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Impacts of Slips on Peristaltic flow and Heat transfer of micropolar fluids in an asymmetric channel

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Abstract

The study of peristaltic motion is an area of increasing research interest in industrial, biological and engineering interest. In this study, effects of slips on the peristaltic flow and heat transfer of micropolar fluids in an asymmetric channel are investigated analytically. The developed non-linear coupled partial differential equations are converted into non-linear coupled ordinary differential equations using similarity transformation. The ordinary differential equations are solved for the cases when the thermal viscosity parameter is zero and non-zero. Exact solutions are gotten for the cases of linear and non-linear when the thermal viscosity parameter is zero and non-zero, respectively. The obtain results depict that viscous and thermal slips enhances the flow of the bolus as it is being transported through the digestive system. Also, the effect of microrotation helps in reducing the pressure gradient for the flow.

Keywords: Peristaltic; Micropolar; Asymmetric channel; Exact solutions.
MSC2010: 76P05.

1 Introduction

Peristalsis is a series of wave-like muscle contractions that moves food to different processing stations in the digestive tract. It is coined from New Latin and is derived from the Greek *peristellein*, "to wrap around". The process of peristalsis begins in the oesophagus when a bolus of food is swallowed. The strong wave-like motions of the smooth muscle in the oesophagus carry food to the stomach, where it is churned into a liquid mixture called the "chime". In much of a digestive tract such as the human gastrointestinal tract, smooth muscle tissue contracts in sequence to produce a peristaltic wave, which propels a ball of food (called a bolus while in the oesophagus and upper gastrointestinal tract and chyme in the stomach) along the tract. Peristaltic movement comprises relaxation of circular smooth muscles, then their contraction behind the chewed material to keep it from moving backward, then longitudinal contraction to push it forward. peristalsis continues in the small intestine where it mixes and shifts the chyme back and forth, allowing nutrients to

be absorbed into the bloodstream through the small intestine walls which contain millions of villi and micro-villi. Peristalsis concludes in the large intestine where water from the undigested food material is absorbed into the bloodstream. Finally, the remaining waste products are excreted from the body through rectum and anus. The circulation of lymph in the lymph capillaries as well as valves in the capillaries is as a results of peristalsis since the human lymphatic system has no central pump. The movement of sperm from the testicles to the urethra is peristalsis. The earthworm is a limbless annelid-worm with a hydrostatic skeleton that moves by peristalsis. Its hydrostatic skeleton consists of a fluid-filled body cavity surrounded by an extensible body wall. The worm moves by radially constricting the anterior portion of its body, resulting in an increase in length via hydrostatic pressure. This constricted region propagates posteriorly along the worm's body. As a result, each segment is extended forward, then relaxes and re-contacts the substrate, with hair-like set preventing backwards slipping.
A peristaltic pump is a positive-displacement pump in which a motor pinches advancing portions of a flexible tube to propel a fluid within the tube. The pump isolates fluid from machinery, which is important if the fluid is abrasive or must remain sterile. Robots have been designed that use peristalsis to achieve locomotion, as the earthworm uses it. Peristaltic pumps are used in a huge number of industries. They can be used in printing inks and colourings, mining slurries, waste water slurries, bleach, sodium bromide and lime slurry pumping. Peristaltic pumps also are excellent for suction lift applications. As with all technologies, they evolve and improve. Early designs were inhibited by the shoe design limitations and inferior rubber technology.

The theory of a micropolar fluid derives from the need to model the flow of fluids that contain rotating micro-constituents. A micropolar fluid is the fluid with internal structures in which coupling between the spin of each particle and the macroscopic velocity field is taken into account. It is a hydro dynamical framework suitable for granular systems which consist of particles with macroscopic size. Eringen [1] was the first pioneer of formulating the theory of micropolar fluids. Various works on micropolar fluids ranging from applications of microrotation fluid, slip effect, magneto-Micropolar fluid and many more have been carried out by [2–7]

The effect of magnetic field on peristaltic mechanisms is important in connection with certain problems of the movement of the conductive physiological fluids, for example, the blood and blood pump machines. A number of researchers have discussed the effects of magnetic field on the peristaltic flow [3–6, 11, 35, 42, 43]. There are few attempts in which the effects of variable viscosity in the peristaltic mechanisms are considered. Mention may be made to the interesting works of [4, 13]. The variable viscosity is considered to be a function of space (height). In a typical situation most of the fluids have temperature dependent viscosity and this property varies significantly when large temperature difference exists. Massoudi and Christie [10] studied the effects of variable viscosity for a simple pipe flow of a third grade fluid. Later on Pakdermirli and Yilbas [34] and Pantokratos [35] considered the temperature dependent viscosity. The aim of the present paper is to investigate the peristaltic flow of Micropolar fluid through a porous non-uniform channel with variable viscosities and thermal conduction. The similarity transformation was used to transform the the governing nonlinear coupled partial differential equations to nonlinear ordinary differential equations under the assumption of long wave-length and low Reynolds number. Exact solutions were obtained for axial velocity, microrotation component, wall shear stress, stream function and pressure gradient. The effects of various physical parameters appear in the problem are discussed graphically.

2 Mathematical Formulation

Consider the flow of an unsteady, incompressible, viscous and electrically conducting micropolar fluid through a non-uniform porous channel of uniform thickness under the action of an external magnetic field Figure: 1. The upper and lower walls satisfy the convective conditions through

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temperature distributions. Let $Y = h(X, t)$ denote the upper and lower wall of the channel is considered to be induced by a sinusoidal wave train propagating with a wave speed c along the length of the channel wall, such that

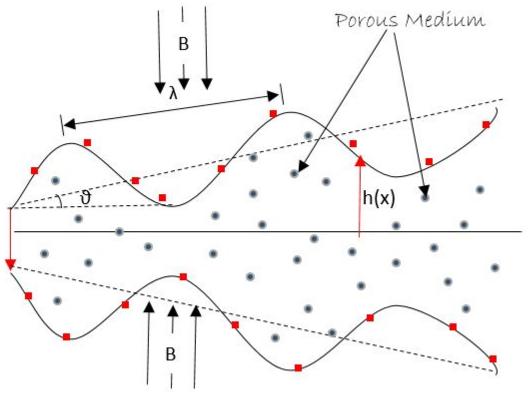

Figure 1: A Physical sketch of the problem

$$h(X, t) = a + tan(\theta)(X - ct) + bsin(\frac{2\pi}{\lambda}(X - ct)) \tag{2.1}$$

where a is the half width of the channel at the inlet, λ is the wave length, b is the amplitude of wave, θ is the angle between the axis of the channel and the walls, $XandY$ represent the rectangular co-ordinates with X measured the axis of the channel and Y the traverse axis perpendicular to X. The system is stressed by an external transverse uniform constant magnetic field of strength and hence total magnetic field induction vector is $B(0, B_0, 0)$, where the induced magnetic field have been neglected due to the assumption of weak electrical conductivity. The equations of motion for unsteady flow through porous medium of an incompressible magneto-micro-polar fluid with externally imposed magnetic field by neglecting the body couples are;

$$\nabla \cdot \mathbf{V} = 0 \tag{2.2}$$

$$\rho\left(\frac{\partial \mathbf{V}}{\partial \bar{t}} + \mathbf{V} \cdot \nabla\ \mathbf{V}\right) = -\nabla \mathbf{p} + (\bar{\mu} + \kappa)\nabla^2 \mathbf{V} + \kappa\left(\nabla \times \bar{\Omega}\right) + \mathbf{J} \times \mathbf{B} - \frac{(\bar{\mu} + \kappa)}{K_p}\mathbf{V} \tag{2.3}$$

$$\rho\bar{j}\left(\frac{\partial \bar{\Omega}}{\partial \bar{t}} + \mathbf{V} \cdot \nabla\ \bar{\Omega}\right) = -2\kappa\bar{\Omega} + \kappa\mathbf{V}\mathbf{p} + (\bar{\mu} + \kappa)\nabla^2 \mathbf{V} \times \mathbf{V} - \gamma\left(\nabla \times \nabla \times \bar{\Omega}\right) + (\alpha + \beta + \gamma)\nabla(\nabla \cdot \bar{\Omega}) \tag{2.4}$$

Along with the generalized ohm's law $\mathbf{J} = \sigma(E + \mathbf{V} \cdot B)$. Where $\mathbf{V} = (\bar{u}, \bar{v}, 0)$ is the velocity vector, $\bar{\Omega} = (0, 0, \bar{N})$ the microrotation vector, $\bar{p}$ the total fluid pressure, $\bar{\mu}$ is the dynamic viscosity, $\frac{\partial}{\partial \bar{t}}$ is the material time derivative, $\bar{t}$ is the time, $\bar{\rho}$ the fluid density, $\bar{j}$ the micro gyration parameter, $\mathbf{J}$ current density vector, σ electrical conductivity of the fluid and E is the electric field vector. The present phenomenon can be transfer from laboratory frame to wave frame via the following relations

$\bar{x} = \bar{X} - c\bar{t}$, $\bar{y} = \bar{Y}$, $\bar{u} = \bar{U} - c$, $\bar{v} = \bar{V}$, $\bar{w} = \bar{W}$, $\bar{p}(\bar{x}, \bar{y}) = \mathbf{p}(\bar{X}, \bar{Y}, \bar{t})$

Where c is the speed of propagation of wave.
Using the assigned values of velocity field, we have the following expressions:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{2.5}$$

$$\rho\left(\frac{\partial}{\partial \bar{t}}+(\bar{u}+c)\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right)(\bar{u}+c)=-\frac{\partial \bar{p}}{\partial \bar{x}}+(\mu+\bar{k})\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\bar{u}+c)+ \left(\frac{\partial \mu}{\partial \bar{x}}\frac{\partial}{\partial \bar{x}}+\frac{\partial \mu}{\partial \bar{y}}\frac{\partial}{\partial \bar{y}}\right)(\bar{u}+c)+\bar{K}\frac{\partial \bar{N}}{\partial \bar{y}}-\sigma B_0^2(\bar{u}+c)-\frac{(\mu+\bar{K})}{k_p}(\bar{u}+c) \tag{2.6}$$

$$\rho\left(\frac{\partial}{\partial \bar{t}}+(\bar{u}+c)\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right)v=-\frac{\partial \bar{p}}{\partial \bar{y}}+(\mu+k)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\bar{v} +\left(\frac{\partial \mu}{\partial \bar{x}}\frac{\partial}{\partial \bar{x}}+\frac{\partial \mu}{\partial \bar{y}}\frac{\partial}{\partial \bar{y}}\right)\bar{v}-\overline{K}\frac{\partial \bar{w}}{\partial \bar{x}}-\frac{(\mu+\bar{K})}{k_p}\bar{v} \tag{2.7}$$

$$\rho\bar{j}\left(\frac{\partial}{\partial \bar{t}}+(\bar{u}+c)\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right)\bar{w}=\gamma\frac{\partial^2 \bar{w}}{\partial \bar{y}^2}-\bar{K}\left(2\bar{w}+\frac{\partial \bar{u}}{\partial \bar{y}}\right) \tag{2.8}$$

The energy equation is

$$\rho Cp\left(\frac{\partial}{\partial \bar{t}}+(\bar{u}+c)\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right)\bar{T}=\frac{\partial}{\partial \bar{x}}\left(k\frac{\partial \bar{T}}{\partial \bar{x}}\right)+\frac{\partial}{\partial \bar{y}}\left(k\frac{\partial \bar{T}}{\partial \bar{y}}\right)+Q_0(\bar{T}-T_0)^{r} \tag{2.9}$$

where C_p the specific heat, $\bar{T}$ the temperature, $\mu\left(\bar{T}\right)$ variable viscosity ,k the variable thermal conductivity, Q_0 the constant heat addition/absorption and T_0 the temperature at the lower and upper walls respectively. Introducing the following dimensionless variables,

$$x=\frac{\bar{x}}{\lambda}, y=\frac{\bar{y}}{a}, u=\frac{\bar{u}}{c}, v=\frac{\bar{v}}{c\delta}, t=\frac{c\bar{t}}{\lambda}, j=\frac{\bar{j}}{a^2}, \delta=\frac{a}{\lambda}, p=\frac{a^2\bar{p}}{\mu_0 c\lambda}, h=\frac{\bar{h}}{a}, \phi=\frac{b}{a} \quad w=\frac{a\bar{w}}{c}, k_p=\frac{\bar{k}_p}{a^2}, k=\frac{\bar{k}}{\mu_0}, \mu(\theta)=\frac{\mu(\bar{T})}{\mu_0}, k(\theta)=\frac{k(\bar{T})}{k_0}, \theta=\frac{\bar{T}-T_0}{T_1-T_0} \tag{2.10}$$

Substituting equation (2.10) into equations (2.5) – (2.9) to obtain
From equation (2.5) we have;

$$\frac{c}{\lambda}\frac{\partial u}{\partial x}+\frac{c\delta}{a}\frac{\partial v}{\partial y}=0$$

$$\frac{c}{\lambda}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

From equation (2.6) we have;

$$\frac{\rho c^2}{\lambda}\left(\frac{\partial}{\partial t}+(u+1)\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)(u+1)=-\frac{\mu_0 c}{a^2}\frac{\partial p}{\partial x}+\frac{\mu_0 c}{a^2}\left(\mu\left(\theta\right)+k\right)\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+ \frac{\mu_0}{a^2}\left(\delta^2\frac{\partial \theta}{\partial x}\frac{\partial u}{\partial x}+\frac{\partial \mu(\theta)}{\partial y}\frac{\partial u}{\partial y}\right)+\frac{\mu_0}{a^2}k\frac{\partial w}{\partial y}-\sigma B_0^2 c\left(u+1\right)-\frac{\mu_0 c}{a^2}\frac{\left(\mu(\theta)+k\right)}{k_p}\left(u+1\right)+\rho g\alpha\left(T_1-T_0\right)\theta \tag{2.11}$$

Simplifying;

$$\frac{\rho c a^2}{\mu_0\lambda}\left(\frac{\partial}{\partial t}+\left(u+1\right)\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)(u+1)=-\frac{\partial p}{\partial x}+\left(\mu\left(\theta\right)+k\right)\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+ \left(\delta^2\frac{\partial \mu(\theta)}{\partial x}\frac{\partial u}{\partial x}+\frac{\partial \mu(\theta)}{\partial y}\frac{\partial u}{\partial y}\right)+k\frac{\partial w}{\partial y}-\frac{\sigma B_0^2 a^2}{\mu_0}\left(u+1\right)+\frac{\rho g\alpha a^2(T_1-T_0)}{\mu_0 c}\theta-\frac{\left(\mu(\theta)+k\right)}{k_p}\left(u+1\right) \tag{2.12}$$

This implies;

$$Re\delta\left(\frac{\partial}{\partial t}+\left(u+1\right)\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)(u+1)=-\frac{\partial p}{\partial x}+\left(\mu\left(\theta\right)+k\right)\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+ \left(\delta^2\frac{\partial \mu(\theta)}{\partial x}\frac{\partial u}{\partial x}+\frac{\partial \mu(\theta)}{\partial y}\frac{\partial u}{\partial y}\right)+k\frac{\partial w}{\partial y}-Ha^2\left(u+1\right)-\frac{\left(\mu(\theta)+k\right)}{k_p}\left(u+1\right)+G_r\theta \tag{2.13}$$

From equation (2.7) we have;

$$\rho\left(\frac{c}{\lambda}\frac{\partial}{\partial t}+(cu+c))\frac{\partial}{\lambda\partial x}+c\delta v\frac{\partial}{a\partial y}\right)c\delta v=-\frac{\mu_0 c\lambda}{a^3}\frac{\partial p}{\partial y}+(\mu_0\mu(\theta)+\mu_0 k)\left(\frac{\partial^2}{\lambda^2\partial x^2}+\frac{\partial^2}{a^2\partial y^2}\right)c\delta v$$
$$+\left(\frac{\mu_0}{\lambda}\frac{\partial\mu(\theta)}{\partial x}\frac{\partial}{\lambda\partial x}+\frac{\mu_0}{a^2}\frac{\partial\mu(\theta)}{\partial y}\frac{\partial}{\partial y}\right)c\delta v-\mu_0\frac{c}{a\lambda}\frac{\partial w}{\partial x}-\frac{(\mu_0\mu(\theta)+\mu_0 k)}{a^2 k_p}c\delta v \tag{2.14}$$

Simplifying we have;

$$\frac{\rho c^2\delta}{\lambda}\left(\frac{\partial}{\partial t}+(u+1)\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)v=-\frac{\mu_0 c\lambda}{a^3}\frac{\partial p}{\partial y}+\frac{\mu_0 c\delta}{a^2}(\mu(\theta)+k)\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$
$$+\frac{\mu_0 c\delta}{a^2}\left(\delta^2\frac{\partial\mu(\theta)}{\partial x}\frac{\partial v}{\partial x}+\frac{\partial\mu(\theta)}{\partial y}\frac{\partial v}{\partial y}\right)-\frac{\mu_0 c}{a\lambda}\frac{\partial w}{\partial x}-\frac{\mu_0 c\delta}{a^2}\frac{(\mu(\theta)+k)}{k_p}v \tag{2.15}$$

$$\frac{\rho ca}{\mu_0\lambda^2}a\delta\left(\frac{\partial}{\partial t}+(u+1))\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)v=-\frac{\partial p}{\partial y}+\frac{a}{\lambda}\delta(\mu(\theta)+k)\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+$$
$$\frac{a}{\lambda}\delta\left(\delta^2\frac{\partial\mu(\theta)}{\partial x}\frac{\partial v}{\partial x}+\frac{\partial\mu(\theta)}{\partial y}\frac{\partial v}{\partial y}\right)-\frac{a^2}{\lambda^2}\frac{\partial w}{\partial x}-\frac{a}{\lambda}\delta\frac{(\mu(\theta)+k)}{k_p}v \tag{2.16}$$

$$Re\frac{a^2}{\lambda^2}\delta\left(\frac{\partial}{\partial t}+(u+1)\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)v=-\frac{\partial p}{\partial y}+\delta^2(\mu(\theta)+k)\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$
$$+\frac{a}{\lambda}\delta\left(\delta^2\frac{\partial\mu(\theta)}{\partial x}\frac{\partial v}{\partial x}+\frac{\partial\mu(\theta)}{\partial y}\frac{\partial v}{\partial y}\right)-\delta^2\frac{\partial w}{\partial x}-\delta^2\frac{(\mu(\theta)+k)}{k_p}v \tag{2.17}$$

$$Re\delta^3\left(\frac{\partial}{\partial t}+(u+1)\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)v=-\frac{\partial p}{\partial y}+\delta^2(\mu(\theta)+k)\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+$$
$$\delta^2\left(\delta^2\frac{\partial\mu(\theta)}{\partial x}\frac{\partial v}{\partial x}+\frac{\partial\mu(\theta)}{\partial y}\frac{\partial v}{\partial y}\right)-\delta^2\frac{\partial w}{\partial x}-\delta^2\frac{(\mu(\theta)+k)}{k_p}v \tag{2.18}$$

From equation (2.8) we have;

$$\rho C_p\left(\frac{c(T_1-T_0)}{\lambda}\frac{\partial\theta}{\partial t}+\frac{c(T_1-T_1)}{\lambda}(u+1)\frac{\partial\theta}{\partial x}+\frac{c\delta(T_1-T_0)}{a}v\frac{\partial\theta}{\partial y}\right)=$$
$$\frac{1}{\lambda}\frac{\partial}{\partial x}\left(k_0 k(\theta)\frac{T_1-T_0}{\lambda}\frac{\partial\theta}{\partial x}+\frac{1}{a}\frac{\partial}{\partial y}(k_0 k(\theta))\frac{(T_1-T_0)}{a}\frac{\partial\theta}{\partial y}\right)+Q_0(T_1-T_0)^r\theta^r \tag{2.19}$$

$$\frac{\rho C_p c(T_1-T_0)}{\lambda}\left(\frac{\partial}{\partial t}+(u+1)\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\theta=\frac{k_0(T_1-T_0)}{\lambda^2}\frac{\partial}{\partial x}\left(k(\theta)\frac{\partial\theta}{\partial x}\right)+$$
$$\frac{k_0(T_1-T_0)}{a^2}\frac{\partial}{\partial y}\left(k(\theta)\frac{\partial\theta}{\partial y}\right)+Q_0(T_1-T_0)^r\theta^r \tag{2.20}$$

$$\frac{\rho C_p ca^2}{k_0\lambda}\left(\frac{\partial}{\partial t}+(u+1)\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\theta=\frac{a^2}{\lambda^2}\frac{\partial}{\partial x}\left(k(\theta)\frac{\partial\theta}{\partial x}\right)+\frac{\partial}{\partial y}\left(k(\theta)\frac{\partial\theta}{\partial y}\right)+\frac{Q_0(T_1-T_0)^{r-1}}{k_0}\theta^r \tag{2.21}$$

$$\frac{\rho ca}{\mu_0}\frac{\mu_0 C_p}{k_0}\frac{a}{\lambda}\left(\frac{\partial}{\partial t}+(u+1)\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\theta=\delta^2\frac{\partial}{\partial x}\left(k(\theta)\frac{\partial}{\partial x}\right)+\frac{\partial}{\partial y}\left(k(\theta)\frac{\partial\theta}{\partial y}\right)+\frac{Q_0(T_1-T_0)^{r-1}}{k_0}\theta^r \tag{2.22}$$

$$\frac{\rho ca}{\mu_0}\frac{a}{\lambda}\left(\frac{\partial}{\partial t}+(u+1)\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\theta=\frac{\delta^2 k_0}{\mu_0 C_p}\frac{\partial}{\partial x}\left(k(\theta)\frac{\partial}{\partial x}\right)+\frac{k_0}{\mu_0 C_p}\frac{\partial}{\partial y}\left(k(\theta)\frac{\partial\theta}{\partial y}\right)+\frac{Q_0(T_1-T_0)^{r-1}a^2}{\mu_0 C_p}\theta^r \tag{2.23}$$

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$$Re\delta\left(\frac{\partial}{\partial t}+(u+1)\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\theta=\frac{\delta^2}{P_r}\frac{\partial}{\partial x}\left(k\left(\theta\right)\frac{\partial\theta}{\partial x}\right)+\frac{1}{P_r}\frac{\partial}{\partial y}\left(k\left(\theta\right)\frac{\partial\theta}{\partial y}\right)+\beta_r\theta^r \tag{2.24}$$

From eq. equation (2.9) we have;

$$\rho a^2 j\left(\frac{c^2}{a\lambda}\frac{\partial w}{\partial t}+\frac{c^2}{a\lambda}(u+1)\frac{\partial w}{\partial x}+\frac{c^2\delta}{a^2}\frac{\partial w}{\partial y}\right)=\frac{c}{a^3}\gamma\frac{\partial^2 w}{\partial y^2}-K\mu_0\left(\frac{2c}{a}w+\frac{c}{a}\frac{\partial u}{\partial y}\right) \tag{2.25}$$

$$\frac{\rho ca}{\mu_0}\delta j\left(\frac{\partial}{\partial t}+(u+1)\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)w=\frac{\gamma}{\mu_0 a^2}\frac{\partial^2 w}{\partial y^2}-K\left(2w+\frac{\partial u}{\partial y}\right) \tag{2.26}$$

$$Re\delta\left(\frac{\partial}{\partial t}+(u+1)\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)w=M\frac{\partial^2 w}{\partial y^2}-K\left(2w+\frac{\partial u}{\partial y}\right) \tag{2.27}$$

where

$$R_e=\frac{\rho ca}{\mu_0}, H_a^2=\frac{\sigma a^2B_0^2}{\mu_0}, \beta_r=\frac{Q_0a^2}{\mu_0C_p(T_1-T_0)^{1-r}}, G_r=\frac{\rho\alpha ga^2(T_1-T_0)}{\mu_0 c},$$
$$p_r=\frac{\mu_0 c_p}{k_0}, M=\frac{\gamma}{\mu_0 a^2} \tag{2.28}$$

R_e is the Reynold's number, H_a the magnetic parameter(Hartman number), G_r Grashof number, P_r Prandtl number, M micropolar parameter and β_r the rate of heat generation/absorption of order r.

Assuming a long wavelength and low Reynolds number in equations (2.24) – (2.28) above and neglecting high powers of δ, we obtain;

From equation (2.24) we have;

$$\frac{\partial p}{\partial x}=(\mu\left(\theta\right)+K)\frac{\partial^2 u}{\partial y^2}+\frac{\partial\mu\left(\theta\right)}{\partial y}\frac{\partial u}{\partial y}+K\frac{\partial w}{\partial y}-\left(Ha^2+\frac{\mu\left(\theta\right)+K}{k_p}\right)(u+1)+G_r\theta \tag{2.29}$$

From equation (2.25) we have;

$$\frac{\partial p}{\partial y}=0 \tag{2.30}$$

$$\frac{1}{P_r}\frac{\partial}{\partial y}\left(k\left(\theta\right)\frac{\partial\theta}{\partial y}\right)+\beta\theta^r=0 \tag{2.31}$$

From equation (2.26) we have;

$$M\frac{\partial^2 w}{\partial y^2}-K\left(2w+\frac{\partial u}{\partial y}\right)=0 \tag{2.32}$$

Introducing the Reynold's models along with linear variation of thermal conductivity

$$\mu\left(\theta\right) \ = \ 1-\varepsilon_1\theta \ \ and \ \ k\left(\theta\right)=1+\varepsilon_2\theta \tag{2.33}$$

Substituting for k(θ) and its associate derivatives in (2.27) to get;

$$k\left(\theta\right)\frac{\partial^2\theta}{\partial y^2}+\frac{\partial k\left(\theta\right)}{\partial y}\frac{\partial\theta}{\partial y}+\beta_r P_r\theta^r=0 \tag{2.34}$$

$$(1+\varepsilon_2\theta)\frac{\partial^2\theta}{\partial y^2}+\varepsilon_2\left(\frac{\partial\theta}{\partial y}\right)^2+\beta_r P_r\theta^r=0 \tag{2.35}$$

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In order for us to solve the problem exactly, we made the assumption of thermal viscosity parameter $\varepsilon_2 = 0$ in (2.35), to obtain,

$$\frac{\partial^2\theta}{\partial y^2} + \beta_r P_r \theta^r = 0 \tag{2.36}$$

where r = 0,1...n When r = 0, equation (2.36) becomes;

$$\frac{\partial^2\theta}{\partial y^2} + \beta_0 P_r = 0 \tag{2.37}$$

The associate boundary conditions;

$$\theta'(0) = 0, \theta(h) = 1 \tag{2.38}$$

Solving equation (2.37) with equation (2.38), we get

$$\theta(y) = 1 - \frac{\beta_0 P_r}{2}\left(y^2 - h^2\right) \tag{2.39}$$

$$\theta(0) = 0, \theta(h) = 1 \tag{2.40}$$

When r = 1

$$\frac{\partial^2\theta}{\partial y^2} + \beta_1 P_r \theta = 0 \tag{2.41}$$

Solving equation (2.42) together with equation (2.40), we get

$$\theta(y) = \sec(\sqrt{\beta_1 P_r} h)\cos(\sqrt{\beta_1 P_r} y) \tag{2.42}$$

For the case of $\varepsilon_2 \neq 0$,in equation (2.35) the problem will be solve by the method of differential transform method which will be treated as the general case in this study.
On differentiating equation (2.32) w.r.t y we have

$$\frac{\partial}{\partial y}\left((\mu(\theta) + k)\frac{\partial^2 u}{\partial y^2}\right) + \frac{\partial}{\partial y}\left(\frac{\partial\mu(\theta)}{\partial y}\frac{\partial u}{\partial y}\right) + k\frac{\partial^2 w}{\partial y^2} - \frac{\partial}{\partial y}\left(Ha^2 + \frac{\mu(\theta) + k}{k_p}\right)(u+1) + G_r\frac{\partial\theta}{\partial y} = 0$$

$$(\mu(\theta) + k)\tfrac{\partial^3 u}{\partial y^3} + \tfrac{\partial\mu(\theta)}{\partial y}\tfrac{\partial^2 u}{\partial y^2} + \tfrac{\partial\mu(\theta)}{\partial y}\tfrac{\partial^2 u}{\partial y^2} + \tfrac{\partial^2\mu(\theta)}{\partial y^2}\tfrac{\partial u}{\partial y} + k\tfrac{\partial^2 w}{\partial y^2} - \left(Ha^2 + \tfrac{\mu(\theta)+k}{k_p}\right)\tfrac{\partial u}{\partial y} - \tfrac{1}{k_p}\tfrac{\partial\mu(\theta)}{\partial y}(u+1) + G_r\tfrac{\partial\theta}{\partial y} = 0$$

$$(\mu(\theta) + K)\tfrac{\partial^3 u}{\partial y^3} + 2\tfrac{\partial\mu(\theta)}{\partial y}\tfrac{\partial^2 u}{\partial y^2} + \tfrac{\partial^2\mu(\theta)}{\partial y^2}\tfrac{\partial u}{\partial y} + K\tfrac{\partial^2 N}{\partial y^2} - \left(Ha^2 + \tfrac{\mu(\theta)+K}{k_p}\right)\tfrac{\partial u}{\partial y} - \tfrac{1}{k_p}\tfrac{\partial\mu(\theta)}{\partial y}(u+1) + G_r\tfrac{\partial\theta}{\partial y} = 0 \tag{2.43}$$

but $\mu(\theta) = 1 - \varepsilon_1\theta$ from Reynolds' model,

$$\begin{aligned}&(1+K)\tfrac{\partial^3 u}{\partial y^3} - \varepsilon_1\tfrac{\partial^3 u}{\partial y^3} - 2\varepsilon_1\tfrac{\partial\theta}{\partial y}\tfrac{\partial^2 u}{\partial y^2} - \varepsilon_1\tfrac{\partial u}{\partial y}\tfrac{\partial^2\theta}{\partial y^2} + K\tfrac{\partial^2 w}{\partial y^2} - \left(Ha^2 + \tfrac{1+K}{k_p}\right)\tfrac{\partial u}{\partial y}\\ &+G_r\tfrac{\partial\theta}{\partial y} + \varepsilon_1\theta\tfrac{\partial u}{\partial y} + \left(\tfrac{\varepsilon_1}{K_p} + G_r\right)\tfrac{\partial\theta}{\partial y} + \tfrac{\varepsilon_1}{K_p}u\tfrac{\partial\theta}{\partial y} = 0\end{aligned}$$

Suppose $\varepsilon_1 = 0$ eq. equation (2.35) becomes;

$$(1+K)\frac{\partial^3 u}{\partial y^3} + K\frac{\partial^2 w}{\partial y^2} - \left(Ha^2 + \frac{1+K}{k_p}\right)\frac{\partial u}{\partial y} + G_r\frac{\partial\theta}{\partial y} = 0 \tag{2.44}$$

From eq. equation (2.33), we obtain;

$$\begin{aligned}\tfrac{\partial u}{\partial y} &= \tfrac{M}{K}\tfrac{\partial^2 w}{\partial y^2} - 2w\\ \tfrac{\partial^2 u}{\partial y^2} &= \tfrac{M}{K}\tfrac{\partial^3 w}{\partial y^3} - 2\tfrac{\partial w}{\partial y}\\ \tfrac{\partial^3 u}{\partial y^3} &= \tfrac{M}{K}\tfrac{\partial^4 w}{\partial y^4} - 2\tfrac{\partial^2 w}{\partial y^2}\end{aligned} \tag{2.45}$$

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Substituting equation (2.45) into equation (2.44) to obtain;

$$(1+K)\left(\frac{M}{K}\frac{\partial^4 w}{\partial y^4}-2\frac{\partial^2 w}{\partial y^2}\right)+K\frac{\partial^2 w}{\partial y^2}-\left(Ha^2+\frac{1+K}{K_p}\right)\left(\frac{M}{K}\frac{\partial^2 w}{\partial y^2}-2w\right)+G_r\frac{\partial\theta}{\partial y}=0$$

$$(1+K)\frac{M}{K}\frac{\partial^4 w}{\partial y^4}-2(1+K)\frac{\partial^2 w}{\partial y^2}+K\frac{\partial^2 w}{\partial y^2}-\left(Ha^2+\frac{1+K}{K_p}\right)\frac{M}{K}\frac{\partial^2 w}{\partial y^2}+2\left(Ha^2+\frac{1+K}{K_p}\right)w+G_r\frac{\partial\theta}{\partial y}=0$$

$$M(1+K)\frac{\partial^4 w}{\partial y^4}-\left((K+2)+M\left(Ha^2+\frac{1+K}{k_p}\right)\right)\frac{\partial^2 w}{\partial y^2}+2K\left(Ha^2+\frac{1+K}{K_p}\right)w=-G_r\frac{\partial\theta}{\partial y}K \tag{2.46}$$

Which simplifies to

$$A\frac{\partial^4 w}{\partial y^4}+B\frac{\partial^2 w}{\partial y2}+Cw=g(y) \tag{2.47}$$

Where

$$\begin{aligned}&A=M(1+K), B=-\{K(2+K)+M\xi\}, C=2K\xi\\&and\ \xi=Ha^2+\tfrac{1+K}{K_p}, g(y)=-KG_r\tfrac{\partial\theta}{\partial y}\end{aligned} \tag{2.48}$$

equation (2.47) is a 4^{th} order linear homogeneous ODE whose solution are dependent on the nature of θ(y).

If $r=0$,

then

$\theta(y)=1-\frac{\beta_0 P_r}{2}\left(y^2-h^2\right)$

$$\Rightarrow g(y)=G_rK\beta_0P_ry \tag{2.49}$$

Thus, equation (2.47) becomes

$$A\frac{\partial^4 w}{\partial y^4}+B\frac{\partial^2 w}{\partial y2}+Cw=G_rK\beta_0P_ry \tag{2.50}$$

Whose solution is;

$$w(y)=c_1\cosh(\lambda_1 y)+c_2\sinh(\lambda_1 y)+c_3\cosh(\lambda_2 y)+c_4\sinh(\lambda_2 y)+\frac{G_r\beta_0P_r}{2\xi}y \tag{2.51}$$

Where

$$\lambda_1=\sqrt{\frac{-B+\sqrt{B^2-4AC}}{2A}}\ and\ \lambda_2=\sqrt{\frac{-B-\sqrt{B^2-4AC}}{2A}}$$

and c_1, c_2, c_3 and c_4 are arbitrary constants. substituting $\mu(\theta)=1-\varepsilon_1\theta$, when $\varepsilon_1=0$

From equation (2.29);

$$\frac{\partial p}{\partial x}=(\mu(\theta)+K)\frac{\partial^2 u}{\partial y^2}+\frac{\partial\mu(\theta)}{\partial y}\frac{\partial u}{\partial y}+K\frac{\partial w}{\partial y}-\left(Ha^2+\frac{\mu(\theta)+K}{k_p}\right)(u+1)+G_r\theta$$

$$\frac{\partial p}{\partial x}=(1+K)\frac{\partial^2 u}{\partial y^2}+K\frac{\partial w}{\partial y}-\xi(u+1)+G_r\theta$$

$$\frac{\partial p}{\partial x}=(1+K)\frac{\partial^2 u}{\partial y^2}+K\frac{\partial w}{\partial y}-\xi(u+1)+G_r\theta$$

$$\xi(u+1)=(1+K)\frac{\partial^2 u}{\partial y^2}+K\frac{\partial w}{\partial y}+G_r\theta-\frac{\partial p}{\partial x} \tag{2.52}$$

Substituting eq. equations (2.45) and (2.46) into equation (2.51) to obtain;

$$u+1=\frac{M\left(1+K\right)}{K\xi}\frac{\partial^3 w}{\partial y^3}-\frac{K+2}{\xi}\frac{\partial w}{\partial y}+\frac{G_r}{\xi}\left(1-\frac{\beta_0 P_r}{2}\left(y^2-h^2\right)\right)-\frac{1}{\xi}\frac{\partial p}{\partial x} \tag{2.53}$$

Differentiating (2.50) with respect to y three times to get;

$$\frac{\partial w}{\partial y}=c_1\lambda_1\sinh\left(\lambda_1 y\right)+c_2\lambda_1\cosh\left(\lambda_1 y\right)+c_3\lambda_2\sinh\left(\lambda_2 y\right)+c_4\lambda_2\cosh\left(\lambda_2 y\right)+\frac{G_r\beta_0 P_r}{2\xi}$$

$$\frac{\partial^2 w}{\partial y^2}=c_1\lambda_1^2\cosh\left(\lambda_1 y\right)+c_2\lambda_1^2\sinh\left(\lambda_1 y\right)+c_3\lambda_2^2\cosh\left(\lambda_2 y\right)+c_4\lambda_2^2\sinh\left(\lambda_2 y\right)$$

$$\frac{\partial^3 w}{\partial y^3}=c_1\lambda_1^3\sinh\left(\lambda_1 y\right)+c_2\lambda_1^3\cosh\left(\lambda_1 y\right)+c_3\lambda_2^3\sinh\left(\lambda_2 y\right)+c_4\lambda_2^3\cosh\left(\lambda_2 y\right) \tag{2.54}$$

Substituting (2.53) into equation (2.52) to obtain;

$$\begin{aligned}u\left(y\right)=&\tfrac{M(1+K)}{K\xi}\left[\ \lambda_1^3\left\{c_1\sinh\left(\lambda_1 y\right)+c_2\cosh\left(\lambda_1 y\right)\right\}+\lambda_2^3\left\{c_3\sinh\left(\lambda_2 y\right)+c_4\cosh\left(\lambda_2 y\right)\right\}\ \right]\\ &-\tfrac{K+2}{\xi}\left[\ \lambda_1\left\{c_1\sinh\left(\lambda_1 y\right)+c_2\cosh\left(\lambda_1 y\right)\right\}+\lambda_2\left\{c_3\sinh\left(\lambda_2 y\right)+c_4\cosh\left(\lambda_2 y\right)\right\}+\tfrac{G_r\beta_0 P_r}{2\xi}y\ \right]\\ &+\tfrac{G_r}{\xi}\left(1-\tfrac{\beta_0 P_r}{2}\left(y^2-h^2\right)\right)-\tfrac{1}{\xi}\tfrac{\partial p}{\partial x}-1\end{aligned}$$

Simplifying we have

$$\begin{aligned}u\left(y\right)=&\tfrac{M(1+K)}{K\xi}\left[\ \lambda_1^3\left\{c_1\sinh\left(\lambda_1 y\right)+c_2\cosh\left(\lambda_1 y\right)\right\}+\lambda_2^3\left\{c_3\sinh\left(\lambda_2 y\right)+c_4\cosh\left(\lambda_2 y\right)\right\}\ \right]\\ &-\tfrac{K+2}{\xi}\left[\ \lambda_1\left\{c_1\sinh\left(\lambda_1 y\right)+c_2\cosh\left(\lambda_1 y\right)\right\}+\lambda_2\left\{c_3\sinh\left(\lambda_2 y\right)+c_4\cosh\left(\lambda_2 y\right)\right\}+\tfrac{G_r\beta_0 P_r}{2\xi}y\ \right]\\ &+\tfrac{G_r}{\xi}\left(1-\tfrac{\beta_0 P_r}{2}\left(y^2-h^2\right)\right)-\tfrac{1}{\xi}\tfrac{\partial p}{\partial x}-1\end{aligned} \tag{2.55}$$

$$\begin{aligned}\tfrac{\partial u}{\partial y}=&\tfrac{M(1+K)}{K\xi}\left[\ \lambda_1^4\left\{c_1\cosh\left(\lambda_1 y\right)+c_2\sinh\left(\lambda_1 y\right)\right\}+\lambda_2^4\left\{c_3\cosh\left(\lambda_2 y\right)+c_4\sinh\left(\lambda_2 y\right)\right\}\ \right]\\ &-\tfrac{2+K}{\xi}\left[\lambda_1^2\left\{\ \left\{c_1\cosh\left(\lambda_1 y\right)+c_2\sinh\left(\lambda_1 y\right)\right\}+\lambda_2^2\left\{c_3\cosh\left(\lambda_2 y\right)+c_4\sinh\lambda_2 y\right\}\ \right\}\right]-\tfrac{G_r\beta_0 P_r}{\xi}y\end{aligned} \tag{2.56}$$

Considering the slip boundary conditions;

$$u\pm\varsigma\frac{\partial u}{\partial y}=-1\ at\ y=\pm\,h\left(x\right),\ \text{w}=0\ at\ y=\pm h\ \ and\ \psi=0\ at\ y=0$$

Where ς is the slip parameter.
Imposing (2.56) on (2.51), (2.54) and (2.55), we have;

$$c_1\cosh\left(\lambda_1 h\right)+c_2\sinh\left(\lambda_1 h\right)+c_3\cosh\left(\lambda_2 h\right)+c_4\sinh\left(\lambda_2 h\right)+\frac{G_r\beta_0 P_r}{2\xi}h=0 \tag{2.57}$$

$$c_1\cosh\left(\lambda_1 h\right)-c_2\sinh\left(\lambda_1 h\right)+c_3\cosh\left(\lambda_2 h\right)-c_4\sinh\left(\lambda_2 h\right)-\frac{G_r\beta_0 P_r}{2\xi}h=0 \tag{2.58}$$

$$\begin{aligned}&\xi_1\left\{\begin{array}{l}\left[c_1\sinh\left(\lambda_1 h\right)+c_2\cosh\left(\lambda_1 h\right)\right]+\alpha\lambda_1\left[c_1\cosh\left(\lambda_1 h\right)+c_2\sinh\left(\lambda_1 h\right)\right]\\ +\left[c_3\sinh\left(\lambda_2 h\right)+c_4\cosh\left(\lambda_2 h\right)\right]+\alpha\lambda_2\left[c_3\cosh\left(\lambda_2 h\right)+c_4\sinh\left(\lambda_2 h\right)\right]\end{array}\right\}\\ &-\tfrac{G_r\beta_0 P_r}{2\xi^2}\left(K+2\right)+\tfrac{G_r}{\xi}-\tfrac{G_r\beta_0 P_r\alpha}{\xi}h-\tfrac{1}{\xi}\tfrac{\partial p}{\partial x}=0\end{aligned} \tag{2.59}$$

$$\begin{aligned}&\xi_1\left[c_1\cosh\left(\lambda_1 h\right)-c_1\sinh\left(\lambda_1 h\right)+\alpha\lambda_1\left(c_2\sinh\left(\lambda_1 h\right)-c_2\cosh\left(\lambda_1 h\right)\right)\right]\\&+\xi_2\left[c_4\cosh\left(\lambda_2 h\right)-c_3\sinh\left(\lambda_2 h\right)+\alpha\lambda_2\left(c_4\sinh\left(\lambda_2 h\right)-c_3\cosh\left(\lambda_2 h\right)\right)\right]\\&\frac{G_r\beta_0 P_r}{2\xi^2}\left(K+2\right)+\frac{G_r}{\xi}h-\frac{1}{\xi}\frac{\partial p}{\partial x}=0\end{aligned}\tag{2.60}$$

Solving equations (2.57)–(2.60) simultaneously to obtain;

$$c_1=c_3=0, c_2=\frac{1}{2\xi^2 L}\left(L_1+2\sinh\left(\lambda_1 h\right)\xi\frac{\partial p}{\partial x}\right)\ and\ c_4=-\frac{1}{2\xi^2 L}\left(L_2+2\sinh\left(\lambda_1 h\right)\xi\frac{\partial p}{\partial x}\right)\tag{2.61}$$

where;

$$\xi_1=\frac{M\left(1+K\right)}{K\xi}\lambda_1^3-\frac{2+K}{\xi}\lambda_1\ and\ \ \xi_2=\frac{M\left(1+K\right)}{K\xi}\lambda_2^3-\frac{2+K}{\xi}\lambda_2\tag{2.62}$$

$$\begin{aligned}&L=\xi_1\left[\cosh\left(\lambda_1 h\right)+\varsigma\lambda_1\sinh\left(\lambda_1 h\right)\right]\sinh\left(\lambda_2 h\right)-\xi_2\left[\cosh\left(\lambda_2 h\right)+\varsigma\lambda_2\sinh\left(\lambda_2 h\right)\right]\sinh\left(\lambda_1 h\right)\\&L_1=\left(\cosh\left(\lambda_2 h\right)+\varsigma\lambda_2\sinh\left(\lambda_2 h\right)\right)G_r\beta_0 P_r h\xi\xi_2+\left(\varsigma\beta_0 P_r h\xi+\left(K+2\right)\beta_0 P_r-2\xi\right)G_r\sinh\left(\lambda_2 h\right)\\&L_2=\left(\cosh\left(\lambda_1 h\right)+\varsigma\lambda_1\sinh\left(\lambda_1 h\right)\right)G_r\beta_0 P_r h\xi\xi_1+\left(\varsigma\beta_0 P_r h\xi+\left(K+2\right)\beta-2\xi\right)G_r\sinh\left(\lambda_1 h\right)\end{aligned}\tag{2.63}$$

Substituting for c_1, c_2, c_3 and c_4 into their constituent equations, we obtain;

$$w\left(y\right)=\frac{1}{2\xi^2 L}\left(L_1\sinh\left(\lambda_1 y\right)-L_2\sinh\left(\lambda_2 y\right)\right)+\frac{1}{L\xi}\frac{\partial p}{\partial x}\left(\sinh\left(\lambda_2 h\right)\sinh\left(\lambda_1 y\right)-\sinh\left(\lambda_1 h\right)\sinh\left(\lambda_2 y\right)\right)\tag{2.64}$$

$$\begin{aligned}&u\left(y\right)=\frac{1}{2\xi^2 L}\left(L_1\xi\cosh\left(\lambda_1 y\right)-L_2\xi_2\cosh\left(\lambda_2 y\right)\right)+\frac{G_r(K+2)}{2\xi^2}+\frac{G_r}{\xi}\left(1-\frac{\beta_0 P_r}{2}\left(y^2-h^2\right)\right)-1\\&+\frac{1}{L\xi}\frac{\partial p}{\partial x}\left(\xi_1\sinh\left(\lambda_2 h\right)\cosh\left(\lambda_1 y\right)-\xi_2\sinh\left(\lambda_1 h\right)\cosh\left(\lambda_2 y\right)-L\right)\end{aligned}\tag{2.65}$$

and

$$\begin{aligned}&\psi\left(y\right)=\frac{1}{2\xi^2 L}\left(\frac{L_1\xi_1}{\lambda_1}\sinh\left(\lambda_1 y\right)-\frac{L_2\xi_2}{\lambda_2}\sinh\left(\lambda_2 y\right)\right)+\frac{G_r\beta_0 P_r(K+2)}{2\xi^2}y+\frac{G_r}{\xi}\left[y-\frac{\beta_0 P_r}{2}\left(\frac{y^3}{3}-h^2 y\right)\right]-y\\&+\frac{1}{L\xi}\frac{\partial p}{\partial x}\left[\frac{\xi_1}{\lambda_1}\sinh\left(\lambda_2 h\right)\sinh\left(\lambda_1 y\right)-\frac{\xi_2}{\lambda_2}\sinh\left(\lambda_1 h\right)\sinh\left(\lambda_2 y\right)-Ly\right]\end{aligned}\tag{2.66}$$

The volumetric rate of flow in the wave frame is given as;

$$q=\int_{-h}^{h}u\left(y\right)dy\tag{2.67}$$

Which simplifies to;

$$q=\frac{1}{L\xi^2}\left(\frac{L_1\xi_1}{\lambda_1}\sinh\left(\lambda h\right)-\frac{L_2\xi_2}{\lambda_2}\sinh\left(\lambda_2 h\right)\right)+\frac{F}{L\xi}\frac{\partial p}{\partial x}+\frac{G_r\beta_0 P_r(K+2)}{\xi^2}h+\frac{2G_r}{\xi}\left(h+\frac{\beta_0 P_r}{3}h^3\right)\tag{2.68}$$

where $F=2\left(\left(\frac{\xi_1}{\lambda_1}-\frac{\xi_2}{\lambda_2}\right)\sinh\left(\lambda_1 h\right)\sinh\left(\lambda_2 h\right)\right)-2Lh$

$$\frac{\partial p}{\partial x}=\frac{1}{F}\left(q+2h\right)L\xi-\frac{1}{F\xi}\left(\frac{L_1\xi}{\lambda_1}\sinh\left(\lambda_1 h\right)-\frac{L_2\xi_2}{\lambda_2}\sinh\left(\lambda_2 h\right)\right)-G_r\left(\frac{\beta_0 P_r(K+2)Lh}{F\xi}\right)-\frac{2L\xi G_r}{F}\left(h+\frac{\beta_r P_r}{3}h^3\right)\tag{2.69}$$

Case 2:

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If $r = 1$ in equation (2.36)

$$\theta(y) = \sec\left(\sqrt{\beta_1 P_r}h\right)\cos\left(\sqrt{\beta_1 P_r}y\right) \tag{2.70}$$

Thus,

$$g(y) = G_r K\sqrt{\beta_1 P_r}\sec\left(\sqrt{\beta_1 P_r}h\right)\sinh\left(\sqrt{\beta_1 P_r}y\right) \tag{2.71}$$

Thus, equation (2.51) becomes;

$$A\frac{\partial^4 N}{\partial y^4} + B\frac{\partial^2 N}{\partial y^2} + CN = G_r K\sqrt{\beta_1 P_r}\sec\left(\sqrt{\beta_1 P_r}h\right)\sinh\left(\sqrt{\beta_1 P_r}y\right) \tag{2.72}$$

Whose solution is;

$$\begin{aligned} &N(y) = c_1\cosh(\lambda_1 y) + c_2\sinh(\lambda_1 y) + c_3\cosh(\lambda_2 y) + c_4\sinh(\lambda_2 y) \\ &+\tfrac{G_r K\sqrt{\beta_0 P_r}}{Z}\sec\left(\sqrt{\beta_0 P_r}h\right)\sin\left(\sqrt{\beta_0 P_r}y\right) \end{aligned} \tag{2.73}$$

Where $Z = A\beta_0^2 - B\beta_0 + C$
And equation (2.52) becomes on substitution of equation (2.53)

$$u + 1 = \frac{M(1+K)}{K\xi}\frac{\partial^3 w}{\partial y^3} - \frac{K+2}{\xi}\frac{\partial w}{\partial y} + G_r\sec\left(\sqrt{\beta_0 P_r}h\right)\cos\left(\sqrt{\beta_0 P_r}y\right) - \frac{1}{\xi}\frac{\partial p}{\partial x} \tag{2.74}$$

Differentiating equation (2.73) 3 times to get

$$\begin{aligned} &\tfrac{\partial w}{\partial y} = \lambda_1\left(c_1\sinh(\lambda_1 y) + c_2\cosh(\lambda_1 y)\right) + \lambda_2\left(c_3\sinh(\lambda_2 y) + c_4\cosh(\lambda_2 y)\right) \\ &+\tfrac{1}{Z}G_r K\beta_1 P_r\sqrt{\beta_1 P_r}\sec\left(\sqrt{\beta_1 P_r}h\right)\cosh\sqrt{\beta_1 P_r}y \\ &\tfrac{\partial^2 w}{\partial y^2} = \lambda_1^2\left(c_1\cosh(\lambda_1 y) + c_2\sinh(\lambda_1 y)\right) + \lambda_2^2\left(c_3\cosh(\lambda_2 y) + c_4\sinh(\lambda_2 y)\right) \\ &-\tfrac{1}{Z}G_r K\beta_1 P_r\sqrt{\beta_1 P_r}\sec\left(\sqrt{\beta_1 P_r}h\right)\sinh\left(\sqrt{\beta_1 P_r}y\right) \\ &\tfrac{\partial^3 w}{\partial y^3} = \lambda_1^3\left(c_1\sinh(\lambda_1 y) + c_2\cosh(\lambda_1 y)\right) + \lambda_2^3\left(c_3\sinh(\lambda_2 y) + c_4\cosh(\lambda_2 y)\right) \\ &-\tfrac{1}{Z}G_r K\left(\beta_1 P_r\right)^2\sqrt{\beta_1 P_r}\sec\left(\sqrt{\beta_1 P_r}h\right)\cosh\left(\sqrt{\beta_1 P_r}y\right) \end{aligned} \tag{2.75}$$

Substituting equation (2.75) into equation (2.74) to obtain;

$$\begin{aligned} u(y) = &\,\xi_1\left(c_1\sinh(\lambda_1 y) + c_2\cosh(\lambda_1 y)\right) + \xi_2\left(c_3\sinh(\lambda_2 y) + c_4\cosh(\lambda_2 y)\right) \\ &+G_r H\cos\left(\sqrt{\beta_1 P_r}y\right) - \tfrac{1}{\xi}\tfrac{\partial p}{\partial x} - 1 \end{aligned} \tag{2.76}$$

where

$$H = \left(\frac{M(1+K)}{K\xi} - \frac{K+2}{\xi}\right)\frac{G_r K\beta_1 P_r\sec\left(\sqrt{\beta_1 P_r}h\right)}{Z}\left(1 - \beta_1 P_r\right) \tag{2.77}$$

$$\begin{aligned} \tfrac{\partial u}{\partial y} = &\,\xi_1\lambda_1\left[c_1\cosh(\lambda_1 y) + c_2\sinh(\lambda_1 y)\right] + \xi_2\lambda_2\left[c_3\cosh(\lambda_2 y) + c_4\sinh(\lambda_2 y)\right] \\ &-G_r H\sqrt{\beta_1 P_r}\sinh\left(\sqrt{\beta_1 P_r}y\right) \end{aligned} \tag{2.78}$$

Invoking the boundary conditions in equations equation (2.19) into equations (2.73), (2.76) and (2.77) to obtain;

$$c_1\cosh(\lambda_1 h) + c_2\sinh(\lambda_1 h) + c_3\cosh(\lambda_2 h) + c_4\sinh(\lambda_2 h) + \frac{1}{Z}G_r K\sqrt{\beta_1 P_r}\tan\left(\sqrt{\beta_1 P_r}h\right) = 0 \tag{2.79}$$

$$c_1\cosh(\lambda_1 h)-c_2\sinh(\lambda_1 h)+c_3\cosh(\lambda_2 h)-c_4\sinh(\lambda_2 h)-\frac{1}{Z}G_rK\sqrt{\beta_1P_r}\tan\left(\sqrt{\beta_1P_r}h\right)=0 \tag{2.80}$$

$$\begin{aligned}&\xi_1\left[(c_1\sinh(\lambda_1 h)+c_2\cosh(\lambda_1 h))+\varsigma\lambda_1(c_1\cosh(\lambda_1 h)+c_2\sinh(\lambda_1 h))\right]\\&+\xi_2\left[(c_3\sinh(\lambda_2 h)+c_4\cosh(\lambda_2 h))+\varsigma\lambda_2(c_3\cosh(\lambda_2 h))+c_4\sinh(\lambda_2 h)\right]\\&+G_r\varsigma\left(\cos\left(\sqrt{\beta_1P_r}h\right)-\sqrt{\beta_1P_r}\sin\left(\sqrt{\beta_1P_r}h\right)\right)-\tfrac{1}{\xi}\tfrac{\partial p}{\partial x}=0\end{aligned} \tag{2.81}$$

$$\begin{aligned}&\xi_1\left[(c_2\cosh(\lambda_1 h)-c_1\sinh(\lambda_1 h))-\varsigma\lambda_1(c_1\cosh(\lambda_1 y)-c_2\sinh(\lambda_1 y))\right]\\&+\xi_2\left[(c_4\cosh(\lambda_2 y)-c_3\sinh(\lambda_2 y))-\varsigma\lambda_2(c_3\cosh(\lambda_2 y))-c_4\sinh(\lambda_2 y)\right]\\&+G_r\varsigma\left(\cos\left(\sqrt{\beta_1P_r}h\right)+\sqrt{\beta_1P_r}\sin\left(\sqrt{\beta_1P_r}h\right)\right)-\tfrac{1}{\xi}\tfrac{\partial p}{\partial x}=0\end{aligned} \tag{2.82}$$

Solving equations equations (2.79)–(2.82) simultaneously to get;

$$c_1=c_3=0, c_2=\frac{1}{L\xi Z}\left(L_1\xi+Z\frac{\partial p}{\partial x}\sinh(\lambda_2 h)\right)\ \mathit{and}\ c_4=-\frac{1}{L\xi Z}\left(L_2\xi+Z\frac{\partial p}{\partial x}\sinh(\lambda_2 h)\right) \tag{2.83}$$

where;

$$L=\xi_1\left[\cosh(\lambda_1 h)+\varsigma\lambda_1\sinh(\lambda_1 h)\right]\sinh(\lambda_2 h)+\xi_2\left[\cosh(\lambda_2 h)+\varsigma\lambda_2\sinh(\lambda_2 h)\right]\sinh(\lambda_1 h)$$

$$L_1=G_r\left(\begin{array}{l}K\tan\left(\sqrt{\beta_1P_r}h\right)\sqrt{\beta_1P_r}\xi_2\left\{\cosh(\lambda_2 h)+\varsigma\lambda_2\sinh(\lambda_2 h)\right\}+\\ \varsigma Z\left\{\sqrt{\beta_1P_r}\sinh\left(\sqrt{\beta_1P_r}h\right)-\cos\left(\sqrt{\beta_1P_r}h\right)\right\}\sinh(\lambda_2 h)\end{array}\right)$$

$$L_2=G_r\left(\begin{array}{l}K\tan\left(\sqrt{\beta_1P_r}h\right)\sqrt{\beta_1P_r}\xi_1\left\{\cosh(\lambda_1 h)+\varsigma\lambda_1\sinh(\lambda_1 h)\right\}+\\ \varsigma Z\left\{\sqrt{\beta_1P_r}\sinh\left(\sqrt{\beta_1P_r}h\right)-\cos\left(\sqrt{\beta_1P_r}h\right)\right\}\sinh(\lambda_2 h)\end{array}\right)$$

Substituting equation (2.83) into equations (2.73) and (2.76) to obtain;
From equation (2.73) we have;

$$\begin{aligned}w(y)=&\tfrac{1}{LZ}\left(L_1\sinh(\lambda_1 y)-L_2\sinh(\lambda y)\right)+\tfrac{G_rK\sqrt{\beta_1P_r}}{Z}\sec\left(\sqrt{\beta_1P_r}h\right)\sinh\left(\sqrt{\beta_1P_r}y\right)\\&+\frac{\partial p/\partial x}{L\xi}\left[\sinh(\lambda_2 h)\sinh(\lambda_1 y)-\sinh(\lambda_1 h)\sinh(\lambda_2 y)\right]\end{aligned} \tag{2.84}$$

From equation (2.76) we have;

$$\begin{aligned}u(y)=&\tfrac{1}{LZ}\left(L_1\xi_1\cosh(\lambda_1 y)-L_2\xi_2\cosh(\lambda_2 y)\right)+\frac{\partial p/\partial x}{L\xi}\left(\begin{array}{l}\xi_1\sinh(\lambda_2 h)\cosh(\lambda_1 y)\\-\xi_2\sinh(\lambda_2 y)\end{array}\right)\\&+G_rH\cos\left(\sqrt{\beta_1P_r}y\right)-\tfrac{1}{\xi}\tfrac{\partial p}{\partial x}-1\end{aligned} \tag{2.85}$$

and

$$\begin{aligned}\psi(y)=&\tfrac{1}{LZ}\left(\tfrac{L_1\xi_1}{\lambda_1}\sinh(\lambda_1 y)-\tfrac{L_2\xi_2}{\lambda_2}\sinh(\lambda_2 y)\right)\\&+\frac{\partial p/\partial x}{L\xi}\left(\begin{array}{l}\frac{\xi_1}{\lambda_1}\sinh(\lambda_2 h)\sinh(\lambda_1 y)\\-\frac{\xi_2}{\lambda_2}\sinh(\lambda_2 y)\end{array}\right)+\tfrac{G_rH}{\sqrt{\beta_1P_r}}\sinh\left(\sqrt{\beta_1P_r}y\right)-\frac{\partial p/\partial x}{\xi}y-y\end{aligned} \tag{2.86}$$

The volumetric rate of flow is given by

$$q=\int_{-h}^{h}u(y)\,dy \tag{2.87}$$

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$$q = \frac{2}{LZ}\left(\frac{L_1\xi_1}{\lambda_1}\sinh(\lambda_1 h) - \frac{L_2\xi_2}{\lambda_2}\sinh(\lambda_2 h)\right) + \frac{\partial p/\partial x}{L\xi}F + \frac{2G_r H}{\sqrt{\beta_1 P_r}}\sin\left(\sqrt{\beta_1 P_r}h\right) - 2h \quad (2.88)$$

where

$$F = 2\left(\frac{\xi_1}{\lambda_1} - \frac{\xi_2}{\lambda_2}\right)\sinh(\lambda_1 h)\sinh(\lambda_2 h) - 2Lh \quad (2.89)$$

$$\frac{\partial p}{\partial x} = \frac{(q+2h)L\xi}{F} - \frac{2\xi}{FZ}\left(\frac{L_1\xi_1}{\lambda_1}\sinh(\lambda_1 h) - \frac{L_2\xi_2}{\lambda_2}\sinh(\lambda_2 h)\right) - \frac{2G_r H L\xi}{F\sqrt{\beta_1 P_r}}\sin\left(\sqrt{\beta_r P_r}h\right) \quad (2.90)$$

3 Graphical Results and Discussions

The analytical solutions for the axial velocity, micro rotation component, pressure gradient , volumetric flow rate and stream function with energy equation were obtained in the previous section for the cases when $r = 0$ *and* 1. This section presents the results obtained graphically using some of the parameters as [3,9,19,25,29,35]. Figure **??** represent the variations of axial velocity with the height when $x = 0$ for different values of all parameters of interest. It can be seen from Figure **??** that an increase in the magnetic parameter $H_a(=\frac{\sigma a^2 B_0^2}{\mu_0})$ reduces the speed of the fluid. It can be observe that at the walls of the channel, the flow started reducing and is more pronounced at the centre of the channel. This is as a result of the external magnetic force that was applied perpendicular to the flow. Figure **??** shows the effect of slip parameters on the axial velocity and it can be seen that the flow stratified increment near the channels as we increase the slip parameter and damping the speed of flow at the centre of the channel. This evident of slips parameter is well pronounce near the walls of the channel. Figure **??** describe the effect of porous permeability parameter (K_p) on the axial velocity. It can be noticed from Figure **??** that as the Grashof number $G_r(=\frac{\rho\alpha g a^2(T_1-T_0)}{\mu_0 c})$ is increasing, the velocity is damping while the reverse is notice for Prandtl number $P_r(=\frac{\mu_0 c_p}{k_0})$ in Figure **??**.

The description of microrotation components were depicted in Figure **??**. It will be noticed from Figures. (**??**, **??**, **??**, **??**) that the microrotation component increases as the governing parameters were increasing while Figures. (**??**, **??**) reduces. Figures. **??** and **??** show the effect of the governing parameters on the shear stresses τ_{xy} and τ_{yx} at the lower and upper walls of the channel. It will be notice that enhancing the magnetic parameter (H_a) and viscosity parameter (K) increases the shear stress at both walls of the channel. The reduction in shear stress at the two walls sets in with increment in porous permeability parameter (K_p), slip parameter (ς), Prandtl number (P_r) and Grasphof number (G_r). The graphs of pressure rise against slip parameter for different Hartmann number (magnetic parameter) explain that the pressure rise decreases as the magnetic parameters is increasing as shown in Figures **??** and **??**. Figure **??** and **??** reveal that the pressure rise ΔP begins to drop as the Grasphof and Prandtl number increases.

Figures [**??** - **??**] describe the stream functions for different parameters of interest. The distribution of stream lines pattern in the presence of magnetic field are shown in Figure **??** in one wave length. We observe that as the Hartmann number H_a increases the formation of bolus at the wall decreases in size. It is interesting to note from Figure **??** that the trapped bolus decreases in size and ultimately vanishes for increasing values of slip parameter ς. Thus, the magnetic field strength and the slip effects helps to restrict the formation of the trapped bolus. Figure **??** gives the distribution of streamlines for different diverging angle θ. We observe that as θ increases the trapped bolus found to increase in size on both sides of the central line of the channel. However, the porous permeability parameter K_p keeps to form more closed streamlines at the wall as shown in Figure **??**.

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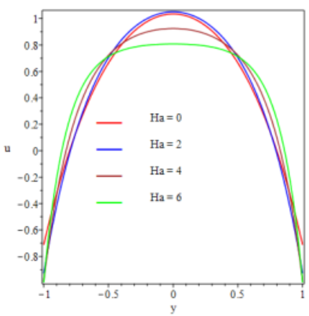
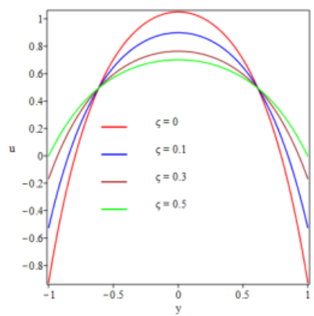
4 Conclusions

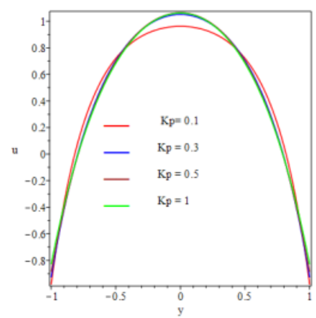
In this paper, an attempt has been made to investigate the effects of slip velocity on the peristaltic transport of physiological fluids represented by a micropolar fluid model passing through a non-uniform porous channel. In this investigation, special emphasis has been paid to the study of velocity distribution, the pumping characteristics and the trapping phenomena.
From the present analysis, one can make an important conclusion that it is possible to increase pumping action (pressure gradient) as often as necessary by applying an external magnetic field and that the bolus formation can be eliminated with a considerable extent. The wall shear stresses τ_{xy} and τ_{yx} increase with the increase of Hartmann number H_a at the lower and upper walls. The slip velocity at the wall has reducing effect on the formation of trapped bolus. Thus the results presented here throws some light on problems associated with fluid movement in the gastrointestinal tract, intra-uterine fluid motion induced by uterine contraction, as well as flow through small blood vessels and intrapleural membranes.



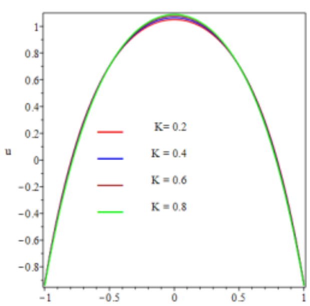
(a) Variation of magnetic parameter H_a



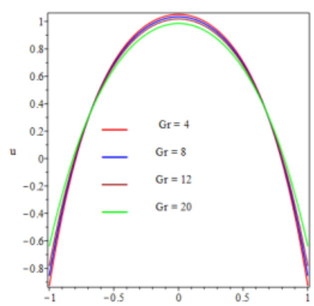
(b) Variation of slip parameter ς



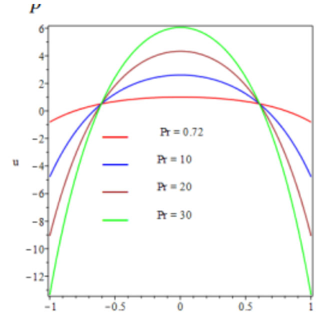
(c) Variations of porous permeability parameter K_p



(d) Variations of viscosity parameter ratio K



(e) Variation of Graphof number G_r



(f) Variations of Prandtl number P_r

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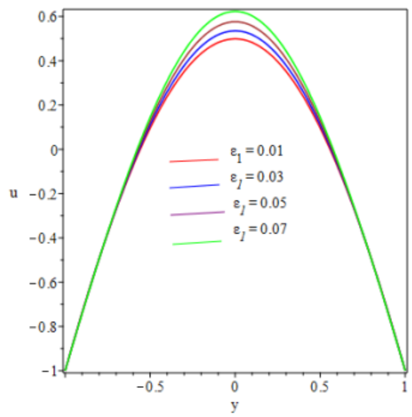
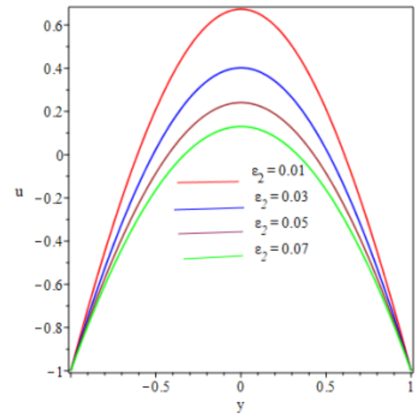
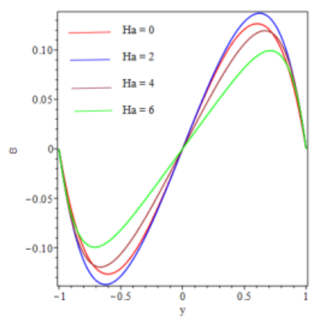


(g) Variations of variable viscosity parameter ε_1

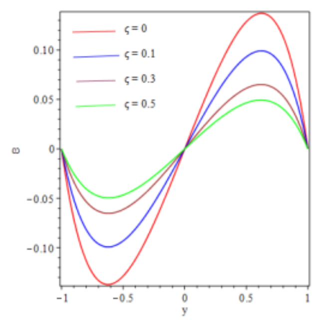


(h) Variations of variable thermal conductivity parameter ε_2

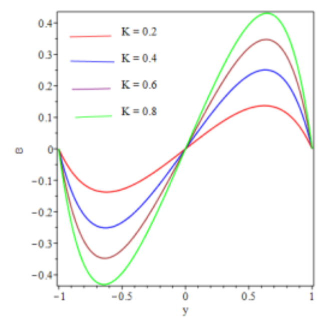
Figure 2: Graphs of axial velocity with varying parameters of interest



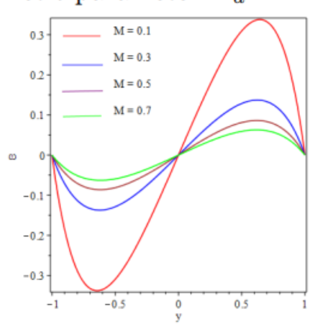
(a) Variation of magnetic parameter H_a



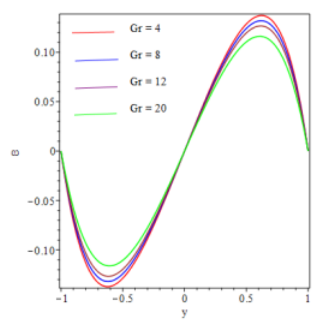
(b) Variation of slip parameter ς



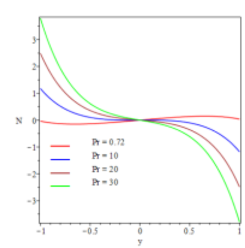
(c) Variations of viscosity ratio parameter K



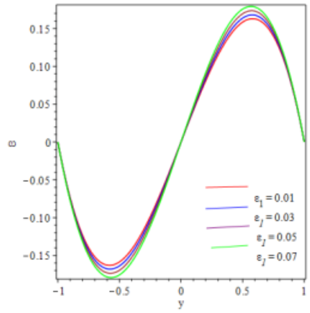
(d) Variations of micropolar parameter M



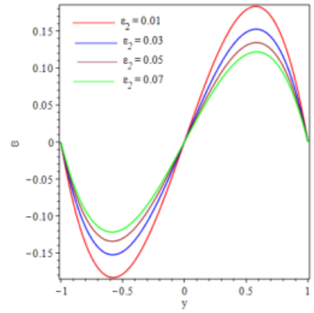
(e) Variation of Graphof number G_r



(f) Variations of Prandtl number P_r

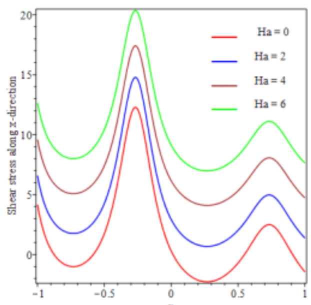


(g) Variations of variable viscosity parameter ε_1

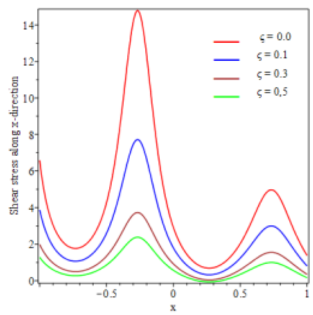


(h) Variations of variable thermal conductivity parameter ε_2

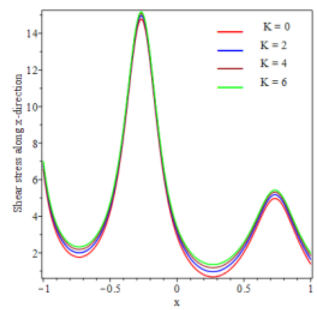
Figure 3: Graphs of micrrotation component with varying parameters of interest



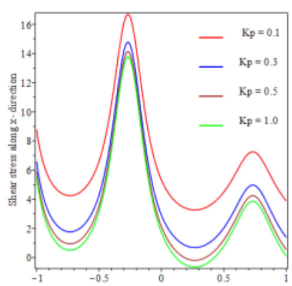
(a) Variation of magnetic parameter H_a



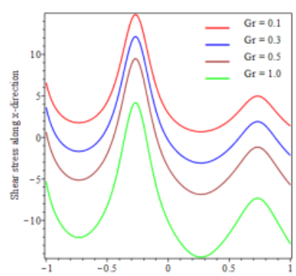
(b) Variation of slip parameter ς



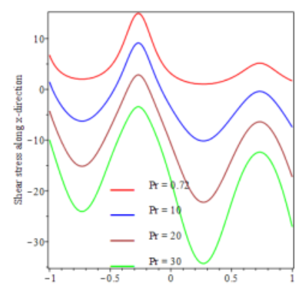
(c) Variations of viscosity ratio parameter K



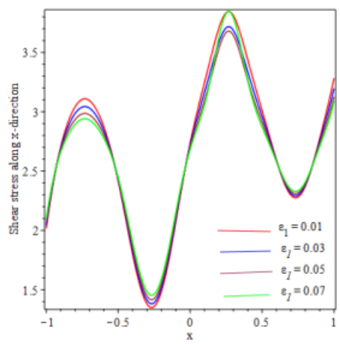
(d) Variations of porosity parameter K_p



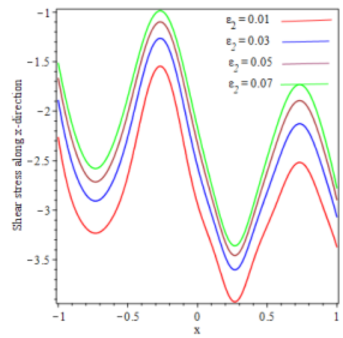
(e) Variation of Graphof number G_r



(f) Variations of Prandtl number P_r

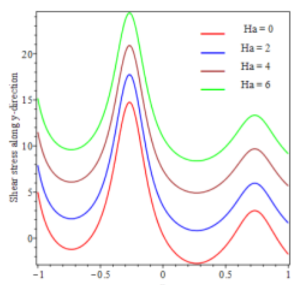


(g) Variation of variable viscosity parameter ε_1

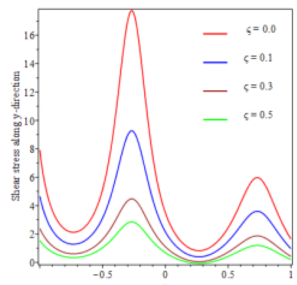


(h) Variation of variable thermal conductivity parameter ε_2

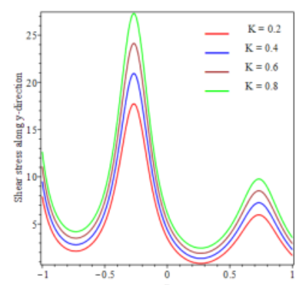
Figure 4: Graphs of wall shear stress τ_{xy} at the lower wall of the channel



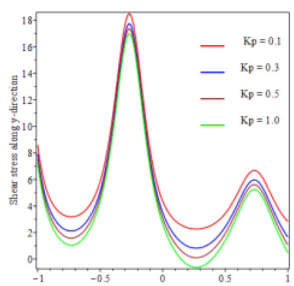
(a) Variation of magnetic parameter H_a



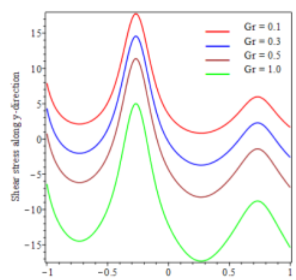
(b) Variation of slip parameter ς



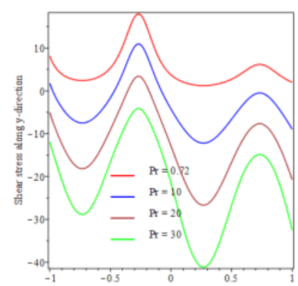
(c) Variations of viscosity ratio parameter K



(d) Variations of porosity parameter K_p



(e) Variation of Graphof number G_r



(f) Variations of Prandtl number P_r

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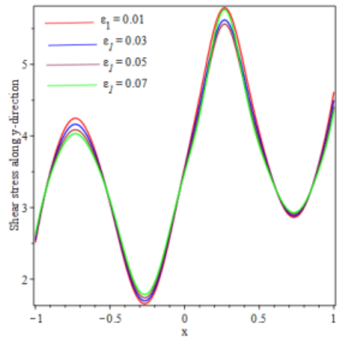
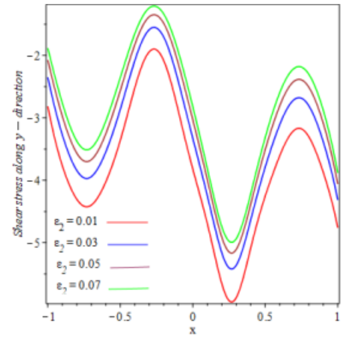
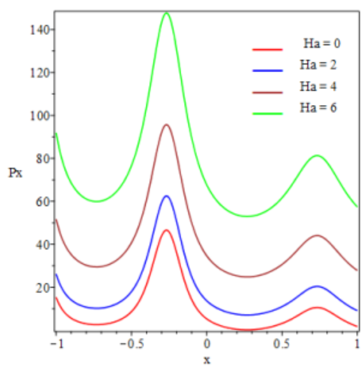


(g) Variations of variable viscosity parameter ε_1

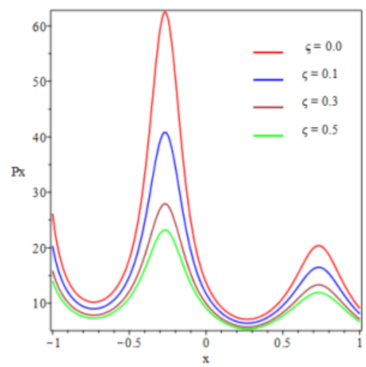


(h) Variations of variable thermal conductivity parameter ε_2

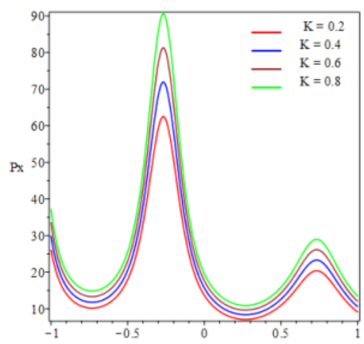
Figure 5: Graphs of wall shear stress τ_{yx} at the lower wall of the channel



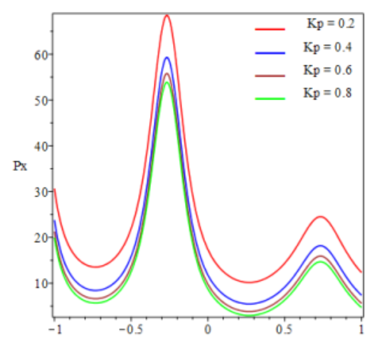
(a) Variation of magnetic parameter H_a



(b) Variation of slip parameter ς



(c) Variations of viscosity ratio parameter K



(d) Variations of porosity parameter K_p

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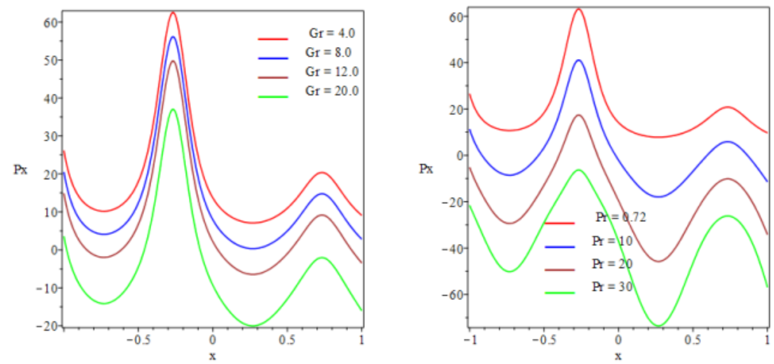
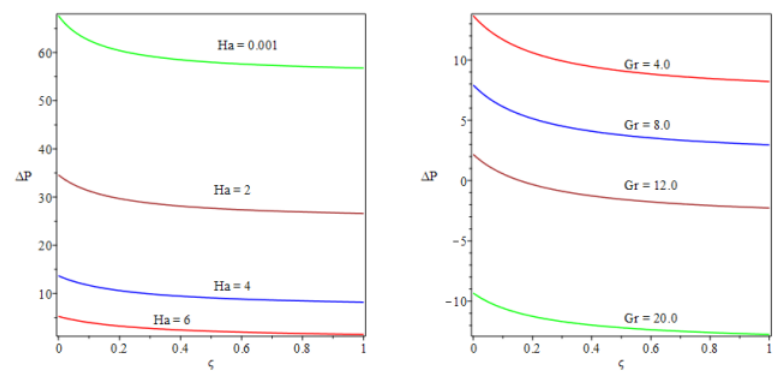


(e) Variation of Graphof number G_r

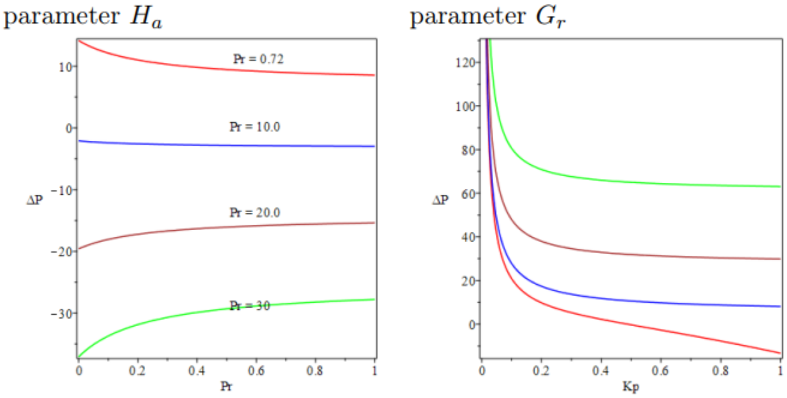
(f) Variations of Prandtl number P_r

Figure 6: Distribution of pressure gradient



(a) Pressure rise against slip parameter with varying magnetic parameter H_a

(b) Pressure rise against slip parameter with varying magnetic parameter G_r



(c) Pressure rise against slip parameter with varying magnetic parameter K_p

(d) Pressure rise against slip parameter with varying magnetic parameter H_a

Figure 7: Graph of pressure rise

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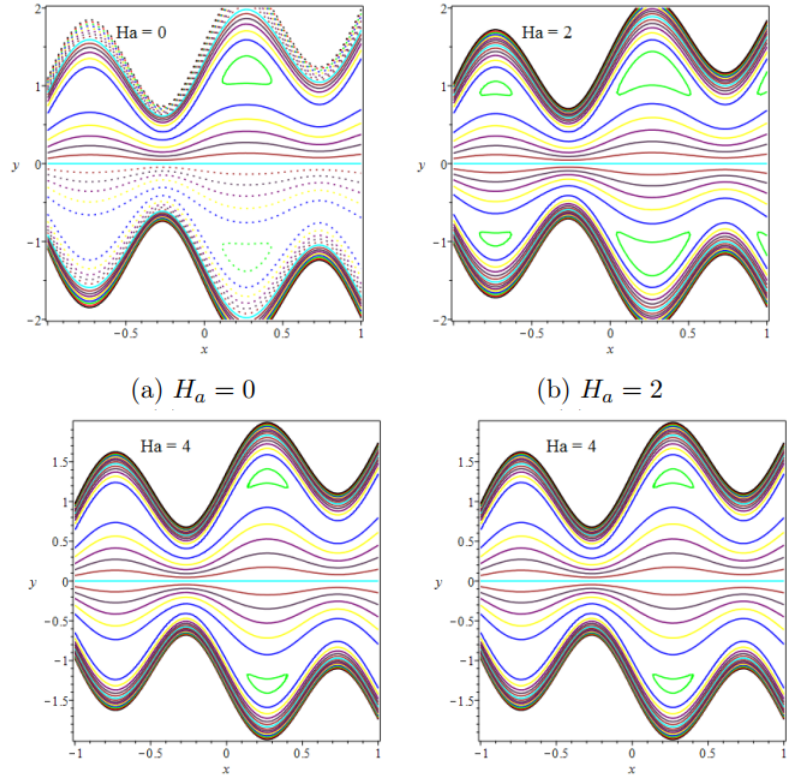
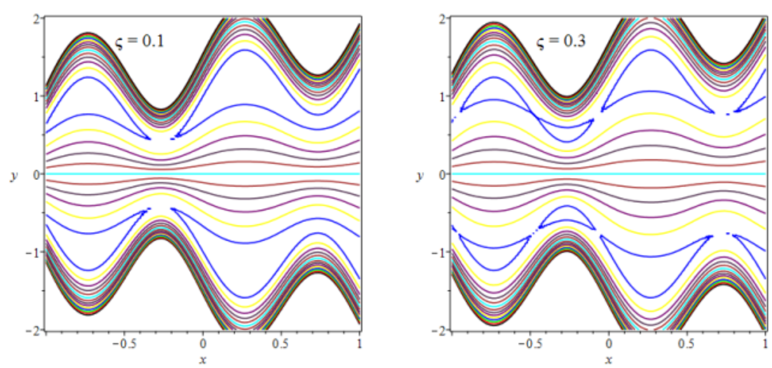


(a) $H_a = 0$ (b) $H_a = 2$

(c) $H_a = 4$ (d) $H_a = 6$

Figure 8: Distribution of streamlines for different Hartman number H_a



(a) $\varsigma = 0.1$ (b) $\varsigma = 0.3$

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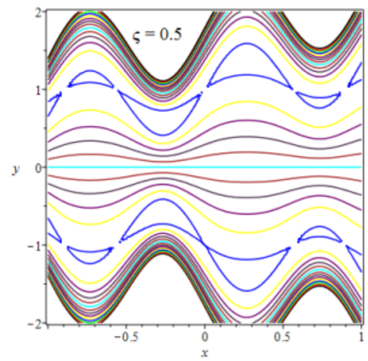
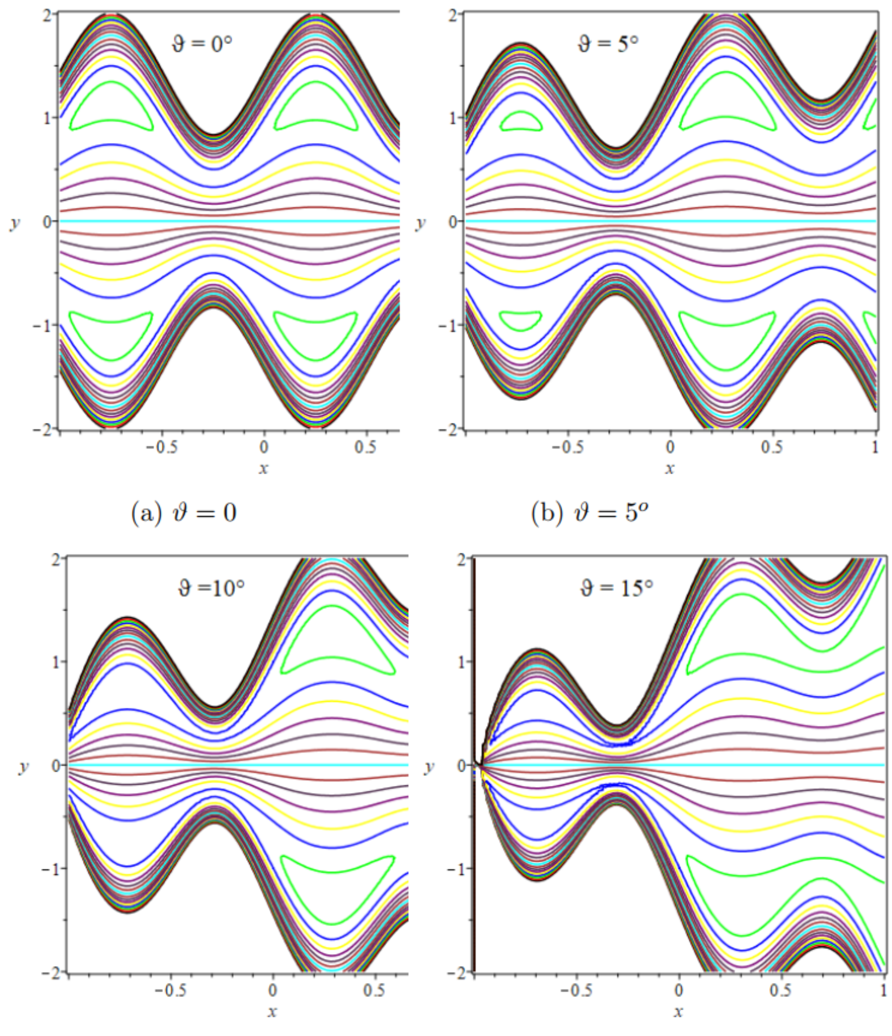


(c) $\varsigma = 0.5$

Figure 9: Distribution of streamlines for different slip parameters ς



(a) $\vartheta = 0$

(b) $\vartheta = 5^o$

(c) $\vartheta = 10^o$

(d) $\vartheta = 15^o$

Figure 10: Distribution of streamlines for different tilt angle ϑ

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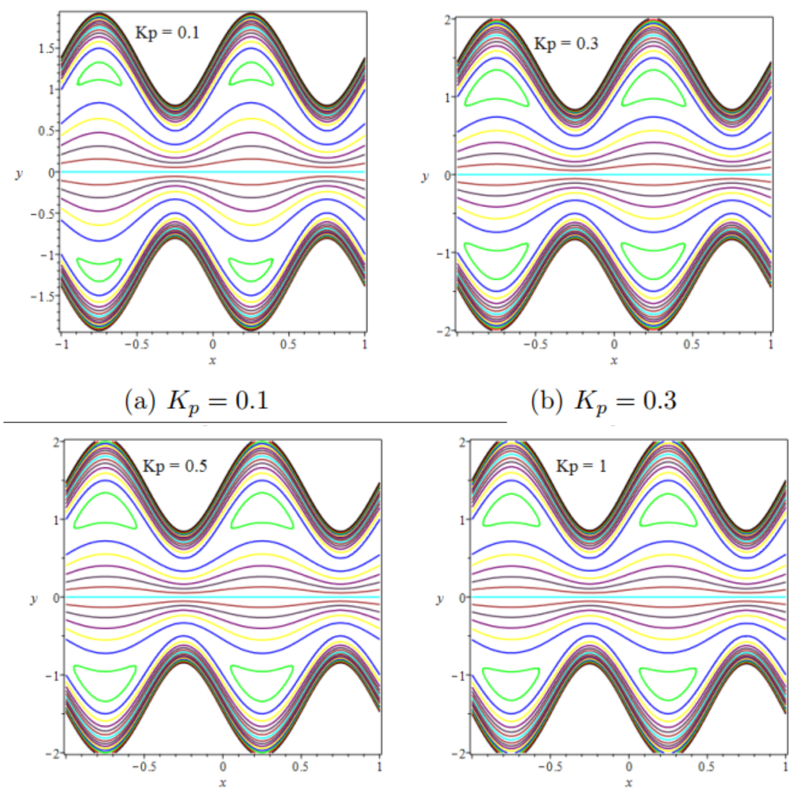
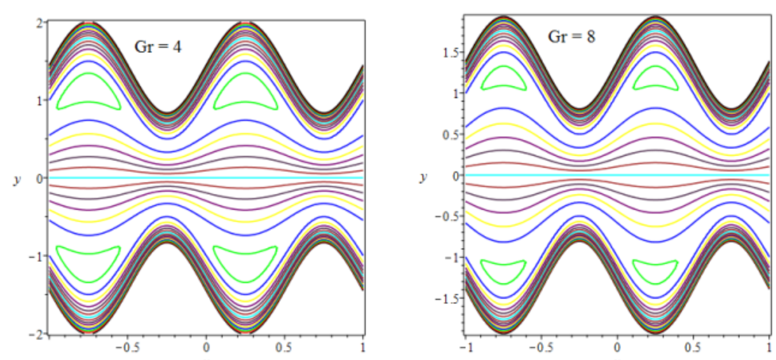


(a) $K_p = 0.1$ (b) $K_p = 0.3$

(c) $K_p = 0.5$ (d) $K_p = 1$

Figure 11: Distribution of streamlines for different permeability K_p



(a) $G_r = 4$ (b) $G_r = 8$

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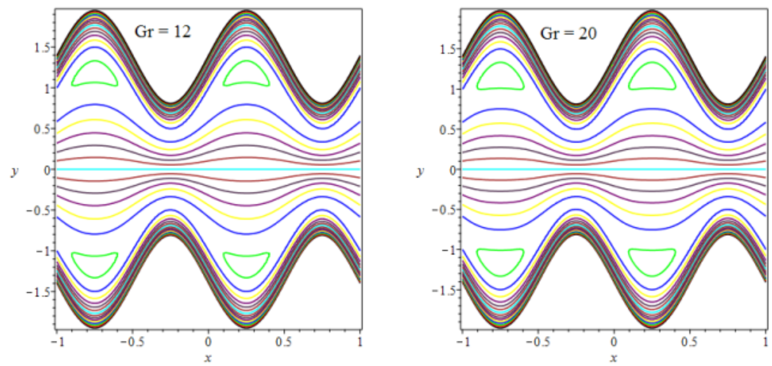
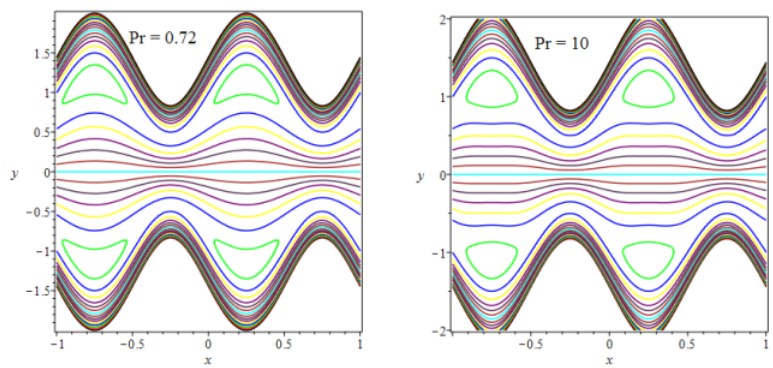
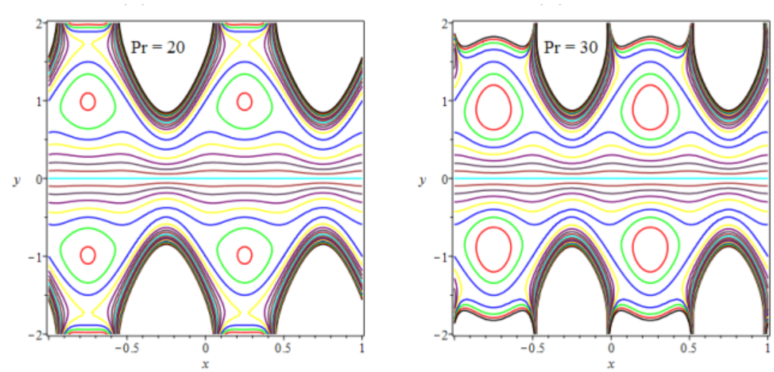


(c) $G_r = 12$ (d) $G_r = 20$

Figure 12: Distribution of streamlines for different Graphof number G_r



(a) $P_r = 0.72$ (b) $P_r = 10$



(c) $P_r = 20$ (d) $P_r = 30$

Figure 13: Distribution of streamlines for different Prandtl number P_r

Conflicts of Interest

No conflict of interest was declared by the authors.

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