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Some algebraic properties of soft quasigroups

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Abstract

Soft set theory, instituted by Molodsov is a robust mathematical tool for dealing with uncertainties and problems that are questionably defined. Our aim in this paper is to broaden soft theory by studying some new algebraic operations (product and product divisions) on soft quasigroups. We define soft quasigroups and introduce different algebraic properties connected with them. It was shown that for soft quasigroups over medial quasigroups, the resulting soft sets based on product and product divisions are also soft quasigroups over medial quasigroups. With the new operations defined on soft quasigroups, it was shown that these new operations are distributive over restricted/extended union and intersection operations.

Keywords and Phrases: Soft sets, quasigroups, parastrophes, soft quasigroups, soft subquasigroups, soft parastrophes.
MSC2010:Primary 20N05, 03E72; Secondary 03E75

1 Introduction

Quasigroups are algebraic structures which are not groups by reason of non-associative properties. The study of the theory of quasigroups and loops started over two centuries ago in an effort to prove Euler's 1782 conjecture that there does not exist certain pairs of mutually orthogonal Latin squares. The theory of soft sets, founded by Molodtsov [1], is a special type of generalisation of set theory for the study and formal modelling of mathematical problems characterized by uncertainties and vagueness due to partial and fragmented information. There exist some mathematical tools for formal modelling, reasoning and computing uncertainties such as fuzzy set theory, rough set theory and neutrosophic set theory (to mention few), but one important property of soft set that makes it different from other tools is that of its flexibility by using parametrisation tool, for instance membership grade that is necessary in fuzzy set and approximation grade in Rough set are not needed in soft set theory. The conclusion for these difficulties is the insufficiency of the parametrization tool of those existing theories. Soft set theory has a rich prospect for applications in many mathematical areas as shown by Molodtsov [1] in his pioneer work.

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Encouraged by the study of algebraic properties of soft sets, our aim in this paper is to institute research on the connections between soft sets and quasigroups structures. To define some related algebraic operations such as union and intersection of soft quasigroups with the introduction of new operations and investigate the distributivity of the latter over the former.

2 Preliminaries

In this section, we review some notions and results concerning quasigroups and soft sets.

2.1 Quasigroup

Definition 2.1 *(Groupoid, Quasigroup, Chein et al. [2]) Let G be a non-empty set. Define a binary operation $(\cdot)$ on G. If $x\cdot y\in G$ for all $x,y\in G$, if there exists $a,b\in G$, then the pair $(G,\cdot)$ is called a groupoid or Magma. If each of the equations:*

$$a\cdot x=b \qquad \textit{and} \qquad y\cdot a=b$$

has unique solutions in G for x and y respectively, then $(G,\cdot)$ is called a quasigroup.

If G has a unique element $e\in G$ such that for all $x\in G$, $x\cdot e=e\cdot x=x$, $(G,\cdot)$, then G is called a loop.

We write xy instead of $x\cdot y$, and stipulate that $\cdot$ has lower priority than juxtaposition among factors to be multiplied. For instance, $x\cdot yz$ stands for $x(yz)$.

If x be a fixed element in a groupoid $(G,\cdot)$, the left and right translation maps of G, L_x and R_x respectively are defined by

$$yL_x=x\cdot y \qquad \text{and} \qquad yR_x=y\cdot x.$$

A groupoid $(G,\cdot)$ is considered a quasigroup if its left and right translation mappings are permutations. Since the left and right translation mappings of a quasigroup are bijective, then the inverse mappings L_x^{-1} and R_x^{-1} exist. Therefore

$$x\backslash y=yL_x^{-1} \qquad \text{and} \qquad x/y=xR_y^{-1}$$

and note that

$$x\backslash y=z\Leftrightarrow x\cdot z=y \qquad \text{and} \qquad x/y=z\Leftrightarrow z\cdot y=x.$$

Definition 2.2 *A quasigroup $(G,\cdot,/,\backslash)$ is a non-empty set G together with three binary operations $(\cdot)$, $(/)$, $(\backslash)$ such that*

(i) *$x\cdot(x\backslash y)=y$, $(y/x)\cdot x=y$ for all $x,y\in G$ and*

(ii) *$x\backslash(x\cdot y)=y$, $(y\cdot x)/x=y$ for all $x,y\in G$.*

For more on quasigroups, readers can check [3, 4, 5, 6, 7, 8].

Definition 2.3 *(Subgroupoid, Subquasigroup, Bruck [3])*
Let $(Q,\cdot,/,\backslash)$ be a groupoid (quasigroup) and $\emptyset\neq H\subseteq Q$. Then, H is called a subgroupoid (subquasigroup) of Q if $(H,\cdot)$ is a groupoid (quasigroup). This is often expressed as $H\le Q$.
Let $\emptyset\neq A,B\subseteq Q$, then

$$A\cdot B=\{a\cdot b|a\in A,\ b\in B\},\ \ A/B=\{a/b|a\in A,\ b\in B\},\ \ A\backslash B=\{a\backslash b|a\in A,\ b\in B\}.$$

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$\star$	1	2	3	4	5	6	7
1	1	4	7	3	6	2	5
2	3	6	2	5	1	4	7
3	5	1	4	7	3	6	2
4	7	3	6	2	5	1	4
5	2	5	1	4	7	3	6
6	4	7	3	6	2	5	1
7	6	2	5	1	4	7	3

Table 1: A medial quasigroup of order 7

Example 2.1 *$(\mathbb{Z}, -, /, \backslash)$ is a non-associative quasigroup with $x/y = x + y$ and $x\backslash y = x - y$.*

Definition 2.4 *(Distributive Quasigroup, Entropic or Medial Quasigroup, Chein et al. [2])*

1. *A quasigroup $(Q, \cdot)$ is called a distributive quasigroup if it satisfies the following relations:*

$$x(yz) = xy \cdot xz \qquad \textit{left distributive law} \tag{2.1.1}$$

$$(yz)x = yx \cdot zx \qquad \textit{right distributive law} \tag{2.1.2}$$

If $(Q, \cdot)$ obeys (2.1.1) or (2.1.2), it is called a left (or right) distributive quasigroup.

2. *A quasigroup $(Q, \cdot)$ is called a medial or entropic quasigroup if it satisfies the following relation:*

$$(xy)(uv) = (xu)(yv) \qquad \textit{medial or entropic law} \tag{2.1.3}$$

3. *A quasigroup $(Q, \cdot)$ is called an idempotent quasigroup if it satisfies the following relation:*

$$xx = x \qquad \textit{idempotent law} \tag{2.1.4}$$

Construction of Medial Quasigroup A medial quasigroup can be constructed using any abelian group. Let $(G, +)$ be an abelian group. Define $\star$ on G as follows

$$x \star y = \alpha(a) + \beta(b) + c, \tag{2.1.5}$$

where c is a fixed element in G and α, β are automorpisms of $(G, +)$, such that $\alpha\beta = \beta\alpha$. Then $(G, \star)$ is a medial quasigroup.
A quasigroup $(G, \star)$ of the form (2.1.4) which does not necessarily obey the condition $\alpha\beta = \beta\alpha$ is said to be affine over $(G, +)$, and referred to as central quasigroup or T-quasigroup. The fundamental Toyoda-Bruck Theorem states that, up to isomorphism, medial quasigroups are precisely central quasigroups with commuting automorphisms.

A finite example of a medial quasigroup is given in Table 1.

Theorem 2.1 *(Pflugfelder [6], Stein [7])*

1. *If $(Q, \cdot, /, \backslash)$ is a distributive quasigroup, the following are true for all $x, y, z \in Q$:*
   (a) $(Q, \cdot)$ is idempotent.
   (b) $L(x)$ and $R(x)$ are automophisms of $(Q, \cdot)$ and $L(x)R(x) = R(x)L(x)$.
   (c) $x(y\backslash z) = (xy)\backslash(xz)$, $(y/z)x = (yx)/(zx)$, $x\backslash(yz) = (x\backslash y)(x\backslash z)$, $(yz)/x = (y/x)(z/x)$.

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(d) $(Q, /)$ and $(Q, \backslash)$ *are distributive quasigroups.*

2. *Let* $(Q, \cdot, /, \backslash)$ *be a medial quasigroup:*

   (a) $(Q, \cdot)$ *is a distributive quasigroup if and only if* $(Q, \cdot)$ *is idempotent.*

   (b) $(Q, /)$ *and* $(Q, \backslash)$ *are medial quasigroups.*

Kinyon and Phillips [8] discovered new equations that axiomatize the variety of trimedial quasigroups (quasigroups in which every subquasigroup generated by three elements is medial). They considered a generalization of trimedial quasigroups i.e. semimedial quasigroups. They showed that right semimedial, left F-quasigroups are trimedial. Fiala [9] improved upon their work by deducing a shorter axiom for trimedial quasigroups.
Some recent works on finite medial quasigroups and their constructions were reported in [10, 11, 12, 13].
Jaiyéọlá [14], Jaiyéọlá,[15], Oyem et al. [16], Wall [17] investigated some other properties of quasigroups and Oyem et al. [16] studied the parastrophy in soft quasigroups and identified distributive quasigroups to be a class of quasigroup in which parastrophy is found to be very meaningful in soft quasigroups.

2.2 Soft Sets and Operations

We begin with the definition of soft set and operations defined on it. We refer readers to [1, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28] for earlier works on soft sets, soft groups and their operations.

Definition 2.5 *(Soft Sets, Soft Subset)*
Let Q be a set (initial universe) and E be a set of parameters. For $A \subset E$, the pair (F, A) is called a soft set over Q if $F(a) \subset Q$ for all $a \in A$, where F is a function mapping A into the set of all non-empty subsets of Q, i.e $F : A \longrightarrow 2^Q \backslash \{\emptyset\}$.
Let (F, A) and (H, B) be two soft sets over a common universe U, then (H, B) is called a soft subset of (F, A) if

1. $B \subseteq A$; *and*

2. $H(x) \subseteq F(x)$ *for all* $x \in B$.

This is usually expressed as $(H, B) \subset (F, A)$ or $(F, A) \supset (H, B)$, and (F, A) is said to be a soft super set of (H, B).

Definition 2.6 *(Restricted Intersection)*
Let (F, A) and (G, B) be two soft sets over a common universe Q such that $A \cap B \neq \emptyset$. Then their restricted intersection is $(F, A) \cap_R (G, B) = (H, C)$ where (H, C) is defined as $H(c) = F(c) \cap G(c)$ for all $c \in C$, where $C = A \cap B \neq \emptyset$.

Definition 2.7 *(Extended Intersection) The extended intersection of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $x \in C$, $H(x)$ is defined as*

$$H(x) = \begin{cases} F(x) & \text{if} \quad x \in A - B \\ G(x) & \text{if} \quad x \in B - A \\ F(x) \cap G(x) & \text{if} \quad x \in A \cap B. \end{cases}$$

This relation is denoted by $(H, C) = (F, A) \cap_E (G, B)$

Definition 2.8 *(Restricted Union)*
The union of two soft sets (F, A) and (G, B) over a common universe U is denoted as $(F, A) \cup_R (G, B) = (H, C)$ where (H, C) is a soft set over U such that $C = A \cap B$ and $H(c) = F(c) \cup G(c)$ for all $c \in C$.

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Definition 2.9 *(Extended Union)*
The union of two soft sets (F,A) and (G,B) over U is denoted by $(F,A) \cup_E (G,B) = (H,C)$ where (H,C) is a soft set over U such that $C = A \cup B$ and

$$H(x) = \begin{cases} F(x) & if \quad x \in A - B \\ G(x) & if \quad x \in B - A \\ F(x) \cup G(x) & if \quad x \in A \cap B. \end{cases}$$

Definition 2.10 *(The AND operation)*
The AND-operation of two soft sets over a common universe U is denoted (F,A) AND (G,B) or $(F,A) \wedge (G,B)$ and is a soft set over U defined by $(F,A) \wedge (G,B) = (H, A \times B)$ such that $H(a,b) = F(a) \cap G(b)$ for $(a,b) \in A \times B$.

Definition 2.11 *(The OR-operation)*
The OR-operation of two soft sets over a common universe U denoted by (F,A) OR (G,B) or $(F,A) \vee (G,B)$ is a soft set over U defined by $(F,A) \vee (G,B) = (H, A \times B)$ such that $H(a,b) = F(a) \cup G(b)$ for $(a,b) \in A \times B$.

3 Main Results

We now present some new results on soft quasigroups by extension and generalization of soft sets and soft groups given by Molodtsov [1], Aktas and Cagman [18], Aslihan et al. [20], Babitha and Sunil [21], Maji et al. [23], Sezgin et al. [29], Pei [30], Ping et al. [31].

3.1 Soft Groupoid and Soft Quasigroup

Definition 3.1 *(Soft: Groupoid and Quasigroup)*
Let Q be a groupoid (quasigroup) and E be a set of parameters. For $A \subset E$, the pair $(F,A)_Q$ will be called a soft groupoid (quasigroup) over Q if $F(a)$ is a subgroupoid (subquasigroup) of Q for all $a \in A$, where $F : A \longrightarrow 2^Q \backslash \{\emptyset\}$.

$\cdot$	i	j	k	l	m	n	o	p
i	i	j	k	l	m	n	o	p
j	j	i	l	k	n	m	p	o
k	k	l	i	j	o	p	n	m
l	l	k	j	i	p	o	m	n
m	n	m	p	o	j	i	l	k
n	m	n	o	p	i	j	k	l
o	p	o	m	n	k	l	i	j
p	o	p	n	m	l	k	j	i

Table 2: Quasigroup $(Q, \cdot)$ of order 8

Example 3.1 *Table 2 represents the Latin square of a finite quasigroup $(Q, \cdot)$, $Q = \{i,j,k,l,m,n,o,p\}$ and let $A = \{\gamma_1, \gamma_2, \gamma_3\}$ be any set of parameters. If $F : A \longrightarrow 2^Q \backslash \{\emptyset\}$ is defined by*

$$F(\gamma_1) = \{i,j\}, \; F(\gamma_2) = \{i,j,k,l\}, \; F(\gamma_3) = \{i,j,o,p\},$$

then, the pair (F,A) is a soft quasigroup over quasigroup $(Q, \cdot)$ because $F(\gamma_i) \leq (Q, \cdot)$, $i = 1,2,3$.

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Example 3.2 *Table 2 represents the Latin square of a finite quasigroup* $(Q, \cdot)$, $Q = \{i, j, k, l, m, n, o, p\}$ *and let* $B = \{\gamma_1, \gamma_2, \gamma_3\}$ *be a set of parameters. If* $F : B \longrightarrow 2^Q \backslash \{\emptyset\}$ *is defined by*

$$G(\gamma_1) = \{i\},\ G(\gamma_2) = \{i, j\},\ G(\gamma_3) = \{i, j, m, n\},\ G(\gamma_4) = Q,$$

then, the pair (G, B) *is a soft quasigroup over quasigroup* $(Q, \cdot)$ *because* $G(\gamma_i) \leq (Q, \cdot),\ i = 1, 2, 3.$

Definition 3.2 *(Soft Subquasigroup and Soft Equality)*
Let (F, A) *and* (G, B) *be two soft quasigroups over a common quasigroup* Q*, then*

1. (F, A) *is a soft subquasigroup of* (G, B)*, denoted* $(F, A) \leq (G, B)$ *, if*
   (a) $A \subseteq B$*, and*
   (b) $F(a) \subseteq G(a)$*, for all* $a \in A$.
2. (F, A) *is soft equal to* (G, B) *denoted* $(F, A) = (G, B)$*, whenever* $(F, A) \subseteq (G, B)$ *and* $(G, B) \subseteq (F, A)$.

Example 3.3 *Table 2 represents a Latin square of a quasigroup* $(Q, \cdot)$, $Q = \{i, j, k, l, m, n, o, p\}$ *on* 8 *symbols. Let* $E = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6\}$ *and let* $A = \{\gamma_1, \gamma_2, \gamma_3\} \subset E$ *and* $\mathfrak{B} = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\} \subset E$ *be two sets of parameters such that*

$$F(\gamma_1) = \{i\},\ F(\gamma_2) = \{i, j\},\ F(\gamma_3) = \{i, j, o, p\} \text{ and}$$
$$G(\gamma_1) = \{i, j\},\ G(\gamma_2) = \{i, j, k, l\},\ G(\gamma_3) = Q = G(\gamma_4).$$

Then, $(F, A) \subseteq (G, B)$ *since* $A \subseteq B$ *and* $F(\gamma) \subseteq G(\gamma)$ *for all* $\gamma \in A$*. But,* $(G, B) \nsubseteq (F, A)$*. Hence,* $(F, A) \neq (G, B)$.

Remark 3.1 *Example 3.3 shows that two soft quasigroups over the same quasigroups need not be equal.*

3.2 Soft Algebraic Operations on Soft Quasigroups

The following example illustrates some algebraic operations on soft quasigroups.

Example 3.4 *Consider Table 2 as a Latin square of a quasigroup* $(Q, \cdot)$, $Q = \{i, j, k, l, m, n, o, p\}$ *on* 8 *symbols. Let* $E = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6\}$ *and let* $A = \{\gamma_1, \gamma_2, \gamma_3\} \subset E$ *and* $B = \{\gamma_2, \gamma_3, \gamma_4, \gamma_5\} \subset E$ *be two sets of parameters such that*

$$F(\gamma_1) = \{i, j\},\ F(\gamma_2) = \{i, j, k, l\},\ F(\gamma_3) = \{i, j, o, p\} \text{ and}$$
$$G(\gamma_2) = \{i, j, k, l\},\ G(\gamma_3) = \{i, j, o, p\},\ G(\gamma_4) = \{i, j\},\ G(\gamma_5) = \{i, j, m, n\}.$$

Extended Intersection:

$$H(c) = \begin{cases} F(c), & \text{if} \quad c \in A - B \\ G(c), & \text{if} \quad c \in B - A \\ F(c) \cap G(c), & \text{if} \quad c \in A \cap B. \end{cases}$$

$(F, A) \cap_E (G, B) = (H, C) = \{H(c)\}_{c \in A-B, B-A, A \cap B} = \Big\{F(\gamma_1) = \{i, j\}; F(\gamma_2) \cap G(\gamma_2) = \{i, j, k, l\}; F(\gamma_3) \cap G(\gamma_3) = \{i, j, o, p\}; G(\gamma_4) = \{i, j\}; G(\gamma_5) = \{i, j, m, n\}\Big\}.$

Restricted Intersection: $(F, A) \cap_R (G, B) = (H, C) = \{H(c)\}_{c \in A \cap B} = \Big\{F(\gamma_2) \cap G(\gamma_2) = \{i, j, k, l\}; F(\gamma_3) \cap G(\gamma_3) = \{i, j, o, p\}\Big\}.$

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Restricted Union: $(F,A)\cup_R(G,B) = (H,C) = \{H(c)\}_{c\in A\cap B} = \Big\{F(\gamma_2)\cup G(\gamma_2) = \{i,j,k,l\}; F(\gamma_3)\cup G(\gamma_3) = \{i,j,o,p\}\Big\}$.

Extended Union: $(F,A)\cup_E(G,B) = (H,C) = \{H(c)\}_{c\in A-B,B-A,A\cap B} = \Big\{F(\gamma_1) = \{i,j\}; F(\gamma_2)\cup G(\gamma_2) = \{i,j,k,l\}; F(\gamma_3)\cup G(\gamma_3) = \{i,j,o,p\}; G(\gamma_4) = \{i,j\}; G(\gamma_5) = \{i,j,m,n\}\Big\}$.

AND-Operation: $(F,A)\wedge(G,B) = (H,kA\times B) = \{H(a,b) = F(a)\cap G(b)\}_{(a,b)\in A\times B} = \Big\{H(\gamma_1,\gamma_2) = F(\gamma_1)\cap G(\gamma_2) = \{i,j\}; H(\gamma_1,\gamma_3) = F(\gamma_1)\cap G(\gamma_3) = \{i,j\}; H(\gamma_1,\gamma_4) = F(\gamma_1)\cap G(\gamma_4) = \{i,j\}; H(\gamma_1,\gamma_5) = F(\gamma_1)\cap G(\gamma_5) = \{i,j\}; H(\gamma_2,\gamma_2) = F(\gamma_2)\cap G(\gamma_2) = \{i,j,k,l\}; H(\gamma_2,\gamma_3) = F(\gamma_2)\cap G(\gamma_3) = \{i,j\}; H(\gamma_2,\gamma_4) = F(\gamma_2)\cap G(\gamma_4) = \{i,j\}; H(\gamma_2,\gamma_5) = F(\gamma_2)\cap G(\gamma_5) = \{i,j\}; H(\gamma_3,\gamma_2) = F(\gamma_3)\cap G(\gamma_2) = \{i,j\}; H(\gamma_3,\gamma_3) = F(\gamma_3)\cap G(\gamma_3) = \{i,j,o,p\}; H(\gamma_3,\gamma_4) = F(\gamma_3)\cap G(\gamma_4) = \{i,j\}; H(\gamma_3,\gamma_5) = F(\gamma_3)\cap G(\gamma_5) = \{i,j\}\Big\}$.

OR-Operation: $(F,A)\vee(G,B) = (H,A\times B) = \{H(a,b) = F(a)\cup G(b)\}_{(a,b)\in A\times B} = \Big\{H(\gamma_1,\gamma_2) = F(\gamma_1)\cup G(\gamma_2) = \{i,j,k,l\}; H(\gamma_1,\gamma_3) = F(\gamma_1)\cup G(\gamma_3) = \{i,j,o,p\}; H(\gamma_1,\gamma_4) = F(\gamma_1)\cup G(\gamma_4) = \{i,j\}; H(\gamma_1,\gamma_5) = F(\gamma_1)\cup G(\gamma_5) = \{i,j,m,n\}; H(\gamma_2,\gamma_2) = F(\gamma_2)\cup G(\gamma_2) = \{i,j,k,l\}; H(\gamma_2,\gamma_3) = F(\gamma_2)\cup G(\gamma_3) = \{i,j,k,l,o,p\}; H(\gamma_2,\gamma_4) = F(\gamma_2)\cup G(\gamma_4) = \{i,j,k,l\}; H(\gamma_2,\gamma_5) = F(\gamma_2)\cup G(\gamma_5) = \{i,j,k,l,m,n\}; H(\gamma_3,\gamma_2) = F(\gamma_3)\cup G(\gamma_2) = \{i,j,k,l,o,p\}; H(\gamma_3,\gamma_3) = F(\gamma_3)\cup G(\gamma_3) = \{i,j,o,p\}; H(\gamma_3,\gamma_4) = F(\gamma_3)\cup G(\gamma_4) = \{i,j,o,p\}; H(\gamma_3,\gamma_5) = F(\gamma_3)\cup G(\gamma_5) = \{i,j,m,n,o,p\}\Big\}$.

Remark 3.2 *Note that going by some of the operations in Example 3.4 on soft quasigroups, the resulting soft sets may not necessarily be a soft quasigroup over the quasigroup. For instance, for $(F,A)\vee(G,B)$, we have $H(\gamma_2,\gamma_5) = F(\gamma_2)\cup G(\gamma_5) = \{i,j,k,l,m,n\}$ and $H(\gamma_3,\gamma_2) = F(\gamma_3)\cup G(\gamma_2) = \{i,j,k,l,o,p\}$ which are not subquasigroups of $(Q,\cdot)$.*

Definition 3.3 *(Product and Divisions of Soft Quasigroups)*
Let (F,A) and (G,B) be any two soft quasigroups over a quasigroup $(Q,\cdot,/,\backslash)$.

1. *The product $(F,A)\cdot(G,B)$ of (F,A) and (G,B) is a soft set (H,C) over $(Q,\cdot)$ such that $H(x) = F(x)\cdot G(x)$ for all $x\in C = A\cap B$.*
2. *The right division product $(F,A)/(G,B)$ of (F,A) and (G,B) is a soft set (H,C) over $(Q,/)$ such that $H(x) = F(x)/G(x)$ for all $x\in C = A\cap B$.*
3. *The left division product $(F,A)\backslash(G,B)$ of (F,A) and (G,B) is a soft set (H,C) over $(Q,\backslash)$ such that $H(x) = F(x)\backslash G(x)$ for all $x\in C = A\cap B$.*

Theorem 3.1 *Let (F,A) and (G,B) be any two soft quasigroups over a medial quasigroup $(Q,\cdot,/,\backslash)$. Then:*

1. *$(F,A)\cdot(G,B)$ is a soft quasigroup over the medial quasigroup $(Q,\cdot)$.*
2. *$(F,A)/(G,B)$ is a soft quasigroup over medial quasigroup $(Q,/)$.*
3. *$(F,A)\backslash(G,B)$ is a soft quasigroup over medial quasigroup $(Q,\backslash)$.*

Proof 3.1 *1. Let $(F,A)\cdot(G,B) = (H,C)$ such that $H(x) = F(x)\cdot G(x)$ for all $x\in C = A\cap B$. For any $x\in C$, let $h_1,h_2\in H(x) = F(x)\cdot G(x)$, then $h_1 = f_1g_1$ and $h_2 = f_2g_2$ for $f_1,f_2\in F(x)$ and $g_1,g_2\in G(x)$. So, going by (2.1.3), $h_1\cdot h_2 = (f_1g_1)(f_2g_2) = (f_1f_2)(g_1g_2)\in F(x)\cdot G(x) = H(x)$. Thus, $H(x)\le(Q,\cdot)$ for any $x\in C$ and so $(F,A)\cdot(G,B)$ is a soft quasigroup over the medial quasigroup $(Q,\cdot)$.*

2. *Let* $(F,A)/(G,B) = (H,C)$ *such that* $H(x) = F(x)/G(x)$ *for all* $x \in C = A \cap B$. *For any* $x \in C$, *let* $h_1, h_2 \in H(x) = F(x)/G(x)$, *then* $h_1 = f_1/g_1$ *and* $h_2 = f_2/g_2$ *for* $f_1, f_2 \in F(x)$ *and* $g_1, g_2 \in G(x)$. *So, going by Theorem 2.1(2b) and (2.1.3) for* $(Q,/)$, $h_1/h_2 = (f_1/g_1)/(f_2/g_2) = (f_1/f_2)/(g_1/g_2) \in F(x)/G(x) = H(x)$. *Thus,* $H(x) \leq (Q,/)$ *for any* $x \in C$ *and so* $(F,A)/(G,B)$ *is a soft quasigroup over the medial quasigroup* $(Q,/)$ *based on Theorem 2.1(2b).*

3. *Let* $(F,A)\backslash(G,B) = (H,C)$ *such that* $H(x) = F(x)\backslash G(x)$ *for all* $x \in C = A \cap B$. *For any* $x \in C$, *let* $h_1, h_2 \in H(x) = F(x)\backslash G(x)$, *then* $h_1 = f_1\backslash g_1$ *and* $h_2 = f_2\backslash g_2$ *for* $f_1, f_2 \in F(x)$ *and* $g_1, g_2 \in G(x)$. *So, going by Theorem 2.1(2b) and (2.1.3) for* $(Q,\backslash)$, $h_1\backslash h_2 = (f_1\backslash g_1)\backslash(f_2\backslash g_2) = (f_1\backslash f_2)\backslash(g_1\backslash g_2) \in F(x)\backslash G(x) = H(x)$. *Thus,* $H(x) \leq (Q,\backslash)$ *for any* $x \in C$ *and so* $(F,A)\backslash(G,B)$ *is a soft quasigroup over the medial quasigroup* $(Q,\backslash)$ *based on Theorem 2.1(2b).*

Theorem 3.2 *Let* $(F,A), (G,B)$ *and* (H,C) *be soft quasigroups over a quasigroup* $(Q,\cdot)$.

1. $(F,A)\cdot\Big((G,B)\cap_R(H,C)\Big) = \Big((F,A)\cdot(G,B)\Big)\cap_R\Big((F,A)\cdot(H,C)\Big)$.
2. $\Big((G,B)\cap_R(H,C)\Big)\cdot(F,A) = \Big((G,B)\cdot(F,A)\Big)\cap_R\Big((H,C)\cdot(F,A)\Big)$.
3. $(F,A)\cdot\Big((G,B)\cap_E(H,C)\Big) = \Big((F,A)\cdot(G,B)\Big)\cap_E\Big((F,A)\cdot(H,C)\Big)$.
4. $\Big((G,B)\cap_E(H,C)\Big)\cdot(F,A) = \Big((G,B)\cdot(F,A)\Big)\cap_E\Big((H,C)\cdot(F,A)\Big)$.
5. $(F,A)\cdot\Big((G,B)\cup_R(H,C)\Big) = \Big((F,A)\cdot(G,B)\Big)\cup_R\Big((F,A)\cdot(H,C)\Big)$.
6. $\Big((G,B)\cup_R(H,C)\Big)\cdot(F,A) = \Big((G,B)\cdot(F,A)\Big)\cup_R\Big((H,C)\cdot(F,A)\Big)$.
7. $(F,A)\cdot\Big((G,B)\cup_E(H,C)\Big) = \Big((F,A)\cdot(G,B)\Big)\cup_E\Big((F,A)\cdot(H,C)\Big)$.
8. $\Big((G,B)\cup_E(H,C)\Big)\cdot(F,A) = \Big((G,B)\cdot(F,A)\Big)\cup_E\Big((H,C)\cdot(F,A)\Big)$.

Proof 3.2 1. *Let* $(G,B)\cap_R(H,C) = (I,D)$ *such that* $I(x) = G(x)\cap H(x)$ *for all* $x \in D = B\cap C$. *Let* $(F,A)\cdot(I,D) = (J,Q)$ *such that* $J(x) = F(x)\cdot I(x)$ *for all* $x \in Q = A\cap D = A\cap B\cap C$. *Hence,*

$$J(x) = F(x)\cdot\big(G(x)\cap H(x)\big)\ \forall\ x \in Q = A\cap B\cap C \tag{3.2.1}$$

Let $(F,A)\cdot(G,B) = (K,E)$ *such that*

$$K(x) = F(x)\cdot G(x)\ \forall\ x \in E = A\cap B \tag{3.2.2}$$

Let $(F,A)\cdot(H,C) = (L,M)$ *such that*

$$L(x) = F(x)\cdot H(x)\ \forall\ x \in M = A\cap C \tag{3.2.3}$$

Let $\Big((F,A)\cdot(G,B)\Big)\cap_R\Big((F,A)\cdot(H,C)\Big) = (K,E)\cap_R(L,M) = (N,P)$. *Then, based on (3.2.2) and (3.2.3), we have* $N(x) = K(x)\cap L(x)$ *for all* $x \in P = E\cap M$ *which gives* $N(x) = (F(x)\cdot G(x))\cap(F(x)\cdot H(x))\ \forall\ x \in P = (A\cap B)\cap(A\cap C)$, *whence*

$$N(x) = F(x)\cdot\big(G(x)\cap H(x)\big)\ \forall\ x \in P = A\cap B\cap C \tag{3.2.4}$$

Comparing (3.2.1) and (3.2.4), we get $(J,Q) = (N,P)$. *Therefore,* $(F,A)\cdot\Big((G,B)\cap_R(H,C)\Big) = \Big((F,A)\cdot(G,B)\Big)\cap_R\Big((F,A)\cdot(H,C)\Big)$.

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2. *Let* $(G,B)\cap_R(H,C)=(I,D)$ *such that* $I(x)=G(x)\cap H(x)$ *for all* $x\in D=B\cap C$. *Let* $(I,D)\cdot(F,A)=(J,Q)$ *such that* $J(x)=I(x)\cdot F(x)$ *for all* $x\in Q=A\cap D=A\cap(B\cap C)$. *Hence,*
$$J(x)=\big(G(x)\cap H(x)\big)\cdot F(x)\ \forall\ x\in Q=A\cap B\cap C \tag{3.2.5}$$
Let $(G,B)\cdot(F,A)=(K,E)$ *such that*
$$K(x)=G(x)\cdot F(x)\ \forall\ x\in E=B\cap A \tag{3.2.6}$$
Let $(H,C)\cdot(F,A)=(L,M)$ *such that*
$$L(x)=H(x)\cdot F(x)\ \forall\ x\in M=C\cap A \tag{3.2.7}$$
Let $\Big((G,B)\cdot(F,A)\Big)\cap_R\Big((H,C)\cdot(F,A)\Big)=(K,E)\cap_R(L,M)=(N,P)$. *Then, based on* (3.2.6) *and* (3.2.7), *we have* $N(x)=K(x)\cap L(x)$ *for all* $x\in P=E\cap M$ *which gives* $N(x)=(G(x)\cdot F(x))\cap(H(x)\cdot F(x))\ \forall\ x\in P=(B\cap A)\cap(C\cap A)$, *whence*
$$N(x)=\big(G(x)\cap H(x)\big)\cdot F(x)\ \forall\ x\in P=A\cap B\cap C \tag{3.2.8}$$
Comparing (3.2.5) *and* (3.2.8), *we get* $(J,Q)=(N,P)$. *Therefore,* $\Big((G,B)\cap_R(H,C)\Big)\cdot(F,A)=\Big((G,B)\cdot(F,A)\Big)\cap_R\Big((H,C)\cdot(F,A)\Big)$.

3. *Let* $(G,B)\cap_E(H,C)=(I,D)$ *such that* $D=B\cup C$ *and*
$$I(x)=\begin{cases}G(x) & \text{if}\quad x\in B-C\\ H(x) & \text{if}\quad x\in C-B\\ G(x)\cap H(x) & \text{if}\quad x\in B\cap C\end{cases}$$
Let $(F,A)\cdot(I,D)=(J,E)$ *such that* $E=A\cap D=A\cap(B\cup C)$ *and*
$$J(x)=F(x)\cdot I(x)=\begin{cases}F(x)\cdot G(x) & \text{if}\quad x\in A\cap(B-C)\\ F(x)\cdot H(x) & \text{if}\quad x\in A\cap(C-B)\\ F(x)\cdot(G(x)\cap H(x)) & \text{if}\quad x\in A\cap(B\cap C)\end{cases} \tag{3.2.9}$$
Let $(F,A)\cdot(G,B)=(K,L)$ *such that* $K(x)=F(x)\cdot G(x)$ *for all* $x\in L=A\cap B$. *Let* $(F,A)\cdot(H,C)=(M,N)$ *such that* $M(x)=F(x)\cdot H(x)$ *for all* $x\in N=A\cap C$. *Let* $(K,L)\cap_E(M,N)=(P,Q)$ *such that* $Q=L\cup N$ *and*
$$P(x)=\begin{cases}K(x) & \text{if}\quad x\in L-N\\ M(x) & \text{if}\quad x\in N-L\\ K(x)\cap M(x) & \text{if}\quad x\in L\cap N\end{cases}.$$ *Thus,* $Q=(A\cap B)\cup(A\cap C)$ *and*
$$P(x)=\begin{cases}F(x)\cdot G(x) & \text{if}\quad x\in(A\cap B)-(A\cap C)\\ F(x)\cdot H(x) & \text{if}\quad x\in(A\cap B)-(A\cap C)\\ (F(x)\cdot G(x))\cap(F(x)\cdot H(x)) & \text{if}\quad x\in(A\cap B)\cap(A\cap C)\end{cases}$$
$$=\begin{cases}F(x)\cdot G(x) & \text{if}\quad x\in A\cap(B-C)\\ F(x)\cdot H(x) & \text{if}\quad x\in C-B\\ F(x)\cdot(G(x)\cap H(x)) & \text{if}\quad x\in A\cap(B\cap C)\end{cases} \tag{3.2.10}$$
Comparing (3.2.9) *and* (3.2.10), *we get* $(J,E)=(P,Q)$. *Therefore,* $(F,A)\cdot\Big((G,B)\cap_E(H,C)\Big)=\Big((F,A)\cdot(G,B)\Big)\cap_E\Big((F,A)\cdot(H,C)\Big)$.

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4. Let $(G,B) \cap_E (H,C) = (I,D)$ such that $D = B \cup C$ and

$$I(x) = \begin{cases} G(x) & \text{if} \quad x \in B - C \\ H(x) & \text{if} \quad x \in C - B \\ G(x) \cap H(x) & \text{if} \quad x \in B \cap C \end{cases}$$

Let $(I,D) \cdot (F,A) = (J,E)$ such that $E = D \cap A = (B \cup C) \cap A$ and

$$J(x) = I(x) \cdot F(x) = \begin{cases} G(x) \cdot F(x) & \text{if} \quad x \in (B - C) \cap A \\ H(x) \cdot F(x) & \text{if} \quad x \in (C - B) \cap A \\ (G(x) \cap H(x)) \cdot F(x) & \text{if} \quad x \in (B \cap C) \cap A \end{cases} \tag{3.2.11}$$

Let $(G,B) \cdot (F,A) = (K,L)$ such that $K(x) = G(x) \cdot F(x)$ for all $x \in L = B \cap A$. Let $(H,C) \cdot (F,A) = (M,N)$ such that $M(x) = H(x) \cdot F(x)$ for all $x \in N = C \cap A$. Let $(K,L) \cap_E (M,N) = (P,Q)$ such that $Q = L \cup N$ and

$$P(x) = \begin{cases} K(x) & \text{if} \quad x \in L - N \\ M(x) & \text{if} \quad x \in N - L \\ K(x) \cap M(x) & \text{if} \quad x \in L \cap N \end{cases}$$. Thus, $Q = (B \cap A) \cup (C \cap A)$ and

$$P(x) = \begin{cases} G(x) \cdot F(x) & \text{if} \quad x \in (B \cap A) - (C \cap A) \\ H(x) \cdot F(x) & \text{if} \quad x \in (C \cap A) - (B \cap A) \\ (G(x) \cdot F(x)) \cap (H(x) \cdot F(x)) & \text{if} \quad x \in (B \cap A) \cap (C \cap A) \end{cases}$$

$$= \begin{cases} G(x) \cdot F(x) & \text{if} \quad x \in (B - C) \cap A \\ H(x) \cdot F(x) & \text{if} \quad x \in (C - B) \cap A \\ (G(x) \cap H(x)) \cdot F(x) & \text{if} \quad x \in (B \cap C) \cap A \end{cases} \tag{3.2.12}$$

Comparing (3.2.11) and (3.2.12), we get $(J,E) = (P,Q)$. Therefore, $\Big((G,B) \cap_E (H,C)\Big) \cdot (F,A) = \Big((G,B) \cdot (F,A)\Big) \cap_E \Big((H,C) \cdot (F,A)\Big)$.

5. This is similar to the proof of 1.

6. This is similar to the proof of 2.

7. This is similar to the proof of 3.

8. This is similar to the proof of 4.

Theorem 3.3 *Let $(F,A), (G,B)$ and (H,C) be soft quasigroups over a quasigroup $(Q, \cdot, /, \backslash)$.*

1. $(F,A)/\Big((G,B) \cap_R (H,C)\Big) = \Big((F,A)/(G,B)\Big) \cap_R \Big((F,A)/(H,C)\Big)$.

2. $\Big((G,B) \cap_R (H,C)\Big)/(F,A) = \Big((G,B)/(F,A)\Big) \cap_R \Big((H,C)/(F,A)\Big)$.

3. $(F,A)/\Big((G,B) \cap_E (H,C)\Big) = \Big((F,A)/(G,B)\Big) \cap_E \Big((F,A)/(H,C)\Big)$.

4. $\Big((G,B) \cap_E (H,C)\Big)/(F,A) = \Big((G,B)/(F,A)\Big) \cap_E \Big((H,C)/(F,A)\Big)$.

5. $(F,A)/\Big((G,B) \cup_R (H,C)\Big) = \Big((F,A)/(G,B)\Big) \cup_R \Big((F,A)/(H,C)\Big)$.

6. $\Big((G,B) \cup_R (H,C)\Big)/(F,A) = \Big((G,B)/(F,A)\Big) \cup_R \Big((H,C)/(F,A)\Big)$.

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7. $(F,A)/\Big((G,B)\cup_E(H,C)\Big) = \Big((F,A)/(G,B)\Big)\cup_E\Big((F,A)/(H,C)\Big)$.

8. $\Big((G,B)\cup_E(H,C)\Big)/(F,A) = \Big((G,B)/(F,A)\Big)\cup_E\Big((H,C)/(F,A)\Big)$.

Proof 3.3 *The arguments of the proof are similar to those of Theorem 3.2.*

Theorem 3.4 *Let $(F,A),(G,B)$ and (H,C) be soft quasigroups over a quasigroup $(Q,\cdot,/,\backslash)$.*

1. $(F,A)\backslash\Big((G,B)\cap_R(H,C)\Big) = \Big((F,A)\backslash(G,B)\Big)\cap_R\Big((F,A)\backslash(H,C)\Big)$.

2. $\Big((G,B)\cap_R(H,C)\Big)\backslash(F,A) = \Big((G,B)\backslash(F,A)\Big)\cap_R\Big((H,C)\backslash(F,A)\Big)$.

3. $(F,A)\backslash\Big((G,B)\cap_E(H,C)\Big) = \Big((F,A)\backslash(G,B)\Big)\cap_E\Big((F,A)\backslash(H,C)\Big)$.

4. $\Big((G,B)\cap_E(H,C)\Big)\backslash(F,A) = \Big((G,B)\backslash(F,A)\Big)\cap_E\Big((H,C)\backslash(F,A)\Big)$.

5. $(F,A)\backslash\Big((G,B)\cup_R(H,C)\Big) = \Big((F,A)\backslash(G,B)\Big)\cup_R\Big((F,A)\backslash(H,C)\Big)$.

6. $\Big((G,B)\cup_R(H,C)\Big)\backslash(F,A) = \Big((G,B)\backslash(F,A)\Big)\cup_R\Big((H,C)\backslash(F,A)\Big)$.

7. $(F,A)\backslash\Big((G,B)\cup_E(H,C)\Big) = \Big((F,A)\backslash(G,B)\Big)\cup_E\Big((F,A)\backslash(H,C)\Big)$.

8. $\Big((G,B)\cup_E(H,C)\Big)\backslash(F,A) = \Big((G,B)\backslash(F,A)\Big)\cup_E\Big((H,C)\backslash(F,A)\Big)$.

Proof 3.4 *The arguments of the proof are similar to those of Theorem 3.2.*

4 Conclusion

Motivated by the study of algebraic properties of soft sets, we extended soft set theory to non-associative algebraic structures known as quasigroup. In this paper, we have introduced soft quasigroups, new soft quasigroup algebraic properties and generalized some notions and properties of soft sets to soft quasigroups. From our results, it can be concluded that for medial quasigroups, soft quasigroups are closed under product and product divisions. Also, with the new operations defined on soft quasigroups, it can be concluded that product and product divisions are distributive over restricted/extended union and intersection operations.

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