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Modelling Nigeria's Domestic Output from select Macroeconomic Variables using Variable Coefficient Functions

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Abstract

This research dwelt on the application of variable coefficient models on Nigeria Domestic Output (proxy by GDP) taking exchange and inflation rate as two selected macroeconomic variables affecting domestic output in Nigeria. Secondary data spanning the period 1999-2019 monthly as extracted from the Central Bank of Nigeria (CBN) statistical bulletin was used in modelling the economic growth. The natural splines model was first fitted to the data to show the significance of the parameter estimates using R software as findings revealed a significant impact of the duo of exchange and inflation rates on economic growth as shown from the F-statistic p-value < 0.05 level of significance. Basis functions such as natural cubic splines and generalized additive models such as smoothing splines and local regression models were also fitted to the data. Generalized Cross-validation and leave-one-out re-sampling techniques were adopted in determining the tuning parameter, effective degrees of freedom and the span for smooth splines and local regression additive models. The model's accuracy was compared using the residual deviances for the GAMs and the Local Regression model was selected through a nested analysis of deviance on the two postulated models. Findings also indicated from the local regression model that the exchange rate and interaction of exchange and inflation rates cause instability in GDP growth in Nigeria.

Keywords: Generalized Additive Models, Local Regression, Natural Splines, Smoothing Splines, GCV

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1. Introduction

Estimation of nonparametric and semi-parametric regression models was developed by econometricians due to the challenges faced in fitting simple and multiple linear regression model specifications in the area of assumption violations such as linearity of regression of functions in explanatory variables, non-robust assumption of the random error terms to identical and independent distribution, thereby circumventing the problem (Anderson, 2009). These non-parametric and semi-parametric regressions are not rigid on these assumptions which its conventional method (parametric) rely on by nature. Some of these estimation techniques are the Generalized Linear Models (GLMs), additive models, Generalized Additive Models (GAMs), and Generalized Additive Mixed Models (GAMMs).

Inflation, exchange rate and Gross Domestic (GDP) are very important variables in assessing the performance of the economy. As a result of this, every nation of the world aspires to have price stability in all these macro and microeconomic indicators. According to Jhingan (2009), Inflation is seen as a persistent and appreciable rise in the general level of prices in an economy. Doguwa (2012) argued that inflation is crucial for economic growth while monetarists posit that it is harmful to economic growth. On the other side, Nell (2000) opined that singledigit inflation may be beneficial; on the other hand, double-digit inflation imposes slower growth. Similarly, Michael (2013) reported that crude oil provided approximately 90 per cent of Nigeria's foreign exchange earnings, about 80% of federal revenue and contributes to the growth rate of Gross Domestic Product (GDP).

Haghiri et al. (2013) adopted the splines smoothing technique and tested the degree of separability among the predictors in the Canadian cable television industry (CATV) using unbalanced panel data spanning 1990-1996 as findings revealed that separability assumption was valid by either parametric or non-parametric methods. The trio of the inflation rate, exchange rate and economic growth are linked economic concepts which numbers of economists are making efforts to interpret their relationships. According to Phillips (1958), the explanations of these relationships can either be in the long term or short term. In the short term, there exists an inverse correlation between the response variable and each of the predictors. Based on this relation, when the exchange rate is on the high side, it triggers inflation. A few other authors who have written extensively on the theoretical properties as well as on the practical applications of variable coefficient models, highlighting their relative benefits are Friedman & Stuetzle (1982), Hastie & Tibshirani (1990), Nell (2002), Cleveland (1979) among others. Also, fewer authors who have written on the direction of inflation rates, exchange rates and economic growths are Mohseni, and Jouzary (2015), Abdulsalam and Abdullahi (2016) & Doguwa (2012).

These variable coefficient models are a very important methodology in the exploration of dynamic patterns in many scientific areas not limited to finance, economics, medicine, epidemiology among others, which thereby serves as a motivation for this study.

2. Materials and Methods

In this section, the procedures for building variable coefficient models are discussed. Natural splines, smoothing splines and local regression models were adopted for this research. These estimation techniques were adopted due to their robustness to parametric assumptions as they were both non-parametric and semi-parametric. The models are fitted on the macroeconomic variables with the gross domestic product (GDP) being the dependent variables, exchange rate and inflation rate the independent covariates.

Model Specification

In this paper, the gross domestic product (gdp) is a function of inflation rate (ir) and exchange rate (er). It is specified as

$$\mathbf{gdp_i = \mathit{f}\,(ir, er)} \quad (1)$$

The functional relationship is specified thus:

$$\mathbf{gdp_i = \beta_0 + \mathit{f}_1\,(ir_i) + \mathit{f}2\,(er_i) + \varepsilon_i} \quad (2)$$

Where

$\boldsymbol{f_1}$ and $\boldsymbol{f2}$ are standardized smooth function and are estimated using very fast non-parametric Additive Backfitting Algorithm with Weights (ARBAW), ε_i represent the error terms is assumed to be distributed normally i.e. $\varepsilon_i \sim N(0, \sigma^2)$.

Natural Cubic Splines Model

The natural cubic spline with K knots and K effective degrees of freedom, represented by K basis functions can be derived by forming cubic splines basis through the imposition of boundary constraints by the use of truncated power series

$$f(x_i) = \beta_0 + \sum_{j=1}^{K+3} \beta_j b_j(x_i) + \varepsilon_i \quad (3)$$

Where b_1, b_2, b_3,…………b_{K+3} are basic functions and the model parameters are estimated using any of the least-squares methods. To fit the cubic spline in equation 4, we add a truncated power basis function to a cubic polynomial at each knot. The truncated power basis function is expressed as

$$h(x,\xi) = (x-\xi)_+^3 = \begin{cases} (x-\xi)^3 & x > \xi \\ 0 & otherwise \end{cases} \quad (4)$$

We estimate a least square regression with intercept and $3 + K$ predictors of form X, X^2, X^3, $h(X,\xi_1)$, $h(X, \xi_2)$, $h(X, \xi_3)$,………….., $h(X, \xi_k)$, where ξ_1,………., ξ_K are the knots. James, Witten, Hastie & Tibshirani (2013) suggested that a natural spline include a further boundary constraint such that the function is linear at the region where X is smaller than the smallest knot or vice versa i.e. $f^{'11}(x) = 0$ and $f^{(3)}(x) = 0$ for $x < \xi_k$ and $x > \xi_k$ which produces more stable estimates. The constraints are $\beta_2 = \beta_3 = 0$, $\sum_{k=1}^{k} \theta_k = 0$, $\sum_{k=1}^{k} \xi_k \theta_k = 0$ where θ_k are the coefficients of the truncated power basis function. The effective number of knots are chosen by cross-validation method such as leave-one-out which involves removing a certain portion of the data, we fit the natural spline with a certain number of knots in the remaining data. Then we predict the held-out data by using the fitted spline.

Smoothing Natural Splines model

Suppose we extend a multiple linear regression such as

$$y_i = \beta_0 + \sum_{j=1}^{p} \beta_j x_{ij} + \varepsilon_i \quad (5)$$

By replacing each $\beta_j x_{ij}$ with a smooth and nonlinear function $s_j x_{ij}$, the model in equation 5 becomes

$$y_i = \beta_0 + \sum_{j=1}^{p} s_j x_{ij} + \varepsilon_i \quad (6)$$

The smoothing natural cubic spline is an approach involving fitting additive models using the backfitting algorithm with weights otherwise known as the ARBAW. This approach involves nonparametric smooth functions in equation 7, which are cubic smoothing splines. According to Alabi & Are (2017), the estimation technique of ARBAW has the advantage that each function of a GAM can be done using any of the smoothing and modelling techniques. Generally, smoothing is the result of the minimization of a penalized residual sum of squares (PRESS) subject to a smoothness penalty. The penalized residual sum of squares is expressed as

$$PRESS = \sum_{t=1}^{T} (y_t - \beta_0 - \sum_{j=1}^{2} s_j(x_{jt}))^2 + \sum_{j=1}^{2} \lambda_j \int g_j^{||}(t_j)^2 dt \quad (7)$$

$$g(x_{jt}) = \beta_0 + \sum_{j=1}^{2} s_j(x_{jt}) \quad (8)$$

Where s_j, j =1, 2 are the smoothing functions on each regressor, λ_j is the tunning or smoothing parameters that control the level of shrinkage in each smooth function s_j. These tuning parameters are chosen efficiently to prevent over flexibility when too low and over-smoothness if too high.

$\sum_{j=1}^{2}\lambda_j \int g_j^{\parallel}(t_j)^2\, dt$ is the smoothness penalty which is dependent on the tuning penalty λ_j. The tunning parameter and corresponding effective degree of freedom are chosen using the generalized cross-validation resampling technique to further improve the accuracy of the smoothing spline model.

Local Regression Model

The local regression is the generalization of the combination of moving average and polynomial regression (Henderson, 1916; Schiaparelli, 1866, Cleveland, 1979; Katkovnik, 1979; Stone, 1977). The methods were developed for scatter plot smoothing known as Locally Estimated Scatter Plot Smoothing (LOESS) and Locally Weighted Scatter Plot Smoothing (LOWESS). It is a memory-based procedure that combines multiple regression models in a k-nearest-neighbour approach. It is quite different from the earlier discussed flexible non-linear functions. The fundamental principle of the model is that given a low degree polynomial in the neighbourhood (with span $\varsigma = k/n$) of any target point x_0, the smooth function can be well approximated as given by the local linear approximation:

$$\psi(x_i) \approx a_o + a_i(x_i - x_0) \qquad x_0 - h \le x_i \le x_0 + h \tag{9}$$

A local quadratic approximation is

$$\psi(x) \approx a_o + a_i(x_i - x_0) + \frac{a_2}{2}(x_i - x_0)^2 \tag{10}$$

The coefficient estimates $\hat{a}_o, \hat{a}_i$ are chosen to minimize

$$\sum_{i=1}^{n} W\left(\frac{x_i - x}{h}\right)(y_i - (a_o + a_i(x_i - x_0))^2) \tag{11}$$

The local linear regression estimate is defined as

$$\hat{\psi}(x_i) = \hat{a}_o \tag{12}$$

Each weighted least squares problem defines $\hat{\psi}(x)$ at one point x_0, the smoothing weights

$$W\left(\frac{x_i - x_0}{h}\right)$$

changes when x_0 change, and so the estimates $\hat{a}_o$ *and* $\hat{a}_i$ change. Since (11) is a weighted least squares problem, the coefficient estimates can be obtained by solving the normal equations (13).

$$X^T W \left[Y - X \begin{pmatrix} \hat{a}_o \\ \hat{a}_1 \end{pmatrix} \right] = 0 \tag{13}$$

Where X is the design matrix

$$X = \begin{bmatrix} 1 & X_1 - X \\ 1 & X_n - X \end{bmatrix}$$

For local linear regression, *W* is a diagonal matrix with entries

$$W\left(\frac{x_i - x_0}{h}\right) \text{ and } \mathrm{Y} = (\mathrm{Y}_1 \ldots \mathrm{Y}_n)^T.$$

When $X^T WX$ is invertible, parameter $\hat{a}_0$ and $\hat{a}_1$ has the explicit representation

$$\begin{pmatrix} \hat{a}_o \\ \hat{a}_1 \end{pmatrix} = \left(X^T W X\right)^{-1} X^T W Y \qquad (14)$$

This shows that the local regression estimate is linear, as defined by

$$\psi(x_i) = \sum_{i-1}^{n} l_i(x) Y_i$$

where the coefficients $l_i(x)$ are given by

$$\mathrm{l(x)} = (\mathrm{l}_1(\mathrm{x}) \ldots \mathrm{l}_\mathrm{n}(\mathrm{x})) = e_1^T \left(X^T W X\right)^{-1} X^T W \qquad (15)$$

where e_1^T is the unit vector. For higher-order fits, additional columns can be added to the design matrix X and vector e_1^T. Just as in smoothing splines where the tuning parameter controls the flexibility and level of shrinkage, span ς is the most important choice in local regression. It plays a role similar to the tuning parameter because the smaller the value of ς, the more local and curvy will the fitted local regression model appear and vice versa. Hence, the value of ς is the first choice to be determined in the local regression. For a more efficient span selection, a cross-validation technique such as the leave-one-out (LOOCV) approach is recommended.

3. Results and Discussion

Monthly data between January 1991 to December 2019 of the Central Bank of Nigeria (CBN) reports on Exchange Rates and Inflation Rates and Gross Domestic Products are analyzed in the study.

Table 1: Descriptive Statistics

Variables	Mean	SD	Min	Max
GDP (Y) (in Billion)	9,510,024	11583221	43180	35230608
Exchange Rate X_1	106.70	43.99392	10.87	180.63
Inflation Rate X_2 *(in %)*	11.6	4.042887	0.9	19.4

Values expressed in Billion Naira

Table 1 showed the descriptive statistics of the analyzed macroeconomic variables and GDP. Analysis indicated the mean value, standard deviation, maximum and maximum for each response and predictor variables over time, recorded on monthly basis.

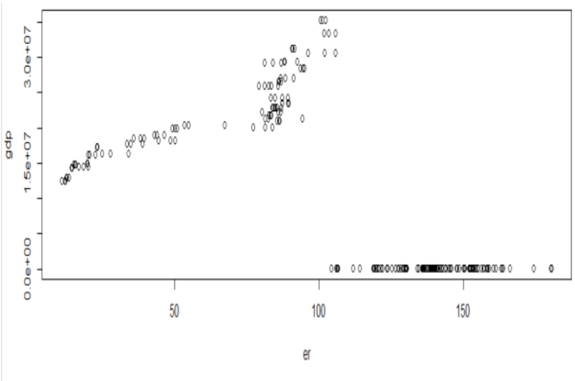

Figure 1: Scatterplot on gross domestic products (GDP) and exchange rate

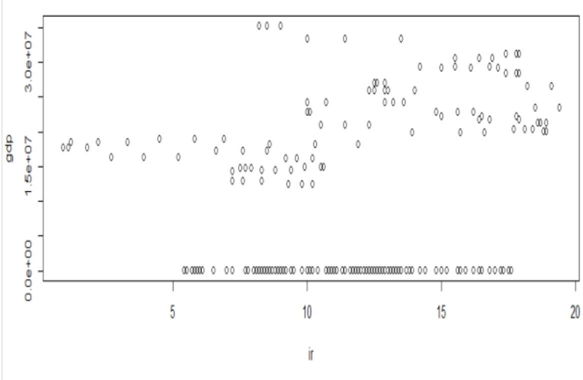

Figure 2: Scatterplot on gross domestic products (GDP) and inflation rate

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Figures 1 and 2 indicated some patterns of non-linearity in the relationship pattern of the response to the set of predictors. As a result of this, smoothing the variables using various smoothing methods to determine the best model that fits the economic growth data well call for serious attention. The natural splines model was first identified to know the behaviour of the estimates in terms of prediction. The interpolated result can be evidenced in **Table** 2.

Natural Splines model (NSM) on Nigeria Gross Domestic Products

Table 2: Parameter estimates of Natural Splines (ns) model

Variables	Estimates	Standard Error	t-value	Pr(>|t|)
Intercept	17.3897	0.5970	29.130	2e-16 ***
ns(log(er),4)1	-6.3173	0.7992	-7.904	63e-14 ***
ns(log(er),4)2	-13.1232	0.8385	-15.650	< 2e-16 ***
ns(log(er), 4)3	-12.5508	1.6203	-7.746	2.63e-13 ***
ns(log(er), 4)4	-10.7600	0.9592	-11.218	< 2e-16 ***
ns(ir, 5)1	-7.0345	1.4666	-4.796	2.83e-06 ***
ns(ir, 5)2	-10.0929	1.9842	-5.087	7.33e-07 ***
ns(ir, 5)3	-10.9623	2.3546	-4.656	5.33e-06 ***
ns(ir, 5)4	-20.6443	3.6249	-5.695	3.57e-08 ***
ns(ir, 5)5	-13.5231	2.7907	-4.846	2.26e-06 ***
log(er):ir	0.2432	0.0418	5.818	1.89e-08 ***

RMSE = 0.7899; R^2 = 0.9279; Adj R^2 = 0.9249; F-statistic = 310; p-value = 0.000

The minimizers of suitable measures of roughness using 4 and 5 for exchange rate and inflation rate respectively showed that the coefficients are statistically significant. However, the interaction effects posed a positive contribution to economic growth while other coefficients posed a negative influence. It can also be evidenced from the R-squared value of 0.9279 that GDP have 92.79% variance in prediction when "er" and "ir" are taken into consideration. The adjusted R^2 of 0.7899 indicated the percentage variance that can be explained by the response variable when other variables are added to the model. The lower RMSE of 0.7899 might find the model as sufficient among several unfitted models of smoothing splines and local regression techniques with special consideration for semi-parametric and non-parametric methods.

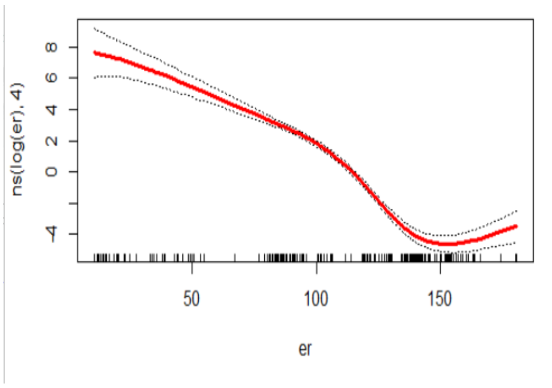

Figure 2a: Natural Splines plots showing relationships between gross domestic products and exchange rate

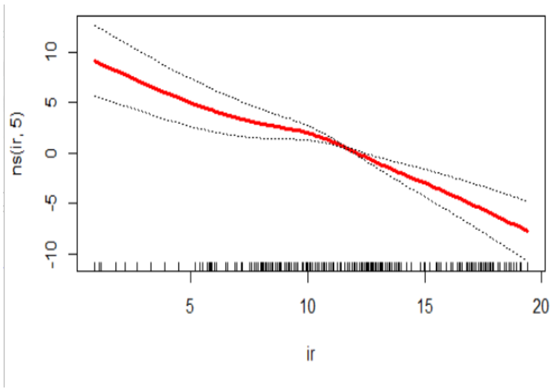

Figure 2b: Natural Splines plots showing relationships between gross domestic products and exchange rate

Each plot of the natural splines model in Figures 2a and 2b displays the relationship between X_j versus Y holding the remaining predictor constant in the model. Also, each plot shows the fitted function and 2 times standard errors with natural splines of 4 and 5 respectively.

Smoothing Splines Model (SSM) on Nigeria Gross Domestic Products

Table 3: Optimal Smoothing Splines

Variables	GCV	Smoothing parameter	Tuning Parameter (λ)	EDF
Exchange Rate (er)	0.0090	1.0957	2.044×10^{-6}	16.90956
Inflation Rate (ir)	4.5235	0.9389	1.5046×10^{-7}	28.07348

The generalized Cross-Validation (GCV) technique was used to confirm the optimal tuning (smoothing) parameter. Predictor variables of Exchange Rate (ER) and Inflation Rate (IR) converge after 14 and 11 iterations respectively with an equivalent degree of freedom (df) of 16.910 and 28.073. The smoothing parameters controlling the tradeoff between goodness-of-fit and smoothness of the model in equation (15) were estimated as $\lambda = 2.044 \times 10^{-6}$ and $\lambda = 1.5046 \times 10^{-7}$

Table 4: ANOVA of semi-parametric effects in smoothing splines model

Effect	df	SS	MS	f-value	p-value
s(log(er), df=1.0957)	1	932.36	932.36	312.9976	0.0000 ***
s(ir, df=0.9389)	1	311.44	311.44	104.5520	0.0000 ***
log(er):ir	1	16.32	16.32	5.4794	0.02004 *
Residuals	247.9	738.46	2.98		

Table 5 ANOVA of non-parametric effects in smoothing splines model

Effect	Non-parametric df	Non-parametric F-value	P-value
s(log(er), 1.0957)	0.1	251.76	0.00000***
s(ir, 0.9389)	0.0	-356.3	

*** *represents statistically significant at 1%;* * *represents statistically significant at 5%*

Null Deviance: 2084.432 on 251 df; **Residual Deviance:** 738.4631 on 247.9043 df

AIC: 996.2722 *reject the null hypothesis of linearity*

$$E(gdp|er,ir) = 20.3950 - 2.2008(\mathrm{er})_i + 0.8754(\mathrm{ir})_i \quad (16)$$

Table 4 and Table 5 show the analysis of variance of semi-parametric and non-parametric effects of the covariate in the smoothing splines model of equation 4.1 respectively. Analysis of the effects of the individual smooth splines models indicates that the semi-parametric and non-parametric effects of the predictors are all statistically significant at a 1% level except for the interaction of the predictors (covariates) having a 5% level of significance. The null model which estimates one parameter for the data set has deviance of 2084.432. The deviance as evidenced from the residual deviance was about 738.463 with 247.904 degrees of freedom, one for the intercept and the remaining for the two predictors. This reduction was achieved with a probability of 0.000. The p-values under the non-parametric ANOVA correspond to the null hypothesis of linearity versus the

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alternative of non-linear relationship. Extremely low p-values for all the smoothing functions indicate non-linear functions are sufficient for all the terms in the model in equation 1.4.

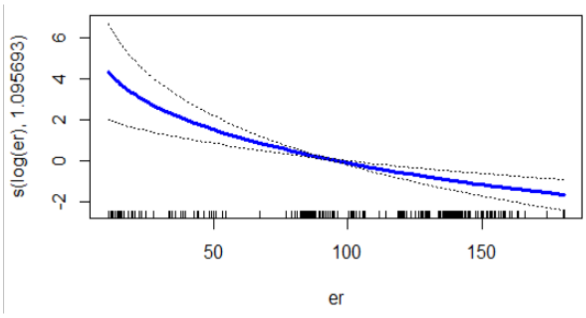

Figure 3a: *Smooth spline plots of the relationships between Exchange Rate and the gross domestic products (GDP) in the fitted model*

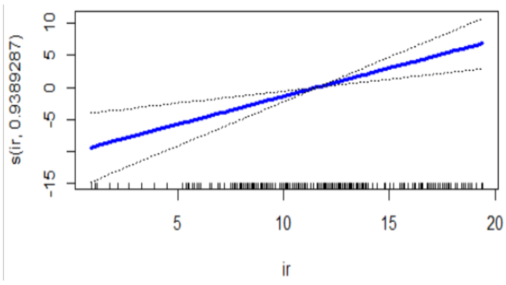

Figure 3b: *Smooth spline plots of the relationships between Inflation Rate and the gross domestic products (GDP) in the fitted model*

In Figures 3a and 3b, the response variable GDP is expressed in terms of mean deviation, thus each smooth function f_j is centred and represents how GDP changes relative to its mean with unit changes in the predictors. Hence, the zero value on the response axis is the mean of GDP. It can be evidenced from figure 3a that keeping the inflation rate fixed, GDP remains unstable for every unit increase in the value of the exchange rate. However, figure 3b showing the effect of inflation rate on economic growth (GDP) indicates that the response variable (GDP) continues to rise for every increase in the inflation rate.

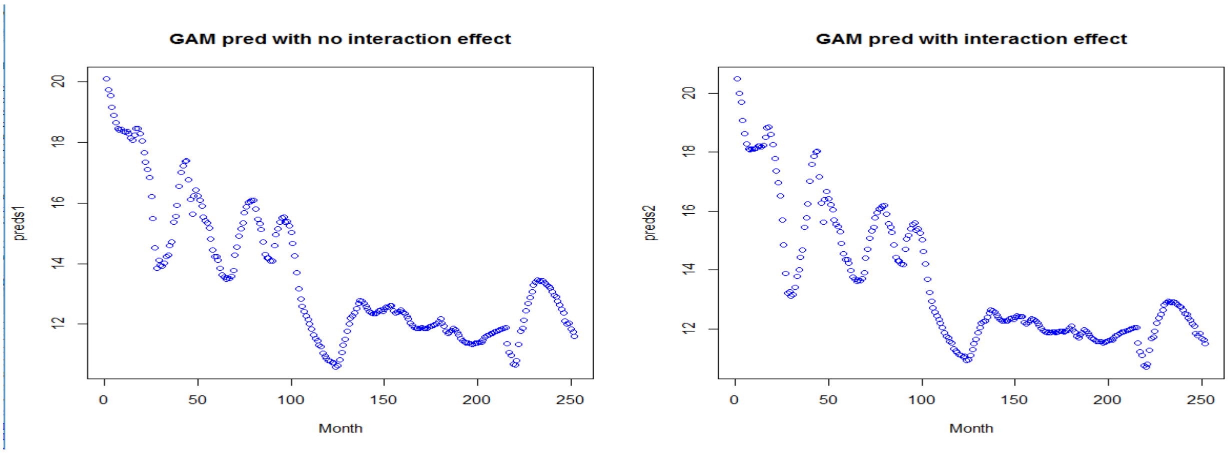

Figure 4: Prediction plots of Smoothing Splines Model without interaction

Prediction plots of figure 4 indicated no compelling evidence that GAM with interaction effect is better than a GAM that does not include interaction effect as the predicted plots showed almost nil variations.

The Local Regression Model (LRM) on Nigeria Gross Domestic Product

Table 6: ANOVA of semi-parametric effects in the local regression model

Effect	df	SS	MS	F-value	p-value
lo(log(er), ir, span = 0.7)	2	1253.05	626.52	428.6288	0.0000 ***
log(er):ir	1	2.65	2.65	2.8126	0.1795
Residuals	237.03	346.47	1.46		

Table 7: ANOVA of non-parametric effects in the local regression model

Effect	Non-parametric df	Non-parametric F-value	P-value
lo(log(er), ir, span = 0.7)	11	28.935	0.0000***
log(er):ir			

** *represents statistically significant at 1%;* * *represents statistically significant at 5%*

Null Deviance: 2084.432 on 251 df; **Residual Deviance:** 346.4716 on 237.0348 df

AIC: 827.305 *reject the null hypothesis of linearity*

The linear form was assumed for the functions and calculated the span *h* using the efficient cross-validation approach. Span selection was done by employing the expensive leave-one-out cross-validation of the deviance or residual sum of squares to obtain an unbiased approximation of the Kullback-Liebler distance. This method selected a span of 0.7 for the local regression functions on the exchange rate and inflation rate. The lower the value of *h*, the smoother the curve. Also, smaller values of *h* imply that the curve is more flexible and "*local*" which reduces the model bias, at the same time minimizing the variance proportionately. The local regression fit on *GDP* results in equation 16 below with the resultant local plots shown in figure 5.

$$E(gdp|er, ir, er * ir) = 23.5309 - 2.9384(\mathrm{er})_i + 0.5297(\mathrm{ir})_i - 0.0534(er * ir) \quad (16)$$

Table 7 shows that the local function on the exchange rate and inflation rate has a significant non-parametric and non-linear effect in the gross domestic products model of equation 4.2. at 1% level of significance. Reduced deviance from 2084.432 to 346.4716 with 237.0348 degrees of freedom was achieved on this model. Similar to the smooth spline model, the deviance test on the local regression model gives a maximum Chi-square *p*-value = 0.00 indicating no evidence of lack of fit.

Table 8: Analysis of Deviance on Postulated Models

Model	Residual df	Residual Deviance	Deviance	Pr(>Chi-square)	AIC
Smoothing Splines	247.900	738.460	2084.432	0.000***	996.272
Local Regression	237.030	553.247	2084.432	0.000***	827.305

******Indicate model is superior to the previous model at 1 per cent level of significance.***

Deviance analysis in table 8 above shows the residual degrees of freedom, residual deviance, model deviance and the chi-square *p*-value of achieving lower deviance for each of the postulated models. This analysis indicates that local regression is better than the smoothing splines model due to its lower residual deviance of 553.2472. Hence, the local regression model of the generalized additive models provides the best estimates for the actual gross domestic products values during the period under study as shown in Figure 5.

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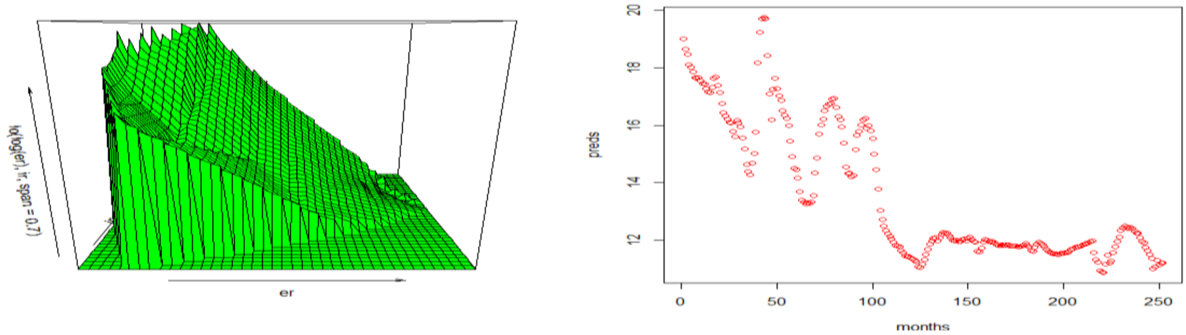

Figure 5: Local regression fit for predicting GDP from the exchange rate and inflation rate.

4. Conclusion

The study examines the impact of exchange rate and inflation rate on GDP using varying coefficient models methodology. In achieving the main objective of this paper, Natural Splines, Gaussian Generalized Additive Models (GAMs) and Local Regression models were applied to data on the inflation rate and exchange rate. These varying coefficient models were applied to data on economic growth. Based on the empirical applications, each of the varying coefficient models provides a much better fit as shown in the lower AICs and lower deviances as exchange rate and inflation rate jointly impact economic growth, but variables of exchange rate and interaction of both exchange and inflation rates cause instability in the prediction of economic growth due to its negative direction which was not found within the a priori expectations. More so, the local regression model was found to be more efficient compared to the generalized additive model as evidenced from their respective deviances. The essence of adopting the respective models is to extend nonparametric and semi-parametric regression to more than one predictor, thereby circumventing the cause of dimensionality. Another merit of the GAM methodology used in the research is that nonlinearities in predictors and interactions among predictors are non-parametrically taken into account. Although, the Generalized Additive Models are not without limitations. The computational algorithms are complex, and interpretations can be difficult as it is biased but efficient due to the inclusion of tuning parameter in estimating the predictors' coefficients. These models are useful mainly when simple models for the linear predictor provide an inadequate fit for the data.

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