MultiCraft

International Journal of Engineering, Science and Technology
Vol. 3, No. 9, 2011, pp. 47-65

INTERNATIONAL JOURNAL OF ENGINEERING, SCIENCE AND TECHNOLOGY

www.ijest-ng.com
www.ajol.info/index.php/ijest

Bayesian estimations in the Kumaraswamy distribution under progressively type II censoring data

Ramin Gholizadeh[1], Mastane Khalilpor[2] and Maghdoode Hadian[3]

[1]Young Researchers Club, Ayatollah Amoli Branch, Islamic Azad University, Amol , IRAN
[2]Department of Statistics, Payame Noor University of Amol, IRAN
[3]Young Researchers Club, Mashhad Branch, Islamic Azad University, Mashhad , IRAN
Corresponding Authors: e-mails: [1]ramin.gholizadh@gmail.com, [2]m.khalilpor@yahoo.com, [3]hadian.maghdoodeh@gmail.com

Abstract

This paper seeks to focus on the study and Bayesian and non-Bayesian estimators for the shape parameter, reliability and failure rate functions of the Kumaraswamy distribution in the cases of progressively type II censored samples. Maximum likelihood estimation and Bayes estimation, reliability and failure rate functions are obtained using symmetric and asymmetric loss functions. Comparisons are made between these estimators using Monte Carlo simulation study. With prior information on the parameter of the Kumaraswamy distribution, Bayes approach under squared error loss function in the reliability function has been suggested based on the pervious observations, this approach can be used for both progressively type II censorings. The study is useful for researchers and practitioners in reliability theory and quality also for scientists in physics and chemistry special hydrological literatare, where Kumaraswamy distribution is widely used.

Keywords: Kumaraswamy distribution; Bayesian estimation; progressively Type II right censoring data; Reliability; Failure rate; simulation study.

DOI: http://dx.doi.org/10.4314/ijest.v3i9.4

1. Introduction

The Kumaraswamy distributions was constructed by Kumaraswamy (1980). Jones (2009) said about it' properties. The probability density function of a Kumaraswamy distributed random variable is given by

$$f_T(t) = \theta \lambda t^{\lambda-1}(1-t^{\lambda})^{\theta-1} \qquad 0 < t < 1, \quad \lambda, \theta > 0\,, \tag{1}$$

where θ and λ are shape parameters, respectively. Here we assume that λ parameter is known. The distribution function $(c.d.f)$ is;

$$F_T(t;\theta) = 1-(1-t^{\lambda})^{\theta} \qquad 0 < t < 1, \quad \lambda, \theta > 0 \tag{2}$$

The reliability and failure rate functions of Kumaraswamy distribution are given respectively, by

$$R(t) = (1-t^{\lambda})^{\theta} \quad ; \quad 0 < t < 1 \;\; , \;\; \lambda, \theta > 0, \tag{3}$$

and

$$H(t) = \frac{\lambda \theta t^{\lambda-1}}{1-t^{\lambda}} \quad ; \quad 0 < t < 1 \ , \ \lambda, \theta > 0 . \tag{4}$$

Figure 1 shows the shape of $f(t;\theta,\lambda)$ for different values of θ and λ.

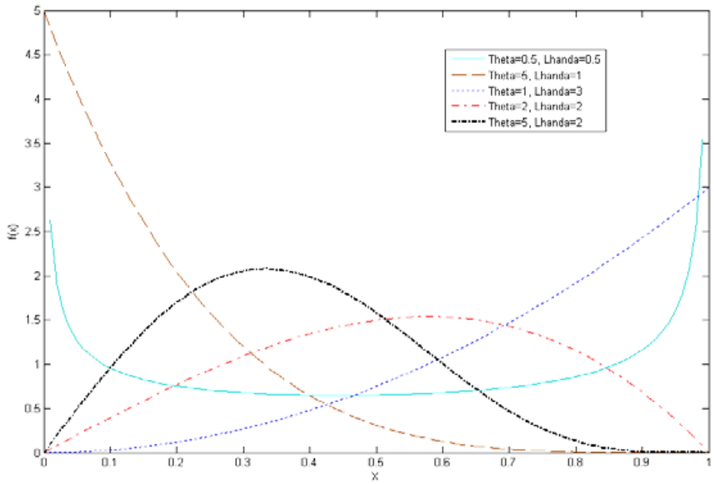

Figure 1: p.d.f. of Kumaraswamy for different values of θ and λ.

In Bayesian estimation, three types of loss functions are considered. The first is the squared error loss function (quadratic loss) which is classified as a symmetric function .The second is the Precautionary loss function which is asymmetric. The Bayes estimator under this asymmetric loss function is denoted by $\hat{\theta}_P$ and may be obtained by solving the following equation, (see Norstrom, 1996),

$$\hat{\theta}_P^2 = \frac{E(\theta \mid \underline{t})}{E(\theta^{-1} \mid \underline{t})} \tag{5}$$

In tertiary case, the LINEX (linear-exponential) loss function which is asymmetric. It was introduced by Varian (1975).

Bayes estimator of θ denoted by $\hat{\theta}_L$ under the LINEX loss is given by

$$\hat{\theta}_L = -\frac{1}{c} \ln\{E_\theta[\exp(-c\theta)]\} \tag{6}$$

provided that the expectation $E_\theta[\exp(-c\theta)]$ exists and is finite, see Calabria and Pulcini (1996).

In many life test studies, it is common that the lifetimes of some test units may not be able to be recorded exactly. For example, in type II censoring, the test ceases after a predetermined number of failures in order to save time or cost. Furthermore, some test units may have to be removed at different stages in the study for various reasons. This would lead to progressive censoring. Progressively type II censored sampling is an important method of obtaining data in lifetime studies. Live units removed early can be readily used in other tests, thereby saving costs to the experimenter, and a compromise can be achieved between time consumption and the observation of some extreme values. Some early works can be found in Cohen (1963), Mann (1971), Thomas and Wilson (1972), Viveros and Balakrishnan (1994), Balakrishnan and Sandhu (1996), Balakrishnan and Aggarwala (2000), and Fernández (2004).

Let us consider the following progressively type II censoring scheme which was generalized by Balakrishnan and Sandhu (1996). Suppose n randomly selected units were placed on a life test; the first r failure times, $Y_1, \ldots, Y_r$ are not observed; at time Y_{r+1}, R_{r+1} units are removed randomly from the test; at time Y_{r+2}, R_{r+2} units are removed randomly from the test and so on.

Finally, at the time of m th failure, Y_m, the experiment is terminated and the remaining R_m units are removed from the test. Therefore, $Y_{r+1} \le \cdots \le Y_m$ are the lifetimes of the completely observed units to fail and $R_{r+1},\ldots,R_m$ are the numbers of units withdrawn from the test at these failure times. At $(i+1)$ th failure, there are n_i units on test where

$$n_i = n - i - \sum_{j=r+1}^{i} R_j, \quad i = r+1,\ldots,m-1.$$

The R_i's, m and r are prespecified integers which must satisfy the conditions: $0 \le r < m \le n, 0 \le R_i \le n_{i-1} - 1$ for $i = r+1,\ldots,m-1$ with $n_r = n - r$ and $R_m = n_{m-1} - 1$. The resulting $(m-r)$ ordered values $Y_{r+1}, Y_{r+2},\ldots,Y_m$ are referred to as general progressively type II censored order statistics.

Note that if $r = 0$, then the general progressive type II censoring scheme reduced to the progressively type II censoring; if $r = 0$ and $R_i = 0$, for $i = r+1,\ldots,m-1$ and $R_m = n - m$ this scheme reduces to conventional type II right censoring; if $R_i = 0$, $i = r+1,\ldots,m-1$ and $R_m = n - m$ the general progressive type II censoring scheme reduces to the case of the type II double censoring.

Progressively type II right censoring is a useful scheme in which a specific fraction of individuals at risk may be removed from the experiment at each of several ordered failure times (see Fernández (2004)). The experimenter can remove units from a life test at various stages during the experiments, possibly resulting in a saving of costs and time (see Sen (1986)). A schematic illustration is depicted in Fig. 2, where $x_{1,n}, x_{2,n},\ldots,x_{m,n}$ denote the failure times and $R_1, R_2,\ldots,R_m$ denote the corresponding numbers of units removed (withdrawn) from the test. Let m be the number of failures observed before termination and $x_{1,n} \le x_{2,n} \le \cdots \le x_{m,n}$ be the observed ordered lifetimes. Let R_i denote the number of units removed at the time of the *i*th failure, $0 \le R_i \le n - \sum R_j - i, i = 2,3,\ldots,m-1$, with $0 \le R_1 \le n-1$ and $R_m = n - \sum_{j=1}^{m-1} R_j - m$, where R_i's and m are pre-specified integers (see (Viveros and Balakrishnan, 1994; Balakrishnan and Aggarwala, 2000; Alimousa and Jaheen, 2002; Marohn, 2002; Soliman, 2005; Li et al., 2007). Note that if $R_1 = R_2 = \ldots = R_{m-1} = 0$, so that $R_m = n - m$, this scheme reduces to the conventional type II right censoring scheme.

Also note that if $R_1 = R_2 = \ldots = R_m = 0$, so that $m = n$, the progressively type II right censoring scheme reduces to the case of no censoring scheme (complete sample case).

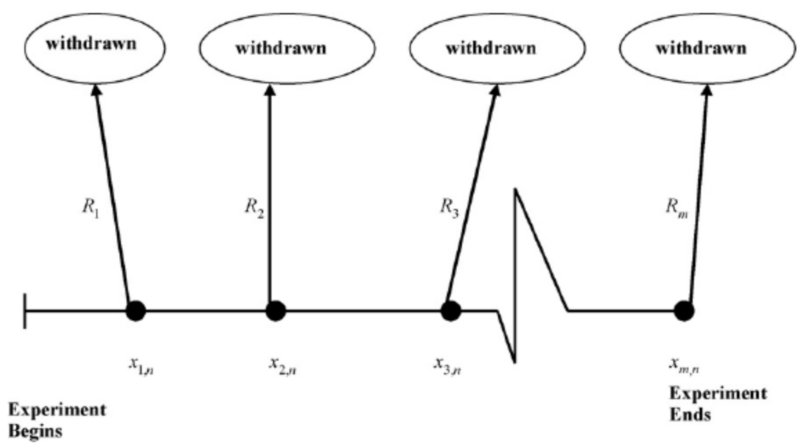

Figure 2: Schematic representation of a progressively type II right censored sample

In this paper, our main object is to study the maximum likelihood estimation and Bayes estimation procedures for the shape parameter, reliability and failure rate functions of the Kumaraswamy distribution based on a progressively type II censored sample. The results obtained in this paper can be specialized to the estimation of the Kumaraswamy distribution based on a complete sample. This paper is organized into five parts. In section 2 the MLE of the parameter θ based on a progressively type II censored sample will be presented. In section3, Bayesian estimators under loss functions different will be introduced. In section 4, Reliability estimators are considered. In section 5, Hazard estimators are obtained. Finally, numerical illustration and comparisons are presented in section 6.

2. Maximum Likelihood Estimation

2.1 Estimation under general progressively type II censoring

Suppose that *n* randomly selected units from a Kumaraswamy population with θ unknown are put on test under a general progressive type II censoring scheme. Let $T=(T_{r+1},T_{r+2},\ldots,T_m)$ denote a general progressively type II censored sample from the population with $(R_{r+1},R_{r+2},\ldots,R_m)$ being the progressive censoring scheme.

The likelihood function for the parameter θ is then

$$L(\theta\mid \underline{t})=\frac{n!}{r!(n-r)!}\left(\prod_{j=r}^{m-1}n_j\right)[F(t_{r+1})]^r\prod_{i=r+1}^{m}f(t_i\mid\theta)[1-F(t_i)]^{R_i} \tag{7}$$

where t is the observed value of T.

In accordance with (1) and (7), the likelihood becomes proportional to

$$\ell(\theta|\underline{t})\propto\theta^{m-r}(1-V^{\theta})^r e^{-\theta W_1} \tag{8}$$

where

$$W_1=W_1(t)=-\sum_{i=r+1}^{m}(R_i+1)\ln(1-t_i^{\lambda})\quad and \quad V=1-t_{r+1}^{\lambda}$$

The logarithm of the LF is given by

$$L=\ln\ell(\underline{t};\theta)=(m-r)\ln\theta-\theta W_1+r\ln(1-V^{\theta}).$$

The MLE of θ, denoted by $\hat{\theta}_{MG}$, is given by

$$\frac{\partial L}{\partial\theta}=\frac{m-r}{\theta}-W_1-\frac{r\ln V}{(V^{-\theta}-1)}=0 \tag{9}$$

The MLE $\hat{\theta}_{MG}$ of θ is the solution to the Eq. (9) which cannot be explicitly solved. A numerical method can be used to solve for the $\hat{\theta}_{MG}$.

Using that $0<z/(\exp(z)-1)<1$ for $z>0$ and $0<1-z/2<z/(\exp(z)-1)<1$, $0<z<2$, it turns out that

$$\max(0,\frac{1}{\theta}+\frac{\ln V}{2})<\frac{-r\ln V}{(e^{-\theta\ln V}-1)}<\frac{1}{\theta}.$$

From the above inequality, the following lemma provides the bounds on the value of $\hat{\theta}_{MG}$.

Lemma 1. The MLE of $\theta, \hat{\theta}_M$ satisfies $\hat{\theta}_L\le\hat{\theta}_M\le\hat{\theta}_U$, where

$$\hat{\theta}_L=\max(\frac{m-r}{W_1},\frac{m}{W_1-\frac{r\ln V}{2}})\ \text{and}\ \hat{\theta}_U=\frac{m}{W_1}.$$

According to Lemma 1, since $\hat{\theta}_M \in (\hat{\theta}_L, \hat{\theta}_U)$, it is convenient to employ the bisection method to determine the MLE. For a given, t the MLE's of $R(t)$ may be obtained by replacing θ by $\hat{\theta}_{MG}$ in Equation (3), then MLE's of $H(t) = -\ln R(t)$ can be obtained.

2.2 Estimation under progressively type II right censoring

Let X denote the lifetime of such a product and X has the Kumaraswamy distribution with the p.d.f. is as (1). With progressively type II right censoring, n units are placed on test. Consider that $X_{1,n} \le X_{2,n} \le \cdots \le X_{m,n}$ is the corresponding progressively type II right censored sample, with censoring scheme $R = (R_1, R_2, \ldots, R_m)$. Since the joint p.d.f. of $X_{1,n}, X_{2,n}, \ldots, X_{m,n}$ is given by

$$A\prod_{i=1}^{m} f_X(x_{i,n}, \lambda, \theta)\,[1 - F_X(x_{i,n}, \lambda, \theta)]^{R_i}, \tag{10}$$

where $A = n(n - R_1 - 1)\ldots(n - R_1 - R_2 \cdots - R_{m-1} - m + 1)$, $f_X(x, \lambda, \theta)$ is the p.d.f of X and $F_X(x, \lambda, \theta)$ is the cumulative distribution function of X.

So, the likelihood function is given by

$$\ell(\theta \mid \underline{t}) \propto \theta^m e^{-\theta W_2} \tag{11}$$

where

$$W_2 = W_2(t) = -\sum_{i=1}^{m} (R_i + 1)\ln(1 - t_{i,n}^{\lambda})$$

The logarithm of the LF is given by

$$L = \ln \ell(\underline{t}; \theta) = m \ln \theta - \theta W_2$$

The MLE of θ, denoted by $\hat{\theta}_{MR}$, is given by

$$\hat{\theta}_{MR} = \frac{m}{W_2} \tag{12}$$

This equation is in implicit form, so it may be solved by using numerical iteration by using MATLAB. For a given, t the MLE's of $R(t)$ may be obtained by replacing θ by $\hat{\theta}_{MR}$ in Equation (3), then MLE's of $H(t) = -\ln R(t)$ can be obtained

3. Bayesian Estimation

The natural family of conjugate prior for θ is a gamma distribution with p.d.f.

$$g(\theta) = \frac{\delta^{\nu}}{\Gamma(\nu)} \theta^{\nu-1} e^{-\delta\theta}, \quad \theta > 0, \nu > 0, \delta > 0. \tag{13}$$

From which the prior mean and variance of θ are given, respectively, by ν/δ and ν/δ^2.

3.1 Estimation under general progressively type II censoring

Applying Bayes theorem, we obtain from Equations (8) and (13), the posterior density of θ as

$$g(\theta\,|\,\underline{t}) = \frac{(\delta+W_1)^{\nu+m-r}}{K\,\Gamma(\nu+m-r)}\theta^{\nu+m-r-1}e^{-\theta(\delta+W_1)}(1-V^{\theta})^{r}, \quad \theta>0, \nu>0, \delta>0, \tag{14}$$

where

$$K = \sum_{j=0}^{r}\omega(j)(1-\frac{j\,p}{\delta+W_1})^{-(\nu+m-r)};\ \omega(j) = (-1)^{j}\binom{r}{j}\ and\ p = \ln V\ .$$

Estimation of θ:

The Bayes estimate $\hat{\theta}_{SG}$ of θ relative to squared error loss function is given by

$$\hat{\theta}_{SG} = \frac{\nu+m-r}{K(\delta+W_1)}\cdot\xi_1 \tag{15}$$

where

$$\xi_1 = \sum_{j=0}^{r}\omega(j)\left(1-\frac{jp}{\delta+W_1}\right)^{-(\nu+m+1-r)}.$$

Under Precautionary loss function, the Bayes estimate $\hat{\theta}_{PG}$ of θ using Equation (5) can be obtained as

$$\hat{\theta}_{PG} = \left[\frac{(\nu+m-r)(\nu+m+1-r)}{(\delta+W_1)^2}\cdot\frac{\xi_1}{\xi_2}\right]^{\frac{1}{2}} \tag{16}$$

where

$$\xi_2 = \sum_{j=0}^{r}\omega(j)\left(1-\frac{jp}{\delta+W_1}\right)^{-(\nu+m-1-r)}.$$

Under LINEX loss function, the Bayes estimate $\hat{\theta}_{LG}$ of θ using Equation (6) can be obtained as

$$\hat{\theta}_{LG} = -\frac{1}{c}\ln(K^{-1}\xi_3) \tag{17}$$

where

$$\xi_3 = \sum_{j=0}^{r}\omega(j)\left(1-\frac{jp-c}{\delta+W_1}\right)^{-(\nu+m-r)}.$$

3.2 Estimation under progressively type II right censoring

Applying Bayes theorem, we obtain from Equations (11) and (13), the posterior density of θ as

$$g(\theta\,|\,\underline{t}) = \frac{(\delta+W_2)^{\nu+m}}{\Gamma(\nu+m)}\theta^{\nu+m-1}e^{-\theta(\delta+W_2)}, \quad \theta>0, \nu>0, \delta>0, \tag{18}$$

Estimation of θ:

The Bayes estimate $\hat{\theta}_{SR}$ of θ relative to squared error loss function is given by

$$\hat{\theta}_{SR} = \frac{\nu+m}{\delta+W_2} \tag{19}$$

Under Precautionary loss function, the Bayes estimate $\hat{\theta}_{PR}$ of θ using Equation (5) can be obtained as

$$\hat{\theta}_{PR} = \frac{\sqrt{(\nu+m-1)(\nu+m)}}{\delta+W_2} \tag{20}$$

Under LINEX loss function, the Bayes estimate $\hat{\theta}_{LR}$ of θ using Equation (6) can be obtained as

$$\hat{\theta}_{LR} = \frac{-(\nu+m)}{c}\ln\left[\frac{\delta+W_2}{\delta+W_2+c}\right] \tag{21}$$

4. Estimation of $R(t)$:

4.1 Estimation under general progressively type II censoring

Consider the reliability $R = R(t)$ is a parameter itself. Replacing θ in terms of R by that of Equation (14), we obtain posterior density function of R as

$$g(R\,|\,\underline{t}) = \frac{Q_1^{\nu+m-r}}{K\Gamma(\nu+m-r)}[\varphi_1(R)]^{\nu+m-r-1} e^{-\varphi_1(R)(Q_1+1)}(1-V^{Z\varphi_1(R)}), \quad 0<R<1, \tag{22}$$

where
Assuming the quadratic loss is appropriate, the Bayes estimate of the reliability function R is

$$\hat{R}_{SG} = K^{-1}\sum_{j=0}^{r}\omega(j)\left(1-\frac{jZP+1}{Q_1}\right)^{-(\nu+m-r)} \tag{23}$$

Under Precautionary loss function, the Bayes estimate of R using Equation (5) is

$$\hat{R}_{PG} = \left(\frac{\sum_{j=0}^{r}\omega(j)\left(1-\frac{jZP+1}{Q_1}\right)^{-(\nu+m-r)}}{\sum_{j=0}^{r}\omega(j)\left(1-\frac{jZP-1}{Q_1}\right)^{-(\nu+m-r)}}\right)^{\frac{1}{2}} \tag{24}$$

Under LINEX loss function, the Bayes estimate $\hat{R}_{LG}$ of R using Equation (6) can be obtained as

$$\hat{R}_{LG} = -\frac{1}{c}\ln(K^{-1}\xi_4) \tag{25}$$

where

$$\xi_4 = \sum_{s=0}^{\infty}\sum_{j=0}^{r}(-1)^{j+s}\binom{r}{j}\frac{c^s}{s!}\left(1-\frac{s+jZP}{Q}\right)^{-(\nu+m-r)}$$

4.2 Estimation under progressively type II right censoring

Consider the reliability $R = R(t)$ is a parameter itself. Replacing θ in terms of R by that of Equation (18), we obtain posterior density function of R as

$$g(R\,|\,\underline{t}) = \frac{Q_2^{\nu+m}}{\Gamma(\nu+m)}[\varphi_2(R)]^{\nu+m-1} e^{-\varphi_2(R)(Q_2+1)}, \quad 0 < R < 1, \tag{26}$$

where

$$Q_2 = Z(\delta + W_2), \varphi_2(R) = \ln(R) \;\; and \;\; Z = \frac{1}{\ln(1-t^{\lambda})}. \tag{27}$$

Assuming the quadratic loss is appropriate, the Bayes estimate of the reliability function R is

$$\hat{R}_{SR} = \left(1 + \frac{1}{Q_2 - 1}\right)^{\nu+m} \tag{28}$$

Under Precautionary loss function, the Bayes estimate of R using Equation (5) is

$$\hat{R}_{PR} = \left(\frac{Q_2 + 1}{Q_2 - 1}\right)^{\frac{\nu+m}{2}} \tag{29}$$

Under LINEX loss function, the Bayes estimate $\hat{R}_{GER}$ of R using Equation (6) can be obtained as

$$\hat{R}_{LR} = -\frac{1}{c}\ln\left[\sum_{s=0}^{\infty}(-1)^s \frac{c^s}{s!}\left(\frac{Q}{Q-s}\right)^{(m+\nu)}\right] \tag{30}$$

5. Estimation of $H(t)$:

5.1 Estimation under general progressively type II censoring

To derive the Bayes estimate of the cumulative failure rate function $H(t) = -\ln R(t)$, we first obtain the posterior density function of $H = H(t)$, which can be given by

$$g(H\,|\,\underline{t}) = \frac{Q_1^{\nu+m-r}}{K\Gamma(\nu+m-r)}(-H)^{\nu+m-r-1} e^{HQ} \, (1 - V^{-ZH})^r \,, \;\; H > 0 \tag{31}$$

The Bayes estimate of H relative to quadratic loss is

$$\hat{H}_{SG} = \frac{K^{-1}(\nu+m-r)}{Q_1} \cdot \xi_5 \tag{32}$$

where

$$\xi_5 = \sum_{j=0}^{r}(-1)^{\nu+m+j-r-1}\binom{r}{j}\left(\frac{jZP}{Q_1} - 1\right)^{-(\nu+m-r+1)}.$$

When the Precautionary loss function is appropriate, the Bayes estimate of H is

$$\hat{H}_{PG} = \left[\frac{(\nu+m-r)(\nu+m-r-1)}{Q_1^2} \cdot \frac{\xi_5}{\xi_6}\right]^{\frac{1}{2}} \tag{33}$$

where

$$\xi_6 = \sum_{j=0}^{r} (-1)^{\nu+m+j-r-1} \binom{r}{j} \left(\frac{jZP}{Q_1} - 1 \right)^{-(\nu+m-r-1)}.$$

When the LINEX loss function is appropriate, the Bayes estimate of H is

$$\hat{H}_{LG} = -\frac{1}{c} \ln(K^{-1}\xi_7) \quad (34)$$

where

$$\xi_7 = \sum_{j=0}^{r} (-1)^{\nu+m+j-r-1} \binom{r}{j} \left(\frac{jZP+c}{Q_1} - 1 \right)^{-(\nu+m-r)}.$$

5.2 Estimation under progressively type II right censoring

To derive the Bayes estimate of the cumulative failure rate function $H(t) = -\ln R(t)$, we first obtain the posterior density function of $H = H(t)$, which can be given by

$$g(H \mid \underline{t}) = \frac{Q_2^{\nu+m}}{\Gamma(\nu+m)} (-H)^{\nu+m-1} e^{HQ_2}, \; H > 0 \quad (35)$$

The Bayes estimate of H relative to quadratic loss is

$$\hat{H}_{SR} = \frac{\nu+m}{Q_2} \quad (36)$$

when the Precautionary loss function is appropriate, the Bayes estimate of H is

$$\hat{H}_{PR} = \frac{\sqrt{(\nu+m)(\nu+m-1)}}{Q_2} \quad (37)$$

when the LINEX loss function is appropriate, the Bayes estimate of H is

$$\hat{H}_{LR} = -\frac{m+\nu}{c} \ln[(-1)^{\nu+m-1} \frac{Q_2}{c-Q_2}] \quad (38)$$

6. Simulation Study

We obtained, in the above Sections, Bayesian and non-Bayesian estimates for the shape parameter θ, reliability, $R(t)$, and failure rate, $H(t)$, functions of the Kumaraswamy distribution. We adopted the squared error loss, Precautionary and LINEX loss functions. The MLE's are also obtained.

In order to assess the statistical performances of these estimates, We conduct a simulation study. The mean square errors (MSE's) using generated random samples of different sizes are computed for each estimator. The random samples are generated as follows:

6.1 Simulation algorithm for general progressively type II censoring

1. Applying the algorithms of Aggarwala and Balakrishnan (1998), the following steps are used to generate a general progressively type II censored sample from the Kumaraswamy distribution.

(A) Generate V_m from the Beta distribution with parameters $n-r$ and $r+1$.

(B) Independently generate Z_{r+i} from $U(0,1)$ for $i=1,\dots,m-r-1$.

(C) Set $V_{r+i}=Z_{r+i}^{1/a_{r+i}}$, $a_{r+i}=i+\sum_{j=m-i+1}^{m}R_j$, $i=1,\dots,m-r-1$.

(D) Set $U_{r+i}=1-V_{m-i+1}V_{m-i+2}\cdots V_m$, $i=1,\dots,m-r$.

(E) Set $Y_i=F^{-1}(U_i), i=r+1,\dots,m$.

This is the desired general progressively type II censored sample from the Kumaraswamy distribution.

2. For given values of the prior parameters (ν,δ), generate a random value for θ from the gamma distribution whose density function given by Equation (13).
3. Using θ, obtained in step (2), generate a general progressive Type II censored sample of size m with given values of $R_i, i=r+1,\dots,m$, from the Kumaraswamy distribution whose PDF is given by (1) according to the above simulation algorithm. and generate random samples of different sizes: n=20 and 30.
4. The MLE of the parameter θ, $\hat{\theta}_{MG}$ is obtained by iteratively solving the Equation (9). The estimators $\hat{R}_{MG}(t_0)$ and $\hat{H}_{MG}(t_0)$ of the functions $R(t)$ and $H(t)$ are then computed at some values t_0.
5. The Bayes estimates relative to squared error loss, $\hat{\theta}_{SG}$, $\hat{R}_{SG}$ and $\hat{H}_{SG}$ given, respectively, by Equations (15), (23) and (32), relative to Precautionary loss $\hat{\theta}_{PG}$, $\hat{R}_{PG}$ and $\hat{H}_{PG}$ given, respectively, by Equations (16), (24) and (33), and relative to LINEX loss $\hat{\theta}_{LG}$, $\hat{R}_{LG}$ and $\hat{H}_{LG}$ given, respectively, by Equations (17), (25) and (34), are all computed.
6. The above steps are repeated 1000 times and the biases and the mean square errors are computed for different sample sizes *n* and $r=3$, where the hat-symbol ^ stands for an estimate $(\hat{\cdot})_M$, $(\hat{\cdot})_S$ and $(\hat{\cdot})_P$.

The computational (our) results were computed by using MATLAB. In all above cases the prior parameters chosen as $\nu=2$ and $\delta=1$, which yield the generated value of $\theta=2$ as the true value. The true values of $R(t)$ and $H(t)$, when $t=t_0=0.5$ are computed to be $R(0.5)=0.5625$ and $H(0.5)=0.2499$. The simulation were carried out for sample sizes $n=20,50,100$. Different choices of the effective sample size m, and different progressive censoring schemes in each case are considered, for simplicity in notation, we will denote the scheme: $(n=20, m=5, \quad R_i=(0,0,0,0,15))$ by $(4^0,15)$. The biases (first entries) and MSE's (second entries) are displayed in Tables 1-3.

6.2 Simulation algorithm for progressively type II right censoring

1. Applying the algorithms of Balakrishnan and Sandhu (1995) and Aggarwala and Balakrishnan (1998), the following steps are used to generate a progressively type II right censored sample from the Kumaraswamy distribution.

(A) Generate m independent $U(0,1)$ random variables $W_1,W_2,\dots,W_m$..

(B) For given values of the progressive Censoring scheme $R_1,R_2,\dots,R_m$.

(C) Set $V_i=W_i^{1/(i+R_m+R_{m-1}+\dots+R_{m-i+1})}$, $\quad i=1,\dots,m$.

(D) Set $U_i=1-V_mV_{m-1}\cdots V_{m-i+1}$, $i=1,\dots,m$; then $U_1,U_2,\dots,U_m$ is a progressive Type II censored sample of size m from $U(0,1)$.

(E) Set $Y_i=F^{-1}(U_i), i=1,\dots,m$, is the required progressive Type II censored sample of size m from Kumaraswamy distribution.

2. For given values of the prior parameters (ν,δ), generate a random value for θ from the gamma distribution whose density function given by Equation (13).
3. Using θ, obtained in step (2), generate a progressive Type II right censored sample of size m with given values of $R_i, i=r+1,\dots,m$, from the Burr Type XII distribution whose PDF is given by (1) according to the above simulation algorithm.

4. The MLE of the parameter θ, $\hat{\theta}_{MR}$ is obtained by iteratively solving the Equation (12). The estimators $\hat{R}_{MR}(t_0)$ and $\hat{H}_{MR}(t_0)$ of the functions $R(t)$ and $H(t)$ are then computed at some values t_0.
5. The Bayes estimates relative to squared error loss, $\hat{\theta}_{SR}$, $\hat{R}_{SR}$ and $\hat{H}_{SR}$ given, respectively, by Equations (19), (28) and (36), relative to Precautionary loss $\hat{\theta}_{PR}$, $\hat{R}_{PR}$ and $\hat{H}_{PR}$ given, respectively, by Equations (20), (29) and (37), and relative to LINEX loss $\hat{\theta}_{LR}$, $\hat{R}_{LR}$ and $\hat{H}_{LR}$ given, respectively, by Equations (21), (30) and (38), are all computed.

The computational (our) results were computed by using MATLAB. In all above cases the prior parameters chosen as $\nu = 2$ and $\delta = 1$, which yield the generated value of $\theta = 2$ as the true value. The true values of $R(t)$ and $H(t)$, when $t = t_0 = 0.5$ are computed to be $R(0.5) = 0.5625$ and $H(0.5) = 0.2499$. The biases (first entries) and MSE's (second entries) are displayed in Tables 4-6.

7. Conclusion

In this paper, the Bayesian and non-Bayesian estimates of parameter θ, reliability, $R(t)$, and failure rate, $H(t)$, functions of the lifetimes following Kumaraswamy distribution have been presented. The estimations are conducted on the basis of progressively type II censored samples. Bayes estimators, under squared error, Precautionary and LINEX loss functions are derived. The MLE's are also obtained. Our observations about the results are stated in the following points:

i. Table 1 shows that the Bayes estimate under LINEX loss function $(c = -5)$ has the smallest estimated MSE's compared with the Bayes estimates under Precautionary and squared error loss functions and MLE's. On the other hand, Bayes estimates under LINEX loss function are underestimation but the Bayes estimates under Precautionary and squared error loss functions and MLE's are overestimation.

ii. Table 2 shows that the MLE estimates have the smallest estimated MSE's as compared with the Bayes estimates. Too, Bayes estimate under squared error loss function is better than the Bayes estimates under Precautionary and LINEX loss functions**.** Bayes estimates under LINEX loss function $(c = -5)$ are overestimation but the Bayes estimates under Precautionary and squared error loss functions and MLE's are underestimation.

iii. Table 3 shows that the Bayes estimates under the Precautionary loss function have the smallest estimated MSE's as compared with the estimates under LINEX and squared error loss functions or MLE's. all Bayes estimates and MLE's are underestimation.

iv. Table 4 shows that the Bayes estimates under the Precautionary loss function have the smallest estimated MSE's as compared with the estimates under LINEX and squared error loss functions or MLE's. Bayes estimates under LINEX loss function $(c = 5)$ are underestimation but others estimates are overestimation.

v. Table 5 shows that the Bayes estimate under squared error loss function is better than the Bayes estimates under Precautionary and LINEX loss functions or MLE's**.** all Bayes estimates and MLE's are underestimation.

vi. Table 6 shows that the Bayes estimate under LINEX loss function $(c = 5)$ have the smallest estimated MSE's as compared with the Bayes estimates under Precautionary and squared error loss functions and MLE's. On the other hand, the Bayes estimates under the LINEX loss function and MLE's are overestimation and, Bayes estimates under Precautionary and squared error loss functions are underestimation.

Bayes approach under squared error loss function for parameter θ estimation of Kumaraswamy distribution in the reliability function has been suggested based on the pervious observations, this approach can be used for both progressively type II censorings.

Table 1. Bias and MSE of different estimators of θ, for different sample size under general progressively type II censoring when $\lambda = 2$ (MSE in parenthesis).

(n,m)	*Scheme*	$\hat{\theta}_{MLE}$	$\hat{\theta}_S$	$\hat{\theta}_P$	$\hat{\theta}_L$	
					-5	5
(20,5)	$(4^0,15)$	0.3814	0.1916	0.3192	-0.5321	-0.8494
		(0.8988)	(0.5152)	(0.6649)	(0.3059)	(1.2435)
	$(3^0,15,0)$	0.3747	0.1805	0.3259	-0.5262	-0.8052
		(0.8294)	(0.5188)	(0.6617)	(0.3015)	(1.2843)
	$(2^0,8,0,7)$	0.4331	0.1386	0.2819	-0.4172	-0.8155
		(0.9411)	0.4361)(	(0.5314)	(0.4298)	(1.3188)
(20,10)	$(9^0,10)$	0.2877	0.1454	0.2964	-0.5821	-0.8275
		(0.7818)	(0. 4185)	0.5432)(	(0.3988)	1.4101)(
	$(8^0,5,5)$	0.2437	0.1446	0.2955	-0.5854	-0.8762
		(0.6881)	0.3968)(	(0.4881)	(0.3707)	(1.7882)
	$(7^0,5,2,3)$	0.2632	0.1278	0.2776	-0.5790	-0.8485
		(0.7102)	(0.4029)	(0.4740)	0.3654)(	(1.6386)
(50,20)	$(19^0,30)$	0.1024	0.0811	0.1432	-0.0691	-0.8293
		(0.2579)	(0.1842)	(0.2089)	(0.0867)	(1.4688)
	$(15^0,8,7,3^5)$	0.1050	0.0804	0.1425	-0.0688	-0.8202
		(0.2304)	(0.1778)	(0.2021)	(0.0868)	(1.3698)
	$(14^0,6^5)$	0.0874	0.1264	0.1899	-0.0816	-0.8256
		(0.2469)	0.2179)(	(0.2502)	(0.0865)	(1.4096)
(50,30)	$(29^0,20)$	0.066	0.0586	0.0973	-0.0869	-0.8122
		(0.1457)	(0. 1279)	0.1387)(	(0.0891)	1.3124)(
	$(26^0,4^5)$	0.0820	0.3313	0.3752	-0.0789	-0.8413
		(0.1696)	0.2708)(	(0.3709)	(0.0919)	(1.6133)
	$(23^0,5^2,2^5)$	0.0753	0.0062	0.0439	-0.1052	-0.7333
		(0.1491)	(0.1219)	(0.1284)	0.0911)(	(1.2374)
(100,50)	$(49^0,50)$	0.0440	0.0447	0.1421	-0.0524	-0.8339
		0.0888	(0.0898)	(0.2023)	(0.0855)	(1.5203)
	$(40^0,10^5)$	0.0397	0.0418	0.1402	-0.0751	-0.8293
		0.0873	(0.0861)	(0.1908)	(0.0821)	(1.4470)
	$(38^0,10^3,2^{10})$	0.0459	0.0476	0.1378	-0.0595	-0.8353
		0.0927	(0.777)	(0.1815)	(0.0744)	(1.5258)
(100,70)	$(69^0,30)$	0.0376	0.0313	0.1392	-0.0824	-0.8407
		0.671	(0.0582)	(0.1881)	(0.0562)	(1.5759)
	$(64^0,6^5)$	0.0334	0.0327	0.1428	-0.0611	-0.8180
		0.0591	(0.0589)	(0.1963)	(0.0571)	(1.3561)
	$(58^0,10^2,2^5)$	0.0215	0.0318	0.1486	-0.0815	-0.8453
		0.0570	(0.0586)	(0.1781)	(0.0569)	(1.4782)

Table 2. Bias and MSE of different estimators of $R(t)$, for different sample size under general progressively type II censoring when $\lambda = 2$ (MSE in parenthesis).

(n,m)	*Scheme*	$\hat{R}_{MLE}$	$\hat{R}_S$	$\hat{R}_P$	$\hat{R}_L$	
					-5	5
(20,5)	$(4^0,15)$	0.0571-	-0.4385	-0.5558	0.5985	-0.6026
		(0.0260)	(0.1677)	(0.3090)	0.3585)(	(1.1823)
	$(4^0,15,0)$	0.0536-	-0.4172	-0.5533	0.6179	-0.6266
		(0.0252)	0.1452)(	0.3062)(	0.4169)(	(1.1997)
	$(2^0,8,0,7)$	-0.1976	-0.4972	0.5125-	0.6742	-0.6411
		(0.0643)	0.2234)(	0.3133)(	0.4856)(	(1.2044)
(20,10)	$(9^0,10)$	-0.0313	-0.5190	-0.5492	0.5385	-0.6655
		(0.0130)	0.2719)(	0.3048)(	0.3339)(	(1.3124)
	$(8^0,5,5)$	0.0260-	-0.5204	-0.5388	0.5343	-0.6590
		(0.0116)	0.2731)(	0.3237)(	0.3278)(	(1.2145)
	$(7^0,5,2,3)$	-0.0285	-0.5195	-0.5441	0.5320	-0.6650
		(0.0122)	0.2721)(	0.3097)(	0.3234)(	(1.2348)
(50,20)	$(19^0,30)$	-0.0110	-0.5625	-0.5718	0.6517	-0.5648
		(0.0056)	(0.3164)	(0.3517)	(0.4523)	(0.9113)
	$(15^0,8,7,3^5)$	-0.0119	-0.5317	-0.5852	0.6286	-0.5711
		0.0052)(	(0.2893)	(0.3613)	(0.4265)	(0.8343)
	$(14^0,6^5)$	-0.0087	-0.5220	-0.5991	0.6772	-0.5887
		(0.0054)	(0.2862)	(0.3929)	(0.4827)	(0.8911)
(50,30)	$(29^0,20)$	-0.0075	-0.5421	-0.5801	-0.7211	-0.5221
		(0.0035)	(0.3252)	(0.3745)	(0.5370)	(0.7875)
	$(26^0,4^5)$	-0.0095	-0.5934	-0.6281	0.7148	-0.5122
		(0.0038)	(0.3633)	(0.4071)	(0.5163)	(0.7714)
	$(23^0,5^2,2^5)$	-0.0088	-0.5785	-0.6816	0.7726	-0.5611
		(0.0036)	(0.3585)	(0.4729)	(0.6058)	(0.8524)
(100,50)	$(49^0,50)$	0.0051-	-0.5547	-0.5721	0.4630	-0.5850
		(0.0022)	(0.3164)	(0.3522)	0.5353)(	(0.7927)
	$(40^0,10^5)$	0.0044-	-0.4816	-0.5435	0.4590	-0.5840
		(0.0021)	0.2253)(	0.3267)(	0.5225)(	(0.7708)
	$(38^0,10^3,2^{10})$	-0.0053	-0.5844	0.6215-	0.4559	-0.5809
		(0.0023)	0.6215)(	0.4801)(	0.5203)(	(0.7450)
(100,70)	$(69^0,30)$	-0.0045	-0.6032	-0.6838	0.4183	-0.5933
		(0.0017)	0.4312)(	0.5537)(	0.4491)(	(0.8185)
	$(64^0,6^5)$	0.0041-	-0.5513	-0.5685	0.4124	-0.6099
		(0.0015)	0.3566)(	0.3584)(	0.4095)(	(0.8486)
	$(58^0,10^2,2^5)$	-0.0022	-0.5117	-0.5835	0.3944	-0.6394
		(0.0014)	0.3046)(	0.4134)(	0.3917)(	(0.8647)

Table 3.Bias and MSE of different estimators of $H(t)$, for different sample size under general progressively type II censoring when $\lambda = 2$ (MSE in parenthesis).

(n,m)	*Scheme*	$\hat{H}_{MLE}$	$\hat{H}_{S}$	$\hat{H}_{P}$	$\hat{H}_{L}$	
					-5	5
(20,5)	$(4^0,15)$	-0.2218	-0.7001	-0.0467	-0.8873	-0.4333
		(0.6616)	0.7381)(	(0.0497)	(0.9180)	(0.7774)
	$(4^0,15,0)$	-0.2499	-0.7279	-0.0503	-0.8915	-0.4280
		(0.6660)	0.7888)(	0.0504)(	(0.9838)	(0.6747)
	$(2^0,8,0,7)$	-0.1431	-0.8183	-0.0307	-0.8941	-0.4795
		(0.1646)	0.8346)(	(0.0817)	(0.9381)	(0.7923)
(20,10)	$(9^0,10)$	-0.2266	-0.8491	-0.1672	-0.9899	-0.8299
		(0.6672)	0.4403)(	0.0526)(	(1.3809)	(1.9111)
	$(8^0,5,5)$	-0.0787	-0.4609	-0.1695	-0.4943	-0.7111
		(0.6595)	0.3891)(	0.0510)(	(0.6778)	(0.4741)
	$(7^0,5,2,3)$	-0.0547	-0.4488	-0.1688	-0.3272	-0.7148
		(0.6583)	(0.3526)	0.0503)(	(0.5256)	(0.5163)
(50,20)	$(19^0,30)$	-0.0752	-0.4297	-0.2204	-0.4832	-0.1166
		(0.6195)	(0.2202)	(0.0574)	(0.2335)	(0.2193)
	$(15^0,8,7,3^5)$	-0.0615	-0.4284	-0.2246	-0.4732	-0.1116
		(0.5943)	(0.2150)	(0.0579)	(0.2336)	(0.2103)
	$(14^0,6^5)$	-0.0837	-0.4211	-0.2263	-0.4833	-0.1145
		(0.5928)	0.1887)(	(0.0580)	(0.2340)	(0.2182)
(50,30)	$(29^0,20)$	-0.1140	-0.4395	-0.1678	-0.4841	-0.1157
		(0.4719)	(0. 2217)	0.0581)(	(0.2344)	0.2113)(
	$(26^0,4^5)$	-0.0886	0.4311-	-0.1791	-0.7837	-0.1261
		(0.5091)	0.2070)(	(0.0576)	(0.2340)	(0.2203)
	$(23^0,5^2,2^5)$	-0.1009	-0.4323	-0.1709	-0.4819	-0.1189
		(0.4962)	(0.2028)	(0.0582)	0.2333)(	(0.2193)
(100,50)	$(49^0,50)$	-0.1423	-0.4403	-0.2178	-0.4771	-0.2227
		(0.3495)	(0.2165)	(0.0613)	(0.2276)	(0.1815)
	$(40^0,10^5)$	-0.1447	-0.4466	-0.2034	-0.4653	-0.2263
		(0.3525)	(0.2430)	(0.0604)	(0.2178)	(0.1872)
	$(38^0,10^3,2^{10})$	-0.1424	-0.4294	-0.2201	-0.4081	-0.2713
		(0.3705)	(0.1857)	(0.0623)	(0.2304)	(0.2019)
(100,70)	$(69^0,30)$	-0.1498	-0.4523	-0.1462	-0.4921	-0.1762
		(0.2595)	(0.2463)	(0.0644)	(0.2381)	(0.2085)
	$(64^0,6^5)$	-0.1616	-0.4403	-0.1404	-0.4733	-0.1276
		(0.2087)	(0.2070)	(0.0643)	(0.2218)	(0.1916)
	$(58^0,10^2,2^5)$	-0.1727	-0.4379	-0.1401	-0.4771	-0.1154
		(0.2669)	(0.2149)	(0.0638)	(0.2277)	(0.1216)

Table 4. Bias and MSE of different estimators of θ, for different sample size under progressively type II right censoring when $\lambda = 2$ (MSE in parenthesis).

m	*Scheme*	$\hat{\theta}_{MLE}$	$\hat{\theta}_S$	$\hat{\theta}_P$	$\hat{\theta}_L$	
					-5	5
5	$(4^0,15)$	0.5169	0.2277	0.0624	0.6389	-0.6915
		(1.9565)	(0.5298)	(0.4136)	(0.4587)	(0.5459)
	$(3^0,15,0)$	0.4595	0.1882	0.0259	0.6475	-0.7085
		(1.9162)	(0.5334)	(0.4275)	(0.4722)	(0.5743)
	$(2^0,8,0,7)$	0.5008	0.1928	0.0302	0.7348	-0.7066
		(2.6665)	0.5562)(	(0.4458)	(0.5306)	(0.5699)
10	$(9^0,10)$	0.1689	0.1334	0.0426	0.9241	-0.4922
		(0.5488)	(0. 3587)	0.3143)(	(0.7539)	0.3291)(
	$(8^0,5,5)$	0.2089	0.1067	0.0170	0.8921	-0.5044
		(0.6749)	0.3086)(	(0.2727)	(0.6346)	(0.3323)
	$(7^0,5,2,3)$	0.2078	0.1365	0.0456	0.9511	-0.4900
		(0.6000)	(0.3418)	(0.2983)	0.7968)(	(0.3251)
20	$(19^0,30)$	0.0966	0.0710	0.0234	0.8772	-0.3113
		(0.2667)	(0.1710)	(0.1589)	(0.5249)	(0.1716)
	$(15^0,8,7,3^5)$	0.1025	0.0728	0.0.251	0.8962	-0.3112
		(0.2336)	(0.1937)	(0.1804)	(0.7484)	(0.1807)
	$(14^0,6^5)$	0.0816	0.0805	0.0327	0.9311	-0.3053
		(0.2485)	(0.1822)	(0.1688)	(0.6584)	(0.1711)
30	$(29^0,20)$	0.0707	0.0625	0.0300	0.5124	-0.2177
		(0.1540)	(0.1306)	(0.1179)	(0.5639)	(0.1237)
	$(26^0,4^5)$	0.0696	0.0729	0.0403	0.5281	-0.2100
		(0.1628)	(0.1346)	(0.1164)	(0.5834)	(0.1268)
	$(23^0,5^2,2^5)$	0.0817	0.0714	0.0388	0.5271	-0.2115
		(0.1686)	(0.1417)	(0.1213)	(0.5967)	(0.1339)
50	$(49^0,50)$	0.0469	0.0371	0.0174	0.2728	-0.1422
		(0.0988)	(0.0764)	(0.0729)	(0.1923)	(0.0732)
	$(40^0,10^5)$	0.0405	0.0273	0.0077	0.2607	-0.1505
		(0.0835)	(0.0774)	(0.0753)	(0.1890)	(0.0757)
	$(38^0,10^3,2^{10})$	0.0502	0.0429	0.0232	0.2808	-0.1376
		(0.0888)	0.0861)(	(0.0770)	(0.2126)	(0.0831)
70	$(69^0,30)$	0.0325	0.0339	0.0198	0.1953	-0.0989
		(0.0635)	(0. 0554)	0.0512)(	(0.1125)	0.0539)(
	$(64^0,6^5)$	0.0246	0.0299	0.0157	0.1905	-0.1024
		(0.0582)	0.0544)(	(0.0514)	(0.1095)	(0.0530)
	$(58^0,10^2,2^5)$	0.0324	0.0341	0.0200	-0.1959	-0.0989
		(0.0623)	(0.0627)	(0.0566)	0.1229)(	(0.0611)

Table 5.Bias and MSE of different estimators of $R(t)$, for different sample size under progressively type II right censoring when $\lambda = 2$ (MSE in parenthesis).

m	*Scheme*	$\hat{R}_{MLE}$	$\hat{R}_S$	$\hat{R}_P$	$\hat{R}_L$	
					-5	5
5	$(4^0,15)$	-0.0489	-0.0115	-0.0272	-0.5562	-0.5564
		(0.0251)	(0.0092)	(0.0110)	1.3574)(	(0.7522)
	$(4^0,15,0)$	0.0401-	-0.0053	-0.0206	-0.5549	-0.5552
		(0.0254)	0.0097)(	0.0114)(	1.3058)(	(0.7329)
	$(2^0,8,0,7)$	-0.0403	-0.0059	0.0212-	0.5547-	-0.5550
		(0.0253)	0.0094)(	0.0111)(	1.2850)(	(0.7323)
10	$(9^0,10)$	-0.0161	-0.0063	-0.0148	-0.5612	-0.5613
		(0.0102)	(0.0068)	(0.0075)	(1.0358)	(0.7744)
	$(8^0,5,5)$	0.0205-	-0.0030	-0.0115	-0.5626	-0.5619
		(0.0114)	(0.0061)	(0.0067)	(1.0415)	(0.7793)
	$(7^0,5,2,3)$	-0.0215	-0.0070	-0.0156	-0.5547	-0.5613
		(0.0113)	(0.0067)	(0.0074)	(1.0199)	(07630)
20	$(19^0,30)$	-0.0100	-0.0090	-0.0135	-0.5629	-0.5623
		(0.0054)	(0.0043)	(0.0047)	0.9453)(	(0.8107)
	$(15^0,8,7,3^5)$	0.0115-	-0.0038	-0.0083	-0.5625	-0.5620
		(0.0052)	0.0039)(	0.0044)(	0.9813)(	(0.8417)
	$(14^0,6^5)$	-0.0078	-0.0051	0.0096-	0.5622-	-0.5628
		(0.0055)	0.0040)(	0.0048)(	0.9721)(	(0.8344)
30	$(29^0,20)$	-0.0080	-0.0053	-0.0083	-0.5623	-0.5625
		(0.0036)	(0.0032)	(0.0034)	(0.9394)	(0.8464)
	$(26^0,4^5)$	0.0077-	-0.0048	-0.0079	-0.5620	-0.5658
		(0.0037)	(0.0029)	(0.0035)	(0.9409)	(0.8485)
	$(23^0,5^2,2^5)$	-0.0095	-0.0037	-0.0069	-0.5627	-0.5623
		(0.0039)	(0.0033)	(0.0037)	(0.9255)	(0.8348)
50	$(49^0,50)$	-0.0054	-0.0038	-0.0056	-0.5621	-0.5627
		(0.0024)	(0.0020)	(0.0022)	(0.9632)	(0.9035)
	$(40^0,10^5)$	-0.0046	0.0033-	-0.0051	-0.5623	-0.5624
		(0.0023)	(0.0019)	(0.0021)	(0.9701)	(0.9100)
	$(38^0,10^3,2^{10})$	-0.0061	-0.0021	-0.0040	-0.5629	-0.5626
		(0.0022)	(0.0018)	(0.0020)	(0.9681)	(0.9085)
70	$(69^0,30)$	-0.0038	-0.0019	-0.0032	-0.5623	-0.5651
		(0.0016)	(0.0013)	(0.0014)	(0.9853)	(0.9410)
	$(64^0,6^5)$	-0.0027	-0.0039	-0.0052	-0.5621	-0.5629
		(0.0015)	(0.0012)	(0.0013)	(1.000)	(0.9546)
	$(58^0,10^2,2^5)$	-0.0037	-0.0036	-0.0049	-0.5622	-0.5625
		(0.0017)	(0.0014)	(0.0015)	(0.9791)	(0.9349)

Table 6. Bias and MSE of different estimators of $H(t)$, for different sample size under progressively type II right censoring when $\lambda = 2$ (MSE in parenthesis).

m	*Scheme*	$\hat{H}_{MLE}$	$\hat{H}_S$	$\hat{H}_P$	$\hat{H}_L$	
					-5	5
5	$(4^0,15)$	0.4754	-0.8918	0.8442-	0.6743	0.2720
		(1.1524)	(0.8350)	(0.7467)	(0.6788)	(0.0919)
	$(3^0,15,0)$	0.5309	0.8819-	-0.8351	0.6674	0.2468
		(1.2209)	(0.8209)	(0.7343)	(0.6720)	(0.0890)
	$(2^0,8,0,7)$	0.5447	-0.8886	-0.8412	0.6720	0.2693
		(1.2675)	0.8327)(	(0.7445)	(0.6689)	(0.0917)
10	$(9^0,10)$	0.5596	-0.8656	-0.8394	0.4742	0.2943
		(1.3786)	(0. 7796)	0.7324)(	(0.2886)	0.1049)(
	$(8^0,5,5)$	0.5461	0.8675-	-0.8412	0.4774	0.2957
		(1.3462)	0.7841)(	(0.7365)	(0.2957)	(0.1063)
	$(7^0,5,2,3)$	0.5132	-0.8589	-0.8329	0.4634	0.2893
		(1.2872)	(0.8650)	(0.7189)	0.2690)(	(0.1005)
20	$(19^0,30)$	0.5163	-0.8517	-0.8379	0.3997	0.3129
		(1.3703)	(0.7407)	(0.7167)	(0.1808)	(0.1095)
	$(15^0,8,7,3^5)$	0.5280	-0.8430	-0.8293	0.3894	0.3052
		(1.4047)	(0.7252)	(0.7018)	(0.1718)	(0.1043)
	$(14^0,6^5)$	0.5709	-0.8531	-0.8392	0.4013	0.3140
		(1.4490)	(0.7433)	(0.7191)	(0.1823)	(0.1105)
30	$(29^0,20)$	0.5032	-0.8451	-0.8357	0.3759	0.3185
		(1.3859)	(0.7251)	(0.7090)	(0.1547)	(0.1104)
	$(26^0,4^5)$	0.5042	-0.8434	-0.8340	0.3739	0.3169
		(1.3889)	(0.7219)	(0.7059)	(0.1529)	(0.1093)
	$(23^0,5^2,2^5)$	0.5010	-0.8465	-0.8371	0.3774	0.3198
		(1.3787)	(0.7274)	(0.7112)	(0.1558)	(0.1112)
50	$(49^0,50)$	0.4600	-0.8382	0.8326-	0.3561	0.3221
		(1.3364)	(0.7098)	(0.7002)	(0.1349)	(0.1101)
	$(40^0,10^5)$	0.4711	0.8390-	-0.8333	0.3569	0.3228
		(1.3747)	(0.7105)	(0.7008)	(0.1348)	(0.1100)
	$(38^0,10^3,2^{10})$	0.4379	-0.8353	-0.8297	0.3530	0.3194
		(1.3079)	0.7037)(	(0.6942)	(0.1314)	(0.1074)
70	$(69^0,30)$	0.4212	-0.8344	-0.8303	0.3469	0.3229
		(1.2955)	(0.7005)	0.6936)(	(0.1250)	0.1082)(
	$(64^0,6^5)$	0.3830	-0.8343	-0.8302	0.3468	0.3228
		(1.2231)	0.7007)(	(0.6939)	(0.1254)	(0.1085)
	$(58^0,10^2,2^5)$	0.3714	-0.8358	-0.8317	0.3485	0.3243
		(1.2025)	(0.7034)	(0.6966)	0.1267)(	(0.1096)

Acknowledgements

Special thanks go to the reviewers for their worthwhile and useful offers.

References

Aggarwala, R., Balakrishnan, N. 1998. Some properties of progressive censored order statistics from arbitrary and uniform distributions with applications to inference and simulation, *J. Statist. Plann. Inference*, Vol. 70, pp. 35 49.

Alimousa M.A.M., Jaheen Z.F. 2002. Statistical inference for the Burr model based on progressively censored data, *Computers and Mathematics with Applications*, Vol. 43, pp. 1441–1449.

Balakrishnan, N., and Aggarwala, R. 2000. Progressive censoring: Theory, methods, and applications. Boston, MA: Birkhauser.

Balakrishnan, N., and Sandhu, R. A. 1996. Best linear unbiased and maximum likelihood estimation for exponential distributions under general progressive Type-II censored samples. *Sankhya B*, Vol. 58, pp. 1-9.

Basu, A. P. and ebrahimi, N. 1991. Bayesian approach to life testing and reliability estimation using asymmetric loss function. *J. Statist. Plann. Infer*., Vol. 29, pp. 21-31.

Calabria, R. and Pulcini, G. 1996. Point estimation under asymmetric loss functions for lift-truncated exponential samples. *Commun. Statist. Theory Meth*, Vol. 25, No. 3, pp. 585-600.

Cohen, A. C. 1963. Progressively censored samples in life testing. *Technometrics*, Vol. 5, pp. 327-339.

Fernández, A. J. 2004. On estimating exponential parameters with general type II progressive censoring. *Journal of Statistical Planning and Inference*, Vol. 121, pp. 135_147.

Jones M.C. (2009) Kumaraswamy's distribution: A beta-type distribution with some tractability advantages, *Statistical Methodology*, Vol. 6, pp. 70–81.

Kumaraswamy P. 1980. A generalized probability density function for double-bounded random processes, *Journal of Hydrology*, Vol. 46, pp. 79–88.

Li X., Shi Y., Wei J., Chai J. 2007. Empirical Bayes estimators of reliability performances using LINEX loss under progressively type-II censored samples, *Mathematics and Computers in Simulation*, Vol. 73, pp. 320-326

Mann, N. R. 1971. Best linear invariant estimation for Weibull parameters under progressive censoring. *Technometrics*, Vol. 13, pp. 521-533.

Marohn F. 2002. A characterization of generalized Pareto distributions by progressive censoring schemes and goodness-of-fit tests, *Communications in Statistics-Theory and Methods*, Vol.31, pp. 1055–1065.

Nassar M. M., and Eissa F. H. 2004. Bayesian estimation for the exponentiated Weibull model, *Commun. Statist.-Theory Meth*., Vol. 33, No. 10, pp. 2343-2362.

Norstrom, J.G. 1996. The use of precautionary loss function in risk analysis. *IEEE Trans. Reliab.*, Vol. 45, No. 3, pp. 400-403.

Pandey B. N. 1997. Testimator of the scale parameter of the exponential distribution using LINEX loss function, *Commun. Statist.-Theory Meth*., Vol. 26, No. 9, pp. 2191- 2202.

Sen P.K. 1986. Progressive censoring schemes, in: S. Kotz, N.L. Johnson (Eds.), Encyclopedia of Statistical Sciences, vol. 7, Wiley, New York, pp. 296–299.

Soliman, A. A. 2000. Inference for reliability and stress-length for a scaled Burr type X distribution. *Commun. Statist. Theory Meth*, Vol. 29, pp. 95-107.

Soliman, A. A. 2005. Estimation of parameters of life from progressively censored data using Burr-XII model, *IEEE Transactions on Reliability* Vol. 54, pp. 34–42.

Surles, J. G. and Padgett, W. J. 2001. Inference for reliability and stress-length for a scaled Burr type X distribution. *Lifetime Data Analysis,* Vol. 7, pp. 187-202.

Thomas, D. R., and Wilson, W. M. 1972. Linear order statistic estimation for the two parameter Weibull and extreme-value distribution under Type II progressively censored samples. *Technometrics*, Vol. 14, pp. 679-691.

Varian, H.R. 1975. A Bayesian Approach to Real Estate Assessment, North Holland, Amsterdam, pp. 195-208.

Viveros, R., and Balakrishnan, N. 1994. Interval estimation of life characteristics from progressively censored samples. *Technometrics*, Vol. 36, pp. 84_91.

Zellner A. 1986, Bayesian estimation and prediction using asymmetric loss functions, *J. A. S. A.*, Vol. 81, pp. 446-451.

Biographical notes

Ramin Gholizadeh was born in Amol, Iran in 1982. He studied BA in Statistics in PNU of Amol in 2006. Then he got his MA in statistics from Azad University of Mashhad in 2008. His MA thesis title was Fussy Reliability of System. He has published 10 articles in different journals since 2008. He is a university lecturer in his country. Now he is working on Statistical Computing, Reliability and Fuzzy.Also he has been invited to be the ISI journal referee such as Journal of Applied Statistics (Taylor & Francis), Mechanism and Machine Theory (Elsevier) and IIE Transactions.

Mastane Khalilpor is with the Department of Statistics, Payame Noor University of Amol, Iran

Maghdoode Hadian is with the Young Researchers Club, Mashhad Branch, Islamic Azad University, Mashhad , Iran

Received December 2011
Accepted March 2012
Final acceptance in revised form August 2012