Application of Games Theory to Crime Investigation of suspects (A Case Study of Delta State Police Divisional Head-quarters, Abraka)

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Abstract

This paper discussed Games Theory and its application to real life situation. Emphasis was on the application of Games Theory to check some unlawful practice and even suspects under investigation such as multiple (or false) confession from suspects, use of physical violence during investigation and wrong conviction of suspects. This research was aimed at using game theory in combating crime in the society. The importance of a crime free society cannot be over-emphasised as a result of the high level of insecurity and hence it is imperative to apply a more tactical approach to foster solution to insecurity. This paper uses the prisoner's dilemma method, a situation where two suspects apprehended for a crime are held in separate rooms and cannot communicate with each other. Also, quantitative method was employed to examine the research problem effectively. Data collection that aided this research were sourced through systematic sampling from previously existing works from authors. This research was developed to check some sharp unlawful Police practice and even from suspect under investigation as well as employing the technique of Game Theory as a way of obtaining true from suspects, administering justice speedily and a strong avenue for prison decongestion due to the alarming conditions in most Nigerian Prisons.

1   Introduction

Game is concerned with interactive decision making involving more than one person. Outcomes are determined by whatever combination of actions resulting from independent choice of several individual decision makers. Games Theory is a bag of analytical tools designed to help us understand the phenomena that we observe when decision-makers interact. The basic assumptions that underlie the theory that decision-makers pursue their well defined exogenous objectives and take into account their knowledge or expectation of other decision makers behaviour (They reason strategically).

The models of games theory are highly abstract representations of classes of real-life situations. Their abstractness allows them to be used to study a wide range of phenomena. For example, the theory of Nash equilibrium has been used to study oligopolistic and political competition. Theory of mixed strategy equilibrium has been used to explain the distribution of tongue length in bees and tubes length in flowers. The theory of repeated games has been used to illustrate social phenomena like threats and promises. The theory of games reveals a sense in which the outcome of trading under a price system is stable in an economy that contains many agents.

Games Theory uses mathematics to express its ideas formally. However, most games theoretical ideas are not inherently mathematical, though a mathematical formation makes it easy to define concepts precisely, to verify the consistency of ideas, to explore the implication assumptions. [1]

The Modern Mathematical approach to game theory is generally attributed to [2] and [3]

A question of priority was raised by Frechet suggestion that several "Zur Theories der Gesellascha fisspielle" by [4] in the early 20's really made grounds for game theory. These theories were then published with comments by Frechet and [2]. Albeit Borel gave a clear statement of an important class of game theoretic problems and introduced its concept of pure and mixed strategy. [2] pointed out that there is no critical result in the minimax theorem without no theory of games can be said to exist. He proved this true under general conditions and in addition he created the conceptual rich theory of games with more than two players.

According to [5] a scientist with limited mathematical teaching could absorb the motivation, the reasoning and conclusion of theory judging by the attention given to it in non-mathematical journals as well as in the mathematical ones. Presumably, the recent war was an important contributing factor to the later rapid development of the theory in the areas such as logistic, submarine search, air defence etc.

2   Motivation

Although, recent advances have been made by various authors in the area of games theory and in particular to security but it is imperative to properly examine games theory with respect to crime investigation of suspects using Delta State Police Divisional Head-quarters Abraka as a case study due to the high level of insecurity in our Nation as well as the world at large and to suggest current trend to tackling these insecurity challenges.

3   Related Works

In 1950, [6] an American mathematician came up with the Nash theory which stated "a situation in which a player will continue with their chosen strategy, having no incentive to deviate from it, after taking into consideration the opponent's strategy (GAMES THEORY-NASH EQULIBRIUM AND ITS APPLICATION 2015b). Games theory booms after this. One of the greatest developments in social science was the creation of non-cooperative game theory by [6].

A British mathematician, [7] proved that game theory is also used in the evaluation of the species. They wrote the book "Game theory and the Evaluation of Fighting".

In her book, "Elementyary Operations Research with Application" [8] said Game Theory is the study of the ways in which strategic interactions among rational players produce outcomes with respect to the preferences (or utilities) of those players, none of which might have been intended by any of them.

Many people desired to live and operate in a secured environment, they want to be sure that their lives and that of their loved ones are secured. The main duty of any government is to secure lives and property [9]. Nigeria is currently having serious security challenges ranging from Boko haram attacks in the North-East to armed robbery and kidnapping in other parts of the country. Data mining has gained recognition in crime analysis [10]. The ability to keep goods in a wharehouse to make it available for sale or future use is called stocking. Control is a process by which events are made to conform to a plan. Demand is dynamic and hence the pertinence to keep goods in stock.

The act of maintaining stock has its associated costs likewise the act of not keeping stock. The latter make the manufacturer\distributors loose customers [11].

Evolutionary game theory is used to study players who adapt their strategies over a period of time to suit rules that need not be rational or forward-looking [12] in "Evolutionary game theory": A renaissance Games. In general, the development of strategies over time based on such rules follows a Markov Chain Model with various conditions as the present strategy profile or how the game has been played recently.

In its simplest form, a signaling game has two players, one of whom has better access to data than the other. It is the player with the improved data that makes the first move. For example, [13] designed a pattern with one supplier and one fabrication. The producer had a better demand prediction than the supplier that the latter must build his capacity for a key component to the processed product.

In screening games, a player lacks more information about the other, for example, the supplier, who is offering a contract. There are a variety of screening games with promises by suppliers, as described by [14]. In fact, the agreement is selected via a list of options from the supplier to get the attention of the manufacturer. In economic literature, the supplier pays a charge for designing a structure to gain information about the manufacture, which is why the design structure is mentioned . The screening game by [14], which closely resemble this one, is one example.

According to [15], games involving two players are primarily used to model project management situations. For example, an investor usually has several investment choices and each choice will probably give rise to a different project. Therefore, an investment choice has to be made before coming up with the project charter.

4   Material and Method

This paper uses the Prisoner's dilemma method designed by [16], a situation where two suspects apprehended for a crime are held in separate rooms and cannot communicate with each other and the prosecutor inform both suspect 1 and suspect 2 individually that if he confesses and testifies against the other, he can go free, but if he does not co-operate and the other does , he will be sentenced to three years in prison. If both confess, they will get a two-year sentence, and if neither confesses, they will be sentenced to one year in prison. Although co-operation is the best strategy for both suspects, research shows that most rational people prefer to confess and testify against the other person than stay silent and take the chance the other party confesses. With the knowledge of the prisoner's dilemma in mind, the investigator uses such technique to obtain true value from suspect arrested for various crimes committed.

In order to properly examine the research problem effectively, it is necessary that the quantitative method be employed since it is best for measuring ranking, categorizing, identifying patterns and making generalizations which is a standard methodology in mathematical science. Data Collection that aided this thesis were sourced through systematic sampling from previously existing works from authors who have distinguished themselves under review i.e. Game Theory. These data were gathered from publications or archive texts, Google scholar etc.

Before analysis, the gathered data was prepared. The data set was checked by transforming variables since it is decision making process and was analysed using Lab Tech software.

5   Two-Person, Zero-Sum Games

A simple way of showing a game situation is to display the anticipated pay off in matrix form. The quantities gained or lost are looked at from the point of view of one of the player rather than from a neutral point of view. Thus positive entries represent gains of player I and negative entries gains of player II. In this type of game, player I's losses are player II gains and vice versa. A game of this sort with 2 players is known as a two-person Zero-sum game, the sum of gains and losses being zero.

Table 5.1: Matrix of Pay-offs

Player I/Player II	B_1	B_2	...	B_n
A_1	g_{11}	g_{12}	...	g_{1n}
A_2	g_{21}	g_{22}	...	g_{2n}
$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
A_n	g_{n_1}	g_{n_2}	...	g_{nn}

Table 5.1, which gives the amount g_{ij} won by player I from player II. When player I plays his ith pure strategy , A_i and player II plays his jth pure strategy, B_j. Thus, the matrix of pay offs (game) to player I is the positive entries while the matrix of the pay-offs (game) of player II is the negative entries of the matrix below.

We examined real life application of games theory to the interrogation and crime detection in combating insecurity in our society using the Delta state Police Divisional Head-quarters, Abraka, as a case study. We examined how the investigator uses the skills of interviewing, questioning, and interrogation of suspects as information gathering techniques police used to aid investigation. They are aimed at the goal discovering the truth, not just getting confession to a crime. This is a contrast to earlier times of policing, when techniques called the "third degree" techniques were identified in the united States by the Wickersham commision in 1931, as being unlawful police practices that caused false confessions and miscarriages of justice, where suspects were sometimes wrongfully convicted and imprisoned.

However, two tables were examined. Table I gives an illustration of the investigation carried out at the Delta State Police Divisional Head-quaters, Abraka. In this case, the techniques of games theory was not applied. Next, the algebra of the game in Table 1 were obtained and the outcome determined. A similar approach is employed in Table II(where Games Theory was applied) and therefore make comparison from the outcome geenrated and then make our conclusion.

6 The Algebra of the Game

Table 1: **Analysis of suspects arrested for various crime according to the Annual Report released by the Delta State Police Divisional Head-quarters, Abraka in 2019**

Months	No of Arrest on Suspect $(A_1 - A_3)$	Theft	Kidnapping	Crime Electoral Violemce	Others
Jan- April	9	3	2	3	1
May- Aug	11	2	3	4	2
Sept- Dec	13	6	4	-	3

Table 2: **Outcome of investigation after interviewing and interrogation of suspects.**

Player I	Player II		
	B_1(Yes)	B_2(No)	B_3(Under Investigation)
A_1	1	3	5
A_2	4	2	5
A_3	4	3	6

Saddle point $\rightarrow$ Maximin = Minimax

$x_1 + 4x_2 + 4x_3 \geq g$

$3x_1 + 2x_2 + 3x_3 \geq g$

Table 3: **Pay-off matrix**

Player I	Player II			
	B_1(Yes)	B_2(No)	Row Minima	Maxima
A_1	1	3	1	
A_2	4	2	2	
A_3	4	3	3	3
Column Maxima	4	3		
Minimax		3		

Recall that $x_1 + x_2 + x_3 = 1$.(expectation of two person non-zero sum games)
Similarly, taking inequalities accross row, we have;

$$
\begin{aligned}
y_1 + 3y_2 &\leq g \\
4y_1 + 2y_2 &\leq g \\
4y_1 + 3y_2 &\leq g
\end{aligned}
$$

Recall that $y_1 + y_2 = 1$(expectation of two person non-zero sum games)

$$
\begin{aligned}
y_2 &= 1 - y_1 \\
y_1 + 3y_2 &\leq g \\
y_1 + 3 - 3y_2 &\leq g \\
3 - 2y_1 &\leq g
\end{aligned}
$$

$$
\begin{aligned}
4y_1 + 2(1 - y_1) &\leq g \\
4y_1 + 2 - 2y_1 &\leq g \\
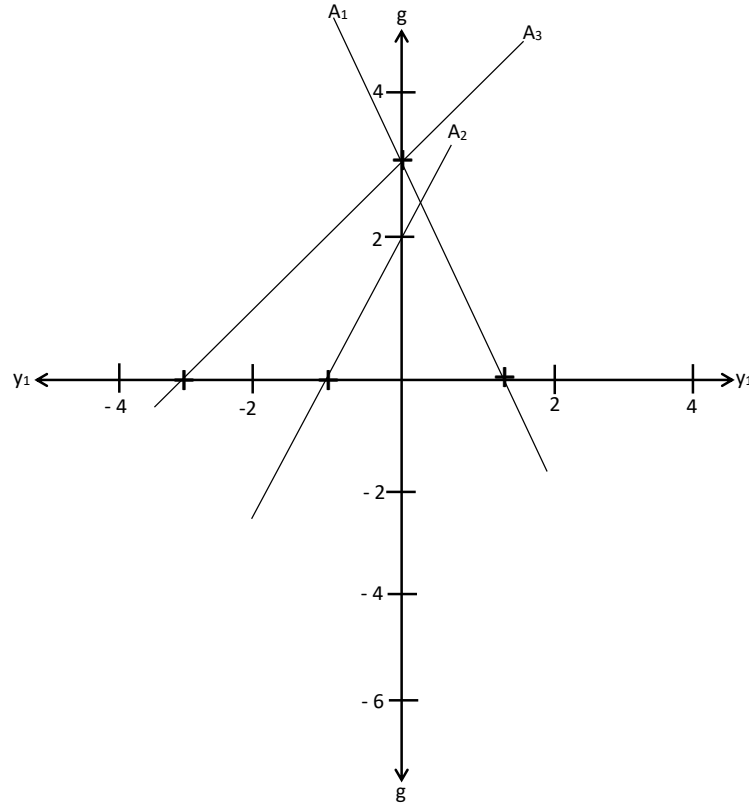2 + 4y_1 - 2y_1 &\leq g \\
2 + 2y_1 &\leq g
\end{aligned}
$$

$$
\begin{aligned}
4y_1 + 3y_2 &\leq g \\
4y_1 + 3(1 - y_1) &\leq g \\
4y_1 + 3 - 3y_1 &\leq g \\
3 + 4y_1 - 3y_1 &\leq g \\
3 + y_1 &\leq g
\end{aligned}
$$

A_1 : If $y_1 = 0$, $g = 3$. If $g = 0$, $y_1 = \frac{-3}{-2} = \frac{3}{2}$.

A_2 : If $y_1 = 0$, $g = 2$. If $g = 0$, $y_1 = \frac{-2}{2} = -1$.

A_3 : If $y_1 = 0$, $g = 3$. If $g = 0$, $y_1 = -3$

$$
\begin{aligned}
x_1 + 4x_2 + 4x_3 &\geq g \\
3x_1 + 2x_2 + 3x_3 &\geq g
\end{aligned}
$$

Graph 1: Arrested Suspects using Games Theory

Recall that $x_1 + x_2 + x_3 = 1$ (Expectation of two person non-zero sum games).
Taking $g = 3$ and $x_2 = 0$, we have;

$$
\begin{aligned}
x_1 + 4x_3 &= 3 \text{......................}(1) \times 3 \\
3x_1 + 3x_3 &= 3 \text{.....................}(2) \times 1 \\
3x_1 + 12x_3 &= 9 \text{.....................}(3) \\
3x_1 + 3x_3 &= 3 \text{......................}(4) \\
(3) - (4) : 9x_3 &= 6
\end{aligned}
$$

So, $x_3 = \frac{6}{9} = \frac{2}{3}$. Now, substitute $x_3 = \frac{2}{3}$ into (4) to get

$$3x_1 + 3x_3 = 3 \implies$$
$$3x_1 + 3(\frac{2}{3}) = 3 \implies$$
$$3x_1 + 2 = 3 \implies$$
$$3x_1 = 3 - 2 \implies$$
$$3x_1 = 1 \implies$$
$$x_1 = \frac{1}{3}.$$

7   Discussion of Results

Using Table 3, we observe that the payoff matrix coincidentally possesses a saddle point at 3 (i.e. Maximin=Minimax). Player A will try to minimize his greatest loss while trying to obtain the truth value from arrested suspects during interrogation. He can only do this by choosing between strategy A_1 and A_3. The biggest loss that can befall him if he chooses A_1 is 1, which occurs when B chooses strategy B_1, implying a decrease in valid confession from suspects during detention. If A chooses strategy A_3, his greatest loss will be 3 which occurs when player B chooses strategy B_2 which indirectly implies an increase in valid confession from suspects. In order to minimize losses during interrogation, player A will choose strategy A_3 , thus limiting his greatest loss to 3 which is the saddle point (i.e. Maximin = Minimax).

Using mixture of strategies but playing strategies A_1 (interview and questioning), $\frac{1}{3}th$ of the time and A3 (interrogation), $\frac{2}{5}th$ of the time, we obtain the best combination of strategy, in which the investigator achieves the task of obtaining the truth from suspects.

Since the investigator operated the strategies randomly, and did achieve the set task, it is therefore expected that it uses the techniques subsequently in other to achieve the desired result.

8   Conclusion

Notice that from Table 3, there is systematic increase in suspects making positive confessions to the various crimes as earlier stated. This as a result of effective use of the techniques of interview, questioning and interrogation thereby implying the truth value was obtained from suspects and decisive sanctions were taken in each case.

Finally, based on the circumstantial evidence provided and prior conclusion from either cases, we conclude that employing the technique of Game theory is an effective way of obtaining truth value from suspects, administering justice speedily and a strong avenue for prison decongestion due to the alarming conditions in most a Nigerian prisons.

This research contributes immensely to knowledge in the following areas of study.

1. It enables owners of firms to make decision in a competitive environment through formulation of proper frame work and analyzing such problems in other to get a larger share of the market and more profits.

2. It enables the security outfits adopt strategies and take decisive steps to checkmate, eradicate to the barest minimum all forms of social vices which tend to disturb internal peace with the country.

3. It provides government with an effective structural frame work through the formulation and implementation of citizenship-oriented policy so as to meet the needs of the masses.

Based on the findings so far, we make a recommendation that the various security arms especially the Police should be made to undergo a continuous compulsory refresher course where they will be exposed to the modern trend of policing.

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