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A Modified Exponential Distribution for Predicting Long Term Unemployment Rate

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Abstract

The segment of those in the labour force who are keenly looking for work but could not find it at least 20 hours during the reference period are considered to be in unemployment state. This is among the biggest threats to social stability in many countries including Nigeria. Several studies have been conducted on the modelling and forecasting of unemployment rate. But these studies gives only short term predictions. This research work modified the exponential distribution that can give a long – term prediction of unemployment rate of a country. The modified distribution satisfied the condition for a distribution. The result shows that if on the average four million persons entered the unemployment state of Nigeria's labour market in 2016, then by the modified exponential distribution, 3814918 persons (7.16 %) will likely join the unemployment state by 2017, 13693214 persons (25.7 %) by 2020, 25969155 persons (48.74 %) by 2025, 34440832 persons (64.64 %) by 2030, 44313800 persons (83.17 %) by 2040, 49013183 persons (91.99 %) by 2050 and so on. In order to avoid the negative effect of unemployment on the Nigeria's economy and even Nigeria as a nation, practical measures must be taken by the government to reduce unemployment to the barest minimum.

Keywords: Exponential distribution, Modified Exponential Distribution, Unemployment Rate, Continuous – Time Stochastic Process and Labour Force.
MSC2010: 91B70

1 Introduction

Unemployment according to International Labour Organisation (ILO) is among the biggest threats to social stability in many countries including Nigeria [1]. There is no universal standard definition of unemployment as various countries adopt definitions that suit their local priorities. Virtually all countries, however, use the ILO definition or a variant of it to compute unemployment. The ILO's definition covers people aged 15 – 64 who during the reference period (which is usually the week preceding the time the survey is administered) were available for work or actively seeking for work but were unable to find work. Like most countries in the world, the Nigerian National Bureau of Statistics (NBS) uses a variant of the ILO definition. It defines unemployment as the proportion of those in the labour force who were actively looking for work but could not find it at least 20 hours during the reference period ([2],[3]). According to [4], unemployment, as one of the key issues that macroeconomic policies address, is a reflection of the mismatch between supply and demand in the labour market. It has a direct impact on consumption, savings, production, and investment. The unemployment rate is used by policy makers to measure economic activities and social stability. [5] examined the factors which determine youth unemployment in Tanzania: such as education system, lack of skills in business, etc., and suggested a way forward that the government and policy makers should review job market laws and regulations in order to promote the smooth transitioning of youths from education to job market, in order to reduce unemployment problems.

There are several causes of youth unemployment in Nigeria; some of which are; rural urban migration, rapid population growth, low standard of education, the rapid expansion of the educational system, lack of steady and sustainable power supply, corruption, poor management practices, neglect of agricultural sector, infrastructural decay, unfavourable government reforms, unfavourable terms and conditions placed on jobs, systemic problems in education etc ([6],[7],[8]).

Several studies have been conducted on the modelling and forecasting of unemployment rate. Most of these studies used the following models for predicting unemployment rate: Autoregressive Integrated Moving Average Model (ARIMA), Self-Exciting Threshold Autoregressive (SETAR) Model, Vector Autoregressive Model (VAR), Autoregressive Moving Average (ARMA), Autoregressive Conditional Heteroscedasticity (ARCH) and the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models. Among these models, the use of ARIMA models are the most flexible tool used in forecasting unemployment rate if there is no government intervention to change this trend ([9],[10],[11],[12]). [13] modelled and forecasted the evolution of unemployment rates in Nigeria using ARIMA model on annual data from 1972 to 2014. The result of the work revealed an increasing rate from 2015 to 2017, while a slight decrease in 2018. During this period of 2015 to 2018 unemployment rate was still very high in Nigeria and suggested that the present administration should focus on capital projects that have the capacity to create employment. However, the models mentioned above gives only short term and not long term predictions which is the motivation for this study. The aim of this paper is to modify the exponential distribution that can give a long – term prediction of unemployment rate of a country.

2 The Model

Unemployment rate (whether decreasing or increasing) is a continuous – time stochastic process. In the study of continuous-time stochastic processes, the exponential distribution is usually used to model the time until something happens in the process. The exponential density function is given by

$$f(t;\lambda) = \begin{cases} \lambda e^{-\lambda t} & , \quad t > 0 \\ 0 & , \quad \text{otherwise} \end{cases} \tag{2.1}$$

The parameter $\lambda > 0$ is called rate parameter. It is the inverse of the expected duration, μ , in a state before the next event. That is $\lambda = \frac{1}{\mu}$.

The Cumulative Distribution Function (CDF) of the exponential function in the time interval is given by (integration (1))

$$f(t;\lambda) = \begin{cases} 1 - e^{-\lambda t} & , \quad t > 0 \\ 0 & , \quad \text{otherwise} \end{cases} \tag{2.2}$$

The Cumulative Distribution Function (CDF) of the exponential function can be written as the probability of lifetime being less than some value, t , as

$$P(T \leq t) = 1 - e^{-\lambda t} \tag{2.3}$$

The rate parameter of the exponential distribution defined as the inverse of the expected stay or duration in a state before transition, has its limitation in application. This is because, in a normal labour market situation, the duration of stay of individuals in a state before transition is not constant; hence, the need for the modification. The labour market has two states namely: unemployment and employment states [14] and the movement from one state to another is the transition.

We start the modification of the exponential rate parameter by introducing some parameters and assumptions. The following parameters for the Modified Distribution were introduced: $k > 0 =$The rate of leaving the present state at a particular time period, $n =$Number of persons in the present state at a particular time period and $J =$Number of persons leaving the present state at a particular time. The assumptions are: The probability that a person will move from his/her present state of persons to the other at a particular time period is $\frac{1}{n}$, the probability that the other persons will remain in the present state at a particular time period is $1 - \frac{1}{n}$, the rate that a person will remain in a state is equal to the probability of remaining in a state at a time period, the stay in a state before transition is exponentially distributed, the rate of leaving a state at a particular time period is equal to the rate parameter of the exponential distribution.

Based on the assumptions and parameters,

$$1 - k = (1 - \frac{1}{n_1})(1 - \frac{1}{n_2})(1 - \frac{1}{n_3})\dots(1 - \frac{1}{n_l}) \tag{2.4}$$

$$1 - k = (1 - \frac{1}{n})^J \tag{2.5}$$

$$1 - k = [(1 - \frac{1}{n})^{-n}]^{-\frac{J}{n}} \tag{2.6}$$

The term in the square bracket on the right hand side of equation (6) can be express as follows

$$(1 - \frac{1}{n})^{-n} = e^{-n\log(1-\frac{1}{n})} \tag{2.7}$$

$$(1 - \frac{1}{n})^{-n} = e^{-n(-\frac{1}{n} - \frac{(\frac{1}{n})^2}{2} - \frac{(\frac{1}{n})^2}{3})} \tag{2.8}$$

If n is large, the right hand side of equation (8) becomes

$$(1 - \frac{1}{n})^{-n} = e^{-n(-\frac{1}{n})} = e \tag{2.9}$$

Substituting equation (9) in equation (6), we have

$$1 - k = e^{-\frac{J}{n}} \tag{2.10}$$

$$k = 1 - e^{-\frac{J}{n}} \tag{2.11}$$

Equation (11) is the rate of leaving one state to another state at a particular time period. Equation (3) now becomes

$$f(t;\lambda) = \begin{cases} 1 - \exp(-J/n))\exp(-(-1 - \exp(-J/n))t) & , \quad x > 0 \\ 0 & , \quad \text{otherwise} \end{cases} \tag{2.12}$$

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The Cumulative Distribution Function (CDF) of the exponential function written as the probability of lifetime being less than some value, t , is

$$P(T \leq t) = 1 - \exp[-(1-\exp(-J/n))t] \tag{2.13}$$

The Mean and Variance of the modified distribution are as follows: The Mean also known as the expected value $E(x)$, is given by

$$\begin{aligned} E(X) &= \int_0^\infty t(1-\exp(-J/n))\exp(-(1-\exp(-J/n))t)dt \\ &= \frac{1}{1-\exp(-J/n)} \end{aligned}$$

The Variance, $Var(X)$, is given by

$$Var(X) = \frac{1}{[1-\exp(-J/n)]^2} \tag{2.14}$$

Equation (13) can be used to give a probable unemployment rate if J (Number of persons entering unemployment state at present) and n (Number of persons from the source of entering unemployment state at present) are known.

3 Validation of the modified distribution

Theorem 1: Condition for existence of a probability distribution function [15]
For any $f(x)$ to be a legitimate probability distribution function for the continuous random variable X, defined over the set of real numbers, it must satisfy the following conditions,
i.

$$f(t) \geq 0 \text{ for all } x \in \mathbb{R} \tag{3.1}$$

ii.

$$\int_{-\infty}^{\infty} f(x)dx = 1 \tag{3.2}$$

iii.

$$P(a < X < b) = \int_a^b f(x)dx \tag{3.3}$$

The verification is to test for the condition for the existence of a probability distribution of theorem (1).
i.

$$f(t) = (1-\exp(-\frac{J}{n}))\exp[-(1-\exp(-\frac{J}{n}))]t \geq 0, \forall J, n > 0, t \in \mathbb{R} \tag{3.4}$$

ii.

$$\begin{aligned} \int_0^t f(t)dt &= \int_0^t (1-\exp(-\frac{J}{n}))\exp[-(1-\exp(-\frac{J}{n}))]dt \\ &= 1-\exp[-(1-\exp(-\frac{J}{n}))]t \end{aligned}$$

For,$J, n > 0$, $n > J$ and as t gets larger, the second term of equation (20) turns to zero. This implies that, the integral of the modified exponential turns to one (1) as t gets large.
iii. Since the integral of the modified exponential distribution turns to one (1), it implies that

$$\begin{aligned} p(0, X < t) &= \int_0^t (1-\exp(-\frac{J}{n}))\exp[-(1-\exp(-\frac{J}{n}))]dt \\ &= 1-\exp[-(1-\exp(-\frac{J}{n}))]t = 1 \end{aligned}$$

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Hence, the proof that the modified exponential meets the condition for a distribution.

4 Unemployment rate flow diagrams

Consider the figure below

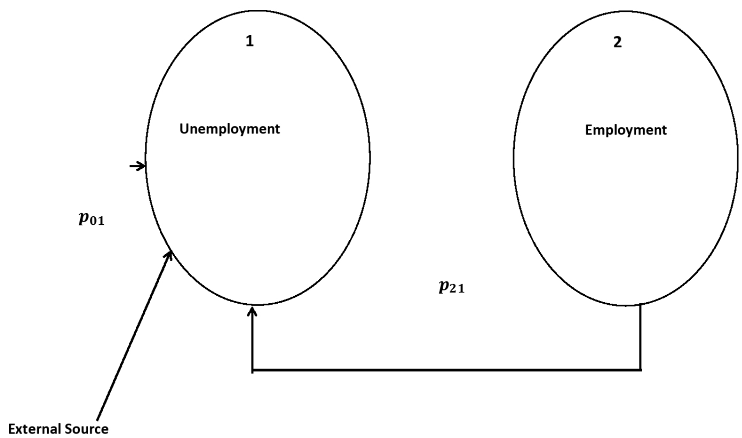

Figure 1: The flow diagram of unemployment rate

5 Application of the modified distribution to the Nigeria's labour market

Prediction of Unemployment Rate.

Using the data of Table 2 (Appendix), [16, 17, 18] the parameter values of the modified exponential distribution is presented in Table 1 below

Table 1: Parameter Values for Unemployment Rate

Parameter	Value
J	4111032
n	53280991

The values generated from fixing the parameter values to equation (13) is presented in Table 3 and the graph obtained from the table is presented in figure 2 below.

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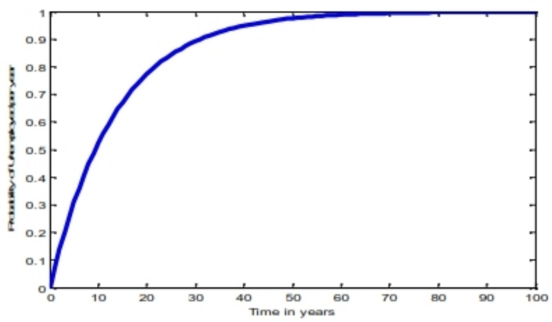

Figure 2: Graph showing the probable unemployment rate in Nigeria on a long-run

6 Discussion and Conclusion

It can be seen from Table 3 and Figure 2 that if on the average about four million persons (both from external source and employment state) are entering the unemployment state each year from 2016, then 3814918 persons (7.16 %) will likely join the unemployment state by 2017, 13693214 persons (25.7 %) by 2020, 25969155 persons (48.74 %) by 2025, 34440832 persons (64.64 %) by 2030, 44313800 persons (83.17 %) by 2040, 49013183 persons (91.99 %) by 2050 and so on. Also, the probable unemployment rate from 2078 upward is slow and at a point can be negligible. This is an indication that if unemployment in Nigeria continue to increase, it will get to a point where the economy of the country will not be able to curtail it. So, the causes of unemployment such as rural urban migration, rapid population growth, low standard of education, the rapid expansion of the educational system, lack of steady and sustainable power supply, corruption, poor management practice, neglect of agricultural sector, infrastructural decay, unfavourable government reforms, unfavourable terms and conditions placed on jobs, systemic problems in education etc, will be at their pick in about 70 years to come from 2016. In order to avoid the negative effect of unemployment on the Nigeria's economy and even Nigeria as a nation, practical measures must be taken by the government to reduce unemployment to the barest minimum.

Competing financial interests

The authors declare no competing financial interests.

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Appendix

Table 2: Nigeria's Labour force, employment and Unemployment Statistics from 2006-2016

Year	Labour Force	Employed (E)	E %	Unemployed (U)	U %
2006	57455701	50388650	87.7	7067051	12.3
2007	59294283	51763909	87.3	7530374	12.7
2008	61191700	52074137	85.1	9117563	14.9
2009	63149835	50709318	80.3	12440517	19.7
2010	65170629	57089471	78.6	8081158	21.4
2011	67256090	51181884	76.1	16074206	23.9
2012	69105775	50170793	72.6	18934982	27.4
2013	71105800	53542667	75.3	17563133	24.7
2014	72931608	55209227	75.7	17722381	24.3
2015	76957923	54486209	70.8	22471714	29.2
2016	81151885	52586421	64.8	28565464	35.2
Total	744771229	579202686	854.3	165568543	245.7
Av.	67706475	52654790	77.7	15051686	22.3

Source: [3]

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Table 3: Unemployment Probabilities

		Unemployed	**%**	**Year**
1	0	0	0	2016
2	0.0716	3814918.956	7.16	2017
3	0.138	7352776.758	13.8	2018
4	0.1997	10640213.9	19.97	2019
5	0.257	13693214.69	25.7	2020
6	0.3101	16522435.31	31.01	2021
7	0.3595	19154516.26	35.95	2022
8	0.4054	21600113.75	40.54	2023
9	0.4479	23864555.87	44.79	2024
10	0.4874	25969155.01	48.74	2025
11	0.5241	27924567.38	52.41	2026
12	0.5582	29741449.18	55.82	2027
13	0.5898	31425128.49	58.98	2028
14	0.6191	32986261.53	61.91	2029
15	0.6464	34440832.58	64.64	2030
16	0.6717	35788841.65	67.17	2031
17	0.6952	37040944.94	69.52	2032
18	0.717	38202470.55	71.7	2033
19	0.7373	39284074.66	73.73	2034
20	0.7561	40285757.3	75.61	2035
21	0.7735	41212846.54	77.35	2036
22	0.7897	42075998.59	78.97	2037
23	0.8048	42880541.56	80.48	2038
24	0.8188	43626475.43	81.88	2039
25	0.8317	44313800.21	83.17	2040
26	0.8438	44958500.21	84.38	2041
27	0.8549	45549919.21	85.49	2042
28	0.8653	46104041.51	86.53	2043
29	0.875	46620867.13	87.5	2044
30	0.8839	47095067.94	88.39	2045

Unemployment Probabilities

		Unemployed	**%**	**Year**
31	0.8922	47537300.17	89.22	2046
32	0.8999	47947563.8	89.99	2047
33	0.9071	48331186.94	90.71	2048
34	0.9137	48682841.48	91.37	2049
35	0.9199	49013183.62	91.99	2050
36	0.9256	49316885.27	92.56	2051
37	0.931	49604602.62	93.1	2052
38	0.9359	49865679.48	93.59	2053
39	0.9405	50110772.04	94.05	2054
40	0.9448	50339880.3	94.48	2055
41	0.9487	50547676.16	94.87	2056
42	0.9524	50744815.83	95.24	2057
43	0.9558	50925971.2	95.58	2058
44	0.959	51096470.37	95.9	2059
45	0.9619	51250985.24	96.19	2060
46	0.9646	51394843.92	96.46	2061
47	0.9671	51528046.4	96.71	2062
48	0.9695	51655920.77	96.95	2063
49	0.9717	51773138.95	97.17	2064
50	0.9737	51879700.94	97.37	2065
51	0.9756	51980934.82	97.56	2066
52	0.9773	52071512.5	97.73	2067
53	0.979	52162090.19	97.9	2068
54	0.9805	52242011.68	98.05	2069
55	0.9819	52316605.06	98.19	2070
56	0.9832	52385870.35	98.32	2071
57	0.9844	52449807.54	98.44	2072
58	0.9855	52508416.63	98.55	2073
59	0.9865	52561697.62	98.65	2074

Unemployment Probabilities

		Unemployed	%	Year
60	0.9875	52614978.61	98.75	2075
61	0.9884	52662931.5	98.84	2076
62	0.9892	52705556.3	98.92	2077
63	0.99	52748181.09	99	2078
64	0.9907	52785477.78	99.07	2079
65	0.9914	52822774.48	99.14	2080
66	0.992	52854743.07	99.2	2081
67	0.9926	52886711.67	99.26	2082
68	0.9931	52913352.16	99.31	2083
69	0.9936	52939992.66	99.36	2084
70	0.994	52961305.05	99.4	2085
71	0.9945	52987945.55	99.45	2086
72	0.9949	53009257.95	99.49	2087
73	0.9952	53025242.24	99.52	2088
74	0.9956	53046554.64	99.56	2089
75	0.9959	53062538.94	99.59	2090
76	0.9962	53078523.23	99.62	2091
77	0.9965	53094507.53	99.65	2092
78	0.9967	53105163.73	99.67	2093
79	0.9969	53115819.93	99.69	2094
80	0.9972	53131804.23	99.72	2095
81	0.9974	53142460.42	99.74	2096
82	0.9976	53153116.62	99.76	2097
83	0.9977	53158444.72	99.77	2098
84	0.9979	53169100.92	99.79	2099
85	0.998	53174429.02	99.8	2100
86	0.9982	53185085.22	99.82	2101
87	0.9983	53190413.32	99.83	2102
88	0.9984	53195741.41	99.84	2103
89	0.9985	53201069.51	99.85	2104
90	0.9987	53211725.71	99.87	2105
91	0.9987	53211725.71	99.87	2106
92	0.9988	53217053.81	99.88	2107
93	0.9989	53222381.91	99.89	2108
94	0.999	53227710.01	99.9	2109
95	0.9991	53233038.11	99.91	2110
96	0.9991	53233038.11	99.91	2111
97	0.9992	53238366.21	99.92	2112
98	0.9993	53243694.31	99.93	2113
99	0.9993	53243694.31	99.93	2114
100	0.9994	53249022.41	99.94	2115
101	0.9994	53249022.41	99.94	2116

Source: [3]