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Hybrid Technique for Optimizing Stock Allocation (HTFOSA)

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Abstract

The quest for optimum distribution of scarce goods from manufacturers to satisfy consumers demand via wholesalers and retailers has caused reasonable drift from ordinary allocation of goods to developing a mathematical model that enhances steady and efficient allocation. Demand is dynamic, hence the need to keep stock. The act of keeping stock has its associated costs, likewise the act of not keeping stock(stockout). This study investigated the different ways by which stock could be allocated. The Lagrangian interpolation polynomial is used to obtain polynomial functions of allocations then the Chebyshev polynomial approximation technique was used to design a hybrid technique that optimizes returns. The features of the hybrid model along with its corresponding algorithm were stated. Relevant theorems are stated with numerical examples. The hybrid model developed has a wide area of applicability as it provides solution to different conditions of stock allocation simultaneously which reduced computational time and overcomes the problem of cause of dimensionality. The technique was compared with the Dynamic programming model of stock allocation and results show a significant and appreciable improvement on stock allocation.

Keywords: Polynomial approximation, Chebyshev polynomial, Stock allocation, Interpolation, Dynamic programming.
MSC2010: 90C39

1 Introduction

Stock can be defined to be goods being held for future use or sale. The ability to keep goods in a warehouse to make it available for sale or future use is called stocking. Control is a process by which events are made to conform to a plan. Demand is dynamic and hence the pertinence to keep goods in stock. The act of maintaining stock has its associated costs likewise the act of not keeping stock. The later makes the manufacturer/distributor lose customers' goodwill thereby incurring shortage cost while the former may increase holding costs. It is therefore often necessary to stock physical goods in order to satisfy demand over a specified time period in a way that the firm minimizes cost and maximizes profit. This work will help in striking a balance between the two costs.

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The act of stocking goods to satisfy future demand gives rise to the problem of designing a very efficient allocation technique so as to minimize cost and maximize profit. Numerous establishments have incurred great loses due to inefficient allocation of goods and resources. Decision regarding quantity allocated and the time at which it is allotted may be based on the minimization of appropriate cost function which balances the total cost resulting from over-stock and stock-out. The major objective of the stock allocation models is to find and obtain an inventory level that minimizes the sum of the shortage cost, the holding cost and other associated costs.

To solve the stock allocation problem, a mathematical model would be developed as a basis for strategic organizational decision. In this work, a recursive polynomial approximation technique for solving multi-stage process would be used. The chebyshev polynomial approximation is used on the resultant function from Lagrangian interpolation polynomial to produce a hybrid technique.. The work used polynomial as the basic means of approximation. The degree of the polynomial for the function is chosen in such a way as to minimize the worst-case error. For instance when using nth degree polynomial centred at a to approximate $f(x)$ the magnitude error is

$$|E| = |f(x) - P_n(x)| \tag{1.1}$$

where

$$P_n(x) = \sum_{i=0}^{n} \frac{f^i(a(x-a)^i)}{i!} \tag{1.2}$$

with the error bounded as

$$|f(x) - P_n(x)| \leq R_n = |\frac{f^{n+1}(c)(x-a)^{n+1}}{(n+1)!}| \tag{1.3}$$

where c lies between a and x, $(a < c < x)$ and f^{n+1} is the $(n+1)th$ derivative of $f(x)$

The Chebyshev polynomials are defined by a three term recursion;
$T_0(x) = 1$, $T_1(x) = x$,...,$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$; $n = 1, 2, \ldots$
which implies that;

$$T_n(\cos\theta) = \cos(n\theta), \theta \in \pi, n = 0, 1, 2, \ldots \tag{1.4}$$

[1] stated that the Chebyshev polynomial $T_n(x)$ is the recurrence relation,

$$T_{n+1} = 2xT_n(x) - T_{n-1}(x), n = 1, 2, 3, \ldots. \tag{1.5}$$

2 Motivation

The channel of distribution is only complete when the products/goods get to the final consumer. The wide gap between the manufacturer and the consumer must be filled through proper and efficient allocation technique. The quest for optimum distribution of goods to both the wholesaler/retailer/consumer has caused the need for developing a mathematical model that enhances steady and efficient allocation. According to [2], over 50% of the populace rely on manufactured goods. If the few goods produced are not distributed equitably, both the company and consumers suffer hence this research work is prompted. According to [3], the growing global economy has caused a dramatic shift in stock management in the twenty-first century. The late 1980's witnessed fears by the computer manufacturing company Hewlett Packard (HP) that inventories running into billions of Dollars and alarming customer dissatisfaction which its order fulfilment process had incurred was a service problem. The management was therefore concerned since order fulfilment was becoming a major battlefield in the high technology industries. Due to their inability to manage stock properly, they incured losses. [4] asserts that the cost associated with storing goods is very large hence the need to reduce storage costs by avoiding unnecessary large inventory. It is therefore

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clear that inventory pervades the business world and hence maintaining stock through product allocation is necessary for any company dealing with physical products. [5] states that the cost associated with maintaining stock and reliability, including inventory cost are on the rise and thus will no doubt affect profit making. The forgoing among others forecloses the need for a viable and efficient stock allocation model to solve allocation problems.

Stock control has therefore become a functional part of every manufacturing and service organization. Most of the existing models are adapted to a particular situation of stock management. The computational time involved in most of the prevailing methods is high. There is therefore need to develop a mathematical model that will help the allocation process.

The aim of the research work is to develop a stock allocation model with wide area of applicability and better approximation properties.We wish to develop a more reliable model using the Lagrange interpolation polynomial and Chebyshev polynomial approximation for n warehouses where $n > 2$.

The Chebyshev polynomial has orthogonal property, which is helpful in implementing approximation methods for iterative computation and has a better convergence property over other standard polynomials ([6]). The study will among other objectives:

1. Provide a decision tool for organizations to make justifiable and cost saving allocation policy.
2. Present a solution with a wide range of applicability.
3. Obtain a method that is time efficient.
4. Find the allocation that maximizes returns

3 Problem Definition

Efficient stock allocation is vital to both the manufacturing and the distribution firms. Methods such as the Dynamic programming model,Inventory models of items with lmperfect quality, Economic Order Quantity (EOQ) models of stock allocation among others. Each of these models has different formular hence inumerable number of formular with each addressing one tiny part of allocation. The computational effort of some of these models is high and the curse of dimensionality is present. In addition the EOQ model is used only when the demand for product is constant. There is therefore the need to develop a mathematical model with a wide area of applicability and more time efficient.

The Lagrangian interpolation polynomial is recursive in nature,computationally expensive and exhibits oscillatory artificets. The Chebyshev polynomial is also recursive in nature, is orthogonal, possesses the minimax property and yields a smaller maximum error over $[a, b]$ [7]. Thia paper therefore uses the Chebyshev polynomial approximation on the Lagrangian interpolation polynomial to obtain an optimal stock allocation.

4 Related Works

[8] stated that a high percentage of the world population depend on manufactured goods which must be moved from their points of manufacture to the points of consumption. These goods would be stored in warehouses near the consumers. These products are therefore stocked in warehouses to meet the timely demand of customers, hence the need for stock management. Demand in most cases is dynamic, therefore policies must be put in place to ensure that the process of distribution is effective and costs less. [9] point out that there is a conflict between the need to give a good service and the need to economize in stock holding. The more the stock held the easier it is to have required items readily available on demand. On the other hand, the more the stocks held the greater the holding cost. It is therefore necessary to seek, find and operate a satisfactory compromise between these two opposing forces. Stock allocation is a system used to ensure that goods and services

reach the ultimate users through efficient stocking in warehouses close to the consumers for easy distribution. [10] outlined the following logical reasons for holding stock; to ensure that sufficient goods are available to meet anticipated demand, to absorb variations in demand and production,to provide a buffer between production processes, to take advantage of bulk purchasing discounts, to absorb seasonal fluctuations in usage or demand,to enable production processes to flow smoothly and efficiently, as deliberate investment policy particularly in times of inflation or possible shortage, as necessary part of the production process.

[11] stated that customer service has become an important dimension of competition along with price and quality. In order to retain a company's current customers and to acquire new customers, prompt service is always considered for which the first requirement is to have service parts readily available. [12] worked on two warehouse inventory, model for perfect quality goods with quantity discount and asserts that the optimal order quantity that minimizes the expected cost per unit time $ECTU(y)$ of the Economic Order Quality (EOQ) is got as

$$y^{\star}_{ij} = \sqrt{\frac{2KD}{I_w c_j(E_2 + 2D)}} \tag{4.1}$$

where $j = 1, 2, \ldots, n$ and

$$y^{\star}_{2j} = \sqrt{\frac{2KD + c_j w^2[2D(I_r + I_w) + E_2(I_r + I_w)]}{I_r c_j(E_2 + 2D)}} \tag{4.2}$$

[13] worked on a two-warehouse inventory model for items with imperfect quality and quantity discount. He considered two cases namely

$y \leq w$ where w is storage capacity of owners warehouse, y is order size and z the storage capacity of the rented warehouse. He ascertained that the expected cost per unit time of Economic Order Quantity is set as y at jth level of purchase for both cases as:

$$ETCU_{1j}(y) = \frac{1}{E_1}(c_j + d)D + \frac{KD}{y} + I_w C_j y \left(\frac{E_2}{2} - \frac{DE[p]}{x}\right); j = 1, 2, \ldots, n \tag{4.3}$$

and $w \leq y \leq w + z$

$$ETCU_{2j}(y) = \frac{KD}{yE_1} + \frac{Dc_j}{E_1} + \frac{dD}{E_1} + \frac{I_r C_j(y-w)^2(xE_2 + 2DE[P])}{2xyE_1} + \frac{I_w c_j w(xE_2(2y-w) + 2DwE[p])}{2xyE_1} \tag{4.4}$$

$j = n, n+1, \ldots, m$

where

$E[p] = \frac{U+L}{2}$, $E_1 = 1 - E[p] = 1 - \frac{U-L}{2}$ and $E_2 = E[(1-p)^2] = (U + L) + \frac{(U^2+UL+L^2)}{3}$

4.1 Lagrangian interpolation polynomial

According to [14],Interpolation refers to the process of constructing an approximating function which agrees with a given function or its derivative at prescribed points. The principle of Lagrangian interpolation is that a function $f(x)$ whose values are given at a collection of points is assumed to be approximately represented by a polynomial $p(x)$ that passes through each and every point.ie;

If $x_0, x_1, x_2, \ldots, x_n$ be distinct values and and $y_0, y_1, y_2 \ldots, y_n$ be values which are not necessarily distinct.

The Lagrangian polynomial problem finds a polynomial of degree at most n such that

$$p(x_0) = y_0, p(x_1) = y_1, \ldots, p(x_n) = y_n \tag{4.5}$$

Now if f is a continuous real function on the real interval $[a, b]$. and the values of this function are known at a finite number of points x one can consider the approximation by a polynomial p such that;

$$f(x_i) = P_n(x_i) \tag{4.6}$$

Then for a given function f defined on $n+1$ points $x_0 < x_1 < \ldots < x_n \in [a, b]$, there exists a unique polynomial of degree smaller than or equal to n such that

$$P_n(x) = \sum_{i=0}^{n} f(x) L_i(x) \tag{4.7}$$

where $L_i(x)$ is defined by

$$L_i(x) = \frac{\pi_{n+1(x)}}{(x - x + i)\pi_{n-1}(x)} = \frac{\pi_{j=0}^{n}(x - x_j)}{\pi_{j=0}^{n}(x_i - x_j)} j \neq i \tag{4.8}$$

$\pi_{n+1}(x)$ being the nodal polynomial,

$$\pi_{n+1}(x) = \pi_{j=0}(x - x_j) \tag{4.9}$$

Let $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$ be a typical polynomial in P_n

Then equation (12) can be written as:

$$Ax = yAx = \Big(\sum_{i=0}^{n} a_i x_j^i\Big) = y_j j = 0, 1, 2, \ldots, n \tag{4.10}$$

where x_i, $i = 0, 1, 2, \ldots, n$ are distinct numbers.

The coefficients of the interpolation polynomial can be found by solving the system of equation above for $a_0, a_1, a_2 \ldots, a_n$.

4.2 Chebyshev Polynomial Approximation (CPA)

Approximation arises due to the difficulty in obtaining the exact area/volume or dimension of some objects. According to [15],the general approximation problem states that if f is an element and S a subset of a norm linear space X, then the approximation theory seeks to find an element $s \in S$ which is as close to f as possible; ie to find an element $s^\star$ of S such that

$$||f - s^\star|| \leq ||f - s|| \forall s^\star \in S. \tag{4.11}$$

The Weierstress theorem states that if f is a continuous function on an interval $[a, b]$ and is given,there exists a polynomial $p(x)$ such that

$$Sup_{x \in C[a,b]} \mid f(x) - p(x) \mid \leq \epsilon, \tag{4.12}$$

[16], The Weierstress theorem therefore implies that "any continuous real-valued function f defined on a bounded interval $[a, b]$ of the real line can be approximated to any degree of accuracy using a polynomial".i.e. for any $\epsilon > 0$, there is a polynomial p such that

$$||f - p||^\infty \equiv \sup_{a \leq x \leq b} |f(x) - p(x)| < \epsilon. \tag{4.13}$$

In other words, there exists a polynomial that approximates any continuous function over a compact domain arbitrarily well. It is therefore possible to use polynomial approximation to obtain an optimal stock allocation if the allocation is shown as a continuous function.

[7] defined Chebyshev's polynomial as a sequence of orthogonal polynomials which are related to De-moivre's formula;

$$[(r\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta) \tag{4.14}$$

and which can be defined recursively.

The Chebyshev's polynomial T_n are polynomials of degree n. The Chebyshev's polynomial of the first kind of order n is defined as

$$T_n(x) = cos[ncos^{-1}(x)], x \in [-1, 1], n = 0, 1, 2, \dots. \tag{4.15}$$

The nth Chebyshev polynomial is defined by

$$T_n(x) = \cos(n\cos^{-1}(x) \tag{4.16}$$

Then the following property is evident, if $x = \cos\theta$,

$$\theta = \cos^{-1} x, \theta \in [0, \pi]. \tag{4.17}$$

and hence

$$T_n(cos\theta) = cos(n\theta)n = 0, 1, 2, \dots \tag{4.18}$$

Hence

$$T_0(x) = cos(0) = 1 \tag{4.19}$$

and

$$T_1(x) = cos[cos^{-1}(x)] = x. \tag{4.20}$$

Therefore the Chebyshev polynomial $T_n(x)$, satisfies the following propositions which follow from $T_n(cos\theta) = cos(n\theta)$;

$$T_0(x) = 1, T_1(x) = x, T_2(x) = 2x^2 - 1, ldots, T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x). \tag{4.21}$$

The Chebyshev polynomial $T_n(x)$, $n \geq 1$, has the following properties that make it plausible for polynomial approximations;

1. Recursion formula ie $T_0(x) = 1$,$T_1(x) = x$,$T_2(x) = 2x^2 - 1, \dots, T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$;The leading coefficient is 2^{n-1} for $n \geq 1$ and 1 for $n = 0$; Symmetric property, $T_n(-x) = (-1)T_n(x)$; ; $T_n(x)$ has n zeros in $[-1, 1]$;Orthogonality property and Minimax property.

4.3 Theorems

The following theorems are relevant to the topic under discussion. These theorems assert the possibility of approximating functions using polynomials,the existence of a best approximating polynomial and the uniqueness of the best approximating polynomials.

4.3.1 Theorem

Theorem 4.1 *([7]) For each $x \in [-1, 1]$ and $n = 1, 2, \dots$ of the Chebyshev polynomial,*

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \tag{4.22}$$

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Theorem 4.2 *([7]) Let $f \in C[-1,1]$ and let S_n be the best approximate to f from P_n relative to*

$$|f| = \left[\int_{-1}^{1} \frac{f^2(x)dx}{\sqrt{1-x^2}}\right]^{\frac{1}{2}} \tag{4.23}$$

then

$$\lim_{n\to\infty} \|f - s_n\| = 0 \tag{4.24}$$

Theorem 4.3 *([17]) If $f \in C[a,b]$, then the best Chebyshev approximation to f from P_n is unique.*

Theorem 4.4 *([18]) If $f \in C[a,b]$, then its best polynomial is a Lagragian interpolation polynomial for f at $n+1$ points in $[a,b]$*

5 Modelling

[6] stated that one of the best polynomials is an interpolation polynomial. To compute best polynomial approximation, it is necessary to use a successive technique. The Remez algorithm is used to produce an optimal polynomial $p(x)$ approximation of a given function $f(x)$ over a given interval. It is an iterative algorithm that converges to a polynomial that has an error function with $N+2$ level extrema. The Remez algorithm is a successive approximation technique for computing the coefficients of the polynomial in P_n which provides the best Chebyshev approximation to a given continuous function f. The Remez algorithm is therefore used as a procedure for the Chebyshev polynomial approximation. In this section, the hybrid of the Lagrangian interpolation polynomial with the Chebyshev polynomial approximation technique (HTFOSA) is presented. The algorithm and mathematical model of the hybrid technique are stated thus;

5.1 Algorithm

The steps taken to arrive at an optimal allocation of stock using the hybrid technique are listed as follows;

Given the allocation(Q) of products;

1. Obtain the functional equation $f(x)$ using the Lagrangian interpolation polynomial method
2. Obtain the corresponding Chebyshev polynomial $p_i(x)$ in reference to $f(x)$ $p_i(x) \in p_n(x)$.
3. Find
$$F(x) = f(x) - p_i(x) \tag{5.1}$$
4. Get F' if it exists. F is the difference between f and p.
5. Stop if F' does not exist
6. Obtain k_i the zeros of $F'(x)$ if F' exists.
7. Test if the $k_i's$ form an oscillation set in $[-1,1]$.
8. If it does,then $p_i(x)$ approximates $f(x)$.
9. Proceed to obtain the best approximate of $f(x)$.

5.2 Hybrid Technique for Optimizins Stock Allocation(HTFOSA)

Let the allocation of products Q of a firm be given.

1. Obtain a functional equation expression $f(Q)$ of the stock using the Lagrangian interpolation.
2. Obtain a corresponding polynomial function P_i of $f_n(Q)$ from the Chebyshev polynomial sequence
$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x). \tag{5.2}$$
3. Choose a polynomial function $P_i(x)$, $i = 1, 2, \ldots, n-1$ in reference to the Chebyshev polynomial $T_n(x)$ to approximate $f_n(Q)$. where
$$P_i(x) \in P_n(x) \tag{5.3}$$
and $degP_i \leq degP_n(x)$ where deg means the degree of the polynomial.
4. Find
$$F(x) = f(x) - P_i(x) \tag{5.4}$$
F is the difference between f and p_i and where
$$|f(x) - P_i(x)| < \epsilon \tag{5.5}$$
5. Obtain $F'(x)$ if it exists
6. Then calculate k_i, the zeros of $F'(x)$ where $k_i \in [-1, 1]$
7. Proceed to find
$$\|F\| = \max[F(a = k_1), F(k_2), \ldots, F(k_{n-1}), F(k_n = b)], k \in [a, b] \tag{5.6}$$
(stop if F' does not exist).
$$= \max[F(-1), F(k_2), \ldots, F(k_{n-1}), F(1)] = 1, k \in [-1, 1] \tag{5.7}$$
8. Then test if the $k's$ form an oscillation set in F.
9. If it does, then P_i approximates f.
10. For the best approximation, find
$$\| f - P_m \| \tag{5.8}$$
where $f - P_m = \lambda_m$ and where $P_m \in P_n$
11. If$|\lambda_m| \leq\| f - P_m \|$, then P_m is the best approximation of f if
$$\max_{x\in[-1,1]} | f(x) - P(x) |< \max_{x\in[-1,1]} | f(x) - q(x) |, \tag{5.9}$$
where $q(x) \in P_n(x)$.

6 Numerical Example

The hybrid technique is now used on the data obtained from a firm to obtain the optimum allocation.

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6.1 Example

The table below is the stock allocation of a company to their six warehouses. Obtain the best approximation of allocation to ensure profitability of the company.

Table 1: Table of allocations

Warehouse(x)	1	2	3	4	5	6
Allocation(000 units)$f(x)$	130	293	8	556	943	43

The corresponding function to the table above using Lagrangian interpolation polynomial method is;

$$f(x) = \frac{130(x-2)(x-3)\cdots(x-6)}{(1-2)(1-3)\cdots(1-6)} + \frac{293(x-1)(x-3)\cdots(x-6)}{(2-1)(2-3)\cdots(2-6)} \tag{6.1}$$

$$+\frac{8(x-1)(x-2)\cdots(x-6)}{(3-1)(3-2)\cdots(3-6)} + \frac{556(x-1)(x-2)\cdots(x-6)}{(4-1)(4-2)\cdots(4-6)} \tag{6.2}$$

$$+\frac{943(x-1)(x-2)\cdots(x-6)}{(5-1)(5-2)\cdots(5-6)} + \frac{43(x-1)(x-2)\cdots(x-6)}{(6-1)(6-2)\cdots(6-5)} \tag{6.3}$$

By simplifying the expression, we have

$$f(x) = 11573x^5 - 200,000x^4 + 395665x^3 - 1617500x^2 - 2926658x - 49560 \tag{6.4}$$

Now,

$$f(x) = 11573x^5 - 200,000x^4 + 395665x^3 - 1617500x^2 - 2926658x - 49560 \tag{6.5}$$

We use $T_5(x)$, as the Chebyshev polynomial in reference to $f(x)$. i.e

$$p(x) = 16x^5 + 20x^3 - 5x \tag{6.6}$$

i.e. to obtain an approximate of $f(x)$ use $f(x) - p(x)$

Therefore

$$F(x) = f(x) - p(x) \tag{6.7}$$

$$F(x) = 11573x^5 - 200,000x^4 + 395665x^3 - 1617500x^2 - 2926658x - 49560 - 16x^5 + 20x^3 - 5x \tag{6.8}$$

$$F(x) = 11557x^5 - 200,000x^4 + 395645x^3 - 1617500x^2 - 2926653x - 49560 \tag{6.9}$$

$$F'(x) = 57785x^4 - 800,000x^3 + 1,186,935x^2 - 3,235,000x - 2,926,653 = 0$$

Hence $x = -0.66566$,$x = 12.5916$, $x = 0.95294 + 2.226328i$ and $x = 0.95294 - 226328i$. $-0.66566 \in [-1, 1]$

Therefore

$$\lambda = -0.66566 \in [-1, 1] \tag{6.10}$$

i.e F' is zero at -0.66566

$$|f| = \max |[F(-1)|, |F(-0.66566)|, |F(1)|] = 1 \tag{6.11}$$

We note that -1, -0.66566 and 1 form an oscillation set of three points.

Therefore P is the approximation for f from P_n.

7 Result

The illustration above shows the way the hybrid method is used to obtain approximate stock allocation. The Lagrangian interpolation polynomial has been applied on the stock allocation to obtain the functional equation. The corresponding Chebyshev polynomial is then applied on the resultant equation and an approximating polynomial obtained. With this hybrid polynomial, corresponding allocation is got which compares favourably with the original allocation.

7.1 Comparison of the original allocations and the HTFOSA model

The tables below show a comparison of the original allocation to the values obtained using the HTFOSA method. In the tables, f is the original allocation to the various depots while p is the estimate obtained using the HTFOSA. f and p are the original allocation from data collected and data obtained using HTFOSA respectively. The comparison table is as follows.

Table 2: Table of comparison

	1	2	3	4	5	6
f	130	293	8	556	943	43
p	99	244	1	485	846	26

Here the spearman's correlation coefficient $r = 0.997$ which shows a very high level of correlation between the allocations.

The allocation problem above is now solved using the Dynamic Programming method.

Table 3: Allocation of products of six depots

products	1	2	3	4	5	6
X	50	100	3	200	350	15
Y	50	100	3	200	350	15
Z	30	93	2	156	243	13

Table 4: Table of returns in millions of Naira

product	0	1	2	3	4	5	6
X	1	0.5	1.0	0.03	2.0	3.5	0.15
Y	2	0.5	1.0	0.03	2.0	3.5	0.15
Z	3	0.3	0.93	0.02	1.56	2.43	0.13

Stage 1

Table 5: Returns corresponding to different product allocations

Product	0	1	2	3
Profit	0	0.5	1.0	0.03

Stage 2

Table 6:

		0	1	2	3
		0	0.5	0.5	0.03
0	0	0	0.5	0.5	0.03
1	1.0	1.0 $\nearrow$	1.5 $\nearrow$	2.0 $\nearrow$	
2	1.0	1.0 $\nearrow$	2.5 $\nearrow$		
3	0.93	0.93 $\nearrow$			

Thus the optimal profit and corresponding allocations of products to the two zones are given by

Table 7: Table of optimal returns

Products	0	1	2	3
$f_2(x_2) + f_1(x_1)$	0	0.5	2.0	2.5
$x_2 + x_1$	$0 + 0$	$0 + 1$	$1 + 2$	$2 + 1$

Table 8: Table of returns in millions of Naira

Products	0	1	2	3	4	5	6
X	1	0.5	1.0	0.03	2.0	3.5	0.15
Y	2	0.5	1.0	0.03	2.0	3.5	0.15
Z	3	0.3	0.93	0.02	1.56	2.43	0.13

Stage 3

Table 9: Table of optimal returns

product $(x_2 + x_1)$		0	1	2	3
$1 + 2$ $f_2(x_2) + f_1(x_1)$					
$x_3 + f_3(x_3)$		0	0.5	0.5	0.03
0	0	0	0.05	0.05	0.03
1	0.03	0.03 $\nearrow$	0.53 $\nearrow$	1.03 $\nearrow$	
2	0.03	0.03 $\nearrow$	0.56 $\nearrow$		
3	0.02	0.02 $\nearrow$			

Therefore the optimal returns and corresponding allocation of products to the three zones are given as

Table 10:

Products	0	1	2	3
$f_3(x_3) + f_2(x_2) + f_1(x_1)$	0	0.5	0.53	$i.03$
$x_3 + (x_2 + x_1)$	$0 + 0$	$0 + 1$	$1 + 2$ 0r $0 + 2$	$2 + 1$

Stage 4

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Table 11: Table of optimal returns

Product $(x_3 + x_2 + x_1)$ $1+3\ \ f_3(x_3)+f_2(x_2)+f_1(x_1)$ $x_4 + f_4(x_4)$		0 0	1 0.5	2 0.5	3 0.03
0	0	0	$0.5^\star$	$0.5^\star$	$0.03^\star$
1	2	2 ↗	2.5 ↗	0.5 ↗	
2	2	2 ↗	4.5 ↗		
3	1.56	1.56 ↗			

Therefore the optimal profits and corresponding allocation of products to the four zones are given as

Table 12:

Products	0	1	2	3
$f_3(x_3)+f_2(x_2)+f_1(x_1)$	0	0.5	2.5	4.5
$x_3 + (x_2 + x_1)$	$0+0$	$0+1$	$1+2$	$2+1$

Stage 5

Table 13:

Product $(x_4 + x_3 + x_2 + x_1)$		0	1	2	3
$1+2+3+4\ \ f_4(x_4)+\cdots+f_1(x_1)$ Zone 5 $x_5 + f_5(x_5)$		0	0.5	0.5	0.03
0	0	0	$0.5^\star$	$0.5^\star$	$0.03^\star$
1	3.5	3.5 ↗	4.0 ↗	4.5 ↗	
2	3.5	3.5 ↗	7.5 ↗		
3	2.43	2.43.5 ↗			

Table 14: Table of optimal returns

Products	0	1	2	3
$f_5(x_5)+f_4(x_4)+f_3(x_3)+f_2(x_2)+f_1(x_1)$	0	0.5	4.5	7.5
$x_5 + (x_4 + x_3 + x_2 + x_1)$	$0+0$	$0+1$	$1+1$	$2+1$

Stage 6

Table 15:

Product $\sum_{j=1}^{5} x_1$ $\sum_{j=1}^{5} f_1(x_1)$ $x_6 + f_6(x_6)$		0 0	1 0.5	2 0.5	3 0.03
0	0	0	$1.7^\star$	$3.1^\star$	$4.5^\star$
1	0.7	0.7 ↗	2.4 ↗	3.8 ↗	
2	0.5	0.5 ↗	2.4 ↗		
3	0.5	0.5 ↗			

Table 16: Table of Optimal returns

Products	0	1	2	3
$f_6(x_6) + f_5(x_5) + \cdots + f_1(x_1)$	0	1.7	3.1	4.5
$x_6 + (x_5 + x_4 + \cdots + x_1)$	$0+0$	$0+1$	$0+2$	$0+3$

Thus the maximum profit is 4.7 if $x_6 = x_5 = x_4 = 0, x_3 = x_2 = x_1 = 1$.

Hence maximum profit can be attained if the three are allocated to the three zones (A, B,C) only on equal basis.

8 Discussion of result

The customer is the focal point of all decisions and actions in every business organization. Products are manufactured and offered to the market place for the purpose of being purchased by the customers. This is the reason why effective and efficient stock management is of paramount importance in the present complex and competitive environment. The Stock allocation, polynomial approximation, Lagrangian interpolation polynomial and Chebyshev polynomial approximation have been discussed. The Lagrangian interpolation polynomial has been used on stock allocation and the Chebyshev polynomial approximation polynomial applied to the resultant function to obtain a hybrid Chebyshev polynomial approximation. The aim is to allocate stock in such a way that returns is optimized. The features of the hybrid method along with its corresponding algorithm is given. The HTFOSA technique has been compared with the Dynamic Programming method and it is clear that the curse of dimensionality problem is overcome. The volume of computation is smaller in the hybrid method than in the DP model. The hybrid technique therefore more time efficient and can be used for any form of stock allocation.

9 Conclusion

[19] assert that the traditional concepts and methods of business management face a limit on the degree of improving the performance of the entire system. They stated that the present complex and competitive global market places efficient distribution and allocation of stock in the forefront. Similarly [19] added that in today's environment every supply chain wants not only to minimize the system wide cost but to keep minimum stock along the supply chain while maximizing service level requirements of the customers. The foregoing shows that stock allocation is a vital exercise to both the manufacturer and the distributor. Stock allocation ensures that goods and services get to the final consumers. Relevant theorems that assert the applicability and uniqueness of this method are stated. Illustrations using the data collected have been made and they proved that the hybrid technique can be used to obtain optimal stock allocation more easily and in shorter time.

Competing financial interests

The author(s) declare no competing financial interests.

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