X_{m+1}, \ldots, X_n)$ is the same as the joint density function of $(e_m, e_{m+1}, \ldots, e_n)$. Thus maximizing the likelihood function is equivalent to minimizing the function, which is as follows:

$$Q(\mathbf{G}) = \sum_{i=m}^{n} e_t^2, \qquad (10)$$

with respect to the parameter $\mathbf{G} = (\psi_1, \ldots, \psi_{p+d}; B_{11}, \ldots, B_{r1})$. For convenience, we shall write $G_1 = \psi_1, G_2 = \psi_2, \ldots\ldots, G_R = B_{r1}$, where R = p +d+r_1. Then the partial derivatives of $Q(\mathbf{G})$ are given by

$$\frac{dQ(\mathbf{G})}{dG_i} = 2 \sum_{t=m}^{n} e_t \frac{de_t}{dG_i} \quad (i = 1, 2, \ldots.., R) \qquad (11)$$

$$\frac{d^2 Q(\mathbf{G})}{dG_i dG_j} = 2\left(\sum_{t=m}^{n} e_t \frac{de_t}{dG_i} \frac{de_t}{dG_j} + \sum_{t=m}^{n} e_t \frac{d^2 e_t}{dG_i dG_j} \right)$$

where the partial derivatives of e_t satisfy the recursive equations

$$\frac{de_t}{d\psi_i} + \sum_{j=1}^{s} W_t(t) \frac{de_{t-j}}{d\psi_i} = \begin{cases} 1, \text{ if } i = 0 \\ X_{t-i}, \text{ if } i = 1, 2, \ldots, p \end{cases} \qquad (12)$$

$$\frac{de_t}{dB_{k1}} + \sum_{j=1}^{s} W_j(t) \frac{de_{t-j}}{dB_{k1}} = -X_{t-k} e_{t-m}$$

(k=1,2,...,r; m_i =1) (13)

$$\frac{d^2 e_t}{d\psi_i d\psi_i'} + \sum_{j=1}^{s} W_j(t) \frac{d^2 e_{t-j}}{d\psi_i d\psi_i'} = 0$$

(i, i' = 1, 2, ..., p) (14)

$$\frac{d^2 e_t}{d\psi_i dB_{k1}} + \sum_{j=1}^{s} W_j(t) \frac{d^2 e_{t-j}}{dB_{k1} d\phi_i} + X_{t-k} \frac{d^2 e_{t-mi}}{d\psi_i} = 0$$

(i=1,2,...,p ; k_i =1,2,...,r; m_i=1) (15)

$$\frac{d^2 e_t}{d\psi_i d\theta_i} + \sum_{j=1}^{s} W_j(t) \frac{d^2 e_{t-j}}{d\psi_i d\theta_i} = 0 \quad (16)$$

$$\frac{d^2 e_t}{dB_{k1} dB_{k1}'} + \sum_{j=1}^{s} W_j(t) \frac{d^2 e_{t-j}}{dB_{k1} dB_{k1}'} + X_{t-k}' \frac{d^2 e_{t-mi}}{dB_{k1}} = -X_{t-k} \frac{de_{t-m}}{dB_{k1}'} \quad (k,\ k' = 1,2,...,r;\ m_i m_i' = 1) \quad (17)$$

$$W_j(t) = \sum_{j=1}^{s} B_{ij} X_{t-j}$$

We assume that e_t = 0 (t = 1, 2, ..., m-1) and

$$\frac{de_t}{dG_i} = 0, \frac{d^2 e_t}{dG_i dG_j} = 0, (i, j = 1, 2, ..., R; t = 1, 2, ..., m\text{-}1)$$

From these assumptions and equation (13) it follows that the second order derivatives with respect to ψ_i (i = 1, 2, ..., p) is zero. For a given set of values $\{\psi_i\}$, and $\{B_{il}\}$ one can evaluate the first and second order derivatives using the recursive equations (12), (13) and (17).

Let

$$V(\mathbf{G}) = \frac{dQ(\mathbf{G})}{dG_1}, \frac{dQ(\mathbf{G})}{dG_2}, ..., \frac{dQ(\mathbf{G})}{dG_R}$$

and let $\mathbf{H(G)} = [d^2 Q(\mathbf{G}) / dG_i dG_j]$ be a matrix of second partial derivatives. Expanding $V(\mathbf{G})$, near $\mathbf{G} = \hat{\mathbf{G}}$ in a Taylor series, we obtain

$$\left[\mathbf{V}(\hat{\mathbf{G}})\right]_{\hat{\mathbf{G}}=\mathbf{G}} = 0 = \mathbf{V(G)} + \mathbf{H(G)}(\hat{\mathbf{G}} - \mathbf{G})$$

Rewriting this equation and following Krzanowski (1998), we have $\hat{\mathbf{G}} - \mathbf{G} = -\mathbf{H}^{-1}(\mathbf{G}) V(\mathbf{G})$, thereby obtaining an iterative equation given by

$$\mathbf{G}^{(k+1)} = \mathbf{G}^{(k)} - \mathbf{H}^{-1}(\mathbf{G}^{(k)}) V(\mathbf{G}^{(k)}),$$

where $\mathbf{G}^{(k)}$ is the set of estimates obtained at the k^{th} stage of iteration. The estimates obtained by the above iterative equations usually converge. For starting the iteration, we need to have good sets of initial values of the parameters. This can be obtained as follows:

Suppose we wish to fit one-dimensional bilinear model BL (p, d, 0, r, 1). We choose the coefficients of the integrated autoregressive models (IAR) part of this model equal to the corresponding best subset IAR model. These coefficients are used as the initial values for starting the iteration of iterative equation.

NUMERICAL EXAMPLE: THE WOLFER SUNSPOT DATA

To present the application of the models proposed, we use a real time series dataset, the Wolfer sunspot, available in Box et al. (1994). The scientists track solar cycles by counting sunspots – cool planet-sized areas on the Sun where intense magnetic loops poke through the star's visible surface. It was Rudolf Wolf who devised the basic formula for calculating sunspots in 1848; these sunspot counts are still continued.

As the Wolfer sunspot data set represent a non-stationary series, the bilinear models proposed in this paper may be applied. The Wolfer sunspot data set is considered in this paper at three levels, namely for t = 50, 150 and 250.

Fitted Model at t=50

$$X_t=0.314548X_{t-1} - 0.458429Xt_{-2} - 0.302114X_{t-4} - 0.220568X_{t-5} - 0.386159X_{t-6} - 0.006533X_{t-1}e_{t-1} - 0.016081X_{t-2}e_{t-1} - 0.006741X_{t-3}e_{t-1} + 0.005995Xt_{-4}e_{t-1} - 0.021399X_{t-4}e_{t-1} - 0.014003X_{t-5}e_{t-1} - 0.014003X_{t-6}e_{t-1} - 0.024585X_{t-7}e_{t-1} + e_t$$

Fitted Model at t=150

$$X_t =0.412820X_{t-1} - 0.271125X_{t-2} - 0.270908X_{t-3} - 0.339150X_{t-5} - 0.293320X_{t-7} - 0.002709X_{t-1}e_{t-1} - 0.006085X_{t-2}e_{t-1} - 0.002411X_{t-3}e_{t-1} - 0.009225X_{t-4}e_{t-1} - 0.006196X_{t-5}e_{t-1} + 0.002575X_{t-6}e_{t-1} - 0.021601X_{t-7}e_{t-1} + 0.010533X_{t-8}e_{t-1} + e_t$$

Fitted Model at t=250

$$X_t =0.239576X_{t-2} - 0.361665X_{t-3} - 0.238746X_{t-4} - 0.325416X_{t-5} - 0.328627X_{t-6} - 0.209789X_{t-7} - 0.365561X_{t-8} - 0.003211X_{t-1}e_{t-2} - 0.0111X_{t-2}e_{t-1} - 0.007729X_{t-3}e_{t-1} - 0.008414X_{t-4}e_{t-1} - 0.006220X_{t-5}e_{t-1} - 0.014280X_{t-6}e_{t-1} - 0.002048X_{t-7}e_{t-1} - 0.007351X_{t-8}e_{t-1} + e_t$$

The derived statistics from the above fitted models are given in Table1.

Table 1. Goodness of Fit of One-Dimensional Integrated Autoregressive Bilinear Models at t = 50, t= 150 and t=250. All models are Significant at $P<0.001$.

	Derived Statistics of one-dimensional IA Bilinear Models		
Pd0r1 at	t=50	t=150	t=250
Residual Variance	210.5	207.5	242.5
AIC	8.40	8.27	8.52
Schwartz Criterion	8.52	8.43	8.79
R^2	0.58	0.63	0.61
Adjusted R^2	0.57	0.60	0.56
F(Statistic)	11.97	27.75	54.70

From Table 1, the bilinear models fitted at t = 50, 150 and 250 are significant with probability level less than 0.001. We could see as well that when t = 150 the derived statistics gave us the best estimate.

CONCLUSION

This study focused on new bilinear models that could handle all non-linear series. Bilinear models at different levels of sample sizes were considered using the non-linear real series.

Moreover, estimation of parameters has witnessed a unique, consistent and convergent estimator that has prevented the models from exploding, thereby making stationarity possible. The introduction of the *d* factor in our new models has made us to capture trend and seasonality in the data, which in turn helps arrive at stationarity easily for any time series data set.

REFERENCES

Akaike, H. 1974. Markovian Representation of Stochastic Processes and its Application to the Analysis of Autoregressive Moving Average Processes. *Ann. Inst.Statist. Math.* 26: 363-383

Bibi A. 2006. Evolutionary transfer functions of bilinear process with time varying coefficients. *Computer and Mathematics with Applications* 52: 3-4, 331-338.

Boonaick K. Stensholt B.K. and Stensholt E. 2005. Multivariate bilinear time series: a stochastic alternative in population dynamics. *Geophysical Research Abstracts, Volume 7, 02219 @Europeans Geosciences Union 2005.*

Box, G.E.P, Jenkins, G.M and Reinsel, G.C. 1994. Time series analysis; forecasting and control. 3rd Edition. Prentice Hall, Inc.

Bruni, C., Dupillo, G. and Koch, G. 1974. Bilinear Systems: An Appealing Class of Nearly Linear System in Theory and Application. *IEEE Trans. Auto Control Ac*-19: 334-338.

Cathy, W. S. 1997. Detection of Additive Outliers in Bilinear Time Series. *Computational Statistics and Data Analysis* 24: 283-294

Doukhan, P., Latour, A., and Oraichi, D., 2006. A simple integer – valued bilinear time series model. *Adv. Appl. Prob.* 38: 559-578.

Drost F.C., van den Akker R., Werker, B.J.M. 2007. Note on integer – valued bilinear time series models. *Econometrics and Finance group CentER, ISSN 0924-7815,* Tilburg University, The Netherlands

Gonclaves, E., Jacob P. and Mendes-Lopes N. 2000. A decision procedure for bilinear time series based on the Asymptotic Separation. *Statistics*, 333-348

Granger, C. W. J. and Anderson, A. P. 1978. *Introduction to Bilinear Time Series Models.* Gottingen, Germany: Vandenhoeck and Ruprecht.

Haggan , V. and Oyetunji, O. B. 1980. On the Selection of Subset Autoregressive Time Series Models. *UMIST Technical Report No. 124* (Dept. of Mathematics).

Krzanowski W.J. 1998. An Introduction to Statistical Modelling. Arnold, the UK.

Liu, J. (1992) On stationarity and Asymptotic Inference of Bilinear time series models. *Statistica Sinica* 2(2): 479-494.

Ojo, J. F. 2009. On Subsetting and Identification of Optimal Models in Bilinear Time Series Modelling. *Ph.D Thesis, University of Ibadan, Ibadan.* pp 1-215

Neudecker H. 1969. Some Theorems on Matrix Differentiation with Special References to Kronecker Matrix Products. *Jour. Amer. Statist. Ass.* 69, 953-963.

Pham, T.D. and Tran, L.T. 1981. On the First Order Bilinear Time Series Model. *Jour. Appli. Prob.* 18 : 617-627.

Rao, M. B., T. S. Rao and Walker, A. M. 1983. On the Existence of some Bilinear Time Series Models. *Journal of Time Series Analysis* 4(2): 60-76.

Shangodoyin, D. K. and Ojo, J. F. 2003. On the Performance of Bilinear Time Series Autoregressive Moving Average Models. *Journal of Nigerian Statistical Association* 16: 1-12.

Subba Rao, T. and M. M. Gabr, 1980. A Test for Linearity of Stationary Time Series. *Jour. Time Series Analysis* 1(2): 145-158.

Subba Rao, T. and Gabr, M. M. 1981. The Estimation and Prediction of Subset Bilinear Time Series Models with Application. *Journal of Time Series Analysis* 2(3), 89-100

Subba Rao, T. 1981. On theory of Bilinear time Series Models .*Jour. R. Sta. Soc.* B. 43: 244-255

Usoro, A. E. and Omekara C. O. 2008. Lowere Diagonal Bilinear Moving Average Vector Models. *Advances in Applied Mathematical Analysis* 3(1), 49-54

Wang H.B. and Wei B.C. 2004. Separable lower triangular bilinear model. *J. Appl. Probab.* 41(1): 221-235

APPENDIX

Here we prove Theorem 1 from section 3. For the sake of simplicity, we will break the proof down into the following steps.

Step 1: For nearly sure convergence, we show that

$$\sum_{r\geq 1} E\left|\left(\prod_{j=1}^{r}(\boldsymbol{\Psi}+\mathbf{B}e_{t-j})\mathbf{C}e_{t-r}\right)_i\right| < \infty \qquad \text{(A1)}$$

for every i=1, 2, ……,p. This then implies that

$$\sum_{r\geq 1}\prod_{j=1}^{r}(\boldsymbol{\Psi}+\mathbf{B}e_{t-j})\mathbf{C}e_{t-r}$$

is absolutely convergent almost surely as well as in the mean.

Step 2: Let us establish (A1) for $i = 1$. The general case is clear. First, we note that for every t in Z, r1 and $s = 1, 2, \ldots, p$, the following inequality holds:

$$E\left|\left((\boldsymbol{\Psi}+\mathbf{B}e_{t-r})\mathbf{C}e_{t-r}\right)_s\right| \leq K_0,$$

where K_0 is a constant that depends only on $\boldsymbol{\Psi}, \mathbf{B}, \mathbf{C}$ and σ_e^2

Step 3: If r≥2, then

$$E\left|\left(\prod_{j=1}^{r}(\boldsymbol{\Psi}+\mathbf{B}e_{t-j})\mathbf{C}e_{t-r}\right)_1\right| \leq K_1\lambda^{(r-1)/2}$$

for some constant $K_1 > 0$. Now observe that

$$E\left|\left(\prod_{j=1}^{r}(\boldsymbol{\Psi}+\mathbf{B}e_{t-j})\mathbf{C}e_{t-r}\right)_1\right| = E\left|\left(\left(\prod_{j=1}^{r-1}(\boldsymbol{\Psi}+\mathbf{B}e_{t-j})\right)(\boldsymbol{\Psi}+B\mathbf{e}_{t-r}\mathbf{C}e_{t-r})\right)_1\right|$$

$$= E\left|\sum_{s=1}^{p}\left(\prod_{j=1}^{r-1}(\boldsymbol{\Psi}+\mathbf{B}e_{t-j})\right)_{1s}\left((\boldsymbol{\Psi}+\mathbf{B}e_{t-r})\mathbf{C}e_{t-r}\right)_s\right|$$

$$\leq \sum_{s=1}^{p}\left(E\left|\left(\prod_{j=1}^{r-1}(\boldsymbol{\Psi}+\mathbf{B}e_{t-j})\right)_{1s}\right|\right)\left(E\left|\left((\boldsymbol{\Psi}+\mathbf{B}e_{t-r})\mathbf{C}e_{t-r}\right)_s\right|\right).$$

(In the above derivation, we have used the fact that $\prod_{j=1}^{r-1}(\boldsymbol{\Psi}+\mathbf{B}e_{t-j})$ and $(\boldsymbol{\Psi}+\mathbf{B}e_{t-r})\mathbf{C}e_{t-r}$ are independently distributed.)

By step 2 and the Cauchy-Schwartz inequality,

$$\sum_{s=1}^{p}\left(E\left|\left(\prod_{j=1}^{r-1}(\boldsymbol{\Psi}+\mathbf{B}e_{t-j})\right)_{1s}\right|\right)\left(E\left|\left((\boldsymbol{\Psi}+\mathbf{B}e_{t-r})\mathbf{C}e_{t-r}\right)_s\right|\right)$$

is not greater than

$$K_0\sum_{s=1}^{p}\left(E\left(\left(\prod_{j=1}^{r-1}(\boldsymbol{\Psi}+\mathbf{B}e_{t-j})\right)_{1s}\right)^2\right)^{1/2}$$

Now, for any $s = 1, 2, \ldots, p$,

$$\left(\left(\prod_{j=1}^{r-1}(\Psi+\mathrm{B}e_{t-j})\right)_{1s}\right)^2=\left(\left(\prod_{j=1}^{r-1}(\Psi+\mathrm{B}e_{t-i})\right)\otimes\left(\prod_{j=1}^{r-1}(\Psi+\mathrm{B}e_{t-j})\right)\right)_{1s:1s}$$

$$=\left(\prod_{j=1}^{r-1}(\Psi+\mathrm{B}e_{t-j})\otimes(\Psi+\mathrm{B}e_{t-i})\right)_{1s:1s}$$

Consequently,

$$E\left(\left(\prod_{j=1}^{r-1}(\Psi+\mathrm{B}e_{t-j})\right)_{1s}\right)^2=\prod_{j=1}^{r-1}(E(\Psi+\mathrm{B}e_{t-i})\otimes(\Psi+\mathrm{B}e_{t-j}))_{1s:1s}$$

$$=((E[(\Psi+\mathrm{B}e_t)\otimes(\Psi+\mathrm{B}e_t)])^{r-1})_{1s:1s}$$

$$=((E(\Psi\otimes\Psi+e_t\Psi\otimes\mathrm{B}+e_t\mathrm{B}\otimes\Psi+e_t^2\mathrm{B}\otimes\mathrm{B}))^{r-1})_{1s:1s}$$

$$=((\Psi\otimes\Psi+\sigma^2\mathrm{B}\otimes\mathrm{B})^{r-1})_{1s:1s}$$

$$\leq k\lambda^{r-1}$$

Hence

$$E\left|\left(\prod_{j=1}^{r}(\Psi+\mathrm{B}e_{t-j})\mathrm{C}e_{t-r}\right)_1\right|\leq K_1 p\lambda^{(r-1)/2}$$

for a suitable choice of K_1.

Step 4: Since $\lambda<1$, we have

$$\sum_{r\geq 1}E\left|\left(\prod_{j=1}^{r}(\Psi+\mathrm{B}e_{t-j})\mathrm{C}e_{t-r}\right)_i\right|<\infty$$

for every $i=1, 2, \ldots\ldots, p$.

Thus (A1) is established. Therefore $\rho(\Psi\otimes\Psi+\sigma^2\mathrm{B}\otimes\mathrm{B})=\lambda<1$ is a sufficient condition for strictly stationary of the model 2.3. Hence the proof.

Remarks on the proof of Theorem 1

For a real valued process conforming to the bilinear model $\psi(\mathrm{B})X_t=\phi(\mathrm{B})\nabla^1 X_t=b_{11}X_{t-1}e_{t-1}+e_t$ for every t in Z under the usual assumptions on the e_t's, a sufficient condition for its existence is given by $\psi^2+b^2\sigma^2<1$.

Ife Journal of Science

Tables of Contents: June Edition 2011; Vol. 13, No. 1

Omotoye Olorode, Sekinat O. Hassan, Olajumoke A. Olabinjo and Idris O. Raimi	Tithonia (Asteraceae) in Nigeria	1
Obuotor E.; Adewumi A. A. and Olaleye V. F.	The Effect of Copper on Some Laboratory Indices of Clarias Gariepinus (Burchell 1822).	11
Salami, B. M. Conte, R. A. and Falebita, D. E.	Geoelectric Evalution of the Groundwater Potential of Parts of Osogbo, Southwestern, Nigeria	17
Ogunfowokan A.O., Akanni M.S., Ajibola R.O and Ayinde F.O.	Trophic Status and Physico-Chemical Parameters of Three Reservoirs in Osun State Nigeria	27
Oláyíwolá M.A[1] and Odébòdé M.O.	Foraminiferal Distribution of Southwestern Nigeria's Offshore Littoral Sediments: Benthic Faunal Diversity Indices and Patterns	45
Chinedu S.N., Okochi V. I. and Omidiji O.	Cellulase Production by Wild Strains of *Aspergillus Niger, Penicillium Chrysogenum* and *Trichoderma Harzianum* Grown on Waste Cellulosic Materials.	57
Bayode S. and Akpoarebe O.	An Integrated Geophysical Investigation of a Spring in Ibuji, Igbara-Oke, Southwestern Nigeria.	63
M. O. Adepoju and J. A. Adekoya	Reconnaissance Geochemical Study of a Part of Igarra Schist Belt, Southwestern Nigeria	75
Adesina, G.O., Akinyemiju, O.A. and Muoghalu, J.I.	Checklist of the Aquatic Macrophytes of Jebba Lake, Nigeria	93
Fasasi, K. A., Malaka, S. L. O. and Amund, O. O.	Studies on the Life Cycle and Morphometrics of Honeybees, *Apis Mellifera Adansonii* (Hymenoptera: Apidae) In A Mangrove Area of Lagos, Nigeria.	103
A.O. Olorunfemi, K.S. Salahudeen and T.A. Adesiyan	Ground Water Quality in Ejigbo Town and Environs, Southwestern Nigeria	111
Govardhan Singh, R.S; Ogunsina, B.S. and Radha, C.	Protein Extractability from Defatted *Moringa Oleifera* Lam. Seeds Flour	121
A. M. A. Sakpere	Identification of ISSR Primers for Genetic Analysis of *Telfairia Occidentalis* Hook F.	129
O. K. Owoade, F. S. Olise, H. B. Olaniyi, I. B. Obioh and E. Bolzacchini	Mass and Energy Audit in a Nigerian Iron and Steel Smelting Factory: An Operational and Efficiency Study.	133
F. A. Oloyede, B. Aponjolosun & A. A. Ogunwole	Reproductive Potentials of a Tropical Fern *Cyclosorus Afer* (Christ.) Ching (Thelypteridaceae: Pteridopyhte) at Obafemi Awolowo University, Ile Ife, Nigeria	143
M.O.Olawole, L. Msimanga, S.A.Adegboyega & F.A. Adesina	Monitoring and Assessing Urban Encroachment into Agricultural Land - A Remote Sensing and GIS Based Study of Harare, Zimbabwe	149
Benjamin, U.K and Nwachukwu, J.I	Model Compaction Equation for Hydrostatic Sandstones of the Niger Delta.	161
J.O. Ojo and C.E. Adeeyinwo	Dependence of Vanadium Recovery on Oxidation State in its Solvent Extraction from Hydrochloric Acid Solutions With TRI N Butyl Phosphate	175
Akintorinwa, O. J., Ojo J. S. and Olorunfemi M. O.	Appraisal of the Causes of Pavement Failure along the Ilesa - Akure Highway, Southwestern Nigeria Using Remotely Sensed and Geotechnical Data	185
O. J. Matthew and O. O. Jegede	Modelling Soil Surface Temperature and Heat Flux Using Force-Restore Method at an Agricultural Site in Ile-Ife, Nigeria.	199
Ojo J.F.	On the Theory of One Dimensional Integrated Autoregressive Bilinear Time Series Modelling	209