MultiCraft

International Journal of Engineering, Science and Technology
Vol. 3, No. 3, 2011, pp. 56-63

INTERNATIONAL JOURNAL OF ENGINEERING, SCIENCE AND TECHNOLOGY
www.ijest-ng.com

An approach for optimal PMU placement using binary particle swarm optimization with conventional measurements

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Abstract

This paper presents an approach to determine the optimal PMU locations in order to render the complete observability to given power system using BPSO. Quadratic programming approach is used in BPSO, and results so obtained are compared with GAMS-MIP solver. A Method for Pseudo observability is also proposed for depth-one and depth-two with and without Zero injection measurements for IEEE-7, IEEE-14, and IEEE-30and IEEE-57 bus systems by BPSO technique.

Keywords: Binary Particle Swarm Optimization, Phasor Measurement Units, Depth-one-observability, Depth-two-observability, Mixed Integer Programming

1. Introduction

Advent of phasor measurement unit (PMU) has given an impetus growth to the modern Energy management systems (EMS) by invoking dynamism to it .Various problems which are encountered by the EMS can be prevented if efficient and accurate state estimators are used .Traditionally a state estimator uses real time analog and status telemetry in order to calculate the system voltages and powers. With involvement of PMU in the system, complex bus voltages and currents can be incorporated dynamically in the state estimation database. Also a single PMU can serve the purpose of many analog measuring devices, as it is capable of measuring voltage and current phasors of its adjacent buses. From view point of full observability of the given system the number of PMU required are very less as compared to other measuring devices.

As strategical placement of PMU is a major concern today. So considering the importance of PMU in present day power system, a suitable methodology to determine optimal location for PMU placement to make system completely observable is considered. Phadke *et.al* (1986) present a procedure by which new PMU locations can be systematically determined in order to render an observable system. In references (Bei *et* al, 2004; Abur, 2009) Integer linear programming is used to formulate the topological observability. Also considerable work is done to shown the effect of conventional measurements on observability. Concept of pseudo–observability and its importance is nicely summarized in (Phadke *et al*, 2009; Reynaldo *et al*, 2005). Concept of conventional measurements are also considered in (Gou, 2008; Gou, 2008). In Chakrabarti *et* al (2008), Binary search approach is applied for placement problem. An Integer quadratic approach to minimize the PMU locations, which ensures observability during normal as well as during loss of single line and PMU, is considered. In Chakrabarti *et* al (2008), Binary particle swarm optimization (BPSO) based quadratic approach deals simultaneously with optimal placement and redundancy issues, however it does not considered the various conventional measurements, which is considered in this paper. In this paper, complete observability of various systems are computed by BPSO and its results are in complete agreement with the results obtained through MIP formulation in GAMS (McCarl *et al*, 2008). Also the concept of incomplete observability is formulated using BPSO considering conventional measurements along with maximizing redundancy. The formulation proposed has been tested on IEEE-7, IEEE-14, and IEEE-30 and IEEE-57bus systems. This paper is arranged in the following manner .Section 2 of the paper discusses details of BPSO formulation. Section 3 deals with MIP formulation used in GAMS .Section 4 deals with various case studies and results obtained and Section 5 concludes the paper.

2. Binary particle swarm optimization

The basic principles of particle swarm optimization (PSO) (de.Valle *et al*, 2008) are taken from the collective movement of a flock of bird, a school of fish, or a swarm of bees. A number of agents or particles are employed in finding the optimal solution for the problem under consideration. The movement of the particles towards finding the optimal solution is guided by both individual and social knowledge of the particles. As shown below, the position of a particle at any instant is determined by its velocity at that instant and the position at the previous instant.

$$x_i(t) = x_i(t-1) + v_i(t) \tag{1}$$

Where $x_i(t)$ and $x_i(t-1)$ are the position vectors of the *i*th particle at the instant *t* and *t*-1 respectively, and $v_i(t)$ is the velocity vector of the particle.

The velocity vector is updated by using the experience of the individual particles, as well as the knowledge of the performance of the other particles in its neighborhood. The velocity update rule for a basic PSO is,

$$v_i(t) = v_i(t-1) + \varphi_1 . r_1(pbest_i - x_i(t-1)) + \varphi_2 . r_2(gbest - x_i(t-1)) \tag{2}$$

Where φ_1 and φ_2 are adjustable parameters called individual and social acceleration constant respectively; $r_1$and r_2 are random numbers in the range[0,1]; $pbest_i$ is the best position vector found by *i*th particle; *gbest* is the best among the position vectors found by all the particles. The vectors *pbest* and *gbest* are evaluated by using a suitably defined fitness function φ_1 and φ_2 are usually defined such that $\varphi_1 + \varphi_2 = 4$ with $\varphi_1 = \varphi_2 = 2$. The maximum and minimum values of the components of velocity are limited by the following constraints to avoid large oscillations around the solution

$$v_{ij} = \begin{cases} -v_{max}, & v_{ij} < -v_{max} \\ v_{max}, & v_{ij}x \geq v_{min} \end{cases} \tag{3}$$

Where v_{max} is the maximum value of the velocity vector and v_{min} is the minimum value of the velocity vector.

However in case of BPSO, each element of the position vector can take only binary values, i.e., 1 or 0. At each stage of iteration, the elements of the position vector x_i are updated according to following rule:

$$x_{ij}(t) = \begin{cases} 1 \text{ if } \rho_{ij} < s(v_{ij}) \\ 0, \text{ otherwise} \end{cases} \tag{4}$$

Where ρ_{ij} is a random number in the range [0,1], $s(v_{ij})$ is a sigmoidal function defined as,

$$s(v_{ij}) = \frac{1}{1 + exp(-v_{ij})} \tag{5}$$

2.1. PMU Placement-- BPSO

The objective of the PMU placement problem in this paper is to minimize the number of PMUs that can make the system observable, and to maximize the measurement redundancy in the system. The fitness function therefore should evaluate following factors, for the position vector of each particle,

- Whether the system is observable or unobservable,
- In both cases, what is the number of PMUs employed,
- And The measurement redundancy.

For PMU placement problem, the position array X of the BPSO algorithm gives the information of PMU installation location. This is a binary bit array with the length of number of buses in the target power system.value1 of one bit means an installation of PMU at corresponding bus, while value 0 means no PMU installation at corresponding bus. The fitness function J (**x**) for using BPSO is given as

$$J(x) = \begin{cases} K, \text{ if system is unobservable} \\ w_1 J_1 + w_2 J_2, \text{ if system is observable} \end{cases} \tag{6}$$

where K is a large number assigned to the fitness function if the position vector representing the PMU placement solution is not able to make the system observable; $w_1$and w_2 are two weights with values such that w_1J_1 and w_2J_2 are comparable in magnitude. The radial buses are excluded from the list of potential locations and therefore, a PMU is pre-assigned to each bus connected to a radial bus. Pre-assigning PMUs to certain buses in this manner reduces the total number of possible combinations of PMU locations, thereby reducing the computational burden.

Various steps followed in BPSO are as follows:

- Initialize the swarm i.e. set initial value for particle position and velocities
- Calculate the value of objective function according to equation (6).and calculate *pbest* and *gbest*.
- Now calculate new values of velocity and positions according to (2) and (4)
- Check for full observability condition, and if not meet replace initial position and velocity vectors with new one and repeat the process from step2.

Various BPSO parameters used in the formulation are given below:

- Number of particles are taken equal to 5*total number of buses in the given system
- Maximum number of iterations is taken equal to 100*number of total buses in the system

2.2. Placement without conventional measurements—BPSO

For complete observability case J_1 and J_2 which are the parts of the fitness function representing the total number of PMUs and the measurement redundancy respectively, and are defined as follows:

$$J_1 = x^T x \tag{7}$$

$$J_2 = (N\text{-}Ax)^T(N\text{-}Ax) \tag{8}$$

The elements of binary vector x are defined as follows:

$$x_i = \begin{cases} 1, \text{ if a PMU is placed at bus } i \\ 0, \text{ otherwise} \end{cases} \tag{9}$$

The elements of binary connectivity vector A are defined as follows:

$$A(i,j) = \begin{cases} 1, & \text{if } i = j \\ 1, \text{ if bus } i \text{ and } j \text{ are connected} & \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

If number of buses in the given system is M , then A is an MXM matrix and N is an $MX1$dimension matrix. Also the vector N can be chosen according to the desired level of measurement redundancy.

The entries of the product Ax in (10) therefore represents the number of times a bus is observed by the PMU placement set defined by x.

So the fitness function formulated for the given problem of placement , not only gives optimal locations for PMUs but also considers the required measurement redundancy i.e. gives system work while loss of single line or PMU.

2.3. Placement considering conventional measurements—BPSO

Unobservability and Zero-injection measurements can also be introduced in the above formulation by making slight changes in the fitness function J_2 , matrix A is replaced by *GA* and *N* is replaced by N_G in (8), where matrix G is the transformation matrix which transforms into matrix the requirements of the cases of full observability, incomplete observability ,or zero injections, N_G indicates the redundancy requirements.

$$J_2 = (N_G - Gx)^T (N_G - Gx) \tag{11}$$

Both G and N_G take different values according to different cases and are well described in (Bei.Gou, 2008).

Case-1 When zero injections are not considered, the transformation matrix *G* and N_G will be:

$$G = \begin{cases} \text{B * A, for depth one} \\ \text{B' * A, for depth two} \end{cases} \tag{12}$$

$$N_G = \begin{cases} \text{b * A, for depth one} \\ \text{b' * A, for depth two} \end{cases} \tag{13}$$

Where *B* is branch to node connectivity matrix and *B*'is matrix, each row of which corresponds to three connecting buses and contains all possible combinations of three connecting buses. Vector *b* will be equal to number of branches present in the system and vector b' is of size equal to total number of possible combinations of three connecting buses.
Case-II When zero injections are considered

$$G = \begin{cases} \text{P1B * A, for depth one} \\ \text{P2B' * A, for depth two} \end{cases} \tag{14}$$

$$N_G = \begin{cases} \text{P1b * A, for depth one} \\ \text{P2b' * A, for depth two} \end{cases} \tag{15}$$

Where P1 is the matrix that keeps the branches which are not associated with zero injection measurements and removes the branches that are associated with zero injection measurements. In similar manner P2 is the matrix that keeps the combinations that are not associated with zero injection measurements and removes branches that are associated with zero injections .The above formulation for PMU placement is described by taking an example, which is taken from Gou (2008).

2.4. Example I

Figure-1 depicts the IEEE-7 bus system consisting of seven nodes (shown by bold letters) and eight branches (shown by circles); if we assume that zero injection measurement is at bus 2.

Then for this system bus connectivity matrix is given as

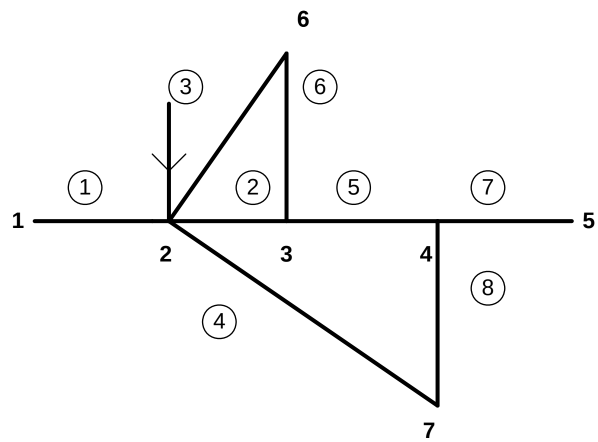

Figure 1. Example for incomplete observability

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

For Depth-1-Observability case
In this case branch- to- node incident matrix *B* is given as

$$B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

Also, since branches 1, 2, 3 and 4 are associated to the zero injection measurement, matrix P1 is

$$P1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

Substituting these values of B and P1 we get our fitness function as

$$J_2 = (P1b\text{-}P1Bx)^T(P1b\text{-}P1Bx) \tag{19}$$

Similarly for Depth-of –Two observability case, the total number of all possible three connecting buses is 15.

$$B' = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \tag{20}$$

Since a zero injection measurement is at bus 2, the matrix P2 is defined as

$$P2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{21}$$

Now fitness function becomes

$$J_2 = (P2b' - P2B'x)^T (P2b' - P2B'x) \tag{22}$$

3. GAMS-MIP solver based formulation

GAMS is a high-level modelling system for mathematical programming problems and Modelling linear, nonlinear and mixed integer optimization problems. Provide an algebraically based high-level language for the compact representation of large and complex models .It allow changes to be made in model specifications simply and safely, unambiguous statements of algebraic relationships. It provides an environment where model development is facilitated by subscript based expandability allowing the modeler to begin with a small data set, then after verifying correctness expand to a much broader context. Be inherently self documenting allowing use of longer variable, equation and index names as well as comments, data definitions etc. GAMS is designed so that model structure, assumptions, and any calculation procedures used in the report writing are documented as a by product of the modeling exercise in a self-contained file. It is also an open system facilitating interface to the newest and best solvers while being solver independent allowing different solvers to be used on any given problem. Mixed integer programming is used for the problem of PMU placement. With the help of connectivity matrix A as given in (10), constraints can be formed, which are directly solved by MIP solver. For IEEE-7 bus system (given in previous section), constraints can be formulated as

$$f(x) = \begin{cases} f_1 = x_1 + x_2 \geq 1 \\ f_2 = x_1 + x_2 + x_3 + x_6 + x_7 \geq 1 \\ f_3 = x_2 + x_3 + x_4 + x_6 \geq 1 \\ f_4 = x_3 + x_4 + x_5 + x_7 \geq 1 \\ f_5 = x_4 + x_5 \geq 1 \\ f_6 = x_2 + x_3 + x_6 \geq 1 \\ f_7 = x_2 + x_4 + x_7 \geq 1 \end{cases} \tag{23}$$

The first constraint f_1 implies that at least one PMU must be placed at either one of buses 1 or 2 (or both) in order to make bus 1 observable. Similarly, the second constraint f_2 indicates that at least one PMU should be installed at any one of the buses 1, 2, 3, 6, or 7 in order to make bus 2 observable.

4. Simulation results

The proposed formulation is tested on various IEEE test systems .and results so obtained are shown in the Table I for complete observability & measurement redundancy is level to 2 (Chakrabarti *et al*., 2008) . The number of radial buses for the given test systems are 1,3&1for IEEE 14,30,57 Bus systems. Also number of constraints becomes too large in GAMS MIP-solver for depth-one and depth-two observability cases, so results obtained through BPSO is only presented in the paper.

Table 1.PMU locations for complete observability using BPSO-Maximizing redundancy

Test Systems	PMU locations
IEEE7	2,4
IEEE14	2,6,7,9
IEEE30	1,2,6,9,10,12,15,19,25,27
IEEE57	1,6,9,15,18,26,29,30,32,35,36,38,42,46,50,54,57

A comparative study is also done by comparing the results so obtained by BPSO and GAMS for complete observability of the various test systems. In GAMS, MIP solver is used and it is found that, though the locations obtained are not unique but the optimized value of objective function and numbers of PMUs obtained are same. As depicted by Table 2. and Table 3.

Table 2. Number of PMUs for complete observability by BPSO and GAMS

Test Systems	Number of PMUs –BPSO	Number of PMUs –GAMS
IEEE7	2	2
IEEE 14	4	4
IEEE30	10	10
IEEE57	17	17

Table 3. PMU locations for complete observability using GAMS-MIP solver

Test Systems	GAMS-MIP solver
IEEE-7	2,4
IEEE-14	2,7,10,13
IEEE-30	1,2,6,9,10,12,15,18,26,27
IEEE-57	1,6,9,15,19,22,25,27,28,32,36,41,45,47,50,53,57.

Results are obtained for incomplete observability without zero injections and with zero injections. Table IV shows results without zero injections, Table V shows results with zero injections with number of zero injections are taken as 1, 7 and 17 in IEEE-14, 30, and 57 bus systems respectively. It is found that the results so obtained are in complete agreement with the results obtained in (Bei.Gou, 2008; Bei.Gou, 2008).

Table 4. Number of PMU for observability cases without zero injections

Test Systems	Complete Observability	Depth-one Observability	Depth-two observability
IEEE14	4	2	2
IEEE30	10	4	3
IEEE57	17	11	8

Table 5. Number of PMU for various observability cases with zero injections

Test Systems	Complete Observability	Depth-one Observability	Depth-two observability
IEEE14	3	2	2
IEEE30	7	4	3
IEEE57	11	9	8

5. Conclusion

This paper presents an approach of finding the optimal number of PMUs by BPSO method. This method computes the optimal location considering conventional measurements and maximizing redundancy simultaneously. Both complete and incomplete observability cases are discussed and number of optimal locations for PMU placement is found. Results for complete observability are crosschecked by GAMS-IMP solver however in case of incomplete observability number of constraints become too large for GAMS-IMP solver, therefore for this case results are obtained only by BPSO method. Results obtained for IEEE-7, IEEE-14, IEEE-30, IEEE-57 bus systems shows that proposed method is effective in calculating optimal locations for PMU placement problem.

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Received February 2011
Accepted March 2011
Final acceptance in revised form April 2011