

Elastodynamics response of Green's function and influence function in fluid saturated incompressible porous medium

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Abstract

The article reports on a methodology to synthesize the response of Green's function and influence function in fluid saturated incompressible porous half space. As an application, the disturbance due to concentrated and distributed loads in normal and tangential direction is investigated by employing the Laplace and Fourier transforms. The integral transforms have been inverted by using a numerical technique to obtain the components of displacement, stress and pore pressure in physical domain. The results concerning these quantities are given and illustrated graphically to depict the effect of pore pressure. A particular case of interest has been deduced from the present investigation.

Key words: Porous, Green's function, pore pressure, Laplace transform, Fourier transform.

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1. Introduction

Most of the modern engineering structures are generally made up of multiphase porous continuum, the classical theory, which represents a fluid saturated porous medium as a single phase material, is inadequate to represent the mechanical behavior of such materials especially when the pores are filled with liquid. In this context the solid and liquid phases have different motions. Due to these different motions, the different material properties and the complicated geometry of pore structures, the mechanical behavior of a fluid saturated porous medium is very complex and difficult. So from time to time, researchers have tried to overcome this difficulty. For more details and for the historical review on the subject of the multiphase continuum mechanics, the reader is referred to the work of de Boer and Ehlers (1988) or to the recently published monograph Boer (2000).

Based on the work of von Terzaghi (1923, 1925), Biot (1941) proposed a general theory of three-dimensional deformations of fluid saturated porous solid. Then the wave propagation and dynamic extensions were done by Biot (1956a, 1956b, 1962). Biot theory is based on the assumption of compressible constituents and till recently, some of his results have been taken as standard references and basis for subsequent analysis in acoustic, geophysics and other such fields.

Based on the work of Fillunger model (1913) (which is further based on the concept of volume fractions combined with surface porosity coefficients) , Bowen (1980) and de Boer Ehlers (1990a, 1990b) developed and used another interesting theory in which all the constituents of a porous medium are assumed to be incompressible . There are reasonable grounds for the assumption that the constituents of many fluid saturated porous media are incompressible. For example, taking the composition of soil, the solid constituents are incompressible and liquid constituents, which are generally water or oils are also incompressible. Moreover in an empty porous solid as a case of classical theory, the change in the volume is due to the changes in porosity during the propagation of a longitudinal wave. The assumption of incompressible constituents does not only meet the properties appearing in many branches of engineering practice, but it also avoids the introduction of many complicated material parameters as considered in the Biot theory. So this model meets the requirements of further scientific developments. Based on this theory de Boer and Ehlers (1993) and Recently, Kumar and Hundal (2002, 2003a, 2003b, 2004, 2005) studied some problems of wave propagation in fluid

saturated incompressible porous media. However, no attempt has been made to study source problem in fluid saturated incompressible porous media.

In the present investigation, the disturbance due to point force in normal and tangential direction have been discussed by the use of integral transforms. Integral transforms have been inverted numerically. Pore pressure on the displacement and stress components are depicted graphically.

2. Problem formulation

We consider a homogenous fluid saturated incompressible porous half space $z \geq 0$ of a rectangular Cartesian coordinate system (x, y, z) having origin on the surface $z=0$ and z axis pointing vertically into the medium. A normal and tangential force is assumed to be acting at the origin of the rectangular Cartesian coordinates.

Following de Boer and Ehlers (1990,1993), the equations governing the deformation of an incompressible porous medium saturated with non-viscous fluid in the absence of body forces are

$$\nabla \cdot (\eta^S \dot{\mathbf{u}}_S + \eta^F \dot{\mathbf{u}}_F) = 0, \quad (1)$$

$$(\lambda^S + \mu^S) \nabla (\nabla \cdot \mathbf{u}_S) + \mu^S \nabla^2 \mathbf{u}_S - \eta^S \nabla p - \rho^S \ddot{\mathbf{u}}_S + S_V (\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S) = 0, \quad (2)$$

$$\eta^F \nabla p + \rho^F \ddot{\mathbf{u}}_F + S_V (\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S) = 0, \quad (3)$$

$$\mathbf{T}_E^S = 2\mu^S \mathbf{E}_S + \lambda^S (\mathbf{E}_S \cdot \mathbf{I}), \quad (4)$$

$$\mathbf{E}_S = \frac{1}{2} (\text{grad } \mathbf{u}_S + \text{grad}^T \mathbf{u}_S), \quad (5)$$

where

$\mathbf{u}_i, \dot{\mathbf{u}}_i, \ddot{\mathbf{u}}_i$, $i = S, F$, denote the displacement, velocities and acceleration of solid and fluid phases respectively and p is the effective pore pressure of the incompressible pore fluid. ρ^S and ρ^F are the densities of the solid and fluid respectively. \mathbf{T}_E^S is the stress in the solid phase and \mathbf{E}_S is the linearized langrangian strain tensor. λ^S and μ^S are the macroscopic Lamé's parameters of the porous solid and η^S and η^F are the volume fractions satisfying $\eta^S + \eta^F = 1$.

The case of isotropic permeability, the tensor S_V describing the coupled interaction between the solid and fluid is given by de Boer and Ehlers (1990) as

$$\mathbf{S}_V = \frac{(\eta^F)^2 \gamma^{FR}}{K^F} \mathbf{I} = S_V \mathbf{I},$$

where γ^{FR} is the effective specific weight of the fluid and k^F is the Darcy's permeability coefficient of the porous medium.

For two dimensional problem, we assume the displacement vector \mathbf{u}_i ($i=F, S$) as

$$\mathbf{u}_i = (u^i, 0, w^i) \quad \text{where } i=F, S. \quad (6)$$

For further consideration it is convenient to introduce the dimensionless quantities defined as:

$$\begin{aligned} x' &= \frac{\omega^*}{C_1} x, \quad z' = \frac{\omega^*}{C_1} z, \quad t' = \omega^* t, \quad u'^S = \left[\frac{\lambda^S + 2\mu^S}{E} \right] \frac{\omega^*}{C_1} u^S, \quad w'^S = \left[\frac{\lambda^S + 2\mu^S}{E} \right] \frac{\omega^*}{C_1} w^S, \\ u'^F &= \left[\frac{\lambda^S + 2\mu^S}{E} \right] \frac{\omega^*}{C_1} u^F, \quad w'^F = \left[\frac{\lambda^S + 2\mu^S}{E} \right] \frac{\omega^*}{C_1} w^F, \quad p' = \frac{p}{E}, \quad t'_{31} = \frac{t_{31}}{E}, \quad t'_{33} = \frac{t_{33}}{E}. \end{aligned} \quad (7)$$

In these relations E is the Young's modulus of the solid phase, ω^* is a constant having the dimensions of frequency, C_1 is the velocity of a longitudinal wave propagating in a fluid saturated incompressible porous medium and is given by de Boer and Ehlers (1993) as

$$C_1 = \sqrt{\frac{(\eta^F)^2(\lambda^S + 2\mu^S)}{(\eta^F)^2\rho^S + (\eta^S)^2\rho^F}}, \quad (8)$$

If pore is absent or gas is filled in the pores then ρ^F is very small as compare to ρ^S and can be neglected so the relation reduce to

$$C_0 = \sqrt{\frac{\lambda^S + 2\mu^S}{\rho^S}}. \quad (9)$$

This gives the velocity of the longitudinal wave propagating in an incompressible empty porous solid where the change in volume is due to the change in porosity and well known result of the classical theory of elasticity. In an incompressible non porous solid $\eta^F \rightarrow 0$, then (8) becomes $C_1 = 0$ and physically acceptable as longitudinal wave cannot propagate in an incompressible medium.

The displacement components u^i and w^i are related to the non dimensional potential ϕ^i and ψ^i as

$$u^i = \frac{\partial \phi^i}{\partial x} + \frac{\partial \psi^i}{\partial z}, \quad w^i = \frac{\partial \phi^i}{\partial z} - \frac{\partial \psi^i}{\partial x}, \quad i=F,S \quad (10)$$

We define the Laplace and Fourier transforms as follows:

$$(\bar{\phi}^i, \bar{\psi}^i)(x, z, s) = \int_0^\infty \{\phi^i, \psi^i\}(x, z, t) e^{-st} dt, \quad (11)$$

$$\{\tilde{\phi}^i, \tilde{\psi}^i\}(\xi, z, s) = \int_{-\infty}^\infty \{\bar{\phi}^i, \bar{\psi}^i\}(x, z, s) e^{ix\xi} dx, \quad (12)$$

With the aids of (1) – (7) and (10) - (12) the values of $\tilde{\phi}^S$, $\tilde{\phi}^F$, \tilde{p} , $\tilde{\psi}^S$, $\tilde{\psi}^F$ satisfying radiation conditions that $\tilde{\phi}^S$, $\tilde{\phi}^F$,

\tilde{p} , $\tilde{\psi}^S$, $\tilde{\psi}^F \rightarrow 0$, $z \rightarrow \infty$ are given by

$$\tilde{\phi}^S = A_1 e^{-az}, \quad (13)$$

$$\tilde{\phi}^F = m_1 A_1 e^{-az}, \quad (14)$$

$$\tilde{p} = n_1 A_1 e^{-az}, \quad (15)$$

$$\tilde{\psi}^S = B_1 e^{-bz}, \quad (16)$$

$$\tilde{\psi}^F = m_2 B_1 e^{-bz}, \quad (17)$$

where

$$a^2 = \xi^2 + r_1^2, \quad b^2 = \xi^2 + r_2^2, \quad n_1 = \frac{\eta^S \rho^F s^2 \delta_1^2 + s \delta_2 \rho^S}{(\eta^F)^2 \rho^S}, \quad m_1 = -\frac{\eta^S}{\eta^F}, \quad m_2 = \frac{\rho^S \delta_2}{\rho^F \delta_1^2 s + \rho^S \delta_2},$$

$$r_1^2 = \frac{s^2 (\eta^F)^2 + s \delta_2}{(\eta^F)^2}, \quad r_2^2 = \frac{\delta_1^4 s^3 \rho^F / \rho^S + \delta_1^2 \delta_2 s^2 + s^2 \delta_2 \delta_1^2 \rho^F / \rho^S}{(\rho^F / \rho^S \delta_1^2 s + \delta_2) \delta^2}, \quad \delta_1 = \frac{C_1}{C_0}, \quad \delta = \frac{\beta_0}{C_0}, \quad \delta_2 = \frac{S_V C_1^2}{w^* \rho^S C_0^2}.$$

3. Boundary conditions

We consider the normal force or tangential force on the surface of the half space $z = 0$, i.e.

$$t_{33}^S - p = -P_1 F(x, t), \quad \text{at } z = 0, \quad (18)$$

$$t_{31}^S = -P_2 F(x, t), \quad \text{at } z = 0, \quad (19)$$

where P_1, P_2 are the magnitude of the forces.

and

1) $P_2 = 0$ for the force in normal direction.

2) $P_1 = 0$ for the force in tangential direction .

Considering $P_1' = \frac{P_1}{E}, P_2' = \frac{P_2}{E}$, and with the aid of (4)-(7) and (10)- (19) we obtain the

the components of displacement and stress and pore pressure as :

$$\tilde{u}^S = \frac{1}{\Delta} (i\xi \Delta_1 e^{-az} - b \Delta_2 e^{-bz}), \quad (20)$$

$$\tilde{w}^S = \frac{1}{\Delta} (a \Delta_1 e^{-az} + i\xi \Delta_2 e^{-bz}), \quad (21)$$

$$\tilde{u}^F = \frac{1}{\Delta} (i\xi m_1 \Delta_1 e^{-az} - b m_2 \Delta_2 e^{-bz}), \quad (22)$$

$$\tilde{w}^F = \frac{1}{\Delta} (a m_1 \Delta_1 e^{-az} + i\xi m_2 \Delta_2 e^{-bz}), \quad (23)$$

$$\tilde{t}_{33}^S = \frac{1}{\Delta} (-hd_1 \Delta_1 e^{-az} - 2i\xi d_2 b \Delta_2 e^{-bz}), \quad (24)$$

$$\tilde{t}_{31}^S = \frac{1}{\Delta} (-2iad_1 \Delta_1 e^{-az} + (\xi^2 + b^2)d_1 \Delta_2 e^{-bz}), \quad (25)$$

$$\tilde{p} = \frac{1}{\Delta} (-n_1 \Delta_1 e^{-az}), \quad (26)$$

where

$$\Delta = (d_1 h - n_1) (\xi^2 + b^2) d_1 - 4ab \xi^2 d_1 d_2, \Delta_1 = [(\xi^2 + b^2) d_1 P_1 \tilde{F}(\xi, s) + 2i\xi b d_2 P_2 \tilde{F}(\xi, s)],$$

$$\Delta_2 = [2ia\xi d_1 P_1 \tilde{F}(\xi, s) - (d_1 h - n_1) P_2 \tilde{F}(\xi, s)], \quad d_1 = \frac{\lambda^S}{\lambda^S + 2\mu^S}, \quad d_2 = \frac{\mu^S}{\lambda^S + 2\mu^S},$$

$$h = \left(\frac{-\lambda^S \xi^2 + a^2 \lambda^S + 2a^2 \mu^S}{\lambda^S} \right).$$

4. Applications

Case 1.Green's function

The general solutions for displacement and stress presented in equations (20)-(26) will be used to yield response of a half space subjected to a concentrated force as

$$F(x, t) = \delta(x) \delta(t) \quad (27)$$

where, $\delta(\)$ is the dirac – delta function .

Applying the Laplace and Fourier transforms defined by (11) and (12) on (27), yield

$$\tilde{F}(\xi, s) = 1 \quad (28)$$

Case 2: Influence functions

$$\text{Here } F(x, t) = \psi(x) \delta(t) \quad (29)$$

Where $\psi(x)$ is a known function and takes two types of values

1. Uniformly distributed force

$$\psi(x) = \begin{cases} 1, & |x| \leq a \\ 0, & |x| > a \end{cases} \quad (30)$$

where $2a$ is non-dimensional width of a strip.

Applying Laplace and Fourier transforms defined by (11) and (12) on (29) and (30), we obtain

$$\tilde{\tilde{F}}(\xi, s) = [(2 \sin \xi a) / \xi] \quad (31)$$

2. Linearly distributed force

$$\psi(x) = \begin{cases} 1 - \frac{|x|}{a}, & |x| \leq a \\ 0, & |x| > a \end{cases} \quad (32)$$

where $2a$ is non-dimensional width of a strip.

Applying Laplace and Fourier transforms defined by (11) and (12) on (29) and (32), we obtain

$$\tilde{\tilde{F}}(\xi, s) = \frac{2[1 - \cos(\xi a)]}{\xi^2 a} \quad (33)$$

The expressions for displacements, and stresses, pore pressure can be obtained for concentrated, uniformly and linearly distributed force by replacing $\tilde{\tilde{F}}(\xi, s)$ from (28), (31) and (33) in (20) - (26).

Taking $P_2 = 0$ in resulting expressions, we obtain the corresponding results for a force in normal direction and $P_1 = 0$ in the resulting equations yield the results for a force in the tangential direction.

5. Particular case

If the pore liquid is absent, we obtain the corresponding expressions for the components of displacement and stress in empty porous elastic half space as:

$$\tilde{\tilde{u}}^s = \frac{1}{\Delta^*} (i\xi \Delta_1^* e^{-a_1 z} - b_1 \Delta_2^* e^{-b_1 z}), \quad (34)$$

$$\tilde{\tilde{w}}^s = \frac{1}{\Delta^*} (a_1 \Delta_1^* e^{-a_1 z} + i\xi \Delta_2^* e^{-b_1 z}), \quad (35)$$

$$\tilde{\tilde{t}}_{33}^s = \frac{1}{\Delta^*} (-d_1 h^* \Delta_1^* e^{-a_1 z} - 2i\xi d_2 b_1 \Delta_2^* e^{-b_1 z}), \quad (36)$$

$$\tilde{\tilde{t}}_{31}^s = \frac{1}{\Delta^*} (-2ia_1 d_1 \Delta_1^* e^{-a_1 z} + (\xi^2 + b_1^2) d_1 \Delta_2^* e^{-b_1 z}), \quad (37)$$

where

$$\Delta^* = d_1^2 h^* (b_1^2 + \xi^2) - 4d_1 d_2 a_1 b_1 \xi^2, \Delta_1^* = [\tilde{\tilde{F}}(\xi, s) d_1 (\xi^2 + b_1^2) P_1 + 2ib_1 \xi d_2 \tilde{\tilde{F}}(\xi, s) P_2]$$

$$\Delta_2^* = [2i\xi a_1 d_1 \tilde{\tilde{F}}(\xi, s) P_1 - d_1 h^* \tilde{\tilde{F}}(\xi, s) P_2], h^* = \left(\frac{-\lambda^s \xi^2 + a_1^2 \lambda^s + 2a_1^2 \mu^s}{\lambda^s} \right), a_1^2 = s^2 + \xi^2,$$

$$b_1^2 = \xi^2 + \frac{s^2}{\delta^2}.$$

6. Inversion of the transform

The transformed displacements, stresses and pore pressures are functions of the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $\tilde{\tilde{f}}(\xi, z, s)$. To obtain the solution of the problem in the physical domain, we must invert the Laplace and Fourier transform by using the method applied by Kumar et. al (2005)

7. Numerical results and discussion

With the view of illustrating the theoretical results and for numerical discussion we take a model for which the values of the various physical parameters are taken from de Boer and Ehlers (1993)

$$\eta^S = .67, \quad \eta^F = .33, \quad \rho^S = 1.34 \text{Mg} / m^3, \quad \rho^F = .33 \text{Mg} / m^3,$$

$$\lambda^S = 5.5833 \text{MN} / m^2, \quad k^F = .01 \text{m} / \text{s}, \quad \gamma^{FR} = 10.00 \text{KN} / m^3, \quad \mu^S = 8.3750 \text{MN} / m^2.$$

The values of vertical solid displacement w^S , vertical fluid displacement w^F , vertical solid stress t^S_{33} , horizontal solid stress t^S_{31} and pore pressure p for fluid saturated incompressible porous half space (FS) and empty porous elastic half space (ES) are shown due to concentrated force (CF), uniformly distributed force (UDF), linearly distributed force (LDF) at $t=0.5$. The variations of these components with distance x are shown by

- (1) The solid lines with and without central symbols to represent the case when CF is applied for ES and FS respectively.
- (2) The long dashed lines with and without central symbols to represent the case when UF is applied for ES and FS respectively.
- (3) The small dashed lines with and without central symbols to represent the case when LDF is applied for ES and FS respectively.

These variations are shown in figs. 1-10. The computations are carried out for $z=1$ in the range $0 \leq x \leq 10$, $a=1$.

7.1 Force in normal direction

Figure 1. Shows the variations of vertical solid displacement component w^S with distance x for FS, ES. For FS, the values of w^S first increase in the range $0 \leq x \leq 5.6$ then decrease in the remaining range of x whereas for ES, the values of w^S initially decrease in the range $0 \leq x \leq 2.5$ then approach to a constant values as x increases further. It is noticed that values of w^S for ES as compared to the values of w^S for FS are more in the range $0 \leq x \leq 3$, less in the range $3 \leq x \leq 8.4$ then more for $x \geq 8.4$. Figure 2. Shows the variations of the vertical fluid displacement component w^F with distance x for FS. The trend of variations of w^F is same due to all the forces (CF, UDF, LDF) but corresponding values are different in magnitude. Figure 3. Depicts the variations of vertical solid stress component t^S_{33} with distance x for both FS, ES. The values of t^S_{33} for FS decrease in the range $0 \leq x \leq 5.5$ then increase in the remaining range of x whereas the values of t^S_{33} for ES decrease gradually in the range $0 \leq x \leq 2.1$, then oscillates about origin for the remaining values of x due to all three forces (CF, UDF, LDF). It is evident that values of t^S_{33} for FS as compared to the values of t^S_{33} for ES are more in the range $0 \leq x \leq 3$, less in the range $3 \leq x \leq 8.1$ then more in the range $8.1 \leq x \leq 10$. Behaviour of horizontal solid stress component t^S_{31} for both FS and ES is shown in the figure 4. For ES, the values horizontal solid stress t^S_{31} decrease sharply in the range $0 \leq x \leq 2.3$ then its behaviour is oscillatory for the whole range of x whereas for FS values of t^S_{31} are distributed in large range but with small magnitude. Figure 5. Depicts the variations of the pore pressure p with distance x for FS. The values of p increase in the range $0 \leq x \leq 5.5$ then decrease as $x \geq 5.5$ due to all the forces (CF, UDF and LDF)

7.2 Force in tangential direction

Figure 6. Shows the variations of vertical solid displacement component w^S with distance x for FS, ES. Values of w^S initially decrease for FS and increase for ES, then their behaviour is opposite oscillatory due to all three forces (CF, UDF, LDF). Figure 7. Shows the variations of the vertical fluid displacement component w^F with distance x for FS. Values of w^F decrease sharply in the range $0 \leq x \leq 2$ then oscillates for the whole range of x due to all three forces (CF, UDF, LDF). Figure 8. Shows the variations of vertical solid stress component t^S_{33} with distance x for FS and ES. The trend of variations of t^S_{33} for both FS and ES due to all the forces is same but corresponding values are different in magnitude. Its values initially decrease in the range $0 \leq x \leq 2$ then increase in the range $2 \leq x \leq 4$, then oscillates as x increase further.

Figure 9. show the variations of horizontal solid stress component t^S_{31} with distance x for both FS, ES. The values of t^S_{31} for FS first decrease monotonically in the range $0 \leq x \leq 5.8$ then increase monotonically in the range $5.8 \leq x \leq 10$ whereas for ES

values of t_{31}^s start with initial increase then oscillates for the whole range of x . Figure10. Depicts the variations of the pore pressure p with distance x for FS. The values of pore pressure p decrease sharply in the range $0 \leq x \leq 2$, then oscillates for the remaining values of x due to all the forces (CF, UDF, LDF)

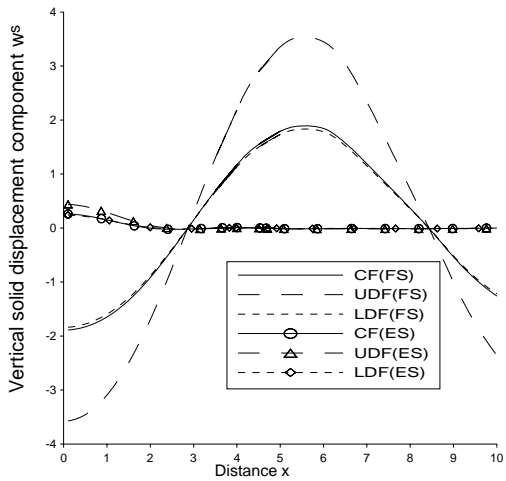


Figure1. Variation of vertical solid displacement component w_s with horizontal distance x (normal direction)

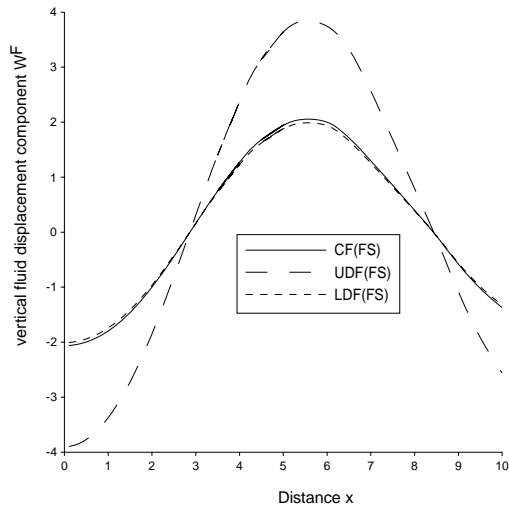


Figure 2. Variation of vertical fluid displacement component w_f with horizontal distance x (normal direction)

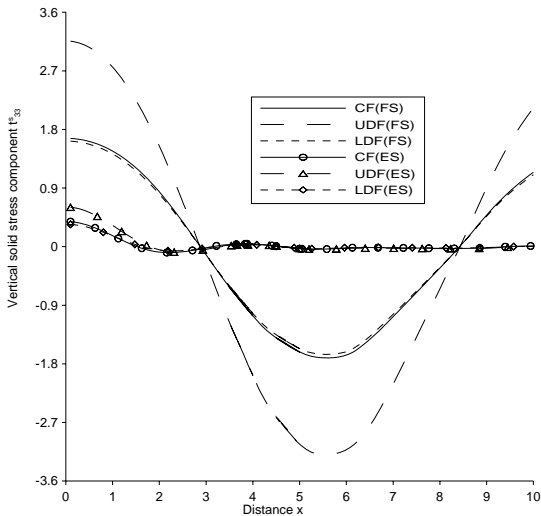


Figure3. Variation of vertical solid stress component t_{33}^s with horizontal distance x (normal direction)

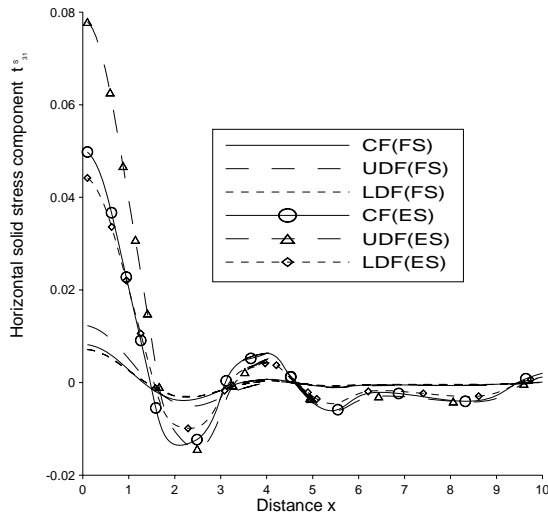


Figure 4. Variation of horizontal solid stress component t_{31}^s with horizontal distance x (normal direction)

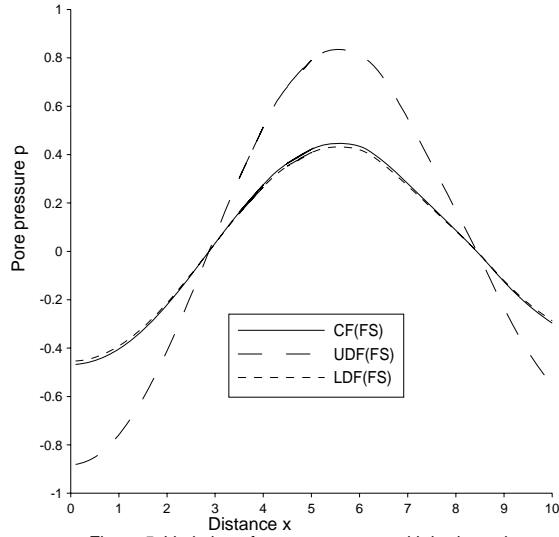


Figure 5. Variation of pore pressure p with horizontal distance x (normal direction)

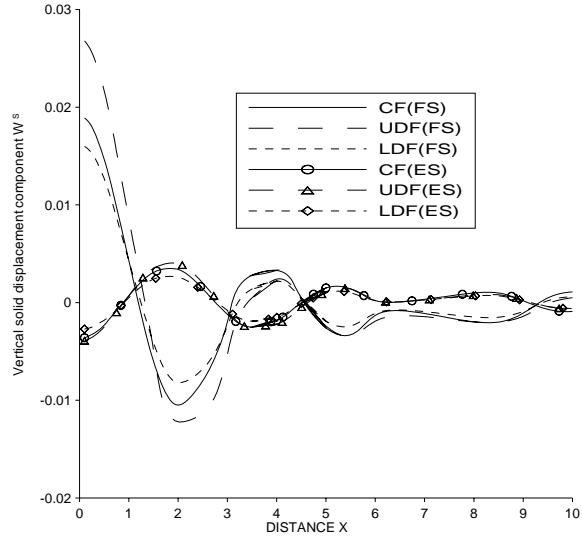


Figure 6. variation of vertical solid displacement component W^s with distance x (Tangential direction)

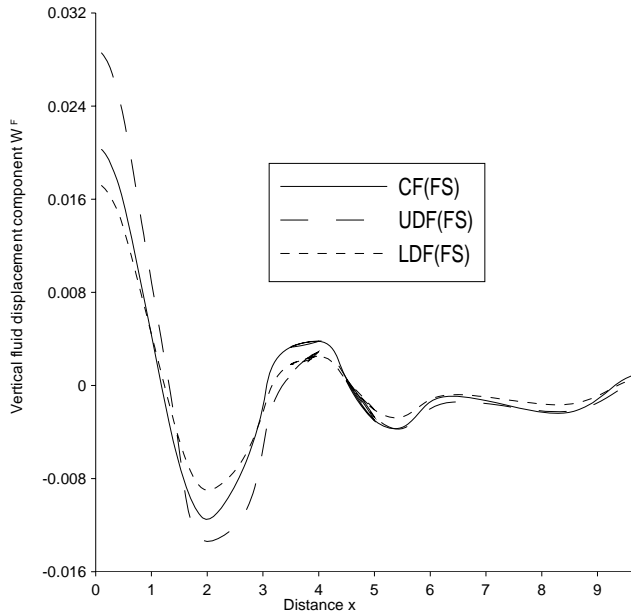


Figure 7. Variation of vertical fluid displacement component W^F with horizontal distance x (Tangential direction)

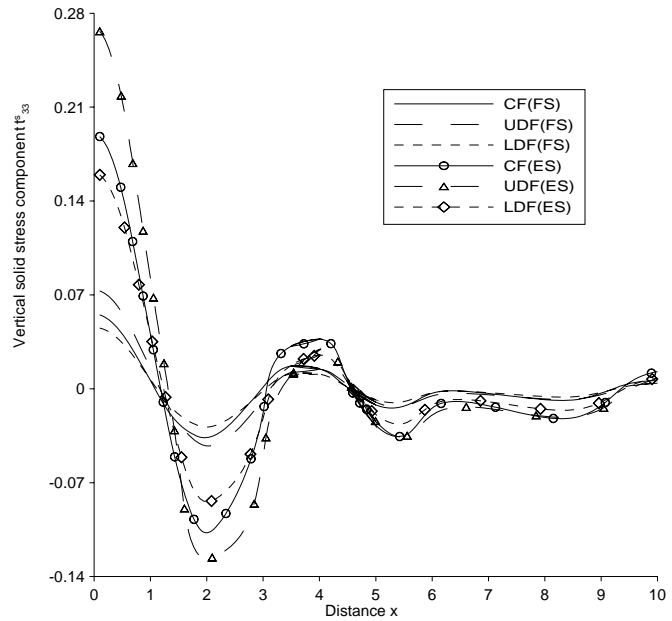


Figure 8: variation of the vertical solid stress component t^s_{33} with horizontal distance x (Tangential direction)

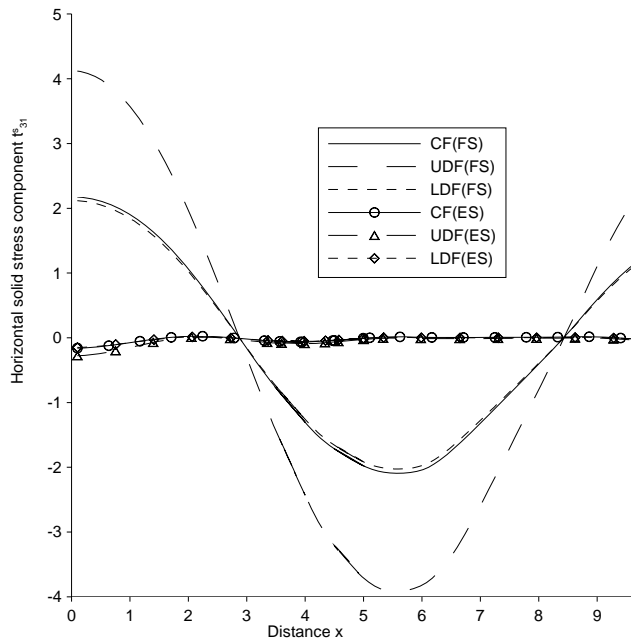


Figure 9: Variation of the horizontal solid stress component t^s_{31} with horizontal distance x (Tangential direction)

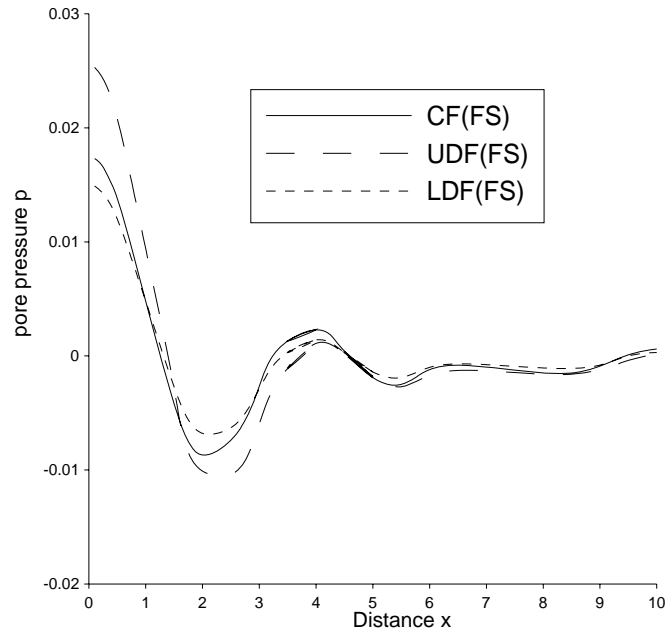


Figure 10: Variation of pore pressure p with horizontal distance x (Tangential direction)

8. Conclusion

1. The Laplace and Fourier transforms are used to derive the components of vertical solid displacement, vertical fluid displacement, vertical stress, horizontal stress and pore pressure.
2. Values of vertical solid displacement, vertical fluid displacement, vertical stress, horizontal stress and pore pressure are close to each other due to CF and LDF as compared to the values due to UDF in both the media.
3. Behaviour of variation of fluid displacement component w^F and pore pressure p is observed similar for all types of forces.
4. Near the point of application of source, the porosity effect increases the values of t^s_{33} and decreases the values of t^s_{31} in normal direction whereas its effect is opposite in tangential direction.

References

- Biot M. A., 1941. General Theory of Three dimensional consolidation . J. Appl. Phys., 12 (2), 155 – 161.
- Biot M.A., 1956a. Theory of Propagation of Elastic Waves in a Fluid-saturated Porous Solid - 1. Low Frequency Range . J. Acoust. Soc. Am. 28 , 168 -178.
- Biot M.A., 1956b. Theory of Propagation of Elastic Waves in a Fluid-saturated Porous Solid - II. Higher Frequency Range . J. Acoust. Soc. Am. 28, 179 - 191.
- Biot M. A., 1962. Mechanics Of Deformation And Acoustic Propagation In Porous Media .J.Appl.Phys. ,33(4), 1482 – 1498 .
- Bowen R. M., 1980. Incompressible Porous Media Models By Use Of The Theory Of Mixtures . Int. J. Engg. Sci. 18, 1129 – 1148.
- de Boer R., 2000. Theory Of Porous Media . Springer –Verleg New York .
- de Boer R. and Ehlers W., 1988. A Historical Review Of The Formulation Of Porous Media Theories . Acta Mechanica 74, 1-8.
- de Boer R. and Ehlers W., 1990a. The Development of the Concept of Effective Stress. Acta Mechanica 83 , 77 - 92.
- de Boer R. and Ehlers W., 1990b. Uplift, Friction and Capillarity – Three Fundamental Effects for Liquid-saturated Porous Solids. Int. J. Solid Structures. 26, 43 - 57.
- de Boer R., Ehlers W. and Liu Z., 1993. One Dimensional Transient Wave Propagation In Fluid Saturated Incompressible Porous Media [J]. Arch. App. Mech. 63, 59 – 72.
- Fillunger P., 1913. Der Auftrieb in Talsperren. Osterr. Wochenschrift fur den offentl. Baudienst. I. Teil 532 - 552, II. Teil 552 - 556, III. Teil 567 – 570.
- Kumar R. and Ailawalia P., 2005. Elastodynamics of inclined loads in micro polar cubic crystal, *Mechanics and Mechanical Engineering*, Vol. 9, No. 2, pp. 57-75.

- Kumar R. and Hundal B.S., 2002. A study of spherical and cylindrical wave propagation in a non-homogenous fluid saturated incompressible porous medium by the method of characteristics. *Currents Trends in Industrial and Applied Mathematics*, Edited By P.Manchanda Et.Al. Ananya Publisher , New Delhi , 181 -194 .
- Kumar R., and Hundal B.S., 2003a. Wave propagation in a fluid saturated incompressible porous medium. *Indian J. Pure and Applied Math.* Vol. 4, pp. 651-65.
- Kumar R.and Hundal B.S., 2003b. One dimensional wave propagation in a non homogenous fluid saturated incompressible porous medium. *Bull. Allahabad Math Soc .* Vol. 18, pp. 1-13.
- Kumar R. and Hundal B.S., 2004. Effect of non homogeneity on one-dimensional wave propagation in a fluid saturated incompressible porous medium. *Bull .Cal. Math Soc.*, Vol. 96, No. 3, pp. 179-188.
- Kumar R.and Hundal B.S., 2005. Symmetric wave propagation in a fluid saturated incompressible porous medium. *J. Sound and Vibration*, 288, pp. 361-373.
- Von Terzaghi K., 1923. Die Berechnung der Durchlässigkeit des Tones aus dem Verlauf der hydromechanischen Spannungserscheinungen. *Sitzungsber Akad. Wiss. (Wien), Math. – Naturwiss. K., Abt. Ila* 132 , pp. 125-138.
- Von Terzaghi K., 1925. *Erdbaumechanik auf Bodenphysikalischer Grundlage*, p. 399. Leipzig – wien : Franz Deuticke .

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