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On Induced Matching Numbers of Stacked-Book Graphs

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Abstract

For a simple undirected graph G, an induced matching in G is a set of edges M no two of which have common vertex or are joined by an edge of G in the edge set $E(G)$ of G. Denoted by $\operatorname{im}(G)$, the maximum cardinal number of M is known as the induced matching number of G. In this work, we probe $\operatorname{im}(G)$ where $G = G_{m,n}$, which is the stacked-book graph obtained by the Cartesian product of the star graph S_m and path P_n.

Keywords: Stacked-Book Graphs, Maximum Induced Matching Number, Cartesian Product of Graphs.

MSC2010: 05C70, 05C15.

1 Introduction

Suppose that G is a graph with $E(G)$ as the edge set of G while $V(G)$ denotes the vertex set of G. Let M be a subset of $E(G)$ such that for every $e_1, e_2 \in M$ there is no such edge in $E(G)$ to which any of the end points of e_1 and e_2 are commonly adjacent. Then M is an *induced matching* in G. Maximum Induced matching (MIM) problem is the generalization of the older graph matching problem, and it was introduced in [1].

Suppose that M is the largest induced matching in G then the cardinal number of M, denoted by $\operatorname{im}(G)$ is called the maximum induced matching number of G. Many investigations have been on this subject. It has attracted interest mostly because it is theoretically interesting and it has a number of direct applications. In [1], the authors described MIM problem as "risk free" marriage where married couples who are perfectly matched are identified while [2] investigated the MIM problem in intersection graphs. Its usefulness in cryptography is also obvious. Its applications can also be found in scheduling and planning, graph coloring [3], secure communication channels [4] neural networks in artificial intelligence [5, 6]. Cameron in her earlier work [7] showed that even though the MIM problem is NP-complete for bipartite graphs, it is easier to resolve for chordal graphs. This was also confirmed for circular graph in [8]. Golumbic and Lewenstein [4] established

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that there is a relationship between MIM number and redundancy number in graphs and also showed that the MIM problem is polynomial-time solvable for tree graphs.

For graphs G and H, the Cartesian product $G\Box H$ have vertex set $V(G)\times V(H)$ and edge set $E(G\Box H)=\{((x_1,x_2),(y_1,y_2)) : (x_1,y_1)\in E(G)$ and $x_2=y_2$ or $(x_2,y_2)\in E(H)$ and $x_1=y_1\}$.

Recent works on MIM problem include [9] where the MIM number was extensively probed for grids, $P_n\Box P_m$, the Cartesian product of paths P_n and P_m. For odd nm, a bound $\text{im}(P_n\Box P_m)\leq\lfloor\frac{nm+1}{4}\rfloor$ was obtained. The bound was tightened in [10] and further in [11]. In [12], investigation was made into obtaining exact algorithm for MIM problem of graphs on $n-$vertices.

In this work, we investigate the Maximum Induced Matching (MIM) problem for stacked-book graph, $G_{m,n}$, class which are graphs obtained from the Cartesian product of star graphs S_m and paths P_n. The MIM numbers are obtained for the initial range of these graphs while lower bounds of MIM number are derived for the general class.

2 Preliminaries

The vertex set of graph G is denoted by $V(G)$ and M is a subset of $E(G)$, the edge set of G, and M is the induced matching of G. A vertex $v\in V(G)$ is called *saturated* if $v\in V(M)$ and *unsaturated* if otherwise. A star graph S_m contains a central vertex v_1 (except if specifically indicated otherwise) with $m-1$ leaves, which are all incident to v_1 as pendants. A path P_n contains n vertices and $n-1$ edges, while a cycle C_m contains m vertices and edges. Suppose that u and v are members of $V(G)$, then $d(u,v)$ is a positive integer, which is the shortest distance between u and v in G. A vertex $v\in V(G)$ is called unsaturable if by its position, cannot be saturated either because of its distance from a saturated vertex or it is at the right distance but still not adjacent to a vertex that can be saturated in other to form an edge in the induced matching. A saturable vertex therefore, is the opposite of an unsaturable vertex. The diameter of a graph G is the maximum distance over all pair vertices u and v in G, and it is denoted by $\text{diam}(G)$. Set $[a,b]$ to denote the set of integers from a to b while $[a]$ is a shortened form of $[1,a]$.

Structure of a Stacked-book graph. The stacked-book graph is the Cartesian product $S_m\Box P_n$ of a star graph S_m and path P_n. Structurally, a $S_m\Box P_n$ contains n number of S_m stars such that there exist $E(G')\in E(S_m\Box P_n)$,
$E(G')=\{v_iu_i : v_i\in V(S_m(i)); u_i\in V(S_m(i+1), 1\leq i<n)\}$ with $S_m(i)$ designated as the star S_m at the i-th position in the stacked-book graph. Clearly,

$$E(S_m\Box P_n)=E(G')\cup E(\cup_{i=1}^{n}S_m(i)).$$

Initial Results.
The following results are obvious

Theorem 2.1. Let P_n be a path graph on n vertices. Then, $\text{im}(P_n)=\lceil\frac{n-1}{3}\rceil$.

Theorem 2.2. Let C_n be a circle graph on n vertices. Then $\text{im}(C_n)=\lfloor\frac{n}{3}\rfloor$.

Theorem 2.3. [9] Suppose that $G_{3,n}$ is a grid graph obtained by the Cartesian product $P_3\Box P_n$, where n is even or odd. Then for a positive integer k,

$$\text{im}(P_3\Box P_n)=\begin{cases}\lceil\frac{3n}{4}\rceil & \text{if } n \text{ is even;}\\ \frac{3(n-1)}{4} & \text{if } n=4k+1\\ \frac{3(n-1)+2}{4} & \text{if } n=4k+3\end{cases}$$

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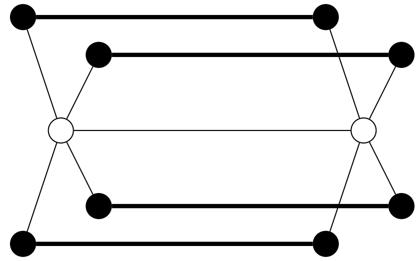

Figure 1: $G_{5,2}$ with 8 saturated vertices and $\text{im}(G_{5,2}) = 4$

3 Results

First we show a result on the MIM number of star graph S_m.

Lemma 3.1. Let S_m be a given star graph. Then $\text{im}(S_m) = 1$

Proof: This follows trivially from the fact that all edges in a star are incident to each other. Thus $\text{im}(S_m) = 1$.

Remark: The implication of this result is that every star graph contains at most one element in its induced matching set.

We present some results on the induced matching of stacked-book graph $G_{m,n}$. Henceforth, for a stacked-booked graph $G_{m,n}$, we refer to $S_m(i)$ and v_i respectively, as the subgraph that induced the star S_m at the ith position in $G_{m,n}$ and its centre vertex, respectively

Lemma 3.2. For a set that has a maximum cardinality of a MIM of $G_{m,2}$, the centers of the subgraphs that induced the star S_m in $G_{m,2}$ cannot be contained in the set.

Proof: Suppose that v_1 and u_1 are the central vertices of $S_m(1)$ and $S_m(2)$ in $G_{m,2}$, respectively. Note that $G_{m,2}$ contain a P_5. By Theorem 2.1, $\text{im}(P_5) = 2$ thus $\text{im}(G_{m,2}) \geq 2$. Assume to the contrary that one of v_1 or u_1 is saturated, say v_1. Then either $v_1u \in M$, where $u \in N(v_1) - u_1$ or $v_1u_1 \in M$. If $v_1u \in M$, then by Lemma 3.1, then at least $m-2$ vertices in $S_m(1)$ will be unsaturated. Thus, for all $v_iu_i \notin M$, $\text{im}(G_{m,2}) = 1$, a contradiction. Now, suppose that $v_1u_1 \in M$ then every vertex in $G_{m,2} \setminus \{v_1, u_1\}$ is a neighbor of either v_1 or u_1. This implies that the vertices in $G_{m,2} \setminus \{v_1, u_1\}$ are unsaturated. That is, $\text{im}(G_{m,2}) = 1$, a contradiction.

The first theorem follows.

Theorem 3.1 For $G_{m,2}$, $\text{im}(G_{m,2}) = m - 1$.

Proof. For $G_{m,2}$, there exist $S_m(1), S_m(2) \subseteq G_{m,2}$ with vertices $v_1, v_2 \ldots v_m$ and $u_1, u_2, \ldots u_m$ and a path $P_5(i) = v_i \to u_i \to u_1 \to u_{i+1} \to v_{i+1}$, for all $i \in [2, m]$. Thus, there exists, the set $\bar{P} = P_5(2), P_5(3), \ldots, P_5(\frac{m-1}{2})$, if m is odd. Therefore, there are $\frac{m-1}{2}$ number of P_5-paths. Now, by Lemma 3.1, $\text{im}(P_5) = 2$. Clearly, $\bar{P}$ consists of all the edges in $E(G_{m,2})$ that can be in M. Therefore, $\text{im}(G_{m.2}) \leq 2\left(\frac{m-1}{2}\right) = m - 1$. Now suppose that m is even. Then, set $P^* = \left\{P_5(2), P_5(3), \ldots, P_5(\frac{m-2}{2}), P_3(t)\right\}$, where $P_3(t) = v_k \to u_k \to u_1$. So, $\text{im}(P^* \setminus P_3(t)) = 2\left(\frac{m-2}{2}\right) = m - 2$. By Theorem 2.1, $\text{im}(P_3(t)) = 1$. Therefore, $\text{im}(P^*) = m - 1$. Hence, for any integer m, $\text{im}(G_{m,2}) \leq m - 1$. Conversely, by definition of induced matching and stacked-book graph,

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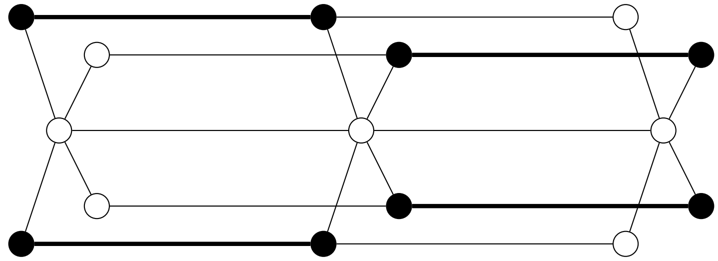

Figure 2: $G_{5,3}$ with 8 saturated vertices and $|M| = 4$

$v_2u_2, v_3u_3, \ldots, v_mu_m$, satisfying the distance conditions belong to M. Thus, $\text{im}(G_{m,2}) \geq m-1$ and hence the claim.

Next we consider the induced matching in $G_{m,3}$, where m is either even or odd and show that the graph contains the same induced matching as $G_{m,2}$.

Theorem 3.2. For $G_{m,3}$, $\text{im}(G_{m,3}) = m - 1$.

To proof Theorem 3.2, we need two results, the first one, which is about the nature of induced matching and distances between vertices of graphs, is more like a folklore because it follows from the definitions of induced matching of graphs.

Lemma 3.3. Let e_1 be in the induced matching of graph G. Then $e_2 \in E(G)$ is also in the induced matching of G if there exists $v_1, v_2 \in e_1$ and $u_1, u_2 \in e_2$ such that $d(v_1, u_1) \geq 3$ and $d(v_2, u_2) \geq 2$.

Proof. The proof follows from the definition of induced matching M of graph G.

Lemma 3.4. Let $G_{m,3}$ be a stacked-book graph with factor star graphs $S_m(1)$, $S_m(2)$ and $S_m(3)$ such that $v_1 \to u_1 \to w_1$ is a P_3 path in $G_{m,3}$, where v_1, u_1 and w_1 are the central vertices of the respective factor star graphs. If u_1 is saturated, and $u_1v_k \in M$ for some $v_k \in V(G_{m,3})$, then M is not the maximum induced matching of $G_{m,3}$.

Proof. For $v_k \in V(G_{m,3})$, $v_k \neq u_1$, for which $v_i \in G_{m,3}$ such that $d(v_k, v_i) = 3$ since the $\text{diam}(G_{m,3}) = 3$. However, suppose that $v_iv_j \in E(G_{m,3})$, for which $d(v_k, v_i) = 3$. It is clear that v_i is a leaf if some $S_m(t)$, $t \in \{1, 3\}$. Thus, $d(u_1, v_j) = 1$, hence a contradiction to Lemma 3.3 and hence the result.

Proof of Theorem 3.2. Suppose that $|M| > m - 1$. Let v_1, u_1 and w_1 be the central vertices of $S_m(1), S_m(2)$ and $S_m(3)$ respectively. Clearly, $v_1u_1, u_1w_1 \notin M$ from Lemma 3.4. Now, first we show that v_1 is not saturable. Suppose that v_1 is saturable, then $v_1v_q \in M$, where v_q is a leaf on $S_m(1)$. By Lemma 3.2, subgraph induced by $S_m(1)$ and $S_m(2)$ does not contain another member of M. Also, let $v_qu_q \in E(G_{m,3})$, with $u_q \in S_m(2)$ and $u_qw_q \in E(G)$, with $w_q \in S_m(3)$. Since $d(v_q, u_q) = 1$, then u_q can not be saturated. Thus, $u_qw_q \notin M$. In like manner, if w_1 is saturated, and $w_1w_q \in M$ no other edge in subgraph of $G_{m,3}$ induced by $S_m(2)$ and $S_m(3)$ is a member of M, and $v_qu_q \notin M$. Without loss of generality, suppose that $v_1, v_q \in M$, then only $\bar{M} := \{u_iw_i : i \in [2, m];\ i \neq q\} \subset E(G_{m,3})$ will be member of M. Thus $|\bar{M}| = m - 2$ and so $|M| = m - 1$, which is a contradiction. Now it has been established that none of the pendants of $S_m(1), S_m(2)$ and $S_m(3)$ can be in M. Thus, the possible members of M are $\{v_iu_i : i \in [2, m]\} \cup \{u_iw_i : i \in [2, m]\} := M'$. Clearly, $|\bar{M}| = 2(m - 1)$. By Lemma 3.3, only half of the members of $\bar{M}$ can be in M. Thus, $\text{im}(G_{m,3}) \leq m-1$. Reasonably,

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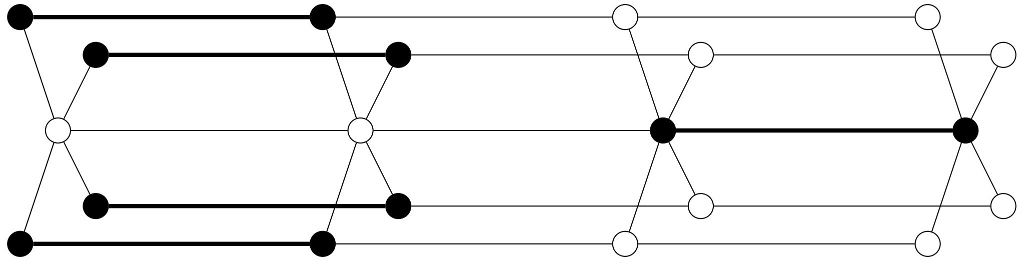

Figure 3: $G_{5,4}$ with 10 saturated vertices and $|M| = 5$

$\text{im}(G_{m,2}) \leq \text{im}(G_{m,3})$. By Theorem 3.1, therefore, $\text{im}(G_{m,3}) \geq m-1$ and thus $\text{im}(G_{m,3}) = m-1$.

Next we investigate the induced matching number of $G_{m,4}$. We start with a lemma that will be employed in the main result.

Lemma 3.5. Let $S_m(1)$, $S_m(2)$, $S_m(3)$ and $S_m(4)$ be the factor stars of $G_{m,4}$. Suppose that $\text{im}(G_{m,4}) \geq m$. Then if $M' = \{u_i w_i : i \in [2, m];\ u_i \in S_m(2), w_i \in S_m(3)\}$, then M' is not a subset of M.

Proof. It is easy to see that $|M'| = m-1$. Now, suppose that $M' \subset M$, then u_i, w_i are saturated for all $i \in [2, m]$. Thus, no vertex $v_i \in S_m(1)$ and $r_i \in S_m(4)$ is saturable, for $i \in [2, m]$, which implies that $\text{im}(G_{m,4}) = m-1$ and thus, a contradiction.

Next we consider the main theorem.

Theorem 3.3. Let $S_m(1), S_m(2), S_m(3)$ and $S_m(4)$ be the factor star graphs $G_{m,4}$. Then, $\text{im}(G_{m,4}) = m$.

Proof. There are at least some edge $M' := \{u_i w_i : i \in [2, m];\ u_i \in S_m(2), w_i \in S_m(3)\}$ not in M, by Lemma 3.5. Suppose therefore that $u_k w_k \notin M$. Then for $v_k \in S_m(1)$, and $r_k \in S_m(4)$, $v_1 v_k, r_1 r_k \in M$, where v_1 and r_1 are the central vertices of $S_m(1)$ and $S_m(4)$ respectively. Thus, $\text{im}(G_{m,4}) \geq m$. Conversely, suppose that $\text{im}(G_{m,4}) = m+1$. Now, let u_1, w_1 be the central vertices of $S_m(2)$ and $S_m(3)$ respectively. Suppose that one of u_i, w_i, say u_i is saturated such that $u_1 u_i \in M$. Then, by Lemma 3.5, no edge in the subgraph of $G_{m,4}$ induced by $S_m(1)$, $S_m(2)$ and $S_m(3)$ is contained in M. Likewise, if $w_1 w_i \in M$, then all other vertices on the subgraph of $G_{m,4}$ induced by $S_m(2)$, $S_m(3)$ and $S_m(4)$ are unstaurable. If any of the pendant of $S_m(2)$ and $S_m(3)$ is in M, then $M = 2$. Now, note as well that if $u_1 w_1 \in M$, then by the distances of u_1 and w_1 to the rest of vertices on $S_m(1), S_m(2), S_m(3)$ and $S_m(4)$, only $u_1 w_1$ will be in M. Thus for optimal M, some members of $M'' := \{v_i u_i; i \in [2, m]\}$ or $M''' := \{w_i r_i : i \in [2, m]\}$ will have to be in M.

Now clearly, it can be seen that $|M' \cup M''| = 2(m-1)$ and only $m-1$ members of $M' \cup M''$ can be in M. Based on this observable fact, at least there will exist a $w_i \in S_m(3)$ that is not saturable. Thus, there exist a saturable vertex $r_i \in S_m(4)$, such that $r_1 r_i \in M$. But since w_1 is saturated, no pendant on $S_m(4)$ is contained in M. Thus, $\text{im}(G_{m,4}) < m+1$ and hence a contradiction. Therefore, $\text{im}(G_{m,4}) \leq m$ and the claim follows.

Now we consider the case of $G_{m,5}$. We shall need some new results to aid the proof.

Lemma 3.6. Suppose that $w_1 \in S_m(3)$ is the central vertex of $S_m(3)$, where $\{S_m(i) : i \in [1, 5]\}$ is the set of factor stars of $G_{m,5}$. If w_1 is saturated, then for M of $G_{m,5}$, $|M| \leq 2m-3$.

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Proof. Suppose that w_1 is the central vertex of $S_m(3)$ and it is saturated. Then one of the w_1w_k, u_1w_1 and w_1r_1 belongs to M where u_1, r_1 are central vertices of $S_m(2)$ and $S_m(4)$ respectively. Suppose that $w_1w_k \in M$, where $k \leq m$. Now for all $i \in [2, m], i \neq k$, $w_i \in S_m(3)$ is unsaturable by Lemma 3.1. Thus members of $\{u_iw_i : i \in [2, m]\}$ and $\{w_ir_i : r_i \in S_m(4), i \in [2, m]\}$ do not belong to M. Also it is clear to see that both edges $v_ku_k, r_kt_k \notin M$, where $t_k \in S_m(5)$. Using similar technique adopted in the proof of Theorem 3.3, it can be deduced that $v_1v_i, t_1t_i \notin M$ for all $i \in [2, m]$. Thus, only $E' = \{v_iu_i : i \in [2, m], i \neq k\}$ and $E'' = \{r_it_i : i \in [2, m], i \neq k\}$ can be in M. Clearly, $|E' \cup E''| = 2(m - 2)$. Thus $|M| = 2m - 3$. Also, if $u_1w_1 \in M$, it can be seen by following the definitions of induced matching that no other edges in the subgraph of $G_{m,5}$ induced by $S_m(1), S_m(2)$ and $S_m(3)$ is a member of M and from Theorem 3.2, only $m - 1$ edges of the subgraph of $G_{m,5}$ induced by $S_m(3), S_m(4)$ and $S_m(5)$ can be in M. Thus, M consists of at most m edges, which is not more than $2m - 3$, since $m \geq 3$. Similar argument above can be employed to show the claim that w_1r_1 does not belong in M.

Lemma 3.7. Suppose that $\text{im}(G_{m,5}) \geq 2(m - 1)$. Then u_1, w_1 and r_1, the central vertices of $S_m(2), S_m(3)$ and $S_m(4)$ respectively are unsaturated.

Proof. This follows from Theorem 3.3 and Lemma 3.2.

We proceed to probe the induced matching of $G_{m,5}$.

Theorem 3.4. For $G_{m,5}$, $\text{im}(G_{m,5}) = 2(m - 1)$.

Proof. From Lemmas 3.6 and 3.7, we see that if u_1, w_1, r_1 are unsaturated, then $|M| \geq 2m-3$. Now we show that $\text{im}(G_{m,5}) \geq 2(m-1)$. Note that there exists a path $P_5(i) = v_i \to u_i \to w_i \to r_i \to t_i$, for all $i \in [2, m]$. Therefore, there are $m - 1$ such paths in $G_{m,5}$. From Theorem 2.1, $\text{im}(P_5) = 2$. Thus, $\text{im}(G_{m,5}) \geq 2(m - 1)$. Conversely, u_1, w_1, r_1 are established not to be saturated for the claim to hold. The edges in $E(G_{m,5})$ left to be members of M the pendants of $S_m(1)$ and $S_m(5)$ and the paths $P_5(i)$. Suppose that a pendant each from $S_m(1)$ and $S_m(5)$ belong to M, then by definition of induced matching, at most one edge on each of the paths $P_5(i)$ can be a member of M. Thus $|M| = m + 1$. The only alternative is if no pendant of $S_m(1)$ and $S_m(5)$ is a member of M. Thus, at most two edges from each member of $P_5(i)$ will be in M. Thus, $|M| \leq 2(m - 1)$ and so, $\text{im}(G_{m,5}) = 2(m - 1)$.

Now we generalize the results.

Theorem 3.5. For $G_{m,n}$ with n even.

$$\text{im}(G_{m,n}) \geq \begin{cases} m\lceil \frac{n}{4} \rceil - 1 & \text{if } n \equiv 2 \mod 4; \\ \frac{mn}{4} & \text{if } n \equiv 0 \mod 4. \end{cases}$$

Proof. The claims follow by combining Theorems 3.1 and 3.3.

Theorem 3.6. For $G_{m,n}$ with n odd

$$\text{im}(G_{m,n}) \geq \begin{cases} m\lfloor \frac{n}{4} \rfloor + 2 & \text{if } n \equiv 3 \mod 4; \\ \frac{mn+3m-8}{4} & \text{if } n \equiv 1 \mod 4. \end{cases}$$

We have established the lower bound for the MIM numbers for the stacked-book graphs. From our preliminary work into establishing the tighter bounds, we have reasons to suggest that the results in the last two theorems may coincide with the upper bounds, and thus we come up with the conjectures below.

Conjecture 3.1. For $G_{m,n}$ with n even

$$\operatorname{im}(G_{m,n}) = \begin{cases} m\lceil \frac{n}{4} \rceil - 1 & \text{if } n \equiv 2 \mod 4; \\ \frac{mn}{4} & \text{if } n \equiv 0 \mod 4. \end{cases}$$

Conjecture 3.2. For $G_{m,n}$ with n odd

$$\operatorname{im}(G_{m,n}) = \begin{cases} m\lfloor \frac{n}{4} \rfloor + 2 & \text{if } n \equiv 3 \mod 4; \\ \frac{mn+3m-8}{4} & \text{if } n \equiv 1 \mod 4. \end{cases}$$

4 Concluding Remarks

We have obtained the MIM number of stacked-book graphs $G_{m,n}$ for all m and for $n \in [1,5]$. These results are building blocks for obtaining the lower bounds for the cases where $n \geq 6$. Conjectures 3.1 and 3.2 suggest that the lower bounds obtained in this work will in fact be equal to the upper bounds, if those can be found. It must be noted that obtaining the lower bounds or the MIM numbers for the complete stacked-book graphs class will take rigorous effort and therefore may be worth considering as a new task.

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