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Pollution-Development Tradeoffs in Nigeria: An Agent-based Model

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Abstract

Like many developing countries, Nigeria, is faced with a number of tradeoffs that pit rapid economic development against environmental preservation. Environmentally sustainable,“green" economic development is slower, more costly, and more difficult than unrestricted, unregulated economic growth. In this paper we develop an agent-based computational model of agents (who can be individuals or organizations) in a population who make successive choices that have economic and environmental impacts. Simulations based on the model suggest that widespread public awareness of environmental issues is insufficient to prevent the tendency towards sacrificing the environment for the sake of growth. Even if people have an understanding of negative impacts and always choose to act in their own self-interest, they may still act collectively in such a way as to bring down the quality of life for the entire society. We conclude that besides raising public awareness, economic intervention by the government (in the form of incentives or penalties) may be the only way to achieve an optimal balance between economic and environmental factors.

Keywords: Agent-based model, developing countries, pollution, environment, economic growth, tradeoff, optimization, Pareto curve.
MSC2010:26A18

1 Introduction

1.1 Environment vs. economic progress in the developing world

In many developing countries, poverty is a basic issue of concern. Poverty fuels other issues, such as crime and political or religious extremism [1]. Economic development is often viewed as a cure for poverty: but development has significant environmental costs. For example, economic development typically requires urbanization, and rapid urban growth which outstrips infrastructure development leads to serious water and air pollution problems [2]. Other development-related circumstances that have significant environmental impact include bush burning for land clearing; clearcutting of forests; overgrazing and intensive farming that degrade land and increase erosion; heavy pesticide use; oil spills; and dumping of untreated industrial waste; dumping of consumer plastic waste; and so on ([3, 4]). There is a high level of awareness and concern about the environment among the Nigerian population ([5, 17]). Nonetheless, environmental conditions are steadily worsening [18]. Nigeria is faced with a tradeoff, as far as its future development is concerned. Sustainable, "green" economic development is slower, more costly, and more difficult than unrestricted, unregulated economic growth. Some authors have advocated environmental education as a cure for Nigeria's environmental problems ([27]). But is effective education a sufficient guarantee that Nigeria will pursue a path that balances economic growth versus environmental preservation? Ideas and beliefs do not always lead to actions. The model that we develop in this paper suggests that widespread public awareness of environmental issues is insufficient to prevent the tendency towards sacrificing the environment for the sake of growth. Even if people have an understanding of negative impacts and always choose to act in their own self-interest, they may still act collectively in such a way as to bring down the quality of life for the entire society.

1.2 Agent-based modeling of socioeconomic behavior

Mathematical modeling and simulation is a foundational tool in economics. Mathematical modeling may also be used to characterize systems in which social behavior and organization are key components. In such cases, agent-based modeling is an effective and popular methodology. Compared to classical mathematical models that rely on algebraic or differential equations, agent-based models are more detailed and require greater computational resources. Indeed, agent-based modeling is impossible without computers, which explains why they have only risen to prominence in the last 50 years. Besides economics, agent-based modeling is widely used in epidemiology, ecology, and other fields in which large populations are modeled. A general overview of agent-based modeling methodology may be found in [19], while anintroduction to agent-based modeling with particular reference to the social sciences is given in [20]. In agent-based modeling, decision-making entities within a social system are designated as "agents" . Depending on the model, agents may be nations, government officials or agencies, companies or corporations, communities, groups, families, individuals, and so on. Agents within the model are treated as mathematical entities, which interact with each other and the environment according to a specified set of mathematically-defined behaviors (which may or may not have a random component).

A prominent early example of an agent-based model of a social system is Schelling's model of segregated housing [22]. In Schelling's model, agents are homeowners of two different races or ethnicities. Homes are represented as squares on a rectangular grid, and the ethnicity of a home's owner is represented by the color of the dot which occupies the square. Based on some simple assumptions about how individuals choose where to live, Schelling showed that neighborhood segregation along ethnic or racial lines can occur simply because of individuals' own preferences, and not because of discriminatory policies or external pressures. Figure 1 shows graphical representations of Schelling's model. Hatna and Benenson in [26] apply Schelling's model to actual neighborhoods in Israeli cities, and show that the model can reproduce the qualitative features of actual housing patterns observed in ethnically-mixed urban areas (see Figure 2.

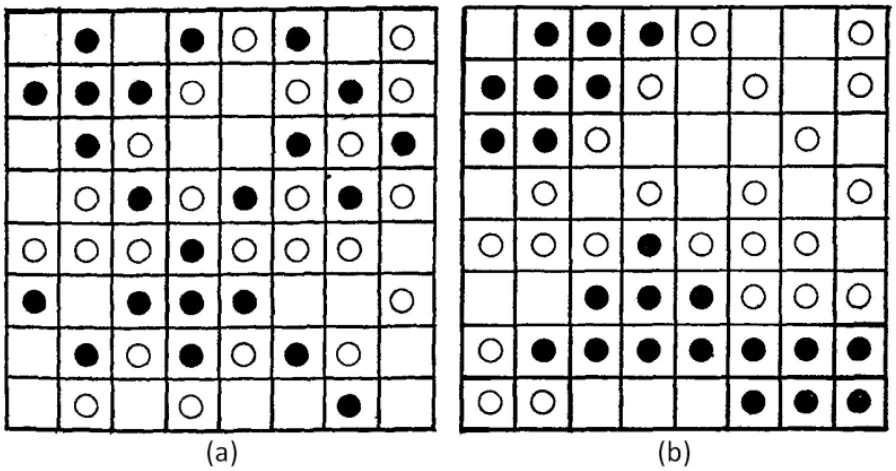

Figure 1: Housing configurations in Schelling's model. Checkerboard squares represent possible housing locations, and black and white dots represent houses belonging to people in two different ethnic groups. (a) shows a random starting configuration, while (b) shows the configuration after several generations in Schelling's model.

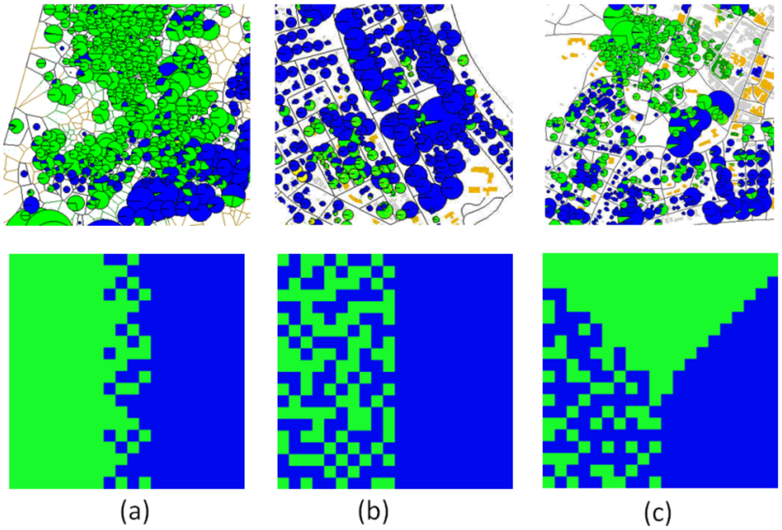

Figure 2: Ethnic composition of neighborhoods in Yaffo and Ramle, Israel (top row), compared to configurations observed in Schelling's model (bottom row), as implemented in [26]. Green and blue denote Jewish and Arab households

Another influential example of an agent-based model is Epstein's model of civil violence [31]. In Epstein's model, a region's population is represented as a set of agents located on a two-dimensional grid. Agents are of two types: cop agents and populace agents. Population agents have varying degrees of disposition towards violence, depending on the agents' level of "grievance" as well as the activities of other cop and populace agents in their vicinities. Based on these factors, agents either become actively violent or remain quiescent: violent agents may be "arrested" and "jailed" by nearby cop agents. Epstein found that such models may exhibit large "bursts" of violence that are localized in time and space (see Figure 3). This behavior can be seen as an example of "self-organized criticality" , a concept introduced by Bak, Tang, and Wiesenfeld to explain random events whose size distributions have heavy tails that obey a power-law distribution [30]. Epstein's model has been analyzed mathematically in [34] and [39], and similar models have been reviewed in [35]. The model has been refined, adapted and applied by other researcher in [32], [33], and [40] (see Figure 4).

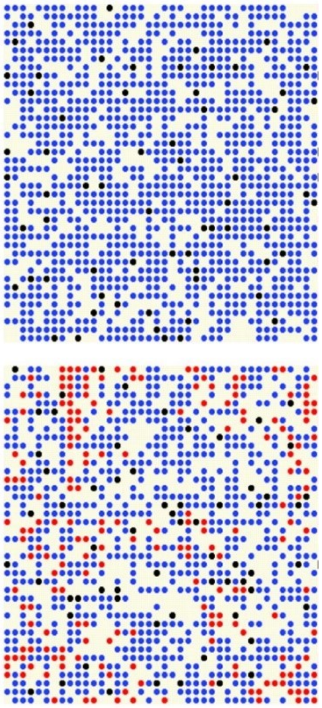

Figure 3: Two configurations of Epstein's model. In the top configuration, populace agents (blue) are all quiescent; black agents are cop agents. In the bottom configuration, a large proportion of populace agents have become violent (indicated by red dots). Epstein found that with certain parameter values, large outbursts of violence interspersed with quiescent periods could be observed. from [31]

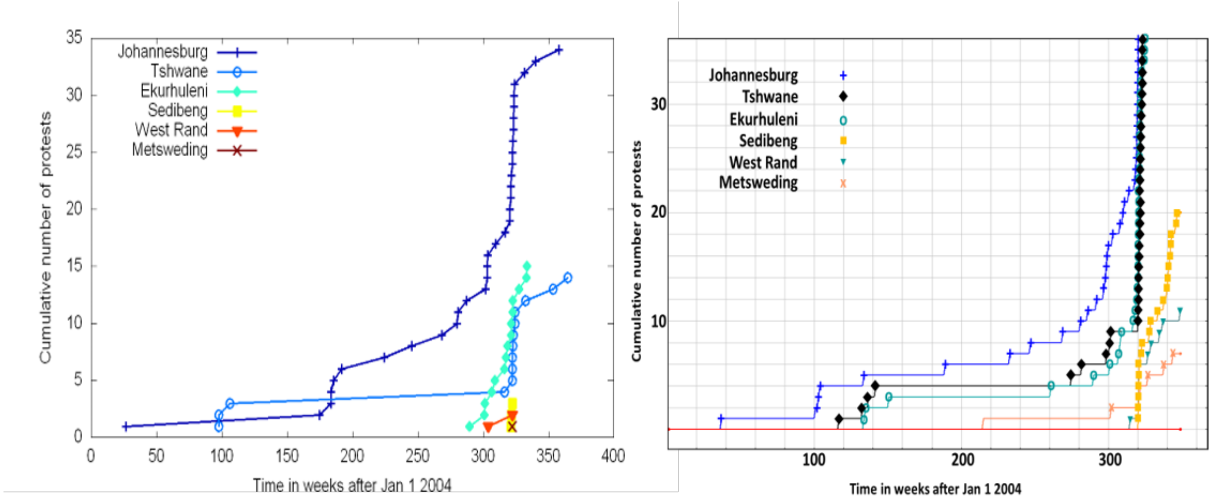

Figure 4: *(Left)* Cumulative plot of service protest occurrence times in different Gauteng Province districts in 2004; *(Right)* National Operational Environment Model simulation output. Figures are taken from [40]

Both Schelling's and Epstein's models exhibit *low-dimensional realism*, which keys in on one or two crucial aspects of the system but leaves out or abstractifies other aspects [28]. Models with low-dimensional realism are typically used to explain trends and tendencies in a qualitative rather than precise quantitative fashion. Such models provide a conceptual framework from which to view the situation from a high-level perspective: their usefulness lies in that and they may suggest practices and policies that can produce beneficial changes in the system being modeled. They have the advantage of possessing few parameters, so that analysis of model behavior is simplified. On the other hand, because they are so simplified these models may fail to capture significant aspects of the system which can strongly affect system behavior. Two contemporary forums in which new agent-based models of social systems are proposed and evaluated are the online Journal of Artificial Societies and Simulation (`jasss.soc.surrey.ac.uk`), and the annual International Conference on Social Computing, Behavioral-Cultural Modeling and Prediction and Behavior Representation in Modeling and Simulation (SBP-BRiMS) [29]. In our model, agents face a choice that involves a tradeoff between two different factors. We have in mind environmental quality and economic development as our two factors, which is a key issue in the developing world and Nigeria in particular. However, the model could be generalized to other tradeoffs faced by modern societies. Other examples include: inflation versus unemployment; social conformity versus individual freedom; traditional versus progressive values; law and order versus civil liberties; healthcare coverage costs versus benefits; free market versus regulated economy; ambition versus contentment; leisure versus productivity; saving versus spending; simplicity versus luxury; and so on. The abstract model presented in this paper is generally applicable to any of these situations. This research builds on an agent-based model first proposed in [24] and further developed in [25]. Other research that applies agent-based methodology to the study of decision-making situations may be found in [23, 36–38].

1.3 Indifference curves and production possibility frontiers

Two of the mathematical constructs used in our model are commonly used in mathematical economics. These two constructs are *indifference curves* and *production possibility frontiers* (or *Pareto frontier*) ([13, 14]). Both of these constructs apply in situations where there are two different offsetting factors (Factor 1 and Factor 2), where the status of each factor is measured by a positive index (the larger the index, the better the status). If we draw a xy plot with Factor 1 on the x

axis and Factor 2 on the y axis, then each agent's index levels corresponds to a point in the plane. An *indifference curve* consists of points in this plane that corresponds to agent states that are equally preferable. In other words, if (f_1, f_2) and (f_1', f_2') are on the same curve, then neither state is preferred to the other. Typically, indifference curves tend to be concave: this reflects the "law of diminishing returns" , i.e. the less of a factor is present, the more valuable it becomes.

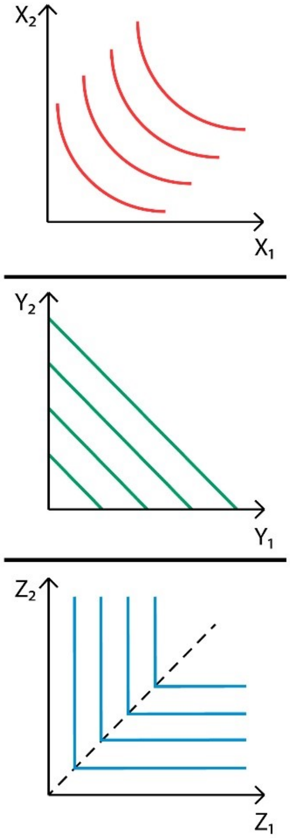

Figure 5: Examples of families of indifference curves. Curves to the upper right have higher utility. (Source: https://policonomics.com/indifference-curves/)

On the other hand, a *production possibility curve* (or *Pareto frontier*) represents the best possible combinations of Factor 1 and Factor 2 that can be achieved by agents. If a point (x_1, x_2) is on the Pareto frontier, then for all other possible choices (x_1', x_2') either $x_1' < x_1$ or $x_2' < x_2$ (see Figure 6).

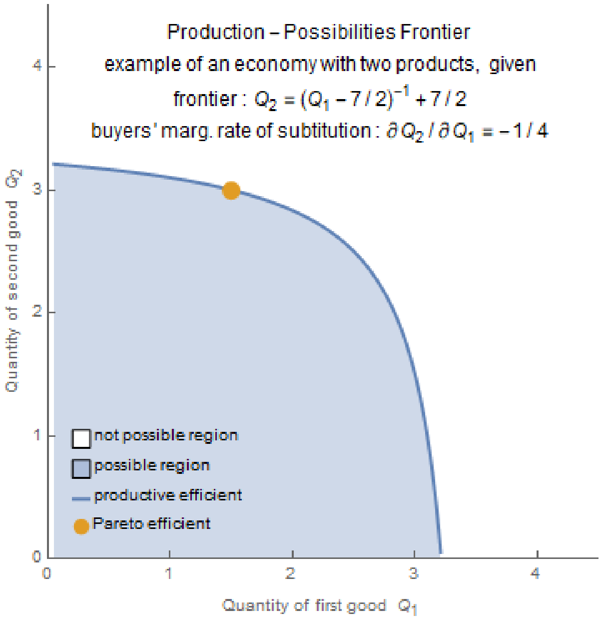

Figure 6: A convex production-possibility (Pareto) frontier (solid line), showing a tradeoff between production of two goods. The shaded region shows possible production levels for the two goods. (source: Namtranhoang1992 - Own work, CC BY-SA 4.0, `https://commons.wikimedia.org/w/index.php?curid=76868820`)

In a system with two factors that is characterized by a Pareto frontier and indifference curves, the optimal system state is the unique point at which the Pareto frontier is tangent to one of the indifference curves. This unique point has the largest possible utility of all (x_1, x_2) pairs within the possible region (see Figure 7).

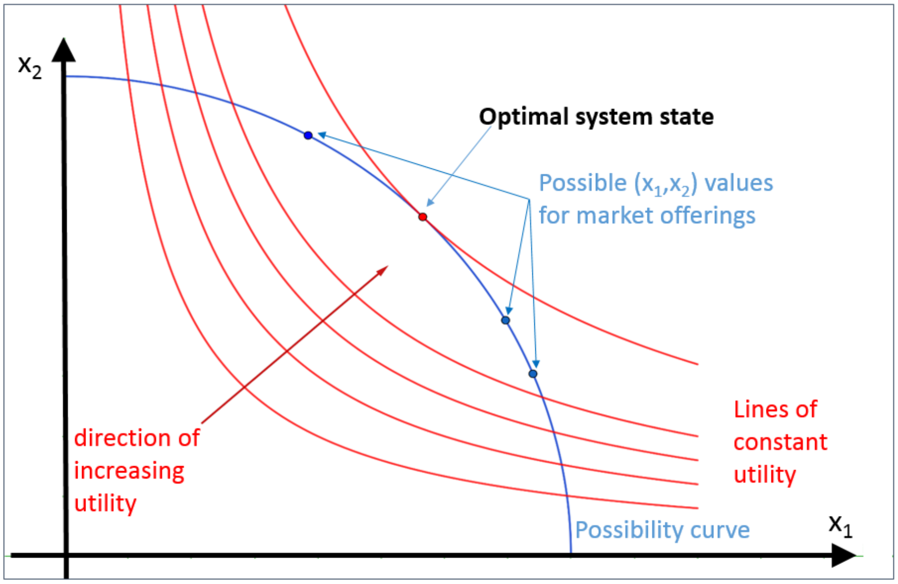

Figure 7: Optimal system state for a system with indifference curves (concave red lines) and a Pareto frontier (convex blue line). Points on the Pareto frontier are Pareto optimal, but do not maximize utility.

2 Methodology

2.1 Structure of the model

We consider an abstract scenario where multiple agents in a population are faced with the same type of choice. This scenario could represent individuals choosing to buy a vehicle or a house; or companies building a factory, or cities choosing to build parks or infrastructure; and so on. Whatever choice an agent makes will affect the agents' status relative to quality of environment and economic development. We suppose that each agent's economic status can be characterized by a numerical value which we call its *economic utility* (ECU); and each agent's environmental status similarly has an *environmental utility* (EVU). The higher the ECU or EVU, the better the agent's status relative to economy or environment respectively. We will associate the variables x_1 and x_2 with ECU and EVU, respectively. Given that the population consists of N agents, we donate the agents' status vectors as $(x_{1n},\ x_{2n}), n = 1 \dots N$. In addition, we make the following basic assumptions concerning agent behavior:

- As time progresses, various agents in the system are presented with new choices with associated ECU, EVU values given by the vector (x'_1, x'_2) .
- When presented with an option, an agent will accept the option if it increases the agent's overall benefit. All combinations of ECU and EVU that produce the same overall benefit lie on a concave indifference curve in the ECU-EVU plane. All agents share the same indifference curves, and the indifference curves form a nested sequence as in Figure 6, where higher curves correspond to greater utility.
- New choices are offered by a competitive market, which responds to the current distribution of agents. At any given time, the (x'_1, x'_2) values for a new choice has a bivariate normal random distribution centered around a point on the Pareto frontier. This central point changes over time in response to agents' current (ECU,EVU) distribution. The market is more responsive

to ECU than EVU, so that the central point's ECU level matches the current average ECU value of agents in the system, while the central point's EVU is calculated so that ECU, EVU lies on the Pareto frontier.

- Agents randomly enter and leave the system, but the number of agents in the system remains constant. A newly entering agent will automatically accept the first choice offered to it.

In an ideal situation where both the agents and the market possess perfect knowledge, the system would move to the point of tangency of the unique indifference curve that is tangent to the Pareto frontier. Our model is intended to show what happens in a non-ideal situation, where agents and the market respond to the immediate situation without omniscience. We will consider indifference curves of the form:

$$x_1^r + x_2^s \; = 1 \, (r, s < 1) \tag{2.1}$$

and possibility curves (Pareto frontiers) of the form:

$$x_1^p + x_2^q \; = 1 \, (p, q \geq 1) \, . \tag{2.2}$$

The possibility curve equation implicitly determines x_2 as a function of x_1 . The functional dependence of x_2 on x_1 is given by the equation

$$x_2 = g\left(x_1\right), \quad \text{where } g\left(x\right) = \left(1 - x^{\frac{1}{p}}\right)^{\frac{1}{q}}$$

As indicated in the algorithm description above, the market offers new choices based on the current distribution of (x_1, x_2) values among agents. The mean ECU for new choices is equal to current mean of agents' ECU values (with the caveat that the value cannot exceed 1, the maximum value on the Pareto frontier). The mean EVU for new choices is then calculated so as to lie on the possibility curve. Letting $(x_1^*,\ x_2^*)$ denote the mean (ECU,EVU) values for new choices, we have:

$$x_1^* = \min\left(1,\ \frac{1}{N}\sum_{n=1}^{N} x_{1n}\right); \qquad x_2^* = g\left(x_1^*\right).$$

Based on these values, a new choice $(x_1', x_2' \,)$ is generated randomly according to the formula:

$$(x_1' \;, x_2' \;) = (x_1^*,\ x_2^*) + \sigma\left(\nu_1, \nu_2\right),$$

Where ν_1 and ν_2 are independent standard normal random variables, and determines the variability of new market offerings. It is possible that x_1' or x_2' could be negative or larger than 1 under this prescription: if this happens, the value is replaced by 0 or 1 respectively. The complete set of model parameters is p, q, r, s, σ, and p_r where p, q determine the shape of the possibility curve; r, s determine the shape of the indifference curve; σ determines the variability of new choices; and p_r is the replacement probability, i.e. the probability that the selected agent leaves the system and is replaced by a new agent. The process of agent choice is depicted in Figure 8. A new choice is generated according to the above prescription, and is offered to a particular agent. Since the choice (in this case) lies above the agent's current indifference curve, the agent accepts the choice. This acceptance slightly increases the mean x_1^* value for the population, which in turn shifts the x_2^* value so that $(x_1^*,\ x_2^*)$ moves along the Pareto frontier.

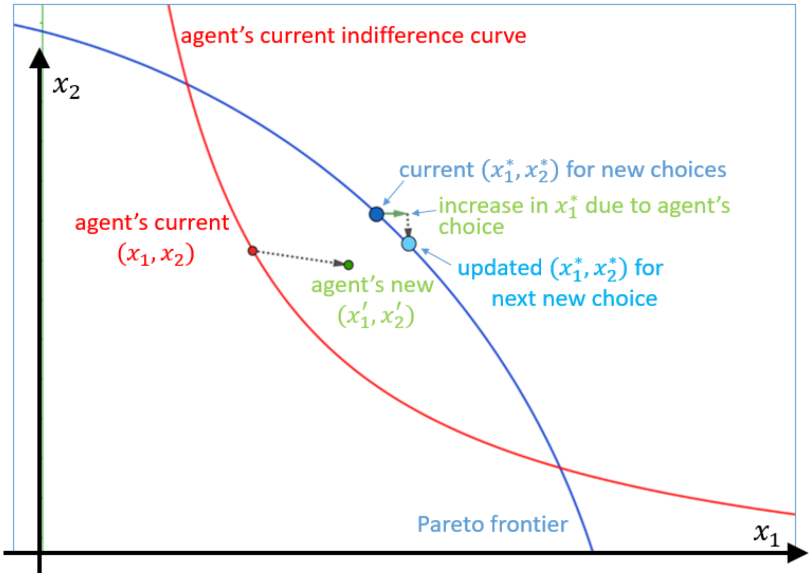

Figure 8: Schematic example of the system's response to agent choice: the agent accepts a new choice, which affects the mean market values (x_1^*, x_2^*).

2.2 Simulations

The algorithm described in Section 2.1 was coded in a Jupyter notebook using the Python3 language. Figure 9 shows the flowchart of the algorithm. The complete code and documentation are available from [6].

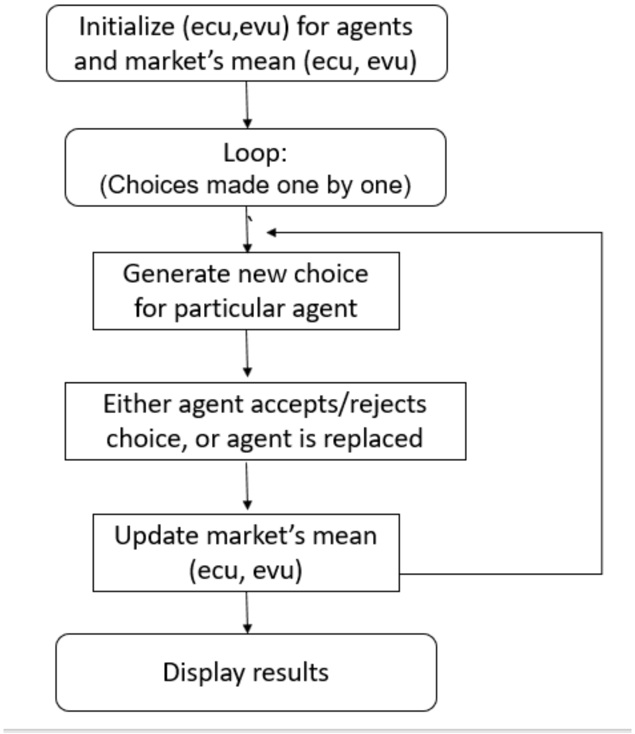

Figure 9: Flowchart for the computer code

Several simulations were conducted to determine the robustness of the model to changes in the parameters that determine the indifference and possibility curves. In these simulations, one parameter was varied while the other parameters were kept at the baseline values given in in Table 1. Note that although the number of time steps ($T = 10^5$) seems very large, each time step represents a single choice made by a single agent: so this value of T corresponds to an average of 100 choices per agent (for 1000 agents).

Possibility curve exponents		Indifference curve exponents		Standard deviation	Population	Time steps	Replacement probability
p	q	r	s	σ	N	T	p_r
2	2	1/2	1/2	.04	1000	10^5	0.02

Table 1: Baseline parameter values used in simulations

3 Results

Figure 10 shows the time evolution of the ECU and EVU indices for the baseline parameter set listed in Table 1. Over time, ECU increases at the expense of EVU . Both curves' slopes are decreasing.

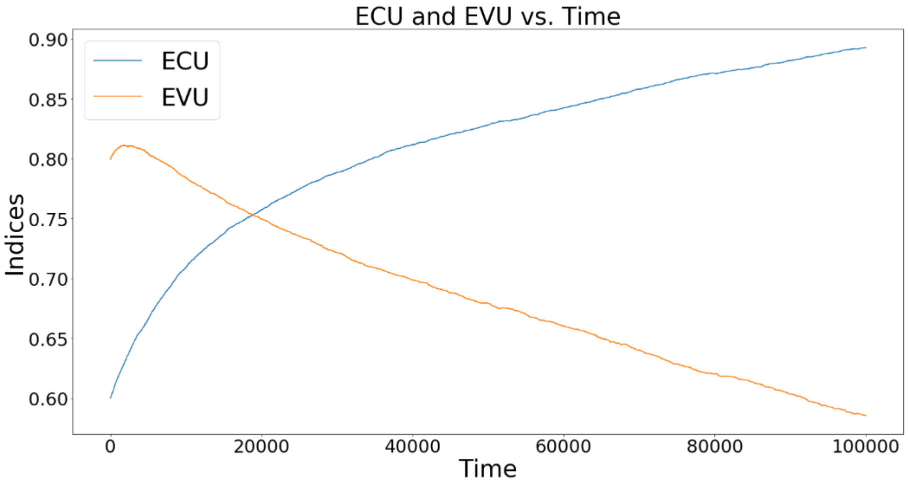

Figure 10: Time evolution of economic utility (ECU) and environmental utility (EVU) indices during simulation, showing continued rise in ECU at the expense of EVU.

Figure 11 shows scatter plots of the initial and final (ECU , EVU) values for a sample of individuals in the population, for the time interval $[0, 10^5]$. The plot reflects the migration to higher ECU and lower EVU values. The population also becomes increasingly spread out over time. Although the bulk of the population has migrated to EVU values close to 0.5, some individuals who are somewhat above the possibility curve remain behind at higher EVU values.

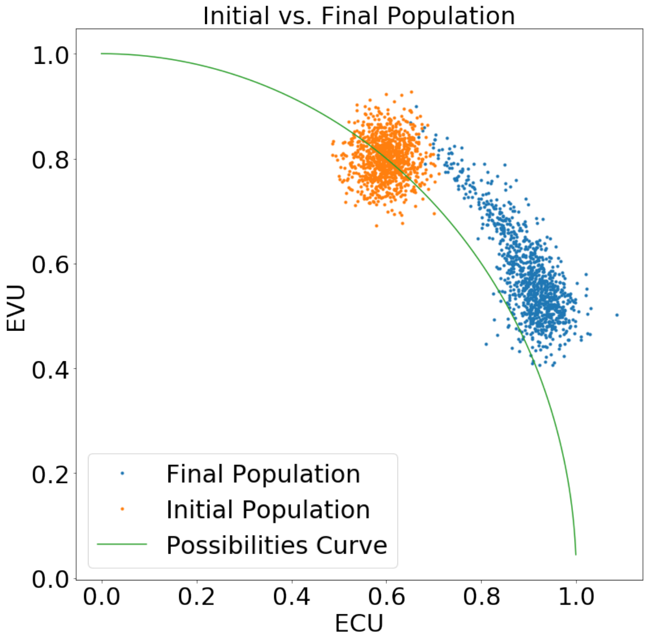

Figure 11: Scatter plot of population samples at the beginning and end of the simulation, showing the smearing out of the population and general migration towards lower environmental utility.

Figure 12 shows how the rate of change of utility (given by the utility curve) depends on the exponents used in the possibility curve. For each set of exponents, the utility rises because the model permits the population to go above the possibility curve. However, once a maximum value is reached, in all cases the utility decreases steadily, showing that the population is not remaining at its optimum operating point. The rate of decrease is larger for larger values of p and q , which corresponds geometrically to a greater curvature in the possibility curve, and practically to cases where a small increases in one variable produce large decreases in the other variable.

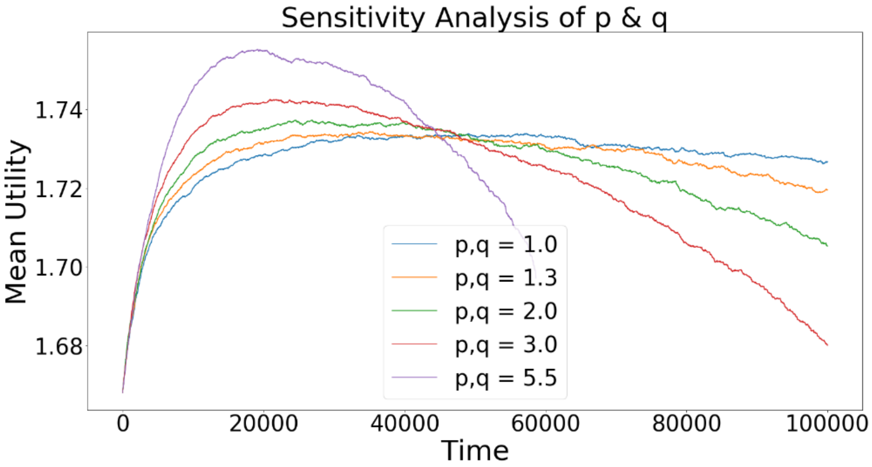

Figure 12: 1 Change in utility over time for different values of the exponents p and q for the possibility equation (2.2).

Figure13 shows how the rate of change of utility (given by the utility curve) depends on the exponents used in the indifference curves. All of the utility curves follow the same pattern of initial rapid rise (as the system reaches the frontier) followed by gradual decline of total utility. The figure shows that the phenomenon of utility degradation is robust to the particular indifference curve exponents used.

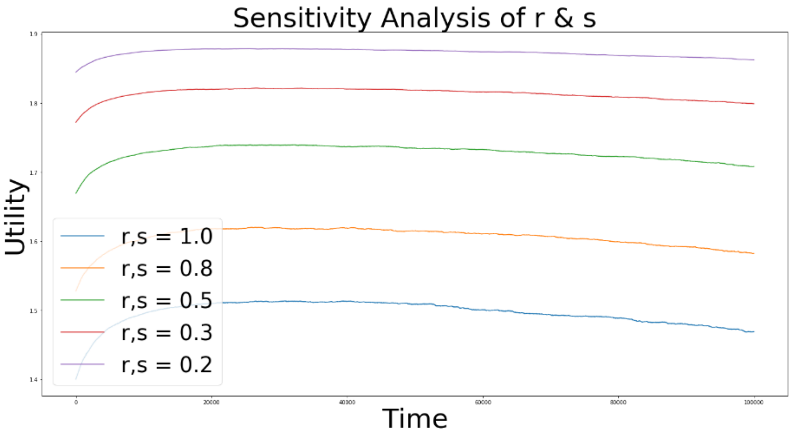

Figure 13: Total utility as a function of time step, for different values of the exponents r, s used in the indifference curve equation (2.1)

4 Discussion

The model shows that even for a well-informed population, an optimal balance between economic and environmental utility is not reached, but instead the balance tilts in favor of economic development at the expense of environmental preservation. Even though agents make informed, rational choices to maximize their utility, the choices available to them on the market are skewed because the market responds only to economic factors and adjusts accordingly.

These results suggest that informing the population of environmental risks (as suggested in [3] and [7]) in itself is not sufficient to reverse the trend towards environmental degradation. This observation is reinforced by research in China that shows that although media coverage of environmental issues has greatly increased, practical efforts to develop a green economy are lacking [15]. Furthermore, fiscal policy in China continues to favor economic growth over preservation [8]. As a result, the environment continues to deteriorate [11], [9].

We conclude that direct economic intervention in order to improve agents' choices is necessary in order to stem the tide towards the worsening environment. Examples of possible interventions include packaging charges, permit fees, profit retention incentives, rebates, and increased quotas [16], [21]. Another example is the Zero Emissions Vehicle (ZEV) credit program that is now in effect in 11 states in the USA [12]. It remains to see whether Nigeria can muster the political will necessary to implement similar economic measures.

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Competing financial interests

The author declares no competing financial interests.

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