Unsteady hydromagnetic Couette flow within a porous channel with Hall effects

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Abstract

   Unsteady hydromagnetic Couette flow of a viscous, incompressible and electrically conducting fluid between two infinitely long parallel porous plates, taking Hall current into account, in the presence of a transverse magnetic field is studied. Fluid flow within the channel is induced due to impulsive movement of the lower plate of the channel. Magnetic lines of force are assumed to be fixed relative to the moving plate. Solution of the governing equations is obtained by Laplace transform technique. The expression for the shear stress at the moving plate due to primary and secondary flows is also derived. Asymptotic behavior of the solution valid for small and large values of time t is analyzed to gain some physical insight into the flow pattern. Numerical values of the primary and secondary velocities are displayed graphically versus non-dimensional channel width variable η for various values of Hall current parameter m, magnetic parameter M^2, suction/injection parameter S and time t whereas the numerical values of shear stress at the moving plate due to primary and secondary flows are presented in tabular form for different values of m, M^2, S and t.

Keywords: Hydromagnetic Couette flow, suction/injection, magnetic field, impulsive movement of the plate, modified Hartmann boundary layer.

1. Introduction

   Theoretical/experimental investigation of unsteady hydromagnetic Couette flow in the presence of a transverse magnetic field is significant from a practical point of view because fluid transients may be expected at the start-up time in so many MHD devices viz. MHD pumps, MHD generators, MHD accelerators, MHD flow meters, nuclear reactors using liquid metal coolant etc. Katagiri (1962) studied unsteady hydromagnetic Couette flow of a viscous, incompressible and electrically conducting fluid in the presence of a uniform transverse magnetic field when the fluid flow within the channel is induced due to impulsive movement of one of the plates of the channel. Muhuri (1963) analyzed this problem in a parallel plate porous channel when the fluid flow within the channel is induced due to uniform accelerated movement of one of the plates of the channel. In their problem Katagiri (1962) and Muhuri (1963) considered that the magnetic lines of force are fixed relative to the fluid. Singh and Kumar (1983) studied the problem considered by Katagiri (1962) and Muhuri (1963) in a non-porous channel when the magnetic lines of force are fixed relative to the moving plate. It may be noted that the study of hydromagnetic flow within a porous channel may find application in designing of cooling systems with liquid metals, geothermal reservoirs, underground energy transport, petroleum and mineral industries, in purification of crude oils etc. Prasad Rao et al. (1982), Makinde and Chinyoka (2001), Makinde and Osalusi (2006), Abbas et al. (2006), Attia (2007), Hayat et al. (2007, 2008), Khan et al. (2009) and Seth et al. (2011) studied hydromagnetic flow within an infinitely long parallel plate channel with porous boundaries considering different aspects of the problem. In all these investigations, effects of Hall current are not taken into account. It is well known that, in an ionized fluid where density is low and/or magnetic field is strong, the effects of Hall current become significant as stated by Cowling (1957) since Hall current induces secondary flow in the flow-field. Hall current on the fluid flow have many applications in MHD power generation, nuclear

power reactors, underground energy storage systems, and in several areas of astrophysical and geophysical interest. Jana and Dutta (1977) investigated effects of Hall current on unsteady MHD Couette flow within a non-porous channel when the magnetic lines of force are fixed relative to the fluid. Bhaskara Reddy and Bathaiah (1982) studied effects of Hall current on MHD Hartmann flow through a porous channel.

   The purpose of the present investigation is to study the effects of Hall current on unsteady hydromagnetic Couette flow of a viscous, incompressible and electrically conducting fluid within an infinitely long parallel plate porous channel in the presence of a uniform transverse magnetic field. Fluid flow within the channel is induced due to impulsive movement of the lower plate of the channel and magnetic lines of force are fixed relative to the moving plate of the channel. Present study may find applications in hybrid MHD energy generators, material processing in petroleum, metallurgy and mineral industries, geophysical and astrophysical problems of interest etc.

   The paper is organized as follows: In Section 2, formulation of the problem and its solution is given. In Section 3, the expression for shear stress at the moving plate is mentioned. In Section 4, asymptotic solutions for small and large values of time t are provided. In Section 5, i.e. in Results and Discussion, the behavior of the primary and the secondary velocities and the non-dimensional shear stress at the moving plate due to the primary and secondary flows with respect to various values of the Hall current parameter m, magnetic parameter M^2, suction/injection parameter S and time t are discussed. The present results are compared with available results of Seth et al. (2011) and Singh and Kumar (1983) and are found to be in good agreement. Finally in Section 6, conclusions of the present study are provided.

2. Formulation of the Problem and its Solution

   Consider unsteady flow of a viscous, incompressible and electrically conducting fluid between two parallel porous plates at $z = 0$ and $z = h$ of infinite length, in x and y-directions, in the presence of a uniform transverse magnetic field H_0 applied parallel to the z-axis (See Fig. 1). Initially (i.e. when time $t' \leq 0$), the fluid and plates of the channel are assumed to be at rest. At time $t' > 0$ the lower plate at $z = 0$ starts moving with uniform velocity U_0 in the x-direction while the upper plate at $z = h$ is kept fixed. The fluid suction/injection takes place through the porous plates of the channel with uniform velocity W_0 ($W_0 > 0$ for suction and $W_0 < 0$ for injection). It is assumed that the no applied or polarization voltage exits so that the effects of polarization of the fluid is negligible (i.e. the induced electric field $\vec{E} = 0$) and the induced magnetic field produced by fluid motion are neglected in comparison to applied one (Cramer and Pai, 1973). Since the plates are of infinite extent in the x and y-directions, all physical quantities, except pressure, depend on z and t' only.
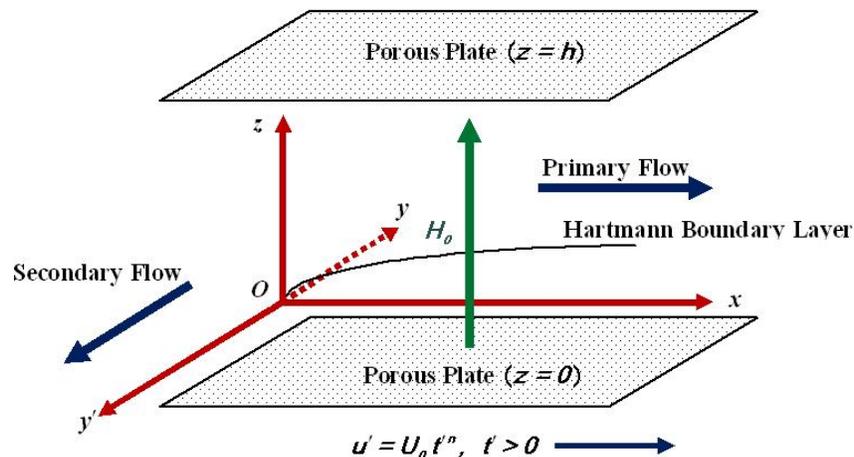
Figure 1. Physical Model of the Problem

Therefore, the fluid velocity $\vec{q}$ and the magnetic field $\vec{H}$ are given by

$$\vec{q} = (u', v', W_0), \qquad \vec{H} = (0, 0, H_0), \tag{1}$$

where u' and v' are fluid velocity in x-direction and y-direction respectively.
Under the above assumptions, the governing equations for the fluid motion are given by

$$\frac{\partial u'}{\partial t'} + W_0 \frac{\partial u'}{\partial z} = \upsilon \frac{\partial^2 u'}{\partial z^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho(1+m^2)}(u' - mv'), \tag{2}$$

$$\frac{\partial v'}{\partial t'} + W_0 \frac{\partial v'}{\partial z} = \upsilon \frac{\partial^2 v'}{\partial z^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho(1+m^2)}(mu' + v'), \tag{3}$$

where υ, σ, ρ, μ_e, $m = \omega_e \tau_e$, ω_e and τ_e are the kinematic coefficient of viscosity, the electrical conductivity of the fluid, the fluid density, the magnetic permeability, the Hall current parameter, the cyclotron frequency and the electron collision time respectively. The initial and boundary conditions for fluid flow are given by

$$u' = v' = 0 \qquad\qquad \text{for } 0 \le z \le h \qquad \text{and } t' \le 0, \tag{4}$$

$$u' = U_0, v' = 0 \qquad\qquad \text{at } z = 0 \qquad\qquad \text{for } t' > 0, \tag{5}$$

$$u' = v' = 0 \qquad\qquad \text{at } z = h \qquad\qquad \text{for } t' > 0. \tag{6}$$

Equation (2) is valid when the magnetic lines of force are fixed relative to the fluid. On the other hand, when the magnetic lines of force are fixed relative to the moving plate (Rossow, 1958), equation (2) is replaced by

$$\frac{\partial u'}{\partial t'} + W_0 \frac{\partial u'}{\partial z} = \upsilon \frac{\partial^2 u'}{\partial z^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho(1+m^2)}(u' - mv' - U_0). \tag{7}$$

Equations (3) and (7) are presented in the compact form as

$$\frac{\partial q'}{\partial t'} + W_0 \frac{\partial q'}{\partial z} = \upsilon \frac{\partial^2 q'}{\partial z^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho(1+m^2)}\left[q'(1+im) - U_0\right], \tag{8}$$

where $q' = u' + iv'$, and $i = \sqrt{-1}$.

The initial and boundary conditions (4) to (6), in compact form, become

$$q' = 0 \qquad\qquad \text{for } 0 \le z \le h \qquad \text{and } t' \le 0, \tag{9}$$

$$q' = U_0 \qquad\qquad \text{at } z = 0 \qquad\qquad \text{for } t' > 0, \tag{10}$$

$$q' = 0 \qquad\qquad \text{at } z = h \qquad\qquad \text{for } t' > 0. \tag{11}$$

Representing equation (8), in non-dimensional form, we obtain

$$\frac{\partial q}{\partial t} + S \frac{\partial q}{\partial \eta} = \frac{\partial^2 q}{\partial \eta^2} - \frac{M^2}{(1+m^2)}\left[q(1+im) - R_e\right], \tag{12}$$

where $\eta = z/h$, $q = q'h/\upsilon$, $t = t'\upsilon/h^2$, $S = W_0 h/\upsilon$ is the suction/injection parameter ($S > 0$ for suction and $S < 0$ for injection), $M^2 = \sigma \mu_e^2 H_0^2 h^2 / \rho\upsilon$ is the magnetic parameter which is the square of Hartmann number and $R_e = U_0 h/\upsilon$ is the Reynolds number.

The initial and boundary conditions (9) to (11), in non-dimensional form, become

$$q = 0 \qquad\qquad \text{for } 0 \le \eta \le 1 \qquad\qquad \text{and } t \le 0, \tag{13}$$

$$q = R_e \qquad\qquad \text{at } \eta = 0 \qquad\qquad \text{for } t > 0, \tag{14}$$

$$q = 0 \qquad\qquad \text{at } \eta = 1 \qquad\qquad \text{for } t > 0. \tag{15}$$

Applying the Laplace transform, equation (12) with the help of the initial condition (13) reduces to

$$\frac{d^2 \bar{q}}{d\eta^2} - S \frac{d\bar{q}}{d\eta} - \left\{p + \frac{(1+im)M^2}{1+m^2}\right\}\bar{q} = -\frac{M^2 R_e}{p(1+m^2)}, \tag{16}$$

where $\bar{q} = \int_0^\infty e^{-pt} q(\eta,t)dt$, $p > 0$ and p being the Laplace transform parameter.

Boundary conditions (14) and (15) after taking the Laplace transform become

$$\bar{q} = R_e / p \qquad\qquad \text{at } \eta = 0 \tag{17}$$

$$\bar{q} = 0 \qquad\qquad \text{at } \eta = 1 \tag{18}$$

The solution of equation (16) subject to the boundary conditions (17) and (18) is given by

$$\bar{q}_j = \frac{M^2}{(1+m^2)p(p+m^*)} + \sum_{k=0}^\infty \left[\frac{1}{p}\left(e^{-ak_1} - e^{-bk_1}\right) - \frac{(-1)^k M^2 \left(e^{-ck_1} + e^{-dk_1}\right)}{(1+m^2)p(p+m^*)}\right], \tag{19}$$

where

$$\overline{q}_j = \overline{q}(z,p)/R_e, \quad a = 2k+\eta, \quad b = 2+2k-\eta, \quad c = k+\eta, d = 1+k-\eta, \\ k_1 = S/2 + \sqrt{p+S^2/4+m^*} \quad \text{and} \quad m^* = \left\{ M^2(1+im)/(1+m^2) \right\}.$$

Taking the inverse Laplace transform of equation (19), we obtain

$$
\begin{aligned}
q_j = & \left(\frac{1-im}{1+m^2}\right)\left(1-e^{-m^*t}\right) + \frac{1}{2}\sum_{k=0}^{\infty}\left[e^{-a\left(\frac{s}{2}-\lambda\right)} erfc\left(\frac{a}{2\sqrt{t}}+\lambda\sqrt{t}\right) + e^{-a\left(\frac{s}{2}+\lambda\right)} erfc\left(\frac{a}{2\sqrt{t}}-\lambda\sqrt{t}\right) \right. \\
& - e^{-b\left(\frac{s}{2}-\lambda\right)} erfc\left(\frac{b}{2\sqrt{t}}+\lambda\sqrt{t}\right) - e^{-b\left(\frac{s}{2}+\lambda\right)} erfc\left(\frac{b}{2\sqrt{t}}-\lambda\sqrt{t}\right) - \frac{(-1)^k(1-im)}{(1+m^2)}\left\{ e^{-c\left(\frac{s}{2}-\lambda\right)} erfc\left(\frac{c}{2\sqrt{t}}+\lambda\sqrt{t}\right) \right. \\
& + e^{-c\left(\frac{s}{2}+\lambda\right)} erfc\left(\frac{c}{2\sqrt{t}}-\lambda\sqrt{t}\right) + e^{-d\left(\frac{s}{2}-\lambda\right)} erfc\left(\frac{d}{2\sqrt{t}}+\lambda\sqrt{t}\right) + e^{-d\left(\frac{s}{2}+\lambda\right)} erfc\left(\frac{d}{2\sqrt{t}}-\lambda\sqrt{t}\right) \\
& \left. \left. -e^{-m^*t}\left(erfc\left(\frac{c}{2\sqrt{t}}+\frac{S\sqrt{t}}{2}\right) + e^{-cS} erfc\left(\frac{c}{2\sqrt{t}}-\frac{S\sqrt{t}}{2}\right) + erfc\left(\frac{d}{2\sqrt{t}}+\frac{S\sqrt{t}}{2}\right) + e^{-dS} erfc\left(\frac{d}{2\sqrt{t}}-\frac{S\sqrt{t}}{2}\right) \right) \right\} \right],
\end{aligned}
$$

(20)

where

$$
\begin{aligned}
& q_j = q(\eta,t)/R_e = u_j + iv_j, \quad \lambda = \alpha + i\beta, \\
& \alpha, \beta = \frac{1}{\sqrt{2}}\left[\left\{ \left(\frac{S^2}{4}+\frac{M^2}{1+m^2}\right)^2 + \frac{m^2M^4}{(1+m^2)^2} \right\}^{1/2} \pm \left(\frac{S^2}{4}+\frac{M^2}{1+m^2}\right) \right]^{1/2}.
\end{aligned}
$$

(21)

Solution (20) is the general solution for unsteady hydromagnetic Couette flow within an infinitely long parallel plate porous channel with Hall effects when the fluid flow within the channel is induced due to impulsive movement of the lower plate of the channel. It clearly demonstrates a unified representation of initial hydromagnetic Couette flow, final steady flow confined within modified Hartmann boundary layer and the decaying oscillations excited by the interaction between Hall current, magnetic field, suction/injection and initial impulsive motion when the magnetic lines of force are fixed relative to the moving plate. This solution is valid for every value of time t. In the absence of Hall current (i.e. $m = 0$), solution (20) is in agreement with the solution obtained by Seth et al. (2011). In the absence of Hall current and suction/injection (i.e. $S = 0$), it agrees with the solution obtained by Singh and Kumar (1983).

3. Shear Stress at the Moving Plate

Non-dimensional shear stress components τ_{xj} and τ_{yj} at the moving plate $\eta = 0$ due to primary and secondary flows are given by

$$\left(\tau_{xj} + i\tau_{yj}\right)\Big|_{\eta=0}$$

$$= \frac{1}{2}\sum_{k=0}^{\infty}\left[-\left(\frac{S}{2}-\lambda\right)\left\{e^{-a'\left(\frac{S}{2}-\lambda\right)}erfc\left(\frac{a'}{2\sqrt{t}}+\lambda\sqrt{t}\right)+e^{-b'\left(\frac{S}{2}-\lambda\right)}erfc\left(\frac{b'}{2\sqrt{t}}+\lambda\sqrt{t}\right)\right\}\right.$$

$$-\left(\frac{S}{2}+\lambda\right)\left\{e^{-a'\left(\frac{S}{2}+\lambda\right)}erfc\left(\frac{a'}{2\sqrt{t}}-\lambda\sqrt{t}\right)+e^{-b'\left(\frac{S}{2}+\lambda\right)}erfc\left(\frac{b'}{2\sqrt{t}}-\lambda\sqrt{t}\right)\right\}-\frac{2}{\sqrt{\pi t}}\left\{e^{-\left(\frac{a'^2}{4t}+\frac{a'S}{2}+\lambda^2 t\right)}+e^{-\left(\frac{b'^2}{4t}+\frac{b'S}{2}+\lambda^2 t\right)}\right\}$$

$$-(-1)^k\left(\frac{1-im}{1+m^2}\right)\left\{-\left(\frac{S}{2}-\lambda\right)\left\{e^{-k\left(\frac{S}{2}-\lambda\right)}erfc\left(\frac{k}{2\sqrt{t}}+\lambda\sqrt{t}\right)-e^{-d'\left(\frac{S}{2}-\lambda\right)}erfc\left(\frac{d'}{2\sqrt{t}}+\lambda\sqrt{t}\right)\right\}\right.$$

$$-\left(\frac{S}{2}+\lambda\right)\left\{e^{-k\left(\frac{S}{2}+\lambda\right)}erfc\left(\frac{k}{2\sqrt{t}}-\lambda\sqrt{t}\right)-e^{-d'\left(\frac{S}{2}+\lambda\right)}erfc\left(\frac{d'}{2\sqrt{t}}-\lambda\sqrt{t}\right)\right\}-\frac{2}{\sqrt{\pi t}}\left\{e^{-\left(\frac{k^2}{4t}+\frac{kS}{2}+\lambda^2 t\right)}-e^{-\left(\frac{d'^2}{4t}+\frac{d'S}{2}+\lambda^2 t\right)}\right\}$$

$$+e^{-m^*t}\left(\frac{2}{\sqrt{\pi t}}\left\{e^{-\left(\frac{k^2}{4t}+\frac{kS}{2}+\frac{S^2 t}{4}\right)}-e^{-\left(\frac{d'^2}{4t}+\frac{d'S}{2}+\frac{S^2 t}{4}\right)}\right\}+S\left\{e^{-kS}erfc\left(\frac{k}{2\sqrt{t}}-\frac{S\sqrt{t}}{2}\right)-e^{-d'S}erfc\left(\frac{d'}{2\sqrt{t}}-\frac{S\sqrt{t}}{2}\right)\right\}\right)\right]\right],$$

$$\tag{22}$$

where $a' = 2k$, $b' = 2+2k$ and $d' = 1+k$.

4. Asymptotic Solutions

To gain physical insight into the flow pattern, we now discuss the asymptotic behavior of the solution (20) for small and for large values of time t.

Case I: When time t is small (i.e. $t < 1$)

For small values of time t, solution (20) assumes the following form

$$u_j = \frac{M^2 t}{(1+m^2)} + \sum_{k=0}^{\infty}\left[\left\{e^{-\frac{aS}{2}}\left\{\left(1+\frac{m_1 a^2}{2}\right)erfc\left(\frac{a}{2\sqrt{t}}\right)-am_1\sqrt{\frac{t}{\pi}}e^{-\frac{a^2}{4t}}\right\}-e^{-\frac{bS}{2}}\left\{\left(1+\frac{m_1 b^2}{2}\right)erfc\left(\frac{b}{2\sqrt{t}}\right)-bm_1\sqrt{\frac{t}{\pi}}e^{-\frac{b^2}{4t}}\right\}\right\}\right.$$

$$-\frac{(-1)^k M^2}{(1+m^2)}\left\{e^{-\frac{cS}{2}}\left\{\left(t+(1+m_1 t)\frac{c^2}{2}+\frac{m_1 c^4}{12}\right)erfc\left(\frac{c}{2\sqrt{t}}\right)-\left(\frac{2m_1 t}{3}+\left(1+\frac{m_1 c^2}{6}\right)\right)c\sqrt{\frac{t}{\pi}}e^{-\frac{c^2}{4t}}\right\}\right.$$

$$\left.+e^{-\frac{dS}{2}}\left\{\left(t+(1+m_1 t)\frac{d^2}{2}+\frac{m_1 d^4}{12}\right)erfc\left(\frac{d}{2\sqrt{t}}\right)-\left(\frac{2m_1 t}{3}+\left(1+\frac{m_1 d^2}{6}\right)\right)d\sqrt{\frac{t}{\pi}}e^{-\frac{d^2}{4t}}\right\}\right\}\right], \tag{23}$$

$$v_j = m_2\sum_{k=0}^{\infty}\left[\left\{e^{-\frac{aS}{2}}\left\{\frac{a^2}{2}erfc\left(\frac{a}{2\sqrt{t}}\right)-a\sqrt{\frac{t}{\pi}}e^{-\frac{a^2}{4t}}\right\}-e^{-\frac{bS}{2}}\left\{\frac{b^2}{2}erfc\left(\frac{b}{2\sqrt{t}}\right)-b\sqrt{\frac{t}{\pi}}e^{-\frac{b^2}{4t}}\right\}\right\}\right.$$

$$-\frac{(-1)^k M^2}{(1+m^2)}\left\{e^{-\frac{cS}{2}}\left\{\left(t+\frac{c^2}{6}\right)\frac{c^2}{2}erfc\left(\frac{c}{2\sqrt{t}}\right)-\left(2t+\frac{c^2}{2}\right)\frac{c}{3}\sqrt{\frac{t}{\pi}}e^{-\frac{c^2}{4t}}\right\}\right.$$

$$\left.+e^{-\frac{dS}{2}}\left\{\left(t+\frac{d^2}{6}\right)\frac{d^2}{2}erfc\left(\frac{d}{2\sqrt{t}}\right)-\left(2t+\frac{d^2}{2}\right)\frac{d}{3}\sqrt{\frac{t}{\pi}}e^{-\frac{d^2}{4t}}\right\}\right\}\right], \tag{24}$$

where $m_1 = \dfrac{S^2}{4}+\dfrac{M^2}{1+m^2}$, $m_2 = \dfrac{mM^2}{1+m^2}$.

It is evident from the expressions (23) and (24) that there arises a Rayleigh boundary layer of thickness $O\left(\sqrt{t}\right)$ near the moving plate at $\eta = 0$ due to the initial impulsive movement of the plate. Also it may be noted from (23) and (24) that the primary velocity

u_j and the secondary velocity v_j are affected by the Hall current, magnetic field and suction/injection. In the absence of Hall current, (i.e. $m = 0$), the secondary velocity vanishes. This is due to fact that Hall current induces secondary flow in the flow-field. There are no inertial oscillations in the flow-field.

Case II: When time t is large (i.e. $t > 1$)

For large values of time t, solution (20) may be represented in the following form

$$u_j(\eta, t) = u_{js} + u_{jt}, \tag{25}$$

where

$$u_{js} = \frac{1}{(1+m^2)} + \sum_{k=0}^{\infty} \left[\left\{ e^{-a\left(\frac{S}{2}+\alpha\right)} \cos \beta a - e^{-b\left(\frac{S}{2}+\alpha\right)} \cos \beta b \right\} \right.$$
$$\left. - \frac{(-1)^k}{(1+m^2)} \left\{ e^{-c\left(\frac{S}{2}+\alpha\right)} (\cos \beta c - m \sin \beta c) + e^{-d\left(\frac{S}{2}+\alpha\right)} (\cos \beta d - m \sin \beta d) \right\} \right], \tag{26}$$

$$u_{jt} = u_{jt_1} + u_{jt_2}, \tag{27a}$$

$$u_{jt_1} = \frac{1}{2} \sum_{k=0}^{\infty} e^{-m_1 t} \left[-\phi_1 + \phi_3 + \frac{(-1)^k}{(1+m^2)} \left\{ (\phi_5 + \phi_7) - m(\phi_6 + \phi_8) - 2\phi_9 \right\} \right], \tag{27b}$$

$$u_{jt_2} = \frac{e^{-\left(M^2/(1+m^2)\right)t}}{1+m^2} \left[\sum_{k=0}^{\infty} \left\{ (-1)^k \left(e^{-cS} + e^{-dS} \right)(\cos(m_2 t) - m \sin(m_2 t)) \right\} - (\cos(m_2 t) - m \sin(m_2 t)) \right], \tag{27c}$$

and

$$v_j(\eta, t) = v_{js} + v_{jt}, \tag{28}$$

where

$$v_{js} = -\frac{1}{(1+m^2)} + \sum_{k=0}^{\infty} \left[\left\{ -e^{-a\left(\frac{S}{2}+\alpha\right)} \sin \beta a + e^{-b\left(\frac{S}{2}+\alpha\right)} \sin \beta b \right\} \right.$$
$$\left. + \frac{(-1)^k}{(1+m^2)} \left\{ e^{-c\left(\frac{S}{2}+\alpha\right)} \left(\sin \beta c + m \cos \beta c + e^{-d\left(\frac{S}{2}+\alpha\right)} (\sin \beta d + m \cos \beta d) \right) \right\} \right], \tag{29}$$

$$v_{jt} = v_{jt_1} + v_{jt_2}, \tag{30a}$$

$$v_{jt_1} = \frac{1}{2} \sum_{k=0}^{\infty} e^{-m_1 t} \left[\phi_2 - \phi_4 - \frac{(-1)^k}{(1+m^2)} \left\{ (\phi_6 + \phi_8) + m(\phi_5 + \phi_7) - 2\phi_{10} \right\} \right], \tag{30b}$$

$$v_{jt_2} = \frac{e^{-\left(M^2/(1+m^2)\right)t}}{1+m^2} \left[\sum_{k=0}^{\infty} \left\{ (-1)^{k+1} \left(e^{-cS} + e^{-dS} \right)(\sin(m_2 t) + m \cos(m_2 t)) \right\} + (\sin(m_2 t) + m \cos(m_2 t)) \right], \tag{30c}$$

$$\phi_1 = \frac{a}{\sqrt{\pi t}} \frac{1}{\xi_1} e^{-\left(\frac{a^2}{4t} + \frac{aS}{2}\right)} \left\{ \left(m_1 t - \frac{a^2}{4t} \right) \cos(m_2 t) - m_2 t \sin(m_2 t) \right\},$$

$$\phi_2 = \frac{a}{\sqrt{\pi t}} \frac{1}{\xi_1} e^{-\left(\frac{a^2}{4t} + \frac{aS}{2}\right)} \left\{ m_2 t \cos(m_2 t) + \left(m_1 t - \frac{a^2}{4t} \right) \sin(m_2 t) \right\},$$

$$\phi_3 = \frac{b}{\sqrt{\pi t}} \frac{1}{\xi_2} e^{-\left(\frac{b^2}{4t} + \frac{bS}{2}\right)} \left\{ \left(m_1 t - \frac{b^2}{4t} \right) \cos(m_2 t) - m_2 t \sin(m_2 t) \right\},$$

$$\phi_4 = \frac{b}{\sqrt{\pi t}} \frac{1}{\xi_2} e^{-\left(\frac{b^2}{4t} + \frac{bS}{2}\right)} \left\{ m_2 t \cos(m_2 t) + \left(m_1 t - \frac{b^2}{4t} \right) \sin(m_2 t) \right\},$$

$$\phi_5 = \frac{c}{\sqrt{\pi t}} \frac{1}{\xi_3} e^{-\left(\frac{c^2}{4t} + \frac{cS}{2}\right)} \left\{ \left(m_1 t - \frac{c^2}{4t} \right) \cos(m_2 t) - m_2 t \sin(m_2 t) \right\},$$

$$\phi_6 = \frac{c}{\sqrt{\pi t}}\frac{1}{\xi_3}e^{-\left(\frac{c^2}{4t}+\frac{cS}{2}\right)}\left\{m_2 t\cos(m_2 t)+\left(m_1 t-\frac{c^2}{4t}\right)\sin(m_2 t)\right\},$$

$$\phi_7 = \frac{d}{\sqrt{\pi t}}\frac{1}{\xi_4}e^{-\left(\frac{d^2}{4t}+\frac{dS}{2}\right)}\left\{\left(m_1 t-\frac{d^2}{4t}\right)\cos(m_2 t)-m_2 t\sin(m_2 t)\right\},$$

$$\phi_8 = \frac{d}{\sqrt{\pi t}}\frac{1}{\xi_4}e^{-\left(\frac{d^2}{4t}+\frac{dS}{2}\right)}\left\{m_2 t\cos(m_2 t)+\left(m_1 t-\frac{d^2}{4t}\right)\sin(m_2 t)\right\},$$

$$\phi_9 = 2\sqrt{\frac{t}{\pi}}\left(\cos(m_2 t)-m\sin(m_2 t)\right)\left\{\frac{c}{\left(S^2 t^2-c^2\right)}e^{-\left(\frac{c^2}{4t}+\frac{cS}{2}\right)}+\frac{d}{\left(S^2 t^2-d^2\right)}e^{-\left(\frac{d^2}{4t}+\frac{dS}{2}\right)}\right\},$$

$$\phi_{10} = 2\sqrt{\frac{t}{\pi}}\left(\sin(m_2 t)+m\cos(m_2 t)\right)\left\{\frac{c}{\left(S^2 t^2-c^2\right)}e^{-\left(\frac{c^2}{4t}+\frac{cS}{2}\right)}+\frac{d}{\left(S^2 t^2-d^2\right)}e^{-\left(\frac{d^2}{4t}+\frac{dS}{2}\right)}\right\},$$

$$\xi_1 = \left\{\left(m_1 t-\frac{a^2}{4t}\right)^2+m_2^2 t^2\right\};\quad \xi_2 = \left\{\left(m_1 t-\frac{b^2}{4t}\right)^2+m_2^2 t^2\right\},$$

$$\xi_3 = \left\{\left(m_1 t-\frac{c^2}{4t}\right)^2+m_2^2 t^2\right\};\quad \xi_4 = \left\{\left(m_1 t-\frac{d^2}{4t}\right)^2+m_2^2 t^2\right\}.$$

From the expressions (25) and (28) we observe that, the fluid flow is in quasi-steady state. The terms u_{js} and v_{js} represent final steady state flow. The steady state flow is confined within a thin boundary layer of thickness $O\left((\alpha + S/2)^{-1}\right)$. This boundary layer may be recognized as the modified Hartmann boundary layer and may be viewed as the classical Hartmann boundary layer modified by Hall current and suction/injection. It is noticed from equation (21) that α increases with increase in either magnetic parameter M^2 or suction/injection parameter S or both. Thus we conclude that the thickness of the modified Hartmann boundary layer decreases with increase in either M^2 or S or both. It is also seen from equations (26) and (29) that steady state flow represents spatial oscillations in the flow-field excited by Hall current, magnetic field and suction/injection. The unsteady part of the flow in equations (25) and (28), represented by u_{jt} and v_{jt}, exhibits inertial oscillations in the flow-field excited by Hall current and magnetic field. The unsteady state flow represented by u_{jt} and v_{jt} is divided into two parts viz. u_{jt_1}, v_{jt_1} and u_{jt_2}, v_{jt_2}.

The inertial oscillations in u_{jt_1} and v_{jt_1} dampen out effectively in dimensionless time of $O\left(\left((S^2/4)+\left(M^2/(1+m^2)\right)\right)^{-1}\right)$, whereas in u_{jt_2} and v_{jt_2}, they dampen out effectively in dimensionless time of $O\left(\left(M^2/(1+m^2)\right)^{-1}\right)$ when the final steady state is developed. This implies that suction/injection reduces the time of decay of inertial oscillations in the major part of the unsteady state flow whereas Hall current increases the time of decay of inertial oscillations in both the parts of the unsteady state flow. In the absence of Hall current, there are no inertial oscillations in the flow-field.

5. Results and Discussion

To study the effects of Hall current, magnetic field, suction/injection and time on the flow-field, the numerical values of the primary velocity u_j and the secondary velocity v_j, computed from the analytical expression (20) mentioned in Section 2 by MATLAB software, are displayed graphically versus non-dimensional channel width variable η in Figs. 2 to 9 for various values of the Hall current parameter m, magnetic parameter M^2, suction/injection parameter S and time t. To compare our results with already existing results of Seth et al. (2011) and Singh and Kumar (1983) we have drawn the profiles of fluid velocity versus non-dimensional channel width variable η for various values of M^2, taking $m=0$, $S=1$ and $S=0$ in Figs. 10 and 11. Figures 2 and 3 illustrate the influence of Hall current on the primary velocity u_j and the secondary velocity v_j. It is seen from Figure 2 that the primary velocity u_j decreases on increasing m. The secondary velocity v_j increases in the lower half of the channel and it decreases in the upper half of the channel on increasing m. This implies that the Hall current tends to retard fluid flow in the primary flow direction throughout the channel and fluid flow in the secondary flow direction in the upper half of the channel. It

has the reverse effect on the secondary flow in the lower half of the channel. Figures 4 and 5 demonstrate the effects of magnetic field on both the primary and secondary velocities. It is noticed from Figures 4 and 5 that both the primary velocity u_j and the secondary velocity v_j increase on increasing M^2 which implies that magnetic field tends to accelerate fluid flow in both the primary and secondary flow directions. This tendency of the magnetic field may be attributed to the movement of the magnetic lines of force along with the moving plate of the channel. Figures 6 and 7 display the influence of suction/injection on the primary and secondary fluid velocities. The primary velocity u_j decreases on increasing $S(>0)$ whereas it increases on increasing $S(<0)$ in the region $0 \leq \eta < 0.8$. The secondary velocity v_j decreases on increasing $S(>0)$ in the region $0 \leq \eta < 0.6$ whereas it increases on increasing $S(<0)$ throughout the channel. This implies that suction retards fluid flow in the primary flow direction in the major part of the channel whereas injection has the reverse effect on it. Suction retards fluid flow in the secondary flow direction in the region $0 \leq \eta < 0.6$ whereas injection accelerates fluid flow in the secondary flow direction throughout the channel. Figures 8 and 9 depict the influence of time on the primary and secondary velocities. Both the primary velocity u_j and the secondary velocity v_j increase on increasing time t which implies that fluid flow in both the primary and secondary flow directions are accelerated due to the passage of time. Figures 10 and 11 show that our results are in good agreement with the results obtained by Seth et al. (2011) and Singh and Kumar (1983).
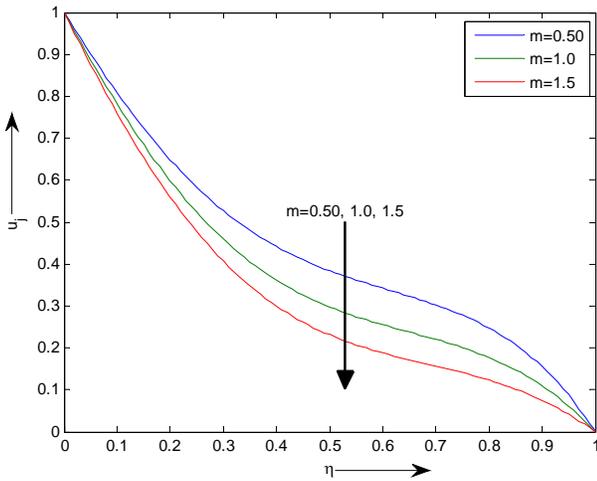
Figure 2. Primary velocity profiles when $M^2 = 10$, $S = 1$ and $t = 0.05$
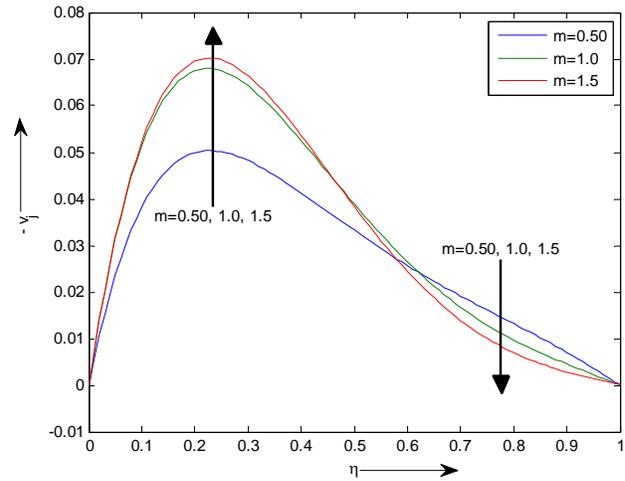
Figure 3. Secondary velocity profiles when $M^2 = 10$, $S = 1$ and $t = 0.05$
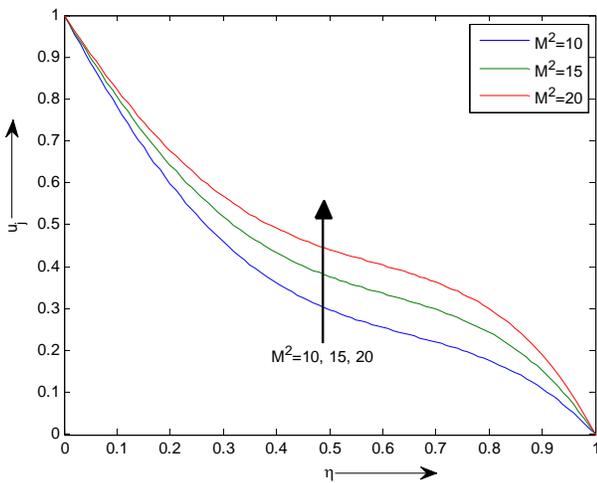
Figure 4. Primary velocity profiles when $m = 1.0$, $S = 1$ and $t = 0.05$
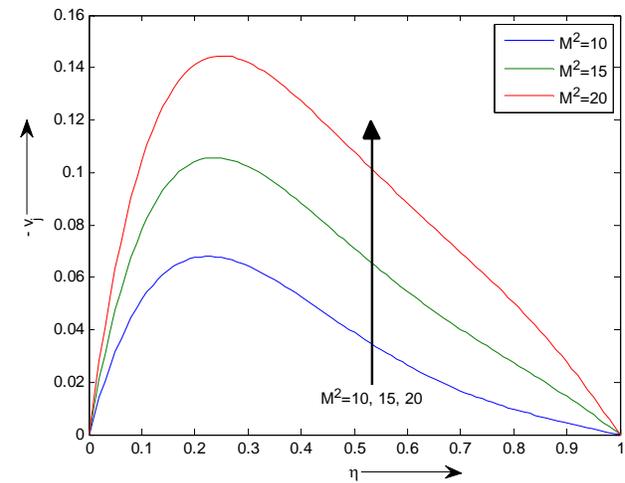
Figure 5. Secondary velocity profiles when $m = 1.0$, $S = 1$ and $t = 0.05$
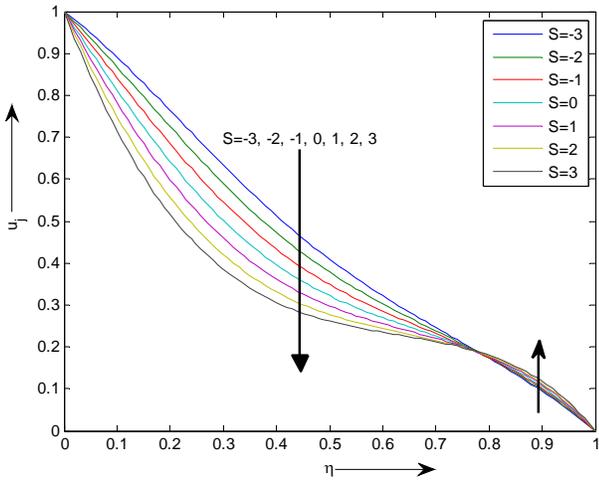
Figure 6. Primary velocity profiles when $m = 1$, $M^2 = 10$ and $t = 0.05$
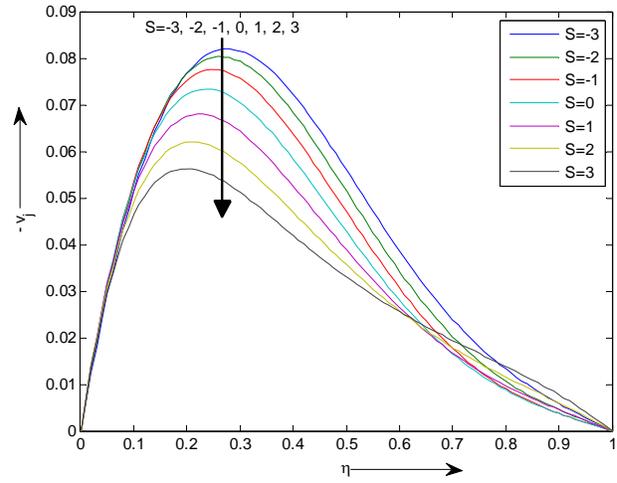
Figure 7. Secondary velocity profiles when $m = 1$, $M^2 = 10$ and $t = 0.05$
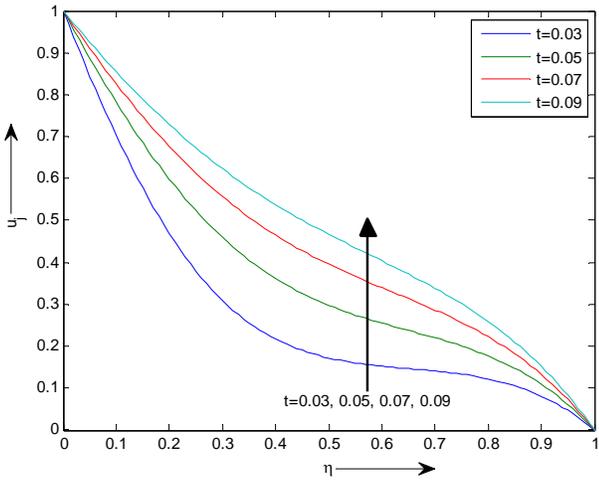
Figure 8. Primary velocity profiles when $m = 1.0$, $M^2 = 10$ and $S = 1$
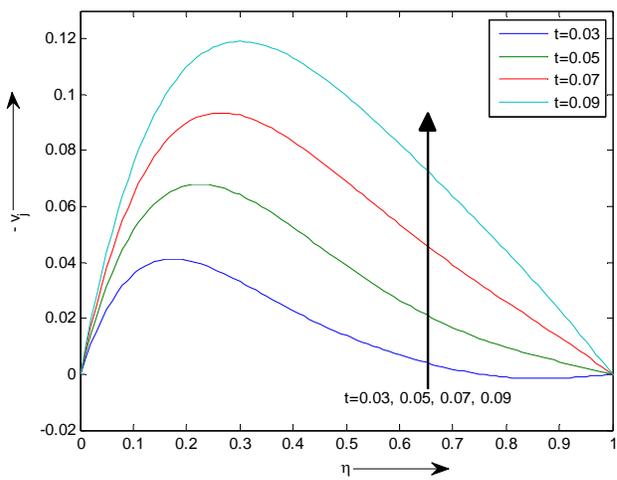
Figure 9. Secondary velocity profiles when $m = 1.0$, $M^2 = 10$ and $S = 1$
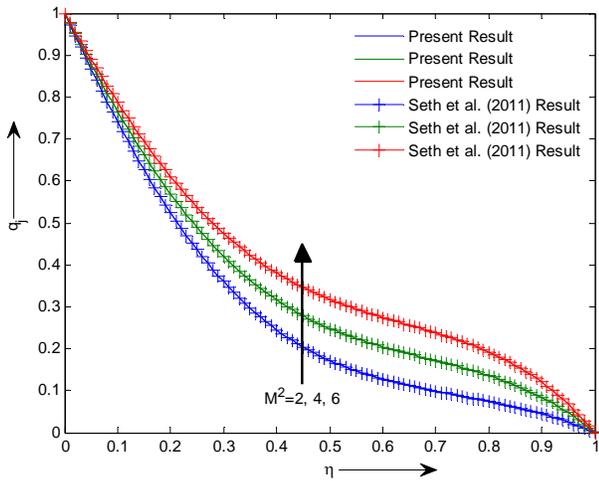
Figure 10. Velocity profiles when $m = 0$, $S = 1$ and $t = 0.05$
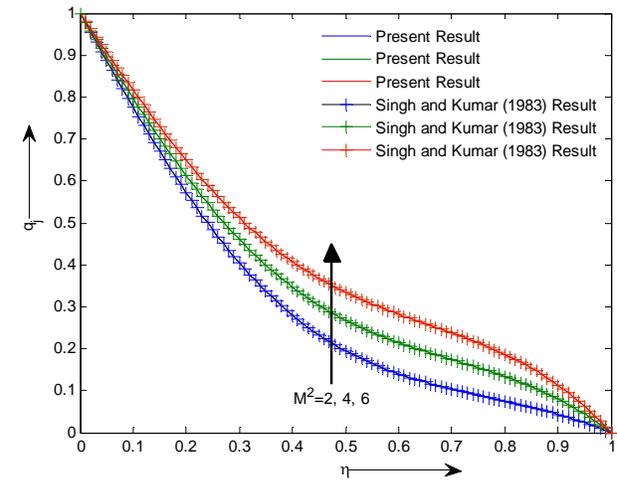
Figure 11. Velocity profiles when $m = 0$, $S = 0$ and $t = 0.05$

The numerical values of the non-dimensional shear stress at the moving plate due to primary and secondary flows, computed from the expression (22) mentioned in Section 3 by MATLAB software, are presented in tabular form in Tables 1 to 3 for various values of m, M^2, S and t. To compare our results with that of existing results of Seth et al. (2011) and Singh and Kumar (1983) we have presented numerical values of shear stress at the moving plate for various values of M^2 and S in Table 4. From Table 1 we see that the primary shear stress τ_{xj} increases on increasing m whereas it decreases on increasing M^2. The secondary shear stress τ_{yj} increases, attains maximum and then decreases on increasing m whereas it increases on increasing M^2. This implies that the Hall current tends to enhance the primary shear stress at the moving plate. Magnetic field tends to reduce the primary shear stress at the moving plate whereas it has the reverse effect on the secondary shear stress at the moving plate. It is found from table 2 that the primary shear stress τ_{xj} decreases on increasing t for all values of S and it increases on increasing $S(>0)$ and decreases on increasing $S(<0)$ in magnitude. This implies that suction tends to enhance primary shear stress at the moving plate whereas injection has the reverse effect on it. Primary shear stress reduces with the passage of time at each time level. It is observed from table 3 that the secondary shear stress τ_{yj} decreases on increasing $S(<0)$ in magnitude for every value of time t considered whereas it decreases on increasing $S(>0)$ when $t \le 0.05$. Secondary shear stress τ_{yj} increases on increasing t when $S(>0)$. This implies that injection tends to reduce the secondary shear stress at the moving plate at each time level whereas suction tends to reduce it when $t \le 0.05$. Secondary shear stress at the moving plate increases with passage of time at each time level in case of suction. It is seen from Table 4 that our results are found to be in good agreement with the results obtained by Seth et al. (2011) and Singh and Kumar (1983).

Table 1. Primary and secondary shear stress at the moving plate when $S = 1$ and $t = 0.05$

$M^2 \downarrow m\rightarrow$	$-\tau_{xj}$			$-\tau_{yj}$		
	0.5	1.0	1.5	0.5	1.0	1.5
10	2.2435	2.8371	3.1870	0.6424	0.7796	0.6833
15	1.9315	2.6982	3.1819	0.9479	1.1280	0.9792
20	1.6755	2.5580	3.1468	1.2421	1.4610	1.2563

Table 2. Primary shear stress $-\tau_{xj}$ at the moving plate when $m = 1.0$ and $M^2 = 10$

$t \downarrow S\rightarrow$	-3	-2	-1	0	1	2	3
0.03	2.0767	2.4182	2.7950	3.2077	3.6562	4.1405	4.6600
0.05	1.4330	1.7265	2.0582	2.4285	2.8371	3.2838	3.7678
0.07	1.1222	1.3729	1.6638	1.9950	2.3655	2.7742	3.2200
0.09	0.9455	1.1572	1.4106	1.7057	2.0410	2.4140	2.8234

Table 3. Secondary shear stress $-\tau_{yj}$ at the moving plate when $m = 1.0$ and $M^2 = 10$

$t \downarrow S\rightarrow$	-3	-2	-1	0	1	2	3
0.03	0.5658	0.5964	0.6202	0.6323	0.6305	0.6171	0.5969
0.05	0.6577	0.7074	0.7474	0.7722	0.7796	0.7723	0.7566
0.07	0.6624	0.7435	0.8103	0.8576	0.8837	0.8921	0.8903
0.09	0.6120	0.7312	0.8323	0.9099	0.9623	0.9940	1.0136

Table 4. Shear stress $-\tau_{xj}$ at the moving plate when $m = 0$ and $t = 0.05$

$M^2 \downarrow S\rightarrow$	-3	-2	-1	0	1	2	3
2	1.1797	1.4926	1.8599	2.2835	2.7642	3.3013	3.8925
4	1.0691	1.3516	1.6836	2.0667	2.5015	2.9873	3.5222
6	0.9689	1.2240	1.5240	1.8704	2.2637	2.7032	3.1871

6. Conclusions

The present investigation deals with the theoretical study of the unsteady hydromagnetic Couette flow within an infinitely long parallel plate porous channel with Hall effects. The significant results are summarized below:

1. For small values of time t, a Rayleigh boundary layer of thickness $O\left(\sqrt{t}\right)$ arises near the moving plate and both the primary and secondary velocities are affected by Hall current, magnetic field and suction/injection.

2. For large values of time t, fluid flow is in quasi-steady state. Steady state flow is confined within a thin modified Hartmann boundary layer. The steady state flow represents spatial oscillations in the flow-field excited by the Hall current, the magnetic field and suction/injection, whereas the unsteady flow exhibits inertial oscillations in the flow-field excited by Hall current and magnetic field.

3. Hall current tends to retard fluid flow in the primary flow direction throughout the channel and fluid flow in the secondary flow direction in the upper half of the channel. It has the reverse effect on the secondary flow in the lower half of the channel.

4. Magnetic field tends to accelerate fluid flow in both the primary and secondary flow directions. This tendency of the magnetic field may be attributed to the movement of the magnetic lines of force along with the moving plate of the channel.

5. Suction retards fluid flow in the primary flow direction in the major part of the channel whereas injection has the reverse effect on it. Suction retards fluid flow in the secondary flow direction in the region $0 \leq \eta < 0.6$ whereas injection accelerates fluid flow in the secondary flow direction throughout the channel.

6. Fluid flow in both the primary and secondary flow directions is accelerated due to the passage of time.

7. Hall current tends to enhance the primary shear stress at the moving plate.

8. Magnetic field tends to reduce the primary shear stress at the moving plate whereas it has the reverse effect on the secondary shear stress at the moving plate.

9. Suction tends to enhance primary shear stress at the moving plate whereas injection has the reverse effect on it.

10. Primary shear stress at the moving plate reduces with the passage of time at each time level.

11. Injection tends to reduce the secondary shear stress at the moving plate at each time level whereas suction tends to reduce it when $t \leq 0.05$.

12. Secondary shear stress at the moving plate increases with passage of time at each time level in case of suction.

Nomenclature

H_0 - uniform transverse magnetic field

h - width of the channel

M^2 - magnetic parameter

m - Hall current parameter

p - Laplace transform parameter

S - suction/injection parameter

t - non-dimensional time

R_e - Reynolds number

U_0 - uniform velocity in *x*-direction

u_j, v_j - non-dimensional primary and secondary velocity respectively

$x,\ y,\ z$ - Cartesian coordinates

ρ - fluid density

σ - electrical conductivity

υ - kinematic coefficient of viscosity

η - channel width variable

μ_e - magnetic permeability

ω_e - cyclotron frequency

τ_e - electron collision time

τ_{xj}, τ_{yj} - primary and secondary shear stress at the moving plate respectively.

Acknowledgement

References

Abbas, Z., Sajid, M. and Hayat, T., 2006. MHD boundary layer flow of an upper convected Maxwell fluid in a porous channel. Theor. Comp. Fluid Dyn., Vol. 20, pp. 229-238.

Attia, H. A., 2007. Time varying Hartmann flow with heat transfer on a Power-Law fluid with uniform suction and injection under constant pressure gradient. Kragujevac J. Sci., Vol. 29, pp. 15-28.

Bhaskara Reddy, N. and Bathaiah, D., 1982. Hall effects on MHD flow through a porous straight channel. Def. Sci. J., Vol. 32, pp. 313-326.

Cowling, T. C., 1957. Magnetohydrodynamics. Interscience Pub., New York.

Cramer, K. R. and Pai, S., 1973. Magnetohydrodynamics for Engineers and Applied Physicists. McGraw Hill Book Company, New York.

Hayat, T., Ahmed, N., Sajid, M. and Asghar, S., 2007. On the MHD flow of second grade fluid in a porous channel. Comp. Math. Appl., Vol. 54, pp. 407-414.

Jana, R. N. and Dutta, N., 1977. Hall effects on unsteady Couette flow, Int. J. Engng. Sci., Vol. 15, pp. 35-43.

Hayat, T., Ahmed, N. and Sajid, M., 2008. Analytic solution for MHD flow of third order fluid in a porous channel. J. Comp. Appl. Maths., Vol. 214, pp. 572-582.

Katagiri, M., 1962. Flow formation in Couette motion in Magnetohydrodynamics. J. Phys. Soc. Jpn., Vol. 17, pp. 393-396.

Khan, M., Rahman, S. and Hayat, T., 2009. Heat transfer analysis and Magnetohydrodynamic flow of a non-Newtonian fluid through a porous medium with slip at the walls. J. Porous Media., Vol. 12, pp. 277-287.

Makinde, O. D. and Chinyoka, T., 2001. Unsteady MHD flow in a porous channel with an exponentially decreasing suction. J. Pure Appl. Math., Vol. 1, No. 1, pp. 1-13.

Makinde, O. D. and Osalusi, E., 2006. MHD steady flow in a channel with slip at permeable boundaries. Romanian J. Phys., Vol. 51, pp. 319-328.

Muhuri, P. K., 1963. Flow formation in Couette motion in Magnetohydrodynamics with suction. J. Phys. Soc. Jpn., Vol. 18, pp. 1671-1675.

Prasad Rao, D. R. V., Krishna, D. V. and Debnath, L., 1982. Combined effect of free and forced convection on MHD flow in a rotating porous channel. Int. J. Math. & Math. Sci., Vol. 5, pp. 165-182.

Rossow, V. J., 1958. On flow of electrically conducting fluid over a flat plate in the presence of a transverse magnetic field. NACA Rept. 1358.

Seth, G. S., Ansari, Md. S. and Nandkeolyar, R., 2011. Unsteady hydromagnetic Couette flow within a porous channel. Tamkang J. Sci. and Eng., Vol. 14, No. 1, pp. 7-14.

Singh, A. K. and Kumar, N., 1983. Unsteady Magnetohydrodynamic Couette flow. Wear, Vol. 89, pp. 125-129.

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