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ANGULAR LOOP MODEL OF 3 DIMENSIONAL ALM TRANSFORMATION SEMIGROUP

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Abstract

In this paper, we considered the Star-like nildempotency square pyramid of Ibrahim et al. in 2023. In particular, we obtained some more useful result on angular-loop model of the square pyramid namely; any distinct non negative integer X_n and $S \in ALM$ with a disk constant point $\phi \in S$ then S is reducible, spinnable and also form an algebraic transitive relation.

Keywords: Reducible, Spinnable, Star-like nildempotency square, pyramid, Transitive relation, Transformation semigroup, 3 Dimensional.
MSC2010: 20N05.

1 Introduction

Many researchers have studied semigroup theory, but few have worked on 3 dimensional star-like nildempotency transformation semigroup been newly established by Ibrahim et al. [?], the study established some useful result that will be helpful to this research.

Definition 1.1: Semigroup: A semigroup in mathematics is an algebraic structure made up of a set and an associative binary operation. The most common multiplicative notation for the binary operation of a semigroup is $x.y$, or just xy, which represents the outcome of applying the semigroup operation to the ordered pair (x, y). Formally, associativity is defined as $(xy)z = x(yz)$ for any x, y, and$z$ in the semigroup.

Definition 1.2: Transformations: Let X and Y be the two non empty sets such that there is some rule F which assigns to each element $y \in Y$, a unique element $x \in X$, then this rule is said to be a transformation or mapping.

Transformation is used instead of mapping, the letter serves as another name for the former. More information on semigroup of mapping are obtained from the work of Howie [?] and Bhattacharya et al. [?]. The domain and image set of any given transformations $\alpha \in \alpha\omega_n$ was denoted by $D(\alpha)$ and $I(\alpha)$ respectively as used by Malik et al. [?]. The study of Umar [?] showed the combinatorial problems in the theory of symmetric inverse semigroup.

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Adeniji and Makanjuols [?] in their work on "a combinatorial property of full transformation semigroup" obtained results on the collapse of the full transformation semigroup which will be very beneficial in this study. Adeshola [?] worked on a research of combinatorial result for certain semigroups of order-preserving full contraction mappings of a finite chain where some result were also generated.

Akinwunmi et al. [?] worked on "Multiplicative Invertibility Characterization on Star-like Cyclicpoid $C_yPW_n^*$ Finite Partial Transformation Semigroups" with well slated axiomatic properties of star-like semigroup which includes Positivity, identity, inverse, linearity and associativity. Mbah et al. [?] worked on some combinatorial results of collapse in partial transformation semigroups were he used triangular array to obtained his results. [?] studied order preserving and other decreasing mappings, idempotent generated and contraction mapping.

2 Preliminary Notes

The study of Ibrahim et al. [?] exhibit some properties which include star-like faces, star-like edges, star-like vertices and those properties are partial symmetric in nature which coincide with the work of Akinwunmi et al. [?], some relevant result from their work are: Let $Mn = \{m_1, m_2, ..., M_n\}$ be n-element distinct non negative integer and let $MIC_n \in S_n$ such that $MIC_n \subseteq CL_n \subseteq S_n$ then MIC_n has at most one constant elementm, Let $\alpha \in S = \begin{pmatrix} n^2 \\ p-(k^-) \end{pmatrix} for\ all\ n \geq 1; k^- \geq 0 \geq p$. and Let $\alpha \in S$ such that $ECL_n \subseteq CI_n \subseteq S$ where $\mid E(S) \mid = \dfrac{n(n-1)}{2} + 1$ then $\mid f(n;p,m) \mid = \sum_{m=2}^{n} \begin{pmatrix} n \\ m \end{pmatrix} \dfrac{n(n-1)}{m}$ for all $n \geq 2; n, m \in M_n$.

Some results obtained by Mogbonju and Azeez [?] which are relevant to the study were: Let $S = SO_n$ then $|S| = 2^n \begin{pmatrix} 2n-1 \\ n-1 \end{pmatrix}$ for all $n \geq 1$, $|H_n| = 2n^{2+n}$ for $n \in \mathbb{N}$, $|S| = \sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} \begin{pmatrix} n+k \\ k \end{pmatrix}$, $|E(S)| = \frac{1}{3n+1} \begin{pmatrix} 4n \\ n \end{pmatrix}$ and $|E(SIO_n)| = 2^n$; $\sum_{r=0}^{n} \begin{pmatrix} n \\ r \end{pmatrix} X^n Y^{(n-r)} = (X+Y)^n = 2^n$, iff $X = Y = 1$.

. An upper bound on the cardinality of a minimal relation for a symmetric semigroups and also the symmetric semigroups with maximum embedding dimension was studied by [?]. However some results are known for good semigroup and a results was proved in [?]. The work of [?] discuss the quality of associated semigroupos for R, how to compute the lent function $\lambda_R(j/I)$ for fractional ideal of R in terms of length of chains of point in their value sets.

However, in this paper we established more useful results from the star-like folding principle of [?], were they use an established 3 dimensional star-like square pyramid as shown below.
The above star-like square pyramid is symmetric in nature and we assumed that the symmetric star-like square pyramid is rigid and solid and has equal dimensions all through, therefore we bisect the top star-like vertex down to A and B so that we can form a triangular model such that we obtained the measurement of the star-like edges as shown in the Figure 2

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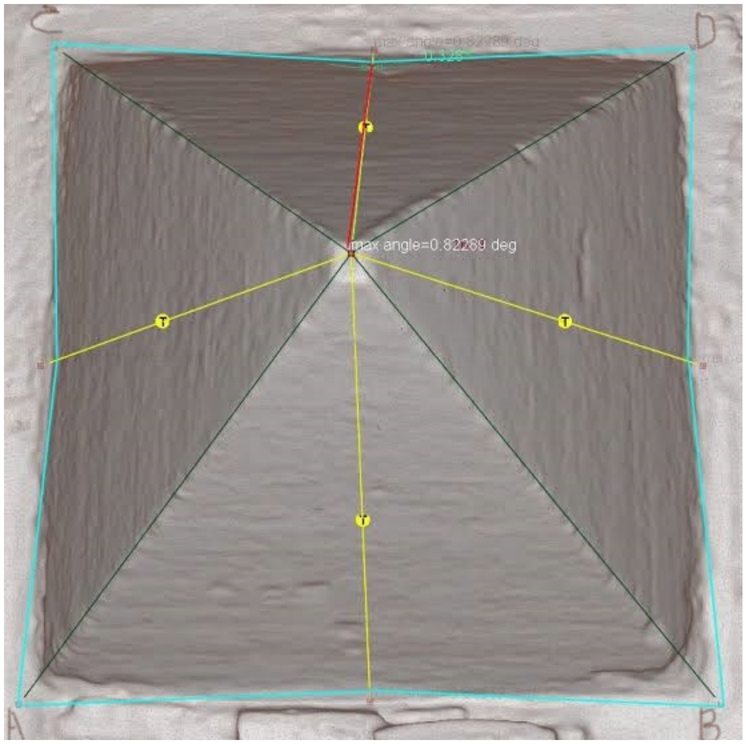

Figure 1: Star-like Square Pyramid

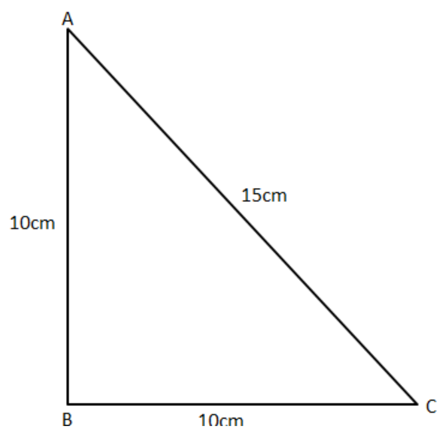

Figure 2: Star-like Right Angle Triangle

Definition 2.1: Star-like Folding Principle: is a physical craft process by which a standard $A4$ paper star-like chain is translated to its native 3Dimensional star-like structure.
Definition 2.2: Star-like Vertices V^*: is a star-like transformation corner where two or more star-like edges E^* meets.
Definition 2.4: Star-like Edges E^* This is a star-like disk points where two or more star-like faces F^* meets.
Definition 2.3: Partial symmetric semigroup: [?] The partial symmetric semigroup S_n is a semigroup in which there is no image $(I(\alpha))$ set that appears more than once such that $S_n = (n+1)!$.
Definition 2.5: [?] Let δ be the chart on $X_n = \{1, 2, 3, \cdots\}$. The map $\alpha : Dom(\alpha) \subseteq X_n \rightarrow Im(\alpha) \subseteq X_n$ is said to be a full transformation semigroup; denoted by T_n, if $Dom(\alpha) = X_n$, and

partial transformation if $Dom(\alpha) \subseteq X_n$; denoted by P_n.
Definition 2.6: Symmetric and Transitive [?] An Ω -valued(binary)relation R on A is an Ω-valued function on A^2, i.e., it is a mapping $R: A^2 \rightarrow$.
R is symmetric if $R(x,y) = R(y,x)$ for all $x, y \in A$;
R is transitive if $R(x,y)R(x,z) \bigwedge R(z,y)$ for all $x, y, z \in A$.
Theorem 2.4 [?] Given a star-like nildempotency transformation $N_c\omega_n^*$, Let R_n^* be a star-like polygon with n star-like vertices and n sides with star-like interior angles, $\alpha_i^*, ...\alpha_n^*$. Then

$$Area(R_n^*) = (\sum_{n=1}^{\infty} \alpha_n) - (n-2)\pi$$

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We are going to introduce some useful results, before then we would like to introduce some of the notations used in this paper in the table below

ϕ	Disk Constant point
E	Angular loop Symmetric Spinnable
$h(\phi)$	high of disk constant point
X_n	The set of all distinct non negative integer
ALM	Angular Loop Model of 3-Dimensional transformation
$D(\phi)$	Value of point in the domain of ϕ
$I(\phi)$	Value of point in the Image of ϕ
$r(\phi)$	Value of point in the rank of ϕ
ϕ_n	Set of all star-like disk constant point
$\mid \phi_n \mid$	The order of the set of all disk constant points

3 Main Results

Let ALM be angular-loop transformation such that $S \in ALM$ then any $\phi \in S$ angular-loop right-angle disk constant point is equal then the following statements are equivalent.

i S is reducible and

ii S is spinnable.

Proof. $i \longrightarrow ii$
Given $S \in ALM$ be the degree of angular-loop transformation,
Suppose S is reducible then, we need to show that it's spinnable. Consider figure 1: (if we carved out each $\phi_n \in S$ with a standard ruling measurement to obtained figure 2: such that for all $\phi \in S$ where$(a, b, c) \in \phi$ $ab = bc$

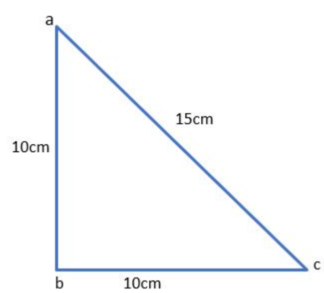

Figure 3:

Since $ab = bc$ with $b \in \phi$ as an angular-loop disk point then $S \in ALM$ is spinnable.
$ii \longrightarrow i$
Suppose $S = ALM$ then $\mid ALM \mid = \emptyset$.
By the angular-loop model and S is a set of angular degree of finite relation such that $\phi \in S$ is a bijection, Then S contain many reducible relation R_i 0f of order n, such that

$$R_{ij} = \omega_0(u)^4 + \omega_1(u)^3 + \omega_2(u)^2 + \omega_3(u) + \omega_4 \tag{3.1}$$

generate a reducible star-like nildempotency triangular system of
Where $R_{ij} = \mid S \mid$;
Therefore, adopting folding principle , we obtained that n order of domain of ALM can be chosen for

$$X_n = \{1, 2, ...\} \quad in \quad \binom{n}{n-1}$$

ways which contain the reducible relation. □

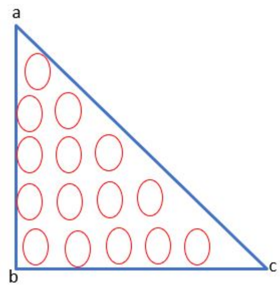

Figure 4:

Let $S \in ALM$ be the set of all angular-loop disk constant relation ϕ such that $\phi_i : a \to b, \phi_j : b \to c, \phi_k : c \to a$ form an algebraic transitive relation

Proof. For any given angular-loop disk constant relation ϕ with coordinate (a,b,c) Let i, j, k be the

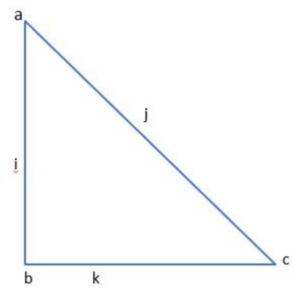

Figure 5:

sum of the other star-like sequence on the three with i on the edges $a - b$, j on edges $c - a$, k on edges $b - c$.
Let ϕ be the disk constant sum of the figure 5: then we have the following relations.

$$\begin{aligned} a + i + b &= \phi \\ b + j + c &= \phi \\ c + k + a &= \phi \end{aligned} \tag{3.2}$$

Adding the three relations of equations 2 we have

$$2(a + b + c) + (i + j + k) = 3\phi \tag{3.3}$$

such that

$$a + b + c \neq i + j + k = 1 + 2 + 3 + \cdots \leq 90^0 \tag{3.4}$$

such that

$$a + b + c = 3\phi - 90^0. \tag{3.5}$$

Therefore, the angular-loop disk constant relation is $3\phi - 90^0$, which show an algebraic transitive relation.

□

Let $S \in ALM$ then the following statement are equivalent

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i Every element $\phi \in S$ is an angular degree

ii $\phi \in S$ has at least $w^{+}(\alpha)$ and $w^{-}(\alpha)$ for all $a, b \in D(\alpha)$

iii There exist a unique $K \in S : \{k = \mid Max(n, w^{+}) \mid . \mid Min(n, w^{-}) \mid\}$ for all $n \geq 1$, then $(n, w^{+}(\alpha), w^{-}(\alpha)) = \sum_{n=1}^{k} \binom{2^{k-1}}{k+n-1}$ such that S is angular degree of ALM

Proof. $i \longrightarrow ii$
Suppose $\phi \in S$ is an angular-loop, such that ϕ is the angular-loop disk point, then $a \in X_n$ is in the range set $b \in X_n : \alpha(a)b$. Since ϕ is a degree ,

$$\alpha(\alpha(a)) = \alpha(a) \tag{3.6}$$

implies

$$\alpha(a) = a \tag{3.7}$$

Such that

$$w^{+} \in X_n$$

then

$$\mid Max(n, w^{+}) \mid = \mid Min(n, w^{-}) \mid \tag{3.8}$$

for some $i < 1, \alpha a, \alpha b \in D(\alpha)$.
For any given angular-loop degree transformation S such that $w^{+}(\alpha)$ *and* is defined by the rank set we have

$$\alpha(a) = b : r(\alpha) \leqslant b \tag{3.9}$$

$ii \longrightarrow iii$
Let $t \in \phi : t = w^{+}(\alpha).w^{-}(\alpha)$ then,
under folding principle and composition of angular-loop transformation.

$$D \mid Max(n, w^{+}) \mid \ replace\ with\ Im \mid Min(n, w^{-}(\alpha) \mid .$$

Thus,

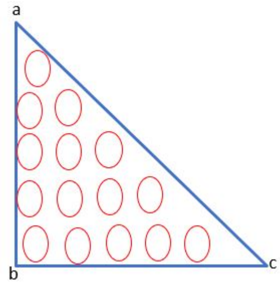

Figure 6:

Given the nildempotency reducible order of $F(n, w^{+}(\alpha), w^{-}(\alpha))$. thus we have a reducible and spinable recurrence angular-loop disk point such that the recurrence relation of the transformation side (a,b,c) equals

$$\sum_{n=1}^{k} \binom{2^{k-1}}{k+n-1} \ for\ all\ n \geq 1 \tag{3.10}$$

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$iii \longrightarrow i$
Assuming $\phi \in ALM$ such that $S \subseteq ALM$ where ϕ_n is an angular-loop disk point for any given angular transformation

$$a, b \in D(\alpha) : r(\phi) \leq X_n \tag{3.11}$$

such that $a_{i+1} - a_i$ is the domain and $b_{i+1} - b_i$ is the rank order of ϕ then

$$\phi_n \mid a_{i+1} - a_i \mid \leq \phi_n \mid b_{i+1} - b_i \mid \tag{3.12}$$

Thus, ϕ is the angular-loop degree order if their exist

$$t \in D(\alpha) : \phi_n^t . \phi_n^t = \phi_n$$

□

Let $E \in ALM$ where E is an angular-loop symmetric spinnable transformation semigroup, then for any $\phi \in E$ $ab \parallel cd$, then $|< abc \mid = \mid < bcd \mid$ for all $a, b, c, d \in \phi$.

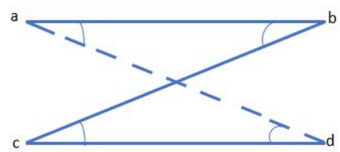

Figure 7:

Proof. Suppose that a and d are in opposite side of the line bc in an angular-loop symmetric spinnable transformation. By folding principle of angular-loop transformation, then

$$|< abc \mid = \mid < bcd \mid \tag{3.13}$$

$$ab \parallel cd \tag{3.14}$$

Therefore, for any given spinnable angular-loop transformation E, the transversal of each $E \in ALM$ makes equal alternative angle on two side. since the line of any spinnable angular-loop transformation E are always spinnable. □

Let $\phi \in ALM$ then $\mid \phi \mid = \binom{u-p}{p-1} = \binom{u-(p-1)}{u-p}$.

Proof. let $X_n = \{1, 2, 3, ...\}$ then $D(\phi) \subseteq X_n$ where $\phi \in ALM$ if

$$F(u, p) = \mid \phi \in ALM : h(\phi) \mid = \mid I(\phi) \mid = p$$

consider $a_0 \in D(\phi)$ such that

$$\phi a_0 \leq \phi b_0 \tag{3.15}$$

$$\phi a_0 \leq b_0$$

implies

$$\phi a_0 = e \ (angular - loop \ constant)$$

So, a_0 has $u - e + 1$ disk point degree of freedom with order

$$\mid \phi \mid = \binom{u-p}{p-1}.$$

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where $u = p = 2$, we have.

$$\binom{u-p}{p-1} = 1 \tag{3.16}$$

for the second formula, since for angular-loop transformation, is a subset of all transformation and that if $\phi \in ALM : h(\phi) = p$, irrespective of the value of $u \geq 2$ whenever $p = (u-1)$, there is exactly two angular-loop elements of height of u order such that

$$\mid \phi \mid = \binom{u-(p-1)}{u-p}.$$

□

Let $L(a, b)$ denote number of sequence path from $(0, 0)$, to (a, b) with x-row and y-column, then any angular-loop transformation $\alpha \in ALM$ from a triangular array of angular-loop sequence.

Proof. Suppose $\alpha \in ALM$, with x-row and y-column of sequence, for all
$N_i = \{i, i+1, i+3, ...\}, (i = \{0, 1, 2, ...\})$.
if L(a,0)=K, L(0,b)=K where (0,0) is the angular origin of all path of the sequence obtained by $\alpha \in ALM$, We have

$$L(a, b) = L(a, b-k) + L(a-k, b) \tag{3.17}$$

such that

$$\binom{a+b}{a} + \binom{a+b}{b} = \frac{a+b}{a!b!}$$

where k is the star-like arbitrary constant. □

4 Conclusion

This paper has showed that for any disk constant point $\phi \in S$ then S is reducible, spinnable and also form an algebraic transitive relation which in turn form a symmetric spinnable relation and the constant disk point ϕ is congruence.

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