IJMSO

International Journal of Mathematical Sciences and Optimization: Theory and Applications
10(1), 2024, Pages 1 - 11
https://doi.org/10.5281/zenodo.10385042

Possibilistic Linear Programming Problem involving Multi-choice Parameters

S. K. Barik [1*]

Department of Mathematics, Shree Jagannath Mahavidylaya, Naugaonhat, Jagatsinghpur, Odisha, India.
* Corresponding author: skb.math@gmail.com

Article Info

Received: 10 October 2023 Revised: 30 November 2023
Accepted: 30 November 2023 Available online: 03 January 2024

Abstract

In this study, a new solution method is proposed for the possibilistic linear programming problem. The right-hand side parameters of the constraint are considered to be multi-choice. The cost coefficient of the objective function is considered to be triangular possibility distribution. In this model, the possibility is characterized by the triangular possibility distribution whereas the multi-choice is handled by the linear combination of binary variable technique. In order to solve the proposed model, a crisp equivalent deterministic multi-objective mixed-integer linear programming problem is established. Then, solve the model using the fuzzy programming approach. Finally, an example numerical model is provided to test the methodology and solution procedure.

Keywords: Possibilistic linear programming problem, Multi-objective mixed-integer linear programming problem, Triangular possibility distribution, Multi-choice parameters, Binary variables, Fuzzy programming approach.
MSC2010: 49M37.

1 Introduction

Real-world input data or parameters of decision making problems are sometimes imprecise or erroneous due to missing or incomplete information. Traditional mathematical programming cannot tackle all problems those have imprecision. Possibilistic mathematical programming problems have provided the platform to deal with such types of real world decision making problems. Theory of possibility includes integration of fuzzy preferences. Initially, Zadeh [1] demonstrated the importance of the theory of possibility, which is connected to the theory of fuzzy sets, by defining a possibility distribution as a fuzzy restriction. It was further developed by Dubois and Prade [2] to play the same roles as the probability distributions in probability theory. Furthermore, various researchers have worked in this direction towards the development of possibilistic mathematical programming problems as follows:

Luhandjula [3] developed a linear program problem where the coefficients of the objective function are imprecision with possibility distributions. Buckley [4] suggested a mathematical programming problem with all the parameters may be fuzzy variables specified by their possibility distribution. Lia and Hwang [5] proposed an auxiliary multiple objective linear programming model to solve a linear programming problem with imprecise objective and/or constraint coefficients. Inuiguchi and Sakawa [6] presented an equivalent condition between a possibilistic linear programming problem with a quadratic membership function and a stochastic linear programming problem with a multivariate normal distribution. Wang and Liang [7] presented a novel interactive possibilistic linear programming approach for solving the multi-product aggregate production planning problem with imprecise forecast demand, related operating costs, and capacity. An interactive possibilistic linear programming model established by Liang [8] for solving multi-objective distribution planning decision problems involving imprecise available supply, forecast demand and unit cost/time coefficients with triangular possibility distributions. Vasant et al. [9] suggested a new method to obtain optimal solution using satisfactory approach in uncertain environment and obtained the optimal solution by using possibilistic linear programming approach and MATLAB software package. Phruksaphanrat [10] proposed a preemptive possibilistic linear programming approach for solving multiobjective Aggregate Production Planning problem with interval demand and imprecise unit price and related operating costs and also attempts to maximize profit and minimize changes of workforce. Kabak and Ülengin [11] developed a possibilistic linear programming model to maximize the total profit of the enterprise and to make strategic resource-planning decisions using fuzzy demand forecasts and fuzzy yield rates and other inputs such as costs and capacities. Bouzembrak et al. [12] presented a possibilistic linear programming model for supply chain network design with imprecise inputs such as market demands, supplied quantities, transportation costs, opening costs, treatment and storage costs are modelled as fuzzy numbers. A new methodology and solution procedure for solving possibilistic linear programming with trapezoidal fuzzy numbers is developed by Wan and Dong [13] and this method is used to capture imprecise or uncertain information for the imprecise objective coefficients and/or the imprecise technological coefficients and/or available resources. A two-phase possibilistic linear programming approach and a fuzzy analytical hierarchical process approach have been developed by Ozgen and Gulsun [14] to optimize two objective functions (minimum cost and maximum qualitative factors benefit) in a four-stage (suppliers, plants, distribution centers, customers) supply chain network within a imprecise optimization framework. Chopra and Saxsena [15] is presented a possibilistic linear programming problem involving multiple objectives functions. A new solution procedure of possibilistic linear programming problem is suggested by Barik and Biswal [16] involving the right hand side parameters of the constraints follows normal distribution and the objective function coefficients as triangular possibility distribution. Gupta et al. [17] presented a weighted possibilistic programming approach to solve the multi-objective multi-item vendor selection-order allocation problem with price-breaks that integrates fuzzy multi-objective integer linear programming and analytic hierarchy process techniques. Hamidieh et al. [18] proposed a new robust possibilistic programming approach model for designing a sustainable closed-loop single-product multi-component multi-level logistics network under uncertainty conditions. Fazli-Khalaf et al. [19] suggested a robust possibilistic programming model for water distribution network design that maximizes the total profit of water distribution as well as maximizing priority of water transferring among water customer zones. Sutthibutr2020 and [20] presented an improved fuzzy programming approach to optimism multi-objective aggregate production planning problem under uncertain environments which integrates the concept of possibilistic linear programming with Beta-Skewness Degree that decision-makers can manipulate the best level of data fuzziness as well as maintain such fuzziness in the optimization process.

The aim of the work is to incorporating triangular possibility distributions in the cost coefficient of the objective functions considering the right hand side parameters of the constraints as multi-choice in nature. Crisp equivalent deterministic model is established and solved by using well known fuzzy programming method.

International Journal of Mathematical Sciences and Optimization: Theory and Applications
10(1), 2024, Pages 1 - 11
https://doi.org/10.5281/zenodo.10385042

2 Mathematical Formulation

Mathematically, a possibilistic linear programming problem with multi-choice parameters can be stated as:

$$\max : z = \sum_{j=1}^{n} \tilde{c}_j x_j \tag{2.1}$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, i = 1, 2, \ldots, m \tag{2.2}$$

$$x_j \geq 0, j = 1, 2, \ldots, n \tag{2.3}$$

where the decision variables $x_j, j = 1, 2, \ldots, n$ and $a_{ij}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$ are assumed to be deterministic. Also, it is assumed that $\tilde{c}_j = (c_j^m, c_j^p, c_j^o), j = 1, 2, \ldots, n$ are imprecise with triangular possibility distributions as shown in Figure 1 where c_j^m is the most possible value (possibility $= 1$ if normalized), c_j^p (the most pessimistic value), and c_j^o (the most optimistic value) are the least possible values. Only $b_i = \{b_i^1, b_i^2, \ldots, b_i^{p_i}\}, i = 1, 2, \ldots, m$ are multi-choice in nature i.e. more than one choice has been assigned for the parameter b_i.

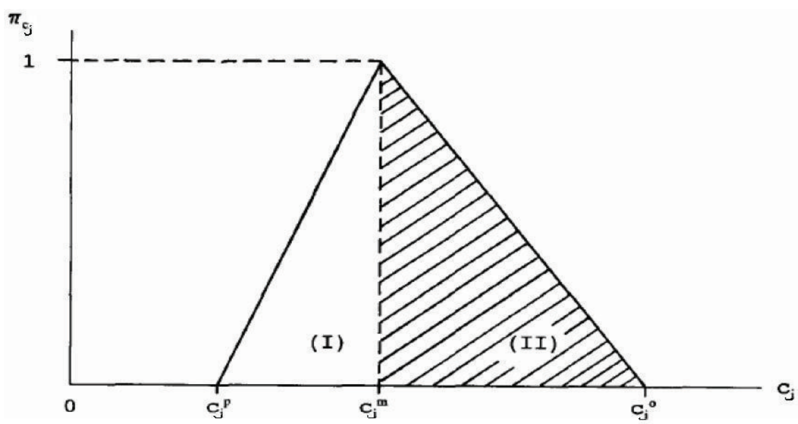

Fig. 1: The triangular possibility distribution π_{c_j} of $\tilde{c}_j$

3 Methodology and Deterministic Models

This Section includes the procedure to find the crisp equivalent deterministic model of the proposed possibilistic linear programming problem and the method to remove the multi-choiceness from the constraints right hand side parameters of the problem.

3.1 When $\tilde{c}_j, j = 1, 2, \ldots, n$ are imprecise with triangular possibility distributions

Since objective coefficients $\tilde{c}_j, j = 1, 2, \ldots, n$ are imprecise with triangular possibility distributions π_{c_j}. Then the objective function of the proposed model can be stated as:

$$\max : z = \sum_{j=1}^{n} (c_j^m x_j, c_j^p x_j, c_j^o x_j) \tag{3.1}$$

or

$$\max : z = ((c^m)^T x, (c^p)^T x, (c^o)^T x) \tag{3.2}$$

where $x = (x_1, x_2, \ldots, x_n)$, $c^m = (c_1^m, c_2^m, \ldots, c_n^m)^T$, $c^p = (c_1^p, c_2^p, \ldots, c_n^p)^T$ and $c^o = (c_1^o, c_2^o, \ldots, c_n^o)^T$.

International Journal of Mathematical Sciences and Optimization: Theory and Applications
10(1), 2024, Pages 1 - 11
https://doi.org/10.5281/zenodo.10385042

Thus, the objective function in equation (3.2) is an imprecise objective function with a triangular possibility distribution. Geometrically, this fuzzy objective is actually defined by three corner points $((c^m)^T x, 1)$, $((c^p)^T x, 0)$ and $((c^o)^T x, 0)$ of the triangle shown in Figure 1. Thus, maximizing the fuzzy objective can be obtained by pushing these three coordinates of the triangle in the right-hand side direction. Since, these vertical coordinates are fixed at either 1 or 0. Then only we consider the three horizontal coordinates $((c^m)^T x, 0)$, $((c^p)^T x, 0)$ and $((c^o)^T x, 0)$, respectively. Therefore, the proposed model with objective function is solved as:

$$\max : z = ((c^m)^T x, (c^p)^T x, (c^o)^T x) \tag{3.3}$$

where $((c^m)^T x, (c^p)^T x, (c^o)^T x)$ is the vector of three objective functions, $(c^m)^T x$, $(c^p)^T x$, and $(c^o)^T x$. To keep the normal shape of the possibility distribution as triangular, we have to make a little change as shown in Figure 2. In order to solve the proposed model with three objective functions as in equation (3.3), maximize $(c^m)^T x$, minimize $[(c^m)^T x - (c^p)^T x]$ and maximize $[(c^o)^T x - (c^m)^T x]$ separately each instead of maximizing these three objectives simultaneously, where the last two objective functions are actually relative measures from $(c^m)^T x$, the first objective function as shown in Figure 2 as follows:

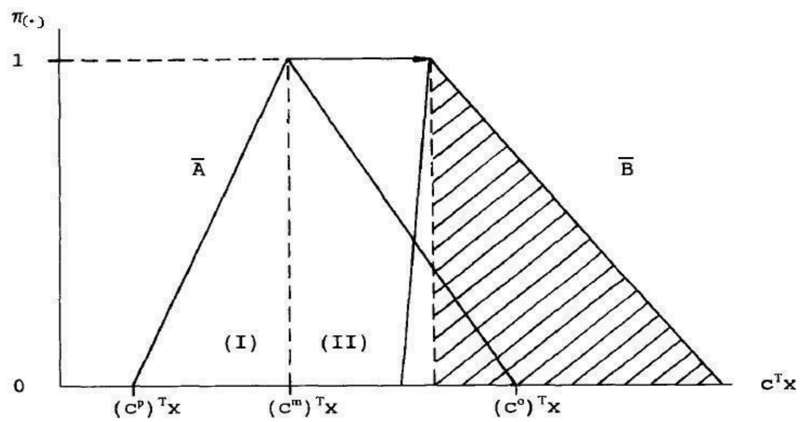

Fig. 2: The strategy to solve "$\max \tilde{c}^T x$"

The three new objectives also pushing the triangular possibility distribution in the direction of the right-hand side. This results the following crisp multi-objective linear programming problem with multi-choice parameters from (2.1)-(2.3) as:

$$\min : z_1 = (c^m - c^p)^T x \tag{3.4}$$

$$\max : z_2 = (c^m)^T x \tag{3.5}$$

$$\max : z_3 = (c^o - c^m)^T x \tag{3.6}$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, i = 1, 2, \ldots, m \tag{3.7}$$

$$x_j \geq 0, j = 1, 2, \ldots, n \tag{3.8}$$

The above crisp multi-objective linear programming problem with multi-choice parameters $b_i, i = 1, 2, \ldots, m$ (3.4)-(3.8) is equivalent to maximizing the most possible value of the imprecise profit (at the point of possibility degree = 1). At the same time, minimize the inferior side of the possibility distribution. It means minimizing the region (I) which is equivalent to "the risk of obtaining lower profit". And, also maximize the region (II) of the possibility distribution, which is equivalent to "the possibility of obtaining higher profit". As shown in Figure 2, prefer the possibility distribution of $\bar{B}$ to that of $\bar{A}$.

International Journal of Mathematical Sciences and Optimization: Theory and Applications
10(1), 2024, Pages 1 - 11
https://doi.org/10.5281/zenodo.10385042

3.2 When $b_1, b_2, ..., b_m$ are multi-choice parameter

According to Khalil et al. [21], the new technique to handle multi-choiceness as follows:

Let $b_i^1, b_i^2, \cdots, b_i^{p_i}$ are p_i number of choices for the right hand side parameter $b_i, i = 1, 2, 3, \cdots, m$ of the i-th constraint (3.7).

Set P= Least common multiple of the number $\{p_1, p_2, ..., p_m\}$ and choose just one value among the p_i number of choices. For this, enclosing P number of binary variables $w_1, w_2, \ldots, w_P$ to construct a set of $s_i = \frac{P}{p_i}$ linear combinations in the following manner as:

$$T_i^1 = b_i^1 w_1 + b_i^2 w_2 + \ldots + b_i^{p_i} w_{p_i} = \sum_{r=1}^{p_i} b_i^r w_r \tag{3.9}$$

$$T_i^2 = b_i^1 w_{p_{i+1}} + b_i^2 w_{p_{i+2}} + \ldots + b_i^{p_i} w_{2p_i} = \sum_{r=1}^{p_i} b_i^r w_{p_i+r} \tag{3.10}$$

$$T_i^3 = b_i^1 w_{2p_{i+1}} + b_i^2 w_{2p_{i+2}} + \ldots + b_i^{p_i} w_{3p_i} = \sum_{r=1}^{p_i} b_i^r w_{2p_i+r} \tag{3.11}$$

$$\vdots$$

$$T_i^{s_i} = b_i^1 w_{(s_i-1)p_{i+1}} + b_i^2 w_{(s_i-1)p_{i+2}} + \ldots + b_i^{p_i} w_{(s_i-1)p_i} = \sum_{r=1}^{p_i} b_i^r w_{(s_i-1)p_i+r} \tag{3.12}$$

Finally, the right hand side of the i-th constraint (3.7) by mathematical expression as

$$\mathbb{T}_i = \sum_{j=1}^{s_i} T_i^j = \sum_{j=1}^{s_i} \left[\sum_{r=1}^{p_i} b_i^r w_{(j-1)p_i+r} \right] \tag{3.13}$$

Thus, the equivalent deterministic model of the crisp multi-objective linear programming problem (3.4)-(3.8) can be stated as:

$$\min : z_1 = (c^m - c^p)^T x \tag{3.14}$$

$$\max : z_2 = (c^m)^T x \tag{3.15}$$

$$\max : z_3 = (c^o - c^m)^T x \tag{3.16}$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq \mathbb{T}_i, i = 1, 2, 3, \ldots, m \tag{3.17}$$

$$\mathbb{T}_i = \sum_{j=1}^{s_i} \left[\sum_{r=1}^{p_i} b_i^r w_{(j-1)p_i+r} \right] \tag{3.18}$$

$$w_u = 0/1, u = 1, 2, 3, \ldots, P \tag{3.19}$$

$$\sum_{u=1}^{P} w_u = 1 \tag{3.20}$$

$$x_j \geq 0, j = 1, 2, \ldots, n \tag{3.21}$$

Since the above model is a multi-objective mixed-integer linear programming problem, use any multi-objective optimization9 technique by [22] such as utility theory, goal programming, or fuzzy programming approaches. In this paper, Zimmermann's fuzzy programming method [23] is used to solve the crisp equivalent multi-objective model. The solution procedure is discussed in the following Section.

IJMSO

International Journal of Mathematical Sciences and Optimization: Theory and Applications
10(1), 2024, Pages 1 - 11
https://doi.org/10.5281/zenodo.10385042

4 Fuzzy Programming Approach

The concept of fuzzy set theory was first introduced by Bellman and Zadeh [24]. Later on Zimmermann [25] used fuzzy set theory with suitable choice of membership function and developed a fuzzy linear programming problem, which is known as maximin problem. He proves that the solutions obtained by fuzzy linear programming technique are more accurate and efficient. Biswal [26] has presented fuzzy programming technique to solve a multi-objective geometric programming problem by introducing a new type of membership function. Hulsurkar [27] presented fuzzy programming technique to solve a multi-objective stochastic linear programming problems. In this study, the equivalent deterministic model (3.14)-(3.21) is a multi-objective mixed-integer linear programming problem. So fuzzy programming technique is applied to find solution of the proposed model.

4.1 Solution Procedures

The steps of fuzzy programming approach is presented below:

Step-1: Select the first objective function (i.e. z_1)and solve it as a single objective linear programming problem subject to the constraints. Find the Positive Ideal Solutions (PIS) and Negative Ideal Solutions (NIS) of the first objective function (i.e. z_1). Similarly, select the second objective function (i.e. z_2) and third objective function (i.e. z_3) and solve it as a single objective linear programming problems subject to the constraints. Find the Positive Ideal Solutions (PIS) and Negative Ideal Solutions (NIS) for both the objective functions (i.e. z_2, z_3).

Step-2: The Positive Ideal Solutions (PIS) and Negative Ideal Solutions (NIS) of the three objective functions (Hwang and Yoon [28]) are as follows:

$z_1^{PIS} = \min : (c^m - c^p)^T x$, $z_1^{NIS} = \max : (c^m - c^p)^T x$

$z_2^{PIS} = \max : (c^m)^T x$, $z_2^{NIS} = \min : (c^m)^T x$

$z_3^{PIS} = \max : (c^o - c^m)^T x$, $z_3^{NIS} = \min : (c^o - c^m)^T x$

Step-3: Form the linear membership functions of these three objective functions using Positive Ideal Solutions (PIS) and Negative Ideal Solutions (NIS) from Step 2 as below:

For the first objective function (z_1):

$$\mu_{z_1} = \begin{cases} 1, & if\ z_1 < z_1^{PIS} \\ \frac{z_1^{NIS} - z_1}{z_1^{NIS} - z_1^{PIS}}, & if\ z_1^{PIS} \leq z_1 \leq z_1^{NIS} \\ 0, & if\ z_1 > z_1^{NIS} \end{cases} \tag{4.1}$$

For the second objective function (z_2):

$$\mu_{z_2} = \begin{cases} 1, & if\ z_2 > z_2^{PIS} \\ \frac{z_2 - z_2^{NIS}}{z_2^{PIS} - z_2^{NIS}}, & if\ z_2^{NIS} \leq z_2 \leq z_2^{PIS} \\ 0, & if\ z_2 < z_2^{NIS} \end{cases} \tag{4.2}$$

For the third objective function (z_3):

$$\mu_{z_3} = \begin{cases} 1, & if\ z_3 > z_3^{PIS} \\ \frac{z_3 - z_3^{NIS}}{z_3^{PIS} - z_3^{NIS}}, & if\ z_3^{NIS} \leq z_3 \leq z_3^{PIS} \\ 0, & if\ z_3 < z_3^{NIS} \end{cases} \tag{4.3}$$

International Journal of Mathematical Sciences and Optimization: Theory and Applications
10(1), 2024, Pages 1 - 11
https://doi.org/10.5281/zenodo.10385042

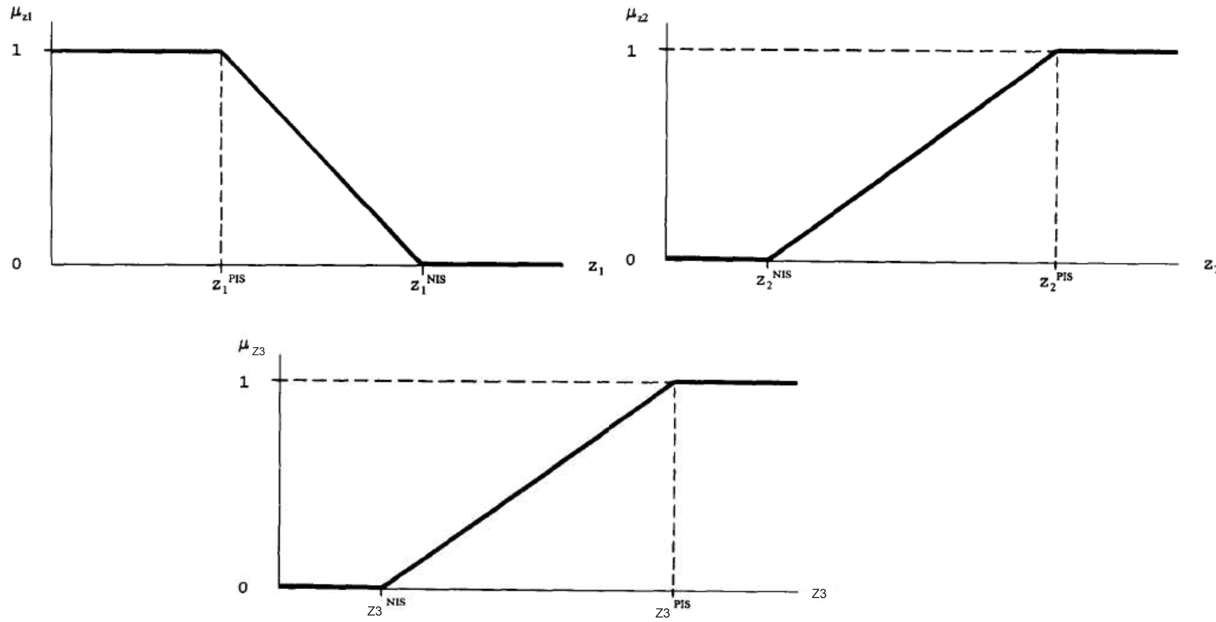

Fig. 3: The membership functions of the three objective functions "z_1, z_2, and z_3"

Step-4: Use max-min operator with an augmented variable λ and formulate a single objective crisp linear programming problem as:

$$\max : \lambda \tag{4.4}$$

subject to

$$\lambda \leq \mu_{z_l}(X),\ l = 1, 2, 3 \tag{4.5}$$

$$\sum_{j=1}^{n} a_{ij} x_j \leq \mathbb{T}_i, i = 1, 2, 3, \ldots, m \tag{4.6}$$

$$\mathbb{T}_i = \sum_{j=1}^{s_i} \left[\sum_{r=1}^{p_i} b_i^r w_{(j-1)p_i + r} \right] \tag{4.7}$$

$$w_u = 0/1, u = 1, 2, 3, \ldots, P \tag{4.8}$$

$$\sum_{u=1}^{P} w_u = 1 \tag{4.9}$$

$$0 \leq \lambda \leq 1, x_j \geq 0,\ j = 1, 2, 3, \ldots, n \tag{4.10}$$

where augmented variable λ represents the overall satisfaction level under the proposed strategy of maximizing the most possible value.

Step-5: Solve the crisp model by using a linear programming algorithm to find an optimal compromise solution x^*. Then evaluate all the objective functions at the optimal compromise solution x^*.

5 Numerical Example

In this Section, consider a mathematical model of possibilistic linear programming with right hand side parameters as multi-choice in nature as:

$$\max z = \tilde{8}x_1 + \tilde{5}x_2 + \tilde{6}x_3 \tag{5.1}$$

Subject to

$$3x_1 + 5x_2 + 8x_3 \leq b_1 \tag{5.2}$$

$$3x_1 + 4x_2 + 3x_3 \leq b_2 \tag{5.3}$$

$$5x_1 + 7x_3 + +8x_3 \geq 34 \tag{5.4}$$

$$x_j \geq 0, j = 1, 2, 3 \tag{5.5}$$

where assume that the coefficients of the objective function $\tilde{8} = (8, 7.5, 9.4)$, $\tilde{5} = (5, 4.6, 6.5)$ and $\tilde{6} = (6, 5.4, 7.2)$ are imprecise number and have a triangular possibility distribution with most possible values 8, 5, 6; most pessimistic values 7.5, 4.6, 5.4; most optimistic values 9.4, 6.5, 7.2, respectively. Also, assume that the right-hand side parameters $b_i, i = 1, 2$ are multi-choice in nature as $b_1 = (18, 20, 22, 25)$, $b_2 = (14, 16, 18)$ respectively.

Now, the deterministic crisp multi-objective mixed-integer linear programming problem of the above mathematical programming model (5.1)-(5.5) can be transformed using linear combination technique to handling multi-choiceness as:

$$\min z_1^* = 0.5x_1 + 0.4x_2 + 0.6x_3 \tag{5.6}$$

$$\max z_2^* = 8x_1 + 5x_2 + 6x_3 \tag{5.7}$$

$$\max z_3^* = 1.4x_1 + 1.5x_2 + 1.2x_3 \tag{5.8}$$

Subject to

$$3x_1 + 5x_2 + 8x_3 \leq \mathbb{T}_1 \tag{5.9}$$

$$3x_1 + 4x_2 + 3x_3 \leq \mathbb{T}_2 \tag{5.10}$$

$$\mathbb{T}_1 = 18w_1 + 20w_2 + 22w_3 + 25w_4 + 18w_5 + 20w_6 + 22w_7 + 25w_8 + 18w_9 + 20w_{10} + 22w_{11} + 25w_{12} \tag{5.11}$$

$$\mathbb{T}_2 = 14w_1 + 16w_2 + 18w_3 + 14w_4 + 16w_5 + 18w_6 + 14w_7 + 16w_8 + 18w_9 + 14w_{10} + 16w_{11} + 18w_{12} \tag{5.12}$$

$$5x_1 + 7x_3 + +8x_3 \geq 34 \tag{5.13}$$

$$\sum_{u=1}^{12} w_u = 1 \tag{5.14}$$

$$w_u = 0/1, x_j \geq 0, u = 1, 2, \ldots, 12, j = 1, 2, 3 \tag{5.15}$$

Further, by using the solution procedure of fuzzy programming method as discussed in Section 4, the above multi-objective programming problem can be transformed into a single objective mixed-integer linear programming problem as:

$$\max : \lambda \tag{5.16}$$

Subject to

$$\mu_{z_1^*} \geq \lambda \tag{5.17}$$

$$\mu_{z_2^*} \geq \lambda \tag{5.18}$$

$$\mu_{z_3^*} \geq \lambda \tag{5.19}$$

$$3x_1 + 5x_2 + 8x_3 \leq \mathbb{T}_1 \tag{5.20}$$

$$3x_1 + 4x_2 + 3x_3 \leq \mathbb{T}_2 \tag{5.21}$$

$$\mathbb{T}_1 = 18w_1 + 20w_2 + 22w_3 + 25w_4 + 18w_5 + 20w_6 + 22w_7 + 25w_8 + 18w_9 + 20w_{10} + 22w_{11} + 25w_{12} \tag{5.22}$$

$$\mathbb{T}_2 = 14w_1 + 16w_2 + 18w_3 + 14w_4 + 16w_5 + 18w_6 + 14w_7 + 16w_8 + 18w_9 + 14w_{10} + 16w_{11} + 18w_{12} \tag{5.23}$$

$$5x_1 + 7x_3 + +8x_3 \geq 34 \tag{5.24}$$

International Journal of Mathematical Sciences and Optimization: Theory and Applications
10(1), 2024, Pages 1 - 11
https://doi.org/10.5281/zenodo.10385042

$$\sum_{u=1}^{12} w_u = 1 \tag{5.25}$$

$$0 \leq \lambda \leq 1, w_u = 0/1, x_j \geq 0, u = 1, 2, \ldots, 12, j = 1, 2, 3 \tag{5.26}$$

where $\mu_{z_1^*}$, $\mu_{z_2^*}$, and $\mu_{z_3^*}$ are the membership functions defined as:
For the first objective function (z_1^*):

$$\mu_{z_1^*} = \begin{cases} 1, & if\ z_1^* < 2.925 \\ \frac{3.14 - z_1^*}{0.215}, & if\ 2.925 \leq z_1^* \leq 3.14 \\ 0, & if\ z_1^* > 3.14 \end{cases} \tag{5.27}$$

For the second objective function (z_2^*):

$$\mu_{z_2^*} = \begin{cases} 1, & if\ z_2^* > 45.33333 \\ \frac{z_2^* - 41.25}{4.08333}, & if\ 41.25 \leq z_2^* \leq 45.33333 \\ 0, & if\ z_2^* < 41.25 \end{cases} \tag{5.28}$$

For the third objective function (z_3^*):

$$\mu_{z_3^*} = \begin{cases} 1, & if\ z_3^* > 8.133333 \\ \frac{z_3^* - 7.875}{0.258333}, & if\ 7.875 \leq z_3^* \leq 8.133333 \\ 0, & if\ z_3^* < 7.875 \end{cases} \tag{5.29}$$

Hence, the above single objective linear programming problem (5.16)-(5.26) can be rewritten as:

$$\max : \lambda \tag{5.30}$$

Subject to

$$0.5x_1 + 0.4x_2 + 0.6x_3 - 3.14 \leq -0.215\lambda \tag{5.31}$$

$$8x_1 + 5x_2 + 6x_3 - 41.25 \geq 4.08333\lambda \tag{5.32}$$

$$1.4x_1 + 1.5x_2 + 1.2x_3 - 7.875 \geq 0.258333\lambda \tag{5.33}$$

$$3x_1 + 5x_2 + 8x_3 \leq \mathbb{T}_1 \tag{5.34}$$

$$3x_1 + 4x_2 + 3x_3 \leq \mathbb{T}_2 \tag{5.35}$$

$$\mathbb{T}_1 = 18w_1 + 20w_2 + 22w_3 + 25w_4 + 18w_5 + 20w_6 + 22w_7 + 25w_8 + 18w_9 + 20w_{10} + 22w_{11} + 25w_{12} \tag{5.36}$$

$$\mathbb{T}_2 = 14w_1 + 16w_2 + 18w_3 + 14w_4 + 16w_5 + 18w_6 + 14w_7 + 16w_8 + 18w_9 + 14w_{10} + 16w_{11} + 18w_{12} \tag{5.37}$$

$$5x_1 + 7x_3 + +8x_3 \geq 34 \tag{5.38}$$

$$\sum_{u=1}^{12} w_u = 1 \tag{5.39}$$

$$0 \leq \lambda \leq 1, w_u = 0/1, x_j \geq 0, u = 1, 2, \ldots, 12, j = 1, 2, 3 \tag{5.40}$$

6 Results Discussion

The above model (5.30)-(5.40) is solved by LINGO 11.0 [29] and compromise solution is obtain as imprecise has a triangular possibility distribution of $(z_2^*, z_2^* - z_1^*, z_2^* + z_3^*)$ where $z_1^* = 3.03080714$, $z_2^* = 43.323819$ and $z_3^* = 8.0062009$, respectively. Hence, the value of the objective function $(43.323819, 40.29301186, 51.3300199)$ with degree of satisfaction level $\lambda = 0.507874$ along with the decision variables $x_1 = 4.215551$, $x_2 = 0.3690946$, $x_3 = 1.292323$. Further, most possible value of the objective function is 43.323819, most pessimistic value of the objective function is 40.29301186 and most optimistic value of the objective function is 51.3300199.

7 Conclusion

In this paper, a new methodology and solution procedure of possibilistic linear programming problem has been developed by taking the right hand side parameters of the constraints as multi-choice in nature and objective function cost coefficients as triangular possibility distributions. After transferring the possibilistic linear programming problem to an auxiliary crisp multi-objective linear programming problem, the proposed model is solved by using Zimmermann's fuzzy programming to obtain the compromise solution. Moreover, our solution has the nature of minimizing the possibility of obtaining lower value of the objective function, maximizing the most possible value of the objective function and maximizing the possibility of higher value of the objective function. This model may applied in real life decision making problems. One can extended this problem by considering the decision variables as possibility distributions.

8 Acknowledgments

The authors would like to acknowledge Internatonal journal of mathematical analysis and optimization: Theory and Applications and the editors whose comments improved the originality of this manuscripts.

References

[1] Zadeh L. A., (1978), Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets and Systems, 1 3-28.

[2] Dubois, D. and H. Prade, D. (1988) Possibility Theory, Plenum Press, New York.

[3] Luhandjula, M. K, Linear programming with a possibilistic objective function, European Journal of Operations Research 31 (1987) 110-117

[4] Buckley, J.J., Possibilistic linear programming with triangular fuzzy numbers (short communication). Fuzzy Sets and Systems 26 (1988) 135138.

[5] Lai, Y-J and Hwang, C-L (1992), A new approach to some possibilistic linear programming problems, Fuzzy Sets and Systems 49: 121-133.

[6] Inuiguchi, M. and Sakawa, M., A possibilistic linear program is equivalent to a stochastic linear program in a special case, Fuzzy Sets and Systems 76 (1995) 309-317.

[7] Wang R.-C. and Liang T.-F. (2005), Applying possibilistic linear programming to aggregate production planning, International Journal of Production Economics, 98 (3), pp. 328-341.

[8] Liang T-F (2007), Application of possibilistic linear programming to multi-objective distribution planning decisions, Journal of the Chinese Institute of Industrial Engineers, 24: 97-109.

[9] Vasant P.M., Barsoum N.N., and Bhattacharya A. (2008), Possibilistic optimization in planning decision of construction industry, International Journal of Production Economics, 111 (2), pp. 664-675.

[10] Phruksaphanrat B. (2011), Preemptive possibilistic linear programming: Application to aggregate production planning, World Academy of Science, Engineering and Technology, 80, pp. 473-480.

[11] Kabak O. and Ülengin F. (2011), Possibilistic linear-programming approach for supply chain networking decisions European Journal of Operational Research, 209 (3), pp. 253-264.

[12] Bouzembrak Y., Allaoui H., Goncalves G., Bouchriha H., and Baklouti M.: A possibilistic linear programming model for supply chain network design under uncertainty, IMA Journal of Management Mathematics, 24 (2): 209-229 (2013).

[13] Wan S.-P., Dong J.-Y., Possibility linear programming with trapezoidal fuzzy numbers, Volume 38, Issues 56, 1 March 2014, Pages 1660-1672.

[14] Ozgen D. and Gulsun B., 2014, Combining possibilistic linear programming and fuzzy AHP for solving the multi-objective capacitated multi-facility location problem, 268: 185-201.

[15] Chopra R and Saxena R, An Approach to Solve a Possibilistic Linear Programming Problem, Applied Mathematics, Vol. 5 No. 2, 2014, pp. 226-233. doi: 10.4236/am.2014.52024.

[16] Barik S.K., Biswal M.P.: Possibilistic linear programming problems involving normal random variables, International Journal of Fuzzy System Applications, Vol 5, No 3, 1-13(2016).

[17] Gupta, P., Govindan, K., Mehlawat, M.K., A weighted possibilistic programming approach for sustainable vendor selection and order allocation in fuzzy environment. The International Journal of Advanced Manufacturing Technology 86, 17851804 (2016).

[18] Hamidieh, A., Naderi, B., Mohammadi, M., Fazli-Khalaf., A robust possibilistic programming model for a responsive closed loop supply chain network design, Cogent Mathematics (2017), 4: pp. 1-22.

[19] M Fazli-Khalaf, K Fathollahzadeh, A Mollaei, B Naderi, M Mohammadi (2019), A robust possibilistic programming model for water allocation problem, RAIRO-Operations Research 53 (1), 323-338.

[20] Sutthibutr, N. and Chiadamrong, N. (2020), Integrated possibilistic linear programming with beta-skewness degree for a fuzzy multi-objective aggregate production planning problem under uncertain environments, Fuzzy Information and Engineering, 12:3, 355-380.

[21] Khalil T.A., Raghav Y.S., Badra N., 2016, Optimal solution of multi-choice mathematical programming problem using a new technique, American Journal of Operations Research, 2016, 6, 167-172.

[22] Lai, Y.-J., Hwang, C.-L., 1996, Fuzzy Multiple Objective Decision Making-Methods and Applications, Springer-Verlag.

[23] Sinha, S.B., Hulsurkar, Suwarna and Biswal, M.P. (2000), Fuzzy programming approach to multi-objective stochastic programming problems when bi's follow joint normal distribution, Fuzzy Sets and Systems 109: 91-96.

[24] Bellman, R.E., Zadeh, L.A.: Decision-making in a fuzzy environment, Management Science, 17: 141-164 (1970).

[25] Zimmarmann, H. L., 1978, Fuzzy programming and linear programnfing with several objective functions, Fuzzy Sets and systems, 1: 45–55.

[26] Biswal M.P.: Fuzzy programming technique to solve multi- objective geometric programming problems, Fuzzy Sets and Systems, 51: 67-71 (1992).

[27] Hulsurkar, S., Biswal, M. P., and Sinha, S. B., 1997, Fuzzy programming approach to multi-objective stochastic linear programming problems, Fuzzy Sets and Systems 88: 173-181.

[28] Hwang, C. L. and K. Yoon, (1981). Multiple Attribute Decision Making. Berlin: Springer-Verlag.

[29] Schrage,L. (2008), LINGO Release 11.0. LINDO System, Inc.