International Journal of Mathematical Sciences and Optimization: Theory and Applications
10(4), 2024, Pages 45 - 54
https://doi.org/10.5281/zenodo.14710043

On Polian Algebras

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Article Info
Received: 16 June 2024 Revised: 20 October 2024
Accepted: 20 October 2024 Available online: 10 November 2024

Abstract

In this paper, polian algebras are introduced. Their properties are investigated. Left absorbing, right absorbing as well as absorbing polian algebras are studied. Manifolds and equipotence are introduced and studied in polian algebras. Moreover, hyperbolic polian algebras are introduced and their properties are investigated.

Keywords: Polian Algebras, Equipotence, Manifolds, Ideals, Annihilators.
MSC2010: 20N02, 20N05, 06F35.

1 Introduction

An algebra of type (2, 0) is a non-empty set, having a constant element, on which is defined a binary operation such that certain axioms are satisfied. BCI algebras and BCK algebras, introduced in [13] and [12], are common varieties of such algebras. There are several other varieties of algebras of type (2, 0). There are also several generalizations of BCI algebras. In [3], BCH algebras were studied. In [18], d algebras were studied. Pre- commutative algebras were studied in [16]. Fenyves algebras were studied in [14], [11] and [15]. In [17], Q algebras were introduced. Homomorphisms of Q algebras were studied in [5].
Recently, it has been shown in [1] that algebras of type (2,0) have diverse applications in coding theory. Motivated by this, more research interest has been given to the study of algebras of type (2,0). Obic algebras were introduced in [6]. In [7], torian algebras were studied. It was shown that the class of torian algebras is a wider class than the class of obic algebras. Ideals of torian algebras were investigated in [10]. The dual and nuclei of ideals as well as congruences developed on ideals of torian algebras were studied. In [8], right distributive torian algebras were studied. Isomorphism Theorems of torian algebras were studied in [9].
In all the aforementioned algebras, when the constant element multiplies a non-constant element x on the right, the product is x. We are therfore interested in an algebra such that the result of the muliplication of a non-constant element by the constant element on the right gives the constant element. Polian algebras satisfy this interesting axiom. In this paper, polian algebras are introduced. Their properties are investigated. Left absorbing, right absorbing as well as absorbing polian algebras are studied. Manifolds and equipotence are introduced and studied in polian algebras. Moreover, hyperbolic polian algebras are introduced and their properties are investigated.

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International Journal of Mathematical Sciences and Optimization: Theory and Applications
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https://doi.org/10.5281/zenodo.14710043

2 Polian Algebras

Definition 2.1. *An algebra $(X;*,0)$; where X is a non-empty set, $*$ a binary operation defined on X, and 0 a constant element of X is called a polian algebra if the following hold for all $x,y,z \in X$:*

1. $0 * x = x$
2. $x * 0 = 0$
3. $x * y = 0,\ y * x = 0 \Rightarrow x = y$
4. $x * (y * z) = y * (x * z)$
5. $x * x = 0$

Example 2.2. *Let $X = \{0,1,2,3,4,5\}$. Define a binary operation $*$ on X by Table 1:*

$*$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	0	0	1	3	3	4
2	0	0	0	3	3	3
3	0	1	2	0	1	2
4	0	0	1	0	0	1
5	0	0	0	0	0	0

Table 1: A polian algebra of order 6

Then $(X;*,0)$ is a polian algebra.

Example 2.3. *Let $X = \{0,1,2,3,4\}$. Define a binary operation $*$ on X by Table 2:*

$*$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	1	0	3	3
3	0	0	2	0	2
4	0	0	0	0	0

Table 2: A polian algebra of order 5

Then $(X;*,0)$ is a polian algebra.

Remark 2.4. *We shall write X for a polian algebra $(X;*,0)$ unless there is the need to emphasize the binary operation and constant element of $(X;*,0)$.*

Definition 2.5. *Let X be a polian algebra. Then X is said to be:*

1. *left absorbing if $x * (y * z) = (x * y) * (x * z)$ for all $x,y,z \in X$.*
2. *right absorbing if $(x * y) * z = (x * z) * (y * z)$ for all $x,y,z \in X$.*
3. *absorbing if it is both left absorbing and right absorbing.*

Example 2.6. *The polian algebra in Example 2.8 is left absorbing but it is not right absorbing.*

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The following Lemmas follow from definition.

Lemma 2.7. *Let X be a left absorbing polian algebra. Then the following hold for all $x, y, z \in X$:*

1. $y * (x * z) = (x * y) * (x * z)$
2. $y * (y * z) = y * z$

Lemma 2.8. *Let X be a right absorbing polian algebra. Then the following hold for all $x, y, z \in X$:*

1. $(z * y) * z = y * z$
2. $(x * z) * z = z$

Proposition 2.9. *Let X be an absorbing polian algebra. Then the following hold for all $x, y, z \in X$:*

1. $y * (x * z) = (x * y) * (x * z)$
2. $y * (y * z) = y * z$
3. $(z * y) * z = y * z$
4. $(x * z) * z = z$
5. $y * (y * z) = (z * y) * z$

Proof. The proof follows from Lemma 2.28 and Lemma 2.8. □

Definition 2.10. *Let S be a non-empty subset of a polian algebra X. S is said to be an ideal of X if the following hold:*

1. $0 \in S$
2. *for any $x, y \in X$ such that $x * y,\ x \in S$, then $y \in S$*

Example 2.11. *Let X be a polian algebra. Then clearly, $\{0\}$ and X are ideals of X. They are called the trivial ideals of X.*

Example 2.12. *Consider the polian algebra X in Example 2.28. Then the set $S = \{0, 1, 2\}$ is an ideal of S.*

Definition 2.13. *Let X be a polian algebra. The subset $B(X) = \{x \in X : a * (b * x) = 0$ for some $a, b \in X\}$ is called the annihilator of X.*

Proposition 2.14. *The annihilator of a left absorbing polian algebra X is an ideal of X.*

Proof. Clearly, $0 \in B(X)$. Now, let $y * z, y \in B(X)$. Notice that for some $a, b \in X$, we have
$0 = a * (b * (y * z)) =$
$a * ((b * y) * (b * z)) =$
$(a * (b * y)) * (a * (b * z)) =$
$0 * (a * (b * z)) =$
$a * (b * z)$. Hence, $z \in B(X)$ as required. □

Definition 2.15. *Let S be an ideal of a polian algebra X. Define a relation $\sim$ on X by $x \sim y$ if and only if $x * y,\ y * x \in S$.*

Proposition 2.16. *Let S be an ideal of a left absorbing polian algebra X. Then the relation $\sim$ is an equivalence relation.*

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Proof. Clearly, $\sim$ is reflexive and symmetric. Now, let $x, y, z \in X$ such that $x \sim y$ and $y \sim z$. Then
$(y * z) * ((x * y) * (x * z)) =$
$(y * z) * (x * (y * z)) =$
$x * ((y * z) * (y * z)) =$
$0 \in S$. Thus, $(x * y) * (x * z) \in S$. Hence, $x * z \in S$.
Similar argument shows that $z * x \in S$. Therefore, $\sim$ is transitive as required. □

Definition 2.17. *An equivalence relation $\sim$ on a polian algebra X is said to be equipotent if whenever $x, y, u, v \in X$ such that $x \sim y$ and $u \sim v$, then $(x * u) \sim (y * v)$.*

Proposition 2.18. *Let S be an ideal of a left absorbing polian algebra X. The relation $\sim$ defined on X by $x \sim y \Leftrightarrow x * y,\ y * x \in S$, is equipotent.*

Proof. We have already shown in Proposition 2.16 that $\sim$ is an equivalence relation on X. Now, let $x, y, u, v \in X$ such that $x \sim y$ and $u \sim v$. Notice that $(x * y) * ((y * u) * (x * u)) =$
$(x * y) * ((x * ((y * u) * u))) =$
$x * (y * ((y * u) * u)) =$
$x * ((y * u) * (y * u)) = 0 \in S$.
So, $(y * u) * (x * u) \in S$.
Similar argument shows that $(x * u) * (y * u) \in S$. Thus, $(x * u) \sim (y * u)$.
Notice also that $(u * v) * ((y * u) * (y * v)) =$
$(u * v) * (y * (u * v)) =$
$y * ((u * v) * (u * v)) = 0 \in S$.
So, $(y * u) * (y * v) \in S$.
Similar argument shows that $(y * v) * (y * u) \in S$.
Thus, $(y * u) \sim (y * v)$. Hence, $(x * u) \sim (y * v)$ as required. □

Definition 2.19. *Let X be a polian algebra. A relation $\sim$ on X is called a manifold if the following hold:*

1. *for each $x \in X$, there exists $y \in X$ such that $x \sim y$*
2. *whenever $x, y, u, v \in X$ such that $x \sim u$ and $y \sim v$, then $(x * y) \sim (u * v)$*

Example 2.20. *Let $X = \{0, 1, 2, 3\}$. Define a binary operation $*$ on X by Table 3:*

$*$	0	1	2	3
0	0	1	2	3
1	0	0	1	1
2	0	0	0	1
3	0	0	1	0

Table 3: A polian algebra of order 4

Then $(X; *, 0)$ is a polian algebra.
The relation $\sim = \{(0,0), (0,1), (0,2), (0,3), (1,0), (1,1), (1,2), (1,3), (2,0), (2,1), (3,0), (3,1), (3,2)\}$ is a manifold on X.
The following proposition is straightforward from definition.

Proposition 2.21. *Let X be a polian algebra, and let $f : X \to X$ such that $f(x * y) = f(x) * f(y)$ for all $x, y \in X$. The f is a manifold on X.*

Definition 2.22. *Let X be a polian algebra, and let $\sim$ be a manifold on X. The set $\{y \in X : x \sim y\}$ is called the manifold class of x. It is denoted by $[x]_\sim$ The set $\{x \in X : x \sim y\}$ is called inverse manifold class of y. It is denoted by $[y]_\sim^{-1}$.*

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https://doi.org/10.5281/zenodo.14710043

Definition 2.23. *A non-empty subset S of a polian algebra X is said to be complete in X if $x * y \in S$ for all $x, y \in S$.*

The following propositions are straightforward from definition.

Proposition 2.24. *Let X be a polian algebra, and let $\sim$ be a manifold on X. Then $[0]_\sim$ and $[0]_\sim^{-1}$ are complete in X.*

Proposition 2.25. *Let X be a polian algebra, and let $\sim$ be a manifold on X. Then the following hold:*

1. $[a]_\sim \cap [b]_\sim \neq \phi \Rightarrow a * b \in [0]_\sim$
2. $[a]_\sim^{-1} \cap [b]_\sim^{-1} \neq \phi \Rightarrow a * b \in [0]_\sim^{-1}$

Definition 2.26. *Let S be a complete subset of a polian algebra X, and let $\sim$ be a manifold on X. Then the collection $\{y \in X : x \sim y,$ for some $x \in S\}$ is called an S-residue of X. It is denoted by $S(X)$.*

Proposition 2.27. *Let S be a complete subset of a polian algebra X, and let $\sim$ be a manifold on X. Then $S(X)$ is complete in X.*

Proof. Let $y, z \in S(X)$. Then $(a * b) \sim (y * z)$ for some $a, b \in S$. Hence, $S(X)$ is complete in X. □

Corollary 2.28. *Let X be a polian algebra, and let $\sim$ be a manifold on X. Then the following hold:*

1. $X(X)$ *is complete in* X
2. $X = \cup [x]_\sim$
3. $[0]_\sim$ *is complete in* $X(X)$

Proof.
1. Clearly, X is complete in X. Hence, by Proposition 2.27, the result follows.
2. Let $y \in X$. Then $a \sim y$ for some $a \in X$. So, $y \in [x]_\sim$ for some $x \in X$. Hence, $X(X) \subseteq \cup [x]_\sim$. Clearly, $\cup [x]_\sim \subseteq X(X)$.
3. Let $a, b \in [0]_\sim$. Then $0 \sim (a * b)$. So, $a * b \in [0]_\sim$.

□

By combining Proposition 2.24, Proposition 2.25, Proposition 2.27 and Corollary 2.28, we have the following theorem:

Theorem 2.29. *Let $\sim$ be a manifold on a polian algebra X, and let S be a complete subset of X. Then the following hold:*

1. $[0]_\sim$ *and* $[0]_\sim^{-1}$ *are complete in* X
2. $[a]_\sim \cap [b]_\sim \neq \phi \Rightarrow a * b \in [0]_\sim$
3. $[a]_\sim^{-1} \cap [b]_\sim^{-1} \neq \phi \Rightarrow a * b \in [0]_\sim^{-1}$
4. $X(X)$ *is complete in* X
5. $X = \cup [x]_\sim$
6. $[0]_\sim$ *is complete in* $X(X)$

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3 Hyperbolic Polian Algebras

Definition 3.1. *Let $(X;*,0)$ be a polian algebra, and let $\circ$ be another binary operation defined on X. Then $\circ$ is said to be hyperbolic over $*$ if $x\circ(y*z)=(x\circ y)*(x\circ z)$ and $(x*y)\circ z=(x\circ z)*(y\circ z)$ for all $x,y,z\in X$. If $(x*y)\circ z=x\circ(y*z)$ for all $x,y,z\in X$, then $(X;*,\circ,0)$ is called a hyperbolic polian algebra.*

Definition 3.2. *Let $(X;*,0)$ be an absorbing polian algebra. If $(X;*,\circ,0)$ is hyperbolic, then $(X;*,\circ,0)$ is called an absorbing hyperbolic polian algebra.*

Example 3.3. *Let $X=\{0,1,2,3\}$. Define the binary operation $*$ on X by Table 4:*

$*$	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	1	0	3
3	0	0	0	0

Table 4: A polian algebra of order 4

Also, define the binary operation $\circ$ on X by Table 5:

$\circ$	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	1	2	3

Table 5: A polian algebra of order 4

Then $(X;*,0)$ is a hyperbolic polian algebra.

Example 3.4. *Let $X=\{0,1,2,3\}$. Define the binary operation $*$ on X by Table 6:*

$*$	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	1	0	3
3	0	0	0	0

Table 6: A polian algebra of order 4

Also, define the binary operation $\circ$ on X by Table 7:

Then $(X;*,0)$ is a hyperbolic polian algebra.

Remark 3.5. *Henceforth, $(X;*,\circ,0)$ will denote a hyperbolic polian algebra.*

The following lemma follows from definition.

Lemma 3.6. *Let $(X;*,\circ,0)$ be a hyperbolic polian algebra. Then the following hold for all $x,y,z\in X$:*

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https://doi.org/10.5281/zenodo.14710043

$\circ$	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	0	2
3	0	0	2	3

Table 7: A polian algebra of order 4

1. $0 \circ x = 0$
2. $x \circ 0 = 0$
3. $x * y = 0 \Rightarrow (x \circ z) * (y \circ z) = 0$
4. $x * y = 0 \Rightarrow (z \circ x) * (z \circ y) = 0$

Definition 3.7. *Let $(X; *, \circ, 0)$ be a hyperbolic polian algebra. A non-empty subset S of X is called a hyperbolic ideal of X if the following hold:*

1. *$a * x \in S \Rightarrow x \in S$ for all $a \in S, x \in X$*
2. *$X \circ S \subseteq S$*

Example 3.8. *Let $X = \{0, 1, 2, 3\}$. Define the binary operation $*$ on X by Table 8:*

$*$	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	0	0	3
3	0	0	0	0

Table 8: A polian algebra of order 4

Also, define the binary operation $\circ$ on X by Table 9:

$\circ$	0	1	2	3
0	0	0	0	0
1	0	1	0	0
2	0	0	2	3
3	0	0	3	2

Table 9: A polian algebra of order 4

Then $(X; *, \circ, 0)$ is a hyperbolic polian algebra.
It is easy to see that the subset $\{0, 1\}$ is a hyperbolic ideal of X. But the subset $\{0, 2\}$ is not.

Lemma 3.9. *Let S be a hyperbolic ideal of a hyperbolic polian algebra $(X; *, \circ, 0)$. Then the annihilator of X is contained in S.*

Proof. Let $x \in B(X), a, b \in X$. Notice that $a * (b * x) = 0 \in S$. Hence, $x \in S$ as required. □

The following proposition is obvious.

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10(4), 2024, Pages 45 - 54
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Proposition 3.10. *Let $(X;*,\circ,0)$ be a polian algebra, and let $\{S_i\}_{i\in I}$ be the collection of hyperbolic ideals of X. Then $\cap S_i$ is also a hyperbolic ideal of X.*

Remark 3.11. *Let $(X;*,\circ,0)$ be a hyperbolic polian algebra, and let T be a non-empty subset of X. Let $\{S_i\}_{i\in I}$ be the collection of hyperbolic ideals of X containing T. By Proposition 3.1, $\{S_i\}_{i\in I}$ is a hyperbolic ideal of X. This hyperbolic ideal is called the hyperbolic ideal generated by T. It is denoted by $< T >$.*

Lemma 3.12. *Let $(X;*,0)$ be an absorbing polian algebra. If $x,y\in X$ such that $x*y=0$, then $(z*x)*(z*y)=0$ and $(y*z)*(x*z)=0$ for all $z\in X$.*

Proof. Notice that $0=x*0=z*(x*y)=(z*x)*(z*y)$; proving the first part.
similar argument gives the second part. □

Theorem 3.13. *Let $(X;*,\circ,0)$ be an absorbing hyperbolic polian algebra, and let T be a non-empty subset of X such that $X\circ T\subseteq T$. Then $< T >= \{a\in X : y_k*(...*(y_1*a)...)=0\}$ for some $y_1,y_2,...,y_k\in T$*

Proof. Let $E=\{a\in X : y_k*(...*(y_1*a)...)=0\}$ for some $y_1,y_2,...,y_k\in T$. Let $a\in X, b\in E$. Then we have $y_1,y_2,...,y_k\in T$ such that $y_k*(...*(y_1*b)...)=0$. Notice that $0=x\circ 0=x\circ(y_k*(...*(y_1*b)...))=$
$(x\circ y_k)*(...((x\circ y_1)*(x\circ b))...)$.
Clearly, $x\circ y_i\in T$ for all $i=1,...,k$. So, $x\circ b\in E$. Now, let $a,b\in X$ such that $a*x\in E$ and $a\in E$. Then we have $y_1,...,y_k,p_1,...,p_l\in T$ such that

$$y_k*(...*(y_1*(a*x))...)=0 \tag{1}$$

and

$$p_l*(...*(p_1*a)...)=0 \tag{2}$$

Now since X is polian, expression (1) gives
$a*(y_k*(...*(y_1*x)...))=0$. Applying Lemma 3.3 to expression (2), we have
$0=(p_l*(...*(p_1*a)...))*(p_l*(...*(p_1*(y_k*(...*(y_1*x)...)))...))$. So, $x\in E$. Hence E is a hyperbolic ideal of X. Clearly, $T\subseteq E$.
Now, let F be an ideal of X containing T. Let $a\in E$. Then we have $y_1,...,y_k\in T$ such that $y_k*(...*(y_1*a)...)=0$. Then since F is a hyperbolic ideal of X, we have $a\in F$. Hence $E\subseteq T$. Therefore $< T >= E$. □

Remark 3.14. *Let E and F be hyperbolic ideals of a left absorbing polian algebra $(X;*,\circ,0)$. It may happen that the union of E and F is not a hyperbolic ideal of X. We see this in the following example.*

Example 3.15. *Let $\{0,1,2,3,4\}$. Define the binary operation $*$ on X by Table 10*

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	2	4
2	0	1	0	1	4
3	0	0	0	0	4
4	0	0	2	2	0

Table 10: A polian algebra of order 5

Also define the binary operation $\circ$ on X by Table 11

$\circ$	0	1	2	3	4
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	4

Table 11: A polian algebra of order 5

Then $(X; *, \circ, 0)$ is a left absorbing hyperbolic polian algebra.

Now, the subsets $E = \{0, 1\}$ and $F = \{0, 2\}$ are hyperbolic ideals of X. But $E \cup F$ is not a hyperbolic ideal of X because $2 * 3 = 1 \in E \cup F$. But $3 \neq E \cup F$.

Theorem 3.16. *Let $(X; *, \circ, 0)$ be an absorbing hyperbolic polian algebra, and let E, F be ideals of X. Then $< E \cup F >= \{a \in X : x * (y * a) = 0\}$ for some $x \in E, y \in F$.*

Proof. Let $V = \{a \in X : x * (y * a) = 0\}$ for some $x \in E, y \in F$. Then clearly, $V \subseteq < E \cup F >$. Now, let $b \in < E \cup F >$. Then we have $y_1, ..., y_k \in < E \cup F >$ such that $y_k * (...(y_1 * b)...) = 0$. Now, if $y_i \in E$ for all $i = 1, ...k$, then $b \in E$. Similarly, we have $b \in F$. Therefore, $b \in V$ since $b * (0 * b) = 0$ and $0 * (b * b) = 0$. Now, if $y_i \in E$ for some $i = 1, ..., k$, and $y_i \in F$ for the remaining i for which y_i is not in E, we may assume that $y_1, ..., y_t \in E$ and $y_{t+1}, ..., y_k \in F$. Now, let $u = y_k * (... * (y_1 * b)...)$. Then we have
$y_k * (... * (y_{t+1} * u)...) =$
$y_k * (... * (y_{t+1} * (y_t * (... * (y_1 * b)...)))...) = 0$. Hence, $u \in F$.
Now, let $v = u * b = (y_t * (... * (y_1 * b)...)) * b$. Then $y_t * (... * (y_1 * v)...) =$
$y_t * (... * (y_1 * ((y_t * (... * (y_1 * b)...)) * b))...) =$
$(y_t * ...(y_1 * b)...) * (y_t * (... * (y_1 * b)...)) = 0$.
Hence, $v \in E$. Now, since $u * (v * b) = v * (u * b) = v * v = 0$, we have that $b \in V$. So, $< E \cup F > \subseteq V$. Therefore, $< E \cup F >= V$ as required. ☐

4 Acknowledgment

The author is thankful to the referees for their valuable comments and suggestions.

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