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Some Geometric Characterization Of Star-like 3D Conjugacy $C^3\omega_n^*$ On Partial One-One Transformation Semigroups

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Abstract

Let $X_n = \{1, 2, 3, ...\}$ be a set of distinct non negative integer then $C^3\omega_n^*$ be star-like conjugacy transformation semigroup for all $D(\alpha^*)$ (domain of α^*) and $I(\alpha^*)$ (Image of α^*) such that an operator $\mid \alpha\omega_i - \omega_{i+1} \mid \leq \mid \alpha\omega_i - \omega_i \mid$ was generated. A star-like transformation semigroup is said to satisfy collapse function if $C^+(\alpha^*) = \mid \cup t\alpha^- : t \in T\alpha\omega_n^* \mid$ while the finding shows that the collapse of 3D star-like conjugacy classes are zero. The geometry model of 3D star-like conjugacy was obtained by using folding principle on a standard A4 paper which shows the star-like 3D conjugacy relation $\alpha(ij) = \frac{\alpha_i+\alpha_{i+1}}{\alpha_i-\alpha_{i+1}} = \frac{\alpha_{i+1}+\alpha_i}{\alpha_{i+1}-\alpha_i}$.Some tables were formed to analyse the structure of star-like derank of $C^3\omega_n^*$ be $\mid n - Im\alpha^* \mid = d$, star-like collapse $C^+(\alpha^*) = \mid \cup t_{\alpha^{-1}} : t \in T\alpha\omega_n^* \mid$, Star-like relapse $C^-(\alpha^*) = \mid n - C^+(\alpha^*)$, Star-like pivot of $C^3\omega_n^*$ be $\mid \dfrac{n.r^+(\alpha^*)}{c^-(\alpha^*) + c^+(\alpha^*} \mid = p$ and Star-like joint of $C^3\omega{*}_n$ be $\mid r^+(\alpha^*) - m^*(\alpha^*) - C^+(\alpha^*) + n \mid = j$. The study conclude that $C^3\omega_n^*$ has n order conjugacy classes and we show that $\phi \in C^3\omega_n^*$.

Keywords: Conjugacy, 3D, Geometric, Partial one-one, Semigroup, Star-like.
MSC2010: 20M20.

1 Introduction

Group theory continues to be an intensively studied matter. There are three historical roots of group theory: the theory of algebraic equation, number theory and geometry. Joseph Louis

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Lagrange, Niels Henrik Abel and Evariste Galois were early researchers in the late 18th century. While semigroup started in the early 1930s with the work of [4]. The star-like partial one-one transformation semigroup denoted as $I\alpha\omega_n^*$ in I_n is also a semigroup in one-one transformation semigroup, see [7].
Transformation is used instead of mapping, the latter serves as another name for the former. More information on semigroup of transformation are obtainable from the works of [5, 6].
The domain and image set of any given transformations $\alpha_i^* \in \alpha\omega_n^*$ was denoted by $D(\alpha^*)$ and $I(\alpha^*)$ respectively as used by [3].
A Star-like transformation semigroup is said to satisfy collapse function if $c^+(\alpha^*) = |\bigcup t\alpha^{-1} : t \in T\alpha\omega_n^*|$ while Relapse function is denoted as $C^-(\alpha) =| n - c^+(\alpha^*) |$ where $n \in N$ see [8] .
Any transformation $\alpha \in \omega_n$ defined in the operator $| \alpha\omega_i - \omega_{i+1} | \leq | \alpha\omega_i - \omega_i |$, is a mapping from a set to itself such that the star-like composition of any two or more transformation of the same set gives the same transformation of this set. Therefore the composition $\alpha \in \alpha\omega_n$ is a special case of $\alpha\omega_n$.

Consider some elements of α such that

$$\alpha = \begin{pmatrix} K_1 & K_2 & K_3 & \dots & K_n \\ \alpha^\star K_1 & \alpha^\star K_2 & \alpha^\star K_3 & \dots & K_n \end{pmatrix} \tag{1.1}$$

The set of all star-like transformation of $\alpha\omega_n$ on X_n would be denoted as α_i. Therefore, the elements of α in the transformation has the form

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ \alpha^\star\omega_1 & \alpha^\star\omega_2 & \alpha^\star\omega^\star 3 & \dots & \alpha\omega_n \end{pmatrix} \tag{1.2}$$

Thus, the transformation to find in succession $\alpha_(i,j)$ special entries of ω_n, was established such that when we consider an element of order four in $\alpha\omega_n \leqslant \omega_n$

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1\alpha & 2\alpha & 3\alpha & 4\alpha \end{pmatrix} \tag{1.3}$$

with domain $D(\alpha) = (1, 2, 3, 4)$ and image set $I(\alpha) = (1\alpha, 2\alpha, 3\alpha, 4\alpha)$ we obtain a general star-like recurrence relations.
The star-like pivot of α^* is denoted and defined as $V^+(\alpha^*) = |\frac{n \cdot r^+(\alpha^*)}{c^+(\alpha^*)+c^-(\alpha^*)}|$. The star-like joint of α^* is denoted and defined as $J^+(\alpha^*) = |r^+(\alpha^*) - m^*(\alpha^*) - C^+ + n|$. The star-like relapse of α^* is denoted and defined as $c^-(\alpha^*) = |n - c^+(\alpha^*)|$.Star-like collapse of α^* is denoted by $c^+(\alpha^*)$ and defined as $c^+(\alpha^*) = |\bigcup_{i=1}^n y_i\alpha^{-1*} : |y_i\alpha^{-1*}| \geq 2|$.

2 Preliminary Notes

The study of [1, 2] exhibit some properties which formed the bases of this research and these properties will be discuss in this section which will help us to formulate our results

3 Generalization of 3-Dimensional star – like sequences through some combinatorial composite functions

Definition 3.1. Conjugacy: *It is a set of element that are connected by an operation that is in group (G)then the element (a) and (b) are conjugate of each other if their is another element (g)*

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in (G) such that $b = gbg^{-}$

Definition 3.2. *Let* $X_n = \{1, 2, 3, \cdots\}$ *be a non empty finite set, and* $C^3\omega_n^*$ *be a 3 D star-like Conjugacy transformation semigroups, such that*

$$|\alpha\omega_i - \omega_{(i+1)}| \leq |\alpha\omega_i - \omega_i| \tag{3.1}$$

For all $\omega_i \in D(\alpha^*)$ *and* $\alpha^*\omega_i \in I(\alpha^*)$*, where* $N_i U \emptyset$; $N_i = i, i+1, i+2, \dots\ i = 0, 1, 2, \dots$.

We investigate the star-like 3 D model using folding principles on A4 paper see definition 2.1 of [1]. The star-like $3D$ model in Fig. 1 represent the star-like conjugacy rectangular prism with the composition of:

1. Star-like faces F^*
2. Star-like edges E^*
3. Star-like vertices V^*

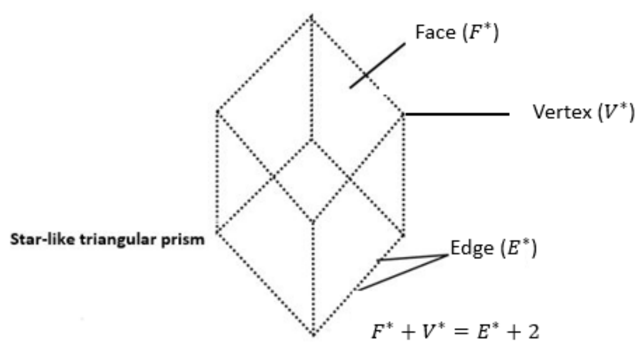

Figure 1: Star-like Conjugacy triangular Prism

By the star-like folding principle structure we unfold the Fig. 1 to obtain the general $3D$ star-like conjugacy equation.

$$F^* + V^* = E^* + 2 \tag{3.2}$$

which is a relation to the unfolded $3D$ star-like conjugacy rectangular prism. Therefore to obtain the volume of a $3D$ star-like conjugacy triangular prism V, we must begin to construct a star-like triangular path with a $3D$ star-like conjugacy array of a control star-like conjugacy disk point which form an n sided star-like conjugacy $3D$ depths. From equation 3.1 combining with 3D general star-like conjugacy, we obtained

$$V = \frac{1}{2} b \times h \times l \tag{3.3}$$

Equivalent to

$$\frac{1}{2} V = |\alpha\omega_i - \omega_{i+1}| \leq |\alpha\omega_i - \omega_i| \tag{3.4}$$

where $\omega_{i+1} \in D(\alpha^*)$ and $\alpha\omega_i \in I(\alpha^*)$, to generate:

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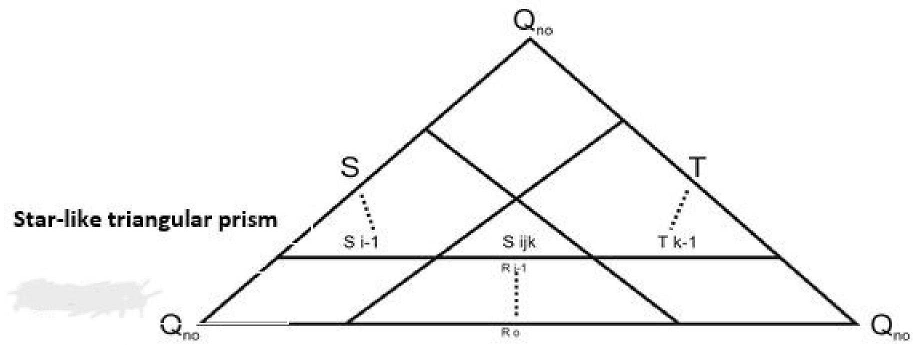

Figure 2: Unfolded Star-like Conjugacy triangular Prism

Lemma 3.1. *Let $C^3\omega_n^*$ be set of star-like conjugacy classes, with $\alpha^* \in \dfrac{E^*+F^*}{2}+\phi^*$ then $D(\alpha^*) \subseteq I(\alpha^*)$ such that $E^* \in D(C^3\omega_n^*)$ and $F^* \in I(C^3\omega_n^*)$.*

Proof. Suppose $C^3\omega_n^*$ be set of star-like conjugacy transformation semigroup with a star-like composite relation.

$$a_i^* + b_j^* + c_k^* = C^3\omega_n^* \tag{3.5}$$

There exist $i(2) = \phi^*$ for $C\omega_n^* \in \alpha\omega_n^*$.
By general 3D conjugacy and star-like operator

$$F^* + V^* = E^* + 2 \tag{3.6}$$

yields

$$\frac{E^* - F^*}{2} = \mid \alpha\omega_i - \omega_{i+1} \mid \leq \mid \alpha_{\omega_{i+1}} - \omega_i \mid \tag{3.7}$$

Then, $C^3\omega_n^*$ satisfy eqn (3.1) and eqn (3.2) we see that $V^*(C^3\omega_n^*) = \frac{E^*-F^*}{2} + \phi^*$ which is the required conjugacy vertices for any $\alpha^* \in C^3\omega_n^*$ with a star-like conjugacy disk constant point $\phi^* \in C^3\omega_n^*$. □

Lemma 3.2. *Let $S^*(x,y)$ represent order of sequences from the star-like origin $(0,0)^*$ to $(x,y)^*$ with star-like row-x and column-y then $\alpha_i^* \in C^3\omega_n^*$ form a star-like triangular array.*

Proof. Suppose $\alpha^* \in C^3\omega_n^*$ with row-x and column-y of triangular star-like sequences for all $N_i = \{i, i+1, i+2, ...\}, (i = 0, 1, 2, ...)$.
Then for any $\alpha_i^* \in C^3\omega_n^*(i = 1, ...)$ we obtain the star-like conjugacy operations
$S^*(x,0) = X_i^*$
$S^*(o,y) = Y_i^*$
Such that $X_i^* = Y_i^* = \phi^* \in C\omega_n^*$
Therefore,

$$S^*(x,y) = S^*(x, y - X_i^*) + S^*(x - Y_i^*, y) \tag{3.8}$$

yields a star-like conjugacy sequential recurrence order

$$\binom{x+y}{x}^* + \binom{x+y}{y}^* = \frac{x+y}{x!\,y!} \tag{3.9}$$

with star-like row-x and column-y ; $x_i \in X_i^*$ and $y_i \in Y_i^* : N_i = \{i, i+1, ...\}$. □

4 Geometry Model on the $3D$ Star-like Transformation

. A star-like $3D$ conjugacy triangular pyramid is a star-like polyhedron with $9(F^*)$, a $12(E^*)$, and all other $5(V^*)$ star-like conjugacy polyhedron meeting at a star-like disk point which was embedded in equation (3.4).

The geometry model of $3D$ star-like conjugacy triangular pyramid was obtained from the generalization of the $3D$ star-like conjugacy sequence of both the bottom and front view respectively as shown in Fig.4

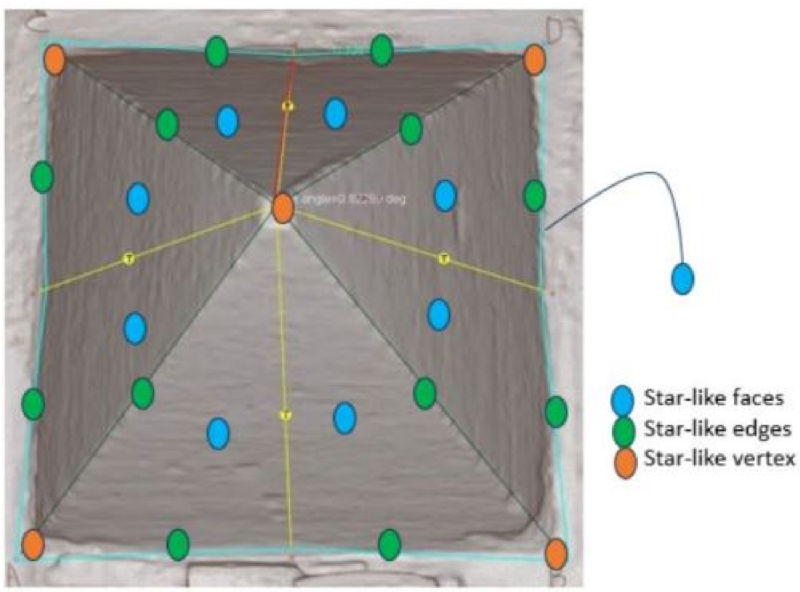

Figure 3: Star-like $3D$ star-like square pyramid of [1]

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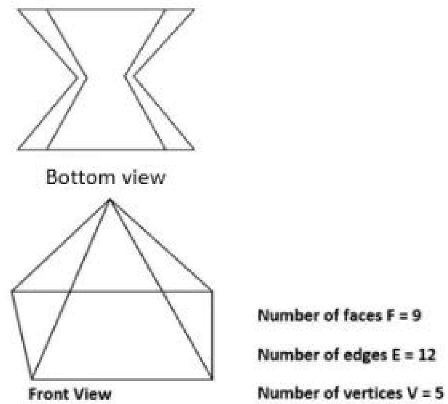

Figure 4: Double View of 3D square pyramid

which shows the star-like $3D$ conjugacy relation

$$\alpha_(i,j) = \frac{\alpha_i + \alpha(i+1)}{\alpha_i - \alpha(i+1)} = \frac{\alpha_(i+1) + \alpha(i)}{\alpha_(i+j) - \alpha(j)} \tag{4.1}$$

That is, by the proof of theorem 3.3 of [1] the star-like $3D$ conjugacy prism is an equivalence relations, so the distinct star-like conjugacy classes transformation, which means that $C^3\omega_n^*$ has n order conjugacy classes.

Lemma 4.1. *Let $\zeta^* \in C^3\omega_n^*$ be a star-like conjugacy spinnable transformation then if $\exists U_n^* \in \zeta$, $pq\overset{\triangle}{\underset{\triangledown}{=}}rs$ such that $|\triangle pqr| \leq |\triangle qrs|$ for all $pqrs \in U_n^*$*

Proof. Suppose $U_n^* \leq \zeta$
Then for any $\zeta^* \in C^3\omega_n^*$ there must exist an equilateral star-like shape such that ζ^* is a conjugacy spinnable.
Consider

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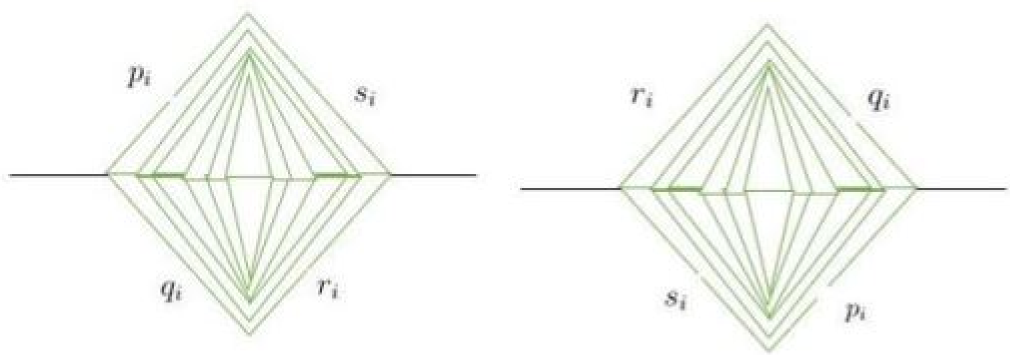

where, the star-like folding principle is adopted and by eqn (3.5) we see that for any $p_i \in \zeta^*$ in above figure there exist a constant disk point $i(2) \in N$ with adjacent equal star-like side $pqrs \in U_n^*$. Then

$$| \triangle pqr | \leq | \alpha\omega_i - \omega_{i+1} | \tag{4.2}$$

$$| \triangle qrs | \leq | \alpha_{i+1} - \omega_i | \tag{4.3}$$

Therefore by conjugacy operator in eqn (3.5)

$$| \triangle pqs | \leq | \triangle qrs | \tag{4.4}$$

Which shows that $\zeta^* \in C^3\omega_n^*$ any conjugate spinnable star-like transformation, the converse makes equal star-like angle on all side. □

5 Main Results

Table 1: Rellapse Table of the Image of $C^3\omega*_n$ $\quad C^-(\alpha^*) = | n - C^+(\alpha) |$

$n/C^-(\alpha)$	1	2	3	4	5	$\sum F(n;d)$
1	2					2
2		3				3
3			5			5
4				6		6
5					6	6

Lemma 5.1. *Given that $\alpha^* \in C^3\omega_n^*$ is spinnable reducible then $| C^+(\alpha^*) | \leq | C^-(\alpha^*) |$ whenever $| (C^3\omega_n^*) | = \binom{(\frac{d^2}{n}) - (2+n)}{d-q}$.*

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Proof. :
Let $X_n = \{1, 2, 3, \cdots\}$ be a non negative star-like set, such that $F(n, d, q) =\mid C^3\omega_n \mid$. Since $D(C^+(\alpha^*)) \subseteq X_n$ and $I(C^-(\alpha^*)) \subseteq X_n$ with $M(\alpha^*) \in C^3\omega_n^*$ of a domain in a star-like point of X_n is chosen from $n\binom{d}{q}$ methods then in each star-like partial conjugacy bijection . We have

$$C^3\omega_n^*(\alpha^*) : DC^+(\alpha^*) \to IC^-(\alpha^*)$$

. Suppose $C^3\omega_n^*$ is rellapsible under the composition of star-like conjugacy mapping where $\alpha^* \in C^3\omega_n^*, f(n, d, q) = \binom{(\frac{d^2}{n}) - (2 + n)}{d - q}$.
Then, by the star-like operator in eqn (3.1) which compels the conjugacy element of a star-like partial one-one is reducible. Therefore by lemme 3.2, a star-like conjugacy spinnable transformation exist and such that $\mid \triangle pqr \mid\leq\mid \triangle qrs \mid$ we show that

$$\mid C^+(\alpha) \mid\leq\mid C^-(\alpha) \mid\leq\mid \alpha\omega_i - \omega_{i+1} \tag{5.1}$$

Which makes every star-like conjugacy transformation $\alpha \in C^3\omega_n^*$ to produce a collapsibble and reducible algebraic structure and makes equal star-like point on all side so that whenever $\mid d \mid=\mid q \mid$ the $\mid C^3\omega_n^* \mid= \binom{(\frac{d^2}{n}) - (2 + n)}{d - q}$. for all $d \geq q \geq n \geq 2$
Hence , the result is complete as shown table 1 ∎

Table 2: Derank Table of the Image of $C^3\omega*_n$ $\quad D(\alpha^*) =\mid n - Im\alpha^* \mid= d$

n/d	1	2	3	4	5	$\sum F(n; d)$
1	1					1
2	1					1
3	3					3
4	4					4
5	4					4

Table 3: Pivot Tab of the Image of $C^3\omega*_n$ $\quad D(\alpha^*) =\mid \frac{n.r^+(\alpha^*)}{C^-(\alpha^*) + C^+(\alpha^*)} \mid= p$

n/p	1	2	3	4	5	$\sum F(n; p)$
1	1					1
2	1	2				3
3		3	2			5
4			4	2		6
5				3	3	6

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Table 4: Joint Table of the Image of $C^3\omega*_n$ $D(\alpha^*) =| r^+(\alpha^*) - m^*(\alpha^*) - C^+(\alpha^*) + n |= j$

n/j	1	2	3	4	5	6	7	8	9	10	$\sum F(n;j)$
1	1										1
2		1	1	1							3
3			1	2	2						5
4				1		2	2	1			6
5	4				1			2	2	1	6

Proposition 5.1
Let X_n be a star-like non-negative generated integer such that $Dom(C^3\omega_n^*) = \sum_{j=1}^n X_n$. Then for any given $\zeta^* \in C^3\omega_n^*$ $| \alpha^* C^3\omega_n^* |= \begin{pmatrix} j+1 \\ n+1 \end{pmatrix} \begin{pmatrix} n+j \\ 2j \end{pmatrix}$.

Proof. :
Given that $\zeta^* \leq Dom(C^3\omega_n^*) \leq X_n$ and $C^3\omega_n^* \subseteq \alpha\omega_n^*$, then
$f(n,j) =| \alpha^* \in \alpha^*\omega_n^* : C^3\omega_n^*(\alpha^*) |=| \alpha C^3\omega_n^* |$.
Consider $j =| j =| r^+(\alpha) - m^*(\alpha) + C^+(\alpha^*) + n |$ such that here exist $k_0 \in Dom(C^3\omega_n^*)$. Equation (3.1) produce a star-like joint $\alpha k_0 = e^0$ so k_0 has $n - e^0 + 1$ star-like order for all $n \geq j \geq 1$. $\begin{pmatrix} n+j \\ 2j \end{pmatrix}$. Since ζ^* is a star-like spinnable transformation, $\varnothing(\zeta^*)$ is a star-like sub-set of all star-like joint $j^* \in C^3\omega_n^*$, irrespective of the value of $n \geq j \geq 1$, whenever $j = (n-1)$ there is exactly finitely many star-like conjugacy composite classes of nth order such that by table 4 and equation (3.5) $| \alpha^* C^3\omega_n^* |= \begin{pmatrix} j_+1 \\ n+1 \end{pmatrix} \begin{pmatrix} n+j \\ 2j \end{pmatrix}$ for all $n \geq j \geq 1$ generate a star-like sequence array. □

Lemma 5.2. *Let $\zeta^* \in C^3\omega_n^*$ then $| C(\alpha^*) |= \begin{pmatrix} a-b \\ b-1 \end{pmatrix} = \begin{pmatrix} a-(b-1) \\ a-b \end{pmatrix} =| r(\alpha^*) |$ for all $a, b, \in \zeta^* \leq C^3\omega_n^*$.*

Proof. Suppose $X_n = \{1, 2, ...\}$ be a non degenerated star-like integers, then $Dom(C(\alpha^*)) = Dom(r(\alpha^*)) = \sum_{i=1} X_n$
If $f(a,b) =| \zeta^* : h(\zeta^*) |=| Im(\zeta^*) |= b$ there exist $K_0 \in X_n$ such that

$$\zeta^* k_0 = \zeta_n^* = \zeta^* k_0 \tag{5.2}$$

so $\zeta^* k_0 = e^0$ (a star-like conjugacy constant).
Since k_0 has $a - e^0 + 1$ disk point degree of freedom with equal order of collapse and rellapse then,

$$| C(\alpha^*) |=| r(\alpha^*) | \tag{5.3}$$

$$\begin{pmatrix} a-b \\ b-1 \end{pmatrix} = \begin{pmatrix} a-(b-1) \\ a-1 \end{pmatrix} = e^0 \tag{5.4}$$

For any star-like conjugacy transformation with $\zeta^* \in C^3\omega_n^*$
$h(\zeta^*) = e^0$ irrespective of the value of $C(\alpha^*)$ and $r(\alpha^*)$ whenever

$$\zeta^* k_0 = \zeta_n^* \tag{5.5}$$

Therefore
$\begin{pmatrix} a-b \\ b-1 \end{pmatrix} = \begin{pmatrix} a-(b-1) \\ a-b \end{pmatrix}$ for all $a, b \in \zeta^* \leq C^3\omega_n^*$. □

6 Conclusion

In this paper, We showed that the geometric characterization of star-like 3D conjugacy classes $C^3\omega_n^*$ on partial one-one transformation semigroups and some results of different functions. The paper conclude that for every 3D star-like conjugacy classes $C^3\omega_n^*$ has n order conjugacy classes and we also show that $\phi \in C^3\omega_n^*$

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