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Invited paper dedicated to late Prof. M. Tajuddin

Phase velocity and attenuation of plane waves in dissipative elastic media: Solving complex transcendental equation using functional iteration method

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Abstract

An iteration method to find the roots of a complex transcendental equation is under scanner. This method identified as functional iteration method is being used mainly in wave propagation problems to calculate the phase velocity and the attenuation of plane harmonic waves in dissipative elastic plates. Few mathematical concerns are explained in this study, which put a question mark on the definition as well as application of this functional iteration method. A condition specified to ensure the convergence of iterative sequence to a valid solution is found to be mathematically invalid. The criterion to conclude an iteration process based on the change in two successive approximations may not yield a valid solution. Another serious concern is that the existence of root is not ensured before starting the iteration process to find it. It must be a necessary requirement as the secular equation derived to study the wave propagation is always a transcendental equation, which may not have a solution at all. Then the absence of solution implies that the mathematical model used does not represent the propagation of defined wave in the medium considered.

Keywords: dissipative elastic medium, wave propagation, complex transcendental equation, iterative method

1. Introduction

Solving a complex transcendental equation $f(z) = 0$ for its one or all roots has always been a problem. For an analytic function f(z), Newton-Raphson method may be used but with a good initial guess. Else, the complex equation is resolved into a set of two real equations in two real variables to be satisfied simultaneously. Such a set of non-algebraic equations may have no (real) solution or may have many solutions. So, to find the solution of a multi-dimensional system is much more difficult than finding roots in one-dimensional case. In fact, there is no general method for solving a system of more than one nonlinear equations. Following are the lines reproduced from the page 340 of a book by Press et al. (1992).

The principal difference between one and many dimensions is that, in one dimension, it is possible to bracket or "trap" a root between bracketing values and then hunt it down like a rabbit. In multidimensions, you can never be sure that the root is there at all until you have found it. This implies that one cannot bracket a root of a complex equation in the complex plane as can be done with a root on real line.

In the studies of vibration analysis in a dissipative elastic medium, the free vibrations are governed by a complex transcendental equation obtained as secular equation. Such equations have been solved by Sharma (2001), Sharma and Sharma (2002) and Sharma et al. (2008f) for complex roots but nothing is mentioned about the mathematical technique used. One may also find some recent papers (Kumar and Pratap, 2008, 2009a), where complex transcendental equations are solved for complex roots without saying a word about the technique used for the same.

There may be hundreds of papers in various journals from different authors using an iteration technique to solve complex transcendental equations while studying the propagation of elastic waves. The present author could go through few of them which are published in international journals devoted to mechanics of elastic materials. These are referred as Kumar and Kansal (2008a,b, 2009), Kumar and Pratap (2006, 2007a,b, 2008, 2009a,b), Pathania et al. (2008), Sharma (2005), Sharma and Kaur (2010), Sharma and Kumar (2009), Sharma and Othman (2007), Sharma and Pal (2004a,b), Sharma and Pathania (2003, 2004, 2005a,b,c), Sharma and Sharma (2009), Sharma and Thakur (2006, 2007), Sharma and Walia (2006, 2007a,b), Sharma et al.(2003a,b, 2004a,b,c, 2005, 2008a,b,c,d,e), Walia et al. (2009). In these papers, the iteration technique is used mainly to study the problems on propagation of elastic waves in dissipative media. In some of the papers, the method has been explained as a part of numerical results while many have used the references of earlier publications. In some recent papers the term 'functional iteration method' has been used to refer to this method.

With the publication of relevant papers in many reputed journals, this iteration method is getting an approval as well as promotion. So an immediate concern of the present discussion is to caution the researchers studying the vibration / dispersion of decaying waves in dissipative elastic media over the use of this erroneous numerical technique. Further, in recent years, an unprecedented increase is noticed in the frequency and the volume of the articles in which complex transcendental equations are said to be solved through this iteration method. Hence, the present study also intends to caution the research journals in the field of solid mechanics so as to restrict the further promotion of this erroneous technique.

2. Functional Iteration Method

An adventure seems to have started with the study of generalized thermoelastic Lamb waves in a plate submerged in liquid, by Sharma and Pathania (2003). However, in this paper, nothing was said about the technique or procedure used to solve the complex transcendental equation obtained as dispersion equation. In general, in the papers mentioned in the previous section, the dispersion equation is claimed to be solved for one or more of its roots, which define the complex velocities for different modes of wave propagation in the dissipative medium considered. In most of these studies, c denotes this complex velocity, which is resolved as $c^{-1} = V^{-1} + i\omega^{-1}Q$. ω, V and Q denote angular frequency, propagation velocity and attenuation coefficient, respectively. The secular or frequency equation $f(c) = 0$ represents the dispersion in velocity and attenuation of the waves propagating in the medium considered. An iterative method is claimed to have used in solving the secular equation defined with a complex transcendental function f. Sharma and Pal (2004) may be the first paper that explains the definition of this iteration method. However, the name for this iteration method, i.e. 'functional iteration method', may have started with the paper of Sharma and Walia (2006). The algorithm of this iteration method may be found a little different in some relevant papers. For example, in Sharma and Walia (2007), it is explained for real propagation velocity V and the complex secular equation involving trigonometric functions of unknowns is identified as an algebraic equation. However, it is not mentioned that how the iterative procedure on V can iterate the other independent variable Q.

Keeping in mind a better explained algorithm, an earlier paper (Sharma and Pal, 2004) referred to as paper-A hereafter) is chosen to discuss the various aspects of the definition and application of this iteration method. For an exact definition of the iteration method, a relevant paragraph on page 597 of paper-A is reproduced below.

The iteration method to solve a transcendental equation $f(c) = 0$*, requires to put this equation into the form* $c = g(c)$*, so that the sequence* $\{c_n\}$ *of iteration for desired root can easily be generated as follows: If* c_0 *be the initial approximation to the root, then we have* $c_1 = g(c_0), c_2 = g(c_1), c_3 = g(c_2)$ *and so on. In general,* $c_{n+1} = g(c_n), n = 0,1,2,3,\ldots$ *If* $|g'(c)| << 1$*, for all* $c \in I$*, then the sequence* $\{c_n\}$ *of approximation to the root will converge to the actual value* $c = \zeta$ *of the root, provided* $c_0 \in I$*,* I *being the interval in which root is expected.*

While using this iteration technique, a root is decided by the convergence criterion based on the difference of two consecutive approximations of the iterated root (paper-A, p. 597). Hypothesis of any theorem that guarantees the convergence of iterated sequence $\{c_n\}$ is not taken into account. This may be the reason that no instance of convergence failure is reported in any of the papers using this technique.

3. Complex Transcendental Equation

A valid procedure for finding a complex root of a complex transcendental equation should remain valid for finding a real root of any real equation. This implies that the functional iteration method, used to solve the complex transcendental equations (e.g., secular equation (22) or (24) in Sharma and Walia (2007)), should be able to solve any real equation. Hence, for simplicity, the examples considered to explain the errors in this numerical technique are restricted to real line.

3.1 Existence of roots

Any general transcendental equation may have no root or may have infinitely many roots. For example, the equation $\sin(x) + \cos(x) - a = 0$, for real x, has infinite roots for $-\sqrt{2} \le a \le \sqrt{2}$ but has no root for any other value of a. Another example may be an equation $exp(z) = 0$, which has no root in the complex plane. This implies that before starting a search to find a root of a transcendental equation, the existence of a root must be checked. The same requirement is observed in the studies on the propagation of elastic waves. In the articles on surface waves in Achenbach (1973), one may find that the existence of a valid root of a secular equation has been proved for Rayleigh waves (p. 189), Love waves (p. 220) and Stoneley waves (p. 195). In dissipative media, the viscoelastic Rayleigh wave modes exist only for certain combinations of complex elastic constants and for a given range of frequencies (Carcione, 2007; p. 117). That means, for the propagation of surface waves, the existence of (a valid) root of secular equation is not something to be expected or assumed but is required to be proved. The secular (dispersion) equations in dissipative elastic media are complex transcendental equations, which need to be solved for appropriate roots. So before deciding any method to solve any of these equations, it is required to ensure the existence of valid or actual roots.

3.2 Initial guess for a root

The sentence '*I being the interval in which root is expected*' in definition may not be correct. For c being a complex number, the I containing c must be a sub-region in the complex plane. It wonders if such a region can be termed as an interval. Further, as discussed in section 3.1, existence of a root of a transcendental equation should not be expected but must be ensured. Else, it implies that the region I is already fixed and the method iterates an approximate initial value to converge to actual root in this region. It indicates to hunting a particular root among all (none to infinite) the roots of secular (transcendental) equation. Then such a special root will have a particular definition as well as an assured existence and uniqueness in the region I. Unfortunately, none of the papers using this functional iteration technique contains an explanation in this regard. Hence, the present author believes that this sentence should have been 'I is the region which contains the only actual root'. Then it demands that there must be some method or procedure to find or fix such a region that contains this particular root of the complex transcendental equation.

3.3 Equivalent form of secular equation

In the studies under scanner, the secular equation, say $f(c) = 0$, involves the periodic (trigonometric) functions. This implies that there will be infinite number of options for the fixed point equation $c = g(c)$. Then, more than one such function may be satisfying the condition $|g'(c)| << 1$, for all $c \in I$. The complex functions $g(c)$ should be analytic in I. Else, it may not be possible to check the condition $|g'(c)| << 1$ at all the points in I. The authors using this technique may have the reasons to choose a $g(c)$ amongst the all valid options. It appears none of them are ready to disclose the same. Moreover, the success of an iterative method $c_{n+1} = g(c_n)$ depends on the proper choice of the function g. But any form $c = g(c)$ may involve inverse trigonometric functions, which are not single-valued. In any case, the expressions of the function $g(c)$ and its derivative $g'(c)$ deserve a space in all the articles using this method. But, these expressions could not be found in any of the studies using this technique.

We consider to solve a real quadratic equation $c^2 - 3c + 2 = 0$ for its two roots $c = 1, 2$. A sequence $\{c_n\}$ from the iterative relation $c_{n+1} = 2/(3 - c_n)$ with any $c_0 \neq 2$ always converges to $c = 1$ and it always converges to $c = 2$ for other iterative relation $c_{n+1} = 3 - 2/c_n$ with any $c_0 \neq 1$. This implies that a desired root may be found only by a particular iterative function. I wonder if there can be some rules to correspond a desired root to an iterative relation. Else, with any arbitrarily chosen function $g(c)$, the proposed method may miss to find the desired root irrespective of how close the initial value c_0 is to the actual root. So, it may be quite likely that a single function $g(c)$ may not be able to find the roots for different values of wave number chosen. Such a requirement may also be felt for studying the propagation of different modes of the wave. Then, in each article using this technique, it becomes inevitable to mention all the functions $g(c)$ used in the equivalent forms of the secular equation..

3.4 Convergence of iterative process

In the proposed method, the root is hunted through the convergence of iterative process in approaching to the equality of two functions (say, $h(c) = g(c)$ and $h(c) = c$) near/at the actual root. So, it is quite likely that the actual root $c = \zeta$ is the point where two curves (corresponding to $g(c)$ and c) touch each other. Then, the common slope implies that $g'(c) = 1$ at that point. Now the continuity of $g'(c)$ implies that there exists a region around ζ where value of $g'(c)$ is around 1. A part of this region will certainly be lying in I if I contains the actual root. But this contradicts the requirement ($|g'(c)| << 1$ for all $c \in I$) for the convergence of sequence $\{c_n\}$. This implies that either the condition '$|g'(c)| << 1$ for all $c \in I'$ can be met or the actual root can be approached but not both simultaneously. So, a sequence following such a condition can never approach (or converge) to the actual root because as and when an iterated member, say c_N, of the sequence approaches the actual root, the value of $|g'(c_N)|$ will be approaching to 1. For example, in the iterative relation $x = e^x - 1$, the two curves $y = x$ and $y = e^x - 1$ touch each other at $x = 0$. For $|g'(x)| = e^x << 1$, the convergence condition $x << 0$ will never allow to approach the actual root at $x = 0$. Similar will be the situation with alternate iterative relation $x = log_e(1+x)$. This situation may also arise when the actual root (or common point of two curves) is a point of intersection of the two curves, e.g. solving $c - \sin c = 0$ as follows.

Let us check the working of functional iteration method on a transcendental equation. Choose $f(c) = c - \sin c$ with $c = 0$ as its actual root. Then an obvious relation $g(c) = \sin c$ with $g'(c) = \cos c$ satisfies the condition $|g'(c)| << 1$ when c is around $\pi/2$. Note that around the actual root (i.e., $c = 0$) the value 1 of $|g'(c)|$ violates the said condition for convergence. So either c_n can approach to 0 or the condition $|g'(c)| << 1$ can be followed. Surprisingly, in this case, the convergence can be obtained by ignoring the said condition but the speed of convergence is very irritating. It is calculated that for $c_0 = 1.57$, the iterative relation $c_{n+1} = \sin c_n$ yields $c_{1000} = 0.0546$ with $c_{1000} - c_{1001} = 2.7195e-5$ and $c_{5000} = 0.0245$ with $c_{5000} - c_{5001} = 2.4454e-6$. The present author wonders, if such a speed of convergence can ever be acceptable. Even an initial value closer to the actual root, say $c_0 = 0.01$, does not improve this slower convergence. In all the studies using functional iteration method, a root is decided by the convergence criterion based on the difference of two consecutive approximations of the iterated root. But, the above approximations of the solution of the equation $c - \sin c = 0$ show that such a criterion may not get a correct value when the speed of convergence is very slow. Then it becomes necessary to verify the approximated root through back substitution before concluding the iterative process. Clearly, the another relation $c_{n+1} = sin^{-1}(c_n)$ is not a valid option to find the actual root with $|c_n| >> 1$, which is required for the condition of convergence ($|g'(c)| << 1$).

The actual proposition describing the sufficient condition for the convergence of an iterative process in one-dimensional case, following Jain et al. (1985), is as follows:

If $g(x)$ is a continuous function in the interval $[a,b]$ that contains the root and $|g'(x)| < 1$ in this interval, then for any choice of $x_0 \in [a,b]$, the sequence $x_{k+1} = g(x_k), k = 0,1,2,3,...$ converges to the actual root of $x = g(x)$.

In the studies under scanner, the respective authors seem to have wrongly applied this proposition to find the root of a multi-dimensional (or, complex) system. It seems the founding authors, while using this technique for complex domain, have replaced the actual condition of convergence $|g'(c)| < 1$ (in one-dimensional case) by a more stringent condition $|g'(c)| << 1$ (for multi-dimensional case). It may, perhaps, have hoped that a condition, more stringent than in one-dimensional case, will ensure the convergence to the actual root in multi-dimensional problems. But, this harsh condition reduced the existence of, otherwise easily available, functions $g(c)$ to near absence. This being the reason that the proposed method became incapable of solving even the obvious simple (one-dimensional) problems discussed as examples in this study. Moreover, in various papers mentioned above, the expressions for the transcendental equations $f(c) = 0$, contains a denominator term, which may obstruct the continuity of the function $f(c)$. But it appears that the authors of these papers did not bother to define / ensure the continuity of $f(c)$ even in the said interval I.

4. Concluding Remarks

The observations in the previous sections raise serious doubts over the success of the proposed method in solving a complex transcendental equation. The functional iteration method as defined above fails in protecting its definition. With convergence condition modified from the one defined for real systems, the method leads to an obvious mathematical contradiction. The convergence algorithm used to conclude the iterative process is prone to failure even in easy situations dealing with real systems and known solutions. The lack of transparency in its use in various studies results in different and conflicting explanations. An iterative method must be specified with its rapidity of convergence, uniqueness of roots of the considered system in a chosen region and the stability of process with respect to the choice of initial guess of the root. For example, following theorem from complex analysis explains this for Newton-Raphson method.

Theorem 1. If a function $f(z)$ is analytic in $\overline{B}(a;R)$ and satisfies the inequalities

$i)\left|\frac{1}{f'(a)}\right| \le A,\quad ii)\left|\frac{f(a)}{f'(a)}\right| \le B \le \frac{R}{2},\quad iii)|f'(z)| \le C$ for $|z-a| < R,\quad iv) 2ABC = \mu \le 1$, then $f(z) = 0$ has a unique root ζ in the domain $\overline{B}(a;R)$ and Newton's process defined by initial guess $'a'$ converges to this root, i.e. $\zeta = lim_{n\to\infty} a_n$. The rapidity of convergence of the process is characterized by the estimate $|\zeta - a_n| \leq B(\frac{1}{2})^{n-1}\mu^{2^n-1}$.

Without above specifications, the functional iterative method can not take an edge over a standard method like Newton-Raphson method. '*Newton direction is a decent direction*' in Press et al. (1992) implies that the derivative of function appearing in the denominator of the iterative formula of Newton-Raphson method accelerates the convergence of the iterative process and it ensures that the incremental step directs the approximated solution towards the actual root. Such a booster as well as rudder may not be available in an arbitrary iterative formula. So, even in one-dimensional case, an iterative method may not always be more suitable or efficient than Newton-Raphson method. In multi-dimensional case such an expectation may lead to a bigger trouble.

In any of the papers using the functional iteration technique, one may not find even a single mention of analyticity or differentiation of complex transcendental functions involved. This implies that the authors of these papers have made no efforts to check the existence of roots of resulting secular equations in complex plane. However, following theorems (Conway, 1973; p. 287-301) from complex analysis could have helped in checking the existence of the roots of a complex equation $f(z) = 0$.

Theorem 2. If f is an entire function of finite order then f assumes each complex number with one possible exception.

Theorem 3. If f is an entire function of order λ and λ is not an integer then f has infinitely many zeros.

Theorem 4. (Great Picard theorem) If an analytic function f has an essential singularity at $z = a$ then (in each neighborhood of a) f assumes each complex number (with one possible exception) an infinite number of times.

Theorem 5. (Little Picard theorem) If f is an entire function that omits two values then f is constant.

The failure in finding the exact roots has been a general concern in any such study. But, in the use of functional iteration technique, the most serious concern is that the existence of a valid root of secular (dispersion) equation is not ensured but assumed. Hence, to study the expected diffusive waves in dissipative elastic medium, one must first ascertain the existence of a root of the resulting secular equation. Only then one may think to start a root-search through iterative or any other procedure. Then, to bracket a region I in complex plane for initial guess is important for the success of the procedure. A theorem from complex analysis may help to estimate the loci of the roots of a complex equation $f(z) = 0$ when function f is a meromorphic (defined and analytic in an open set except for poles). This theorem known as Rouche's theorem is defined as follows (Conway, 1973; p. 125).

Let two complex functions f and g are meromorphic in a neighborhood of in a neighborhood of $\overline{B}(a;R)$ with no zeros or poles on its circular boundary γ. Z_f, Z_g and P_f, P_g denote numbers of zeros and poles of the two functions inside the circle γ. If $|f(z)+g(z)|<|f(z)|+|g(z)|$ on γ then $Z_f - P_f = Z_g - P_g$. According to a simplified version of this theorem, if analytic functions $f(z)$ and $g(z)$ in a simply connected domain satisfy $|g(z)|<|f(z)|$ on its boundary then $f(z)$ and $f(z)+g(z)$ have same number of zeros in the domain. Then Argument Principle (Conway, 1973; p. 123) is always there, which implies that for a simple closed curve γ, $\frac{1}{2\pi i}\int_\gamma [f'(z)/f(z)]dz = Z_f - P_f$. Hence, for complex transcendental analytic function f (i.e. no poles), the integral $\frac{1}{2\pi i}\int_\gamma [f'(z)/f(z)]dz$ provides the number of zeros of f inside the contour γ.

The ultimate aim of the discussion presented is to draw the attention of the relevant research journals and expect them to be cautious while dealing with the papers involving this technique and the research groups frequently using this technique. This effort is also a call to the researchers in the field of mechanics to respect the mathematical sensitivity of any numerical procedure they use.

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Biographical notes

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