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On some three-dimensional problems of piezoelectricity

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Abstract

The problem of an elliptical crack embedded in an unbounded transversely isotropic piezoelectric medium and subjected to remote normal loading is considered first. The integral equation method developed by Roy and his coworkers has been applied suitably with proper modifications to solve the problem. The method has been further applied to solve the problem of a rigid elliptical punch indenting a transversely isotropic piezoelectric half space.

Keywords: Transversely isotropic piezoelectric media; elliptical crack/punch; normal loading; integral equation method.

1. Introduction

Mechanics of piezoelectric materials have become an emerging topic of research due to its wide application in piezoelectric ceramics and composites used in many important technologies. To improve the performance and to predict the reliable service life of piezoelectric components, it is necessary to obtain a detailed analysis of the fracture mechanical behavior of such materials with embedded cracks subjected to mechanical and electric loading. Three-dimensional analysis of cracks in piezoelectric media, when subjected to mechanical and electric loading, is essential for having a better understanding of the fracture behavior of such materials. In three-dimensional analysis, cracks may often be modeled as penny-shaped or elliptical cracks. It is essential to analyze the coupling effect of the loads when piezoelectric solids with such type of embedded cracks are subjected to mechanical as well as electric loading.

The problem of an elliptical crack embedded in an unbounded transversely isotropic piezoelectric medium and subjected to remote normal loading is considered first. The problem has been successfully reduced to a pair of coupled integral equations that are suitable for the application of the integral equation method of Roy and his coworkers (see Mukhopadhyay 1990, Roy and Chatterjee 1992, Roy and Saha 2000). The crack plane is considered to be parallel to the plane of isotropy of the medium and both the mechanical and electric loadings are considered to be arbitrary polynomial functions of the crack-plane coordinates. Solution to the mechanical displacement and electric potential are obtained for prescribed uniform loadings. Variation of both the non-dimensional intensity factors with variation in eccentric angle and aspect ratio is shown graphically. The above method has been further applied to solve the problem of a rigid flat-ended elliptical punch indenting a transversely isotropic piezoelectric half space surface with the plane of isotropy parallel to the surface. Solution to mechanical stress and electric displacement are obtained for prescribed constant normal displacement and constant electric potential interior to the elliptical region.

2. Formulation of the problem

Let the elliptic crack/punch occupy the region

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$$S : \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1, \quad z = 0. \tag{1}$$

The plane of the crack/punch coincides with the plane of isotropy of the medium. The crack/punch center is taken as the origin of the coordinate system and z-axis along the axis of symmetry. A pair of identical mechanical loads *p*(x,y) and a pair of identical electrical loads *q*(x,y) are applied to the upper and lower surface of the crack. On the other hand, in case of punch, normal indentation p' (x,y) and electric potential q' (x,y) are prescribed within the punch area.

The boundary conditions on the crack and punch plane are written separately as follows:

2.1 Boundary conditions on the crack plane z=0:

$$\sigma_{zx}(x, y, 0) = \sigma_{zy}(x, y, 0) = 0, \forall x, y, \tag{2}$$

$$\left.\begin{aligned} \sigma_{zz}(x, y, 0) &= -p(x, y) \\ D_z(x, y, 0) &= q(x, y) \end{aligned}\right\} \forall (x, y) \in S, \tag{3}$$

and,

$$\left.\begin{aligned} u_z(x, y, 0) &= 0 \\ \phi(x, y, 0) &= 0 \end{aligned}\right\} \forall (x, y) \notin S, \tag{4}$$

where all the notations have usual meaning.

2.2 Boundary conditions on the contact surface z=0:

$$\sigma_{zx}(x, y, 0) = \sigma_{zy}(x, y, 0) = 0, \forall x, y, \tag{5}$$

$$\left.\begin{aligned} u_z(x, y, 0) &= -p'(x, y) \\ \phi(x, y, 0) &= q'(x, y) \end{aligned}\right\} \forall (x, y) \in S, \tag{6}$$

and,

$$\left.\begin{aligned} \sigma_{zz}(x, y, 0) &= 0 \\ D_z(x, y, 0) &= 0 \end{aligned}\right\} \forall (x, y) \notin S, \tag{7}$$

and in both the above cases,

as $\sqrt{\left(x^2 + y^2 + z^2\right)} \rightarrow \infty$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = \sigma_{yz} = \sigma_{zx} \rightarrow 0,$$

and, $D_x = D_y = D_z \rightarrow 0.$

2.3 The Coupled Dual Integral Equations

Using pseudo-potential function representation for mechanical displacement components and electric potential, Wang and Huang (1995) obtained four quasi-harmonic equations as follows:

$$\frac{\partial^2 \chi_j}{\partial x^2} + \frac{\partial^2 \chi_j}{\partial y^2} + \lambda_j \frac{\partial^2 \chi_j}{\partial z^2} = 0, (j = 1, 2, 3, 4). \tag{8}$$

Here λ_4 is given by

$$\lambda_4 = \frac{2c_{44}}{c_{11} - c_{12}}, \ (c_{11} \neq c_{12}). \tag{9}$$

Also, λ_j (j = 1, 2, 3) are the three roots of the cubic equation

$$A\lambda^3 + B\lambda^2 + C\lambda + D = 0, \tag{10}$$

where,

$$A = e_{15}^2 + c_{44}\zeta_{11},$$

$$B = \left[2e_{15}^2 c_{13} - c_{44}e_{31}^2 + 2e_{15}e_{31}c_{13} - 2e_{15}c_{11}e_{33} + \zeta_{11}c_{13}^2 + 2c_{13}c_{44}\zeta_{11} - c_{33}c_{11}\zeta_{11} - c_{44}c_{11}\zeta_{33}\right]/c_{11},$$

$$C = \Big[(e_{15} + e_{31})^2 c_{33} - 2(c_{13} + c_{44})(e_{15} + e_{31})e_{33} + \zeta_{11}c_{44}c_{33} + c_{11}e_{33}^2 - \zeta_{33}(c_{13} + c_{44})^2 + \zeta_{33}\left(c_{44}^2 + c_{33}c_{11}\right) + 2e_{15}e_{33}c_{44}\Big]/c_{11},$$

$$D = -\left[c_{44}e_{33}^2 + \zeta_{33}c_{44}c_{33}\right]/c_{11}. \tag{11}$$

Here, c_{ik}, e_{ik}, ζ_{ik} (i,k = 1, 2, 3) are Voigt two-index notation for elastic stiffness constants, piezoelectric constants and dielectric constants.

Renaming λ_i by s_i (i = 1, 2, 3, 4), a suitable solution of equation (8) is given as (Rahaman, 2002) :

$$\chi_j(x,y,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(P,Q,R,T)(\xi,\eta)exp\left[-i(\xi x + \eta y) - m_j z\right]d\xi d\eta \tag{12}$$

where,

$$m_j = \sqrt{(\xi^2 + \eta^2)/s_j}, \ (j = 1,2,3,4). \tag{13}$$

Substituting (12) in the expression for stress-components of piezoelectric media we see that the boundary condition (2) or (5) is satisfied if

$$T(\xi, \eta) = 0, \tag{14}$$

and,

$$R(\xi,\eta) = -\frac{m_1[c_{44}(1+K_{11}) + e_{15}K_{21}]}{m_3[c_{44}(1+K_{13}) + e_{15}K_{23}]}P(\xi,\eta) - \frac{m_2[c_{44}(1+K_{12}) + e_{15}K_{22}]}{m_3[c_{44}(1+K_{13}) + e_{15}K_{23}]}Q(\xi,\eta) \tag{15}$$

where, for j = 1, 2, 3,

$$K_{1j} = \frac{\lambda_j\left[(\zeta_{33} - \lambda_j\zeta_{11})(c_{13} + c_{44}) + (e_{33} - \lambda_j e_{15})(e_{15} + e_{31})\right]}{\left[(\zeta_{33} - \lambda_j\zeta_{11})(c_{33} - \lambda_j c_{44}) + (e_{33} - \lambda_j e_{15})^2\right]},$$

$$K_{2j} = \frac{\lambda_j[(e_{33}-\lambda_j e_{15})(c_{13}+c_{44})-(c_{33}-\lambda_j c_{44})(e_{15}+e_{31})]}{[(\zeta_{33}-\lambda_j\zeta_{11})(c_{33}-\lambda_j c_{44})+(e_{33}-\lambda_j e_{15})^2]}. \tag{16}$$

Using (12), (15) and the remaining boundary conditions for crack/punch, one obtains a coupled integral equation each for the crack and punch that may be expressed in a combined form as follows :

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int\int_S (\xi^2+\eta^2)^{1/2} K(x',y') \times$$

$$exp\left[-i\{\xi(x-x')+\eta(y-y')\}\right]dx'dy'd\xi d\eta = 2\pi \iota P(x,y),\ \forall (x,y)\in S \tag{17}$$

and,

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int\int_S (\xi^2+\eta^2)^{1/2} T(x',y') \times$$

$$exp\left[-i\{\xi(x-x')+\eta(y-y')\}\right]dx'dy'd\xi d\eta = -2\pi LQ(x,y),\ \forall (x,y)\in S \tag{18}$$

where,

$$\iota = \begin{cases} 1, & \text{for a crack} \\ -1, & \text{for a punch} \end{cases}, \tag{19}$$

$$P(x,y) = \begin{cases} p(x,y), & \text{for a crack} \\ p'(x,y), & \text{for a punch} \end{cases}, \tag{20}$$

$$Q(x,y) = \begin{cases} q(x,y), & \text{for a crack} \\ q'(x,y), & \text{for a punch} \end{cases}, \tag{21}$$

$$K(x',y') = \begin{cases} [M_2 u_z(x',y') - M_1\phi(x',y')], & \text{for a crack} \\ [M_3 g_z(x',y') - M_4 d_z(x',y')], & \text{for a punch} \end{cases}, \tag{22}$$

$$T(x',y') = \begin{cases} [N_2 u_z(x',y') - N_1\phi(x',y')], & \text{for a crack} \\ [N_3 g_z(x',y') - N_4 d_z(x',y')], & \text{for a punch} \end{cases}, \tag{23}$$

and,

$$L = \begin{cases} -H, & \text{for a crack} \\ B, & \text{for a punch} \end{cases}. \tag{24}$$

Also, in the above, for the case of a crack, $u_z(x, y)$ is the unknown crack opening displacement normal to the crack face and $\phi(x,y)$ is the electric potential across the crack face, so that

$$u_z(x, y) = 0 = \phi(x, y),\ \forall (x, y) \notin S,$$

and,

$$\begin{pmatrix} M_1 & M_2 \\ N_1 & N_2 \end{pmatrix} = \begin{pmatrix} -B_{32}^a & B_{31}^a \\ -B_{32}^c & B_{31}^c \end{pmatrix} \times \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}^T, \tag{25}$$

where,

$$\begin{pmatrix} B_{31}^a & B_{32}^a \\ B_{31}^c & B_{32}^c \end{pmatrix} = \begin{pmatrix} B_3A_1\sqrt{s_1} - B_1A_3\sqrt{s_3} & B_3A_2\sqrt{s_2} - B_2A_3\sqrt{s_3} \\ B_3C_1\sqrt{s_1} - B_1C_3\sqrt{s_3} & B_3C_2\sqrt{s_2} - B_2C_3\sqrt{s_3} \end{pmatrix}, \tag{26}$$

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \begin{pmatrix} K_{11}B_3 - K_{13}B_1 & K_{12}B_3 - K_{13}B_2 \\ K_{21}B_3 - K_{23}B_1 & K_{22}B_3 - K_{23}B_2 \end{pmatrix}, \tag{27}$$

$$H = \begin{vmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{vmatrix}, \tag{28}$$

and, for $j = 1, 2, 3$,

$$A_j = \frac{c_{33}K_{1j} + e_{33}K_{2j}}{s_j} - c_{13},$$

$$C_j = \frac{e_{33}K_{1j} - \zeta_{33}K_{2j}}{s_j} - e_{31},$$

$$B_j = c_{44}(1 + K_{1j}) + e_{15}K_{2j}, \tag{29}$$

whereas, for the case of a punch, $g_z(x, y)$ is the unknown normal stress component developed on the punch region due to normal indentation and $d_z(x, y)$ is the unknown normal component of electric displacement of the punch area, so that

$g_z(x, y) = 0 = d_z(x, y),\ \forall\ (x, y) \notin S,$

and,

$$\begin{pmatrix} M_3 & M_4 \\ N_3 & N_4 \end{pmatrix} = \begin{pmatrix} -N_1 & -M_1 \\ -N_2 & -M_2 \end{pmatrix} \tag{30}$$

and,

$$B = \begin{vmatrix} B_{31}^a & B_{32}^a \\ B_{31}^c & B_{32}^c \end{vmatrix}. \tag{31}$$

2.4 Infinite systems of Fredholm integral equation of second kind

After carrying out the necessary steps following Roy and his coworkers (Mukhopadhyay, 1990; Roy and Chatterjee, 1992; Roy and Saha, 2000), one obtain the following infinite systems of Fredholm integral equation of second kind :

$\forall s = 0, 1, 2, \ldots, \infty$, $(n + s)$ even; and $\xi, r \in [0, 1]$,

$$\varepsilon_s \mathbb{I}_{s,s}^c \begin{pmatrix} \mathcal{K}_s(\xi) \\ \mathcal{T}_s(\xi) \end{pmatrix} + \varepsilon_s \sum_{\substack{n=0 \\ n \neq s}}^{\infty} \mathbb{I}_{n,s}^c \int_0^1 \mathrm{L}_{n,s}(\xi, t) \begin{pmatrix} \mathcal{K}_n(t) \\ \mathcal{T}_n(t) \end{pmatrix} dt = \begin{pmatrix} \mathcal{F}_s(\xi) \\ \mathcal{G}_s(\xi) \end{pmatrix} \tag{32}$$

and,

$\forall s = 1, 2, \ldots, \infty$, $(n + s)$ even; and $\xi, r \in [0, 1]$,

$$\varepsilon_s \mathbb{I}^s_{s,s} \begin{pmatrix} \bar{\mathcal{K}}_s(\xi) \\ \bar{\mathcal{T}}_s(\xi) \end{pmatrix} + \varepsilon_s \sum_{\substack{n=1 \\ n \neq s}}^{\infty} \mathbb{I}^s_{n,s} \int_0^1 \mathrm{L}_{n,s}(\xi,t) \begin{pmatrix} \bar{\mathcal{K}}_n(t) \\ \bar{\mathcal{T}}_n(t) \end{pmatrix} dt = \begin{pmatrix} \bar{\mathcal{F}}_s(\xi) \\ \bar{\mathcal{G}}_s(\xi) \end{pmatrix} \tag{33}$$

where,

$$\mathcal{K}_s(\xi) = \begin{cases} \left[M_2 \psi_s^C(\xi) - M_1 \theta_s^C(\xi)\right], & \text{for a crack} \\ \left[M_3 \psi_s^P(\xi) - M_4 \theta_s^P(\xi)\right], & \text{for a punch} \end{cases}, \tag{34}$$

$$\mathcal{T}_s(\xi) = \begin{cases} \left[N_2 \psi_s^C(\xi) - N_1 \theta_s^C(\xi)\right], & \text{for a crack} \\ \left[N_3 \psi_s^P(\xi) - N_4 \theta_s^P(\xi)\right], & \text{for a punch} \end{cases}, \tag{35}$$

and, $\bar{\mathcal{K}}_s(\xi)$ and $\bar{\mathcal{T}}_s(\xi)$ are similar expressions with $\psi_s^{C(P)}(\xi)$, $\theta_s^{C(P)}(\xi)$ replaced by $\bar{\psi}_s^{C(P)}(\xi)$, $\bar{\theta}_s^{C(P)}(\xi)$.

Also,

$$\mathrm{L}_{n,s}(\xi,t) = (\xi t)^{\frac{1}{2}} \int_0^\infty k J_{n+\frac{1}{2}}(kt) J_{s+\frac{1}{2}}(k\xi)\, dk, \tag{36}$$

$$\begin{pmatrix} \mathcal{F}_s(\xi) \\ \mathcal{G}_s(\xi) \end{pmatrix} = b^1 L_1 \xi^{-s} \mathcal{D} \int_0^\xi \frac{r^{s+1}}{(\xi^2 - r^2)^{1/2}} \begin{pmatrix} P_s(r) \\ -Q_s(r) \end{pmatrix} dr, \tag{37}$$

where,

$$L_1 = \begin{cases} -\pi H, & \text{for a crack} \\ 2B, & \text{for a punch} \end{cases}, \tag{38}$$

and,

$$\mathcal{D} = \begin{cases} 1, & \text{for a crack} \\ \frac{d}{d\xi}, & \text{for a punch} \end{cases}. \tag{39}$$

$\bar{\mathcal{F}}_s(\xi)$ and $\bar{\mathcal{G}}_s(\xi)$ are similar expressions with $P_s(r)$, $Q_s(r)$ replaced by $\bar{P}_s(r)$, $\bar{Q}_s(r)$.

3. Exact Solution for particular cases

In the following we present exact solution to the transformed Fourier coefficients of mechanical displacement and electric potential in case of crack and, normal stress and normal electric displacement coefficients in case of punch for prescribed constant mechanical and electrical loading in case of crack and prescribed constant normal indentation and electric potential in case of punch.

Let constant mechanical and electric loading be prescribed normally on the crack face, whereas constant normal indentation and electric potential be prescribed on the punch area. Then,

$$P(x,y) = P \text{ (constant)}, \text{ and } Q(x,y) = Q \text{ (constant)}, \tag{40}$$

where,

$$P = \begin{cases} p, & \text{for a crack} \\ p', & \text{for a punch} \end{cases} ; \quad Q = \begin{cases} q, & \text{for a crack} \\ q', & \text{for a punch} \end{cases} . \tag{41}$$

Therefore,

$$\begin{aligned} P_0(r) = P,\ P_s(r) = \bar{P}_s(r) = 0,\ \forall s \geq 1, \\ Q_0(r) = Q,\ Q_s(r) = \bar{Q}_s(r) = 0,\ \forall s \geq 1. \end{aligned} \tag{42}$$

Hence,

$$\mathcal{F}_0(\xi) = \begin{cases} -\pi bHp\xi, & \text{for crack} \\ \frac{2Bp'}{b}, & \text{for punch} \end{cases} ; \quad \mathcal{G}_0(\xi) = \begin{cases} \pi bHq\xi, & \text{for crack} \\ \frac{-2Bq'}{b}, & \text{for punch} \end{cases} ; \tag{43}$$

and,

$$\mathcal{F}_s(\xi) = \bar{\mathcal{F}}_s(\xi) = 0,\ \forall s \geq 1.$$

Therefore, the infinite systems of integral equation (32), now reduce to a single pair of coupled equations in each case obtained for $s = 0$. For $s = 0$, only first term of equation (32) contributes to the solution which is :

$$\varepsilon_0 \mathbb{I}^c_{0,0} \begin{pmatrix} \mathcal{K}_0(\xi) \\ \mathcal{T}_0(\xi) \end{pmatrix} = \begin{pmatrix} \mathcal{F}_0(\xi) \\ \mathcal{G}_0(\xi) \end{pmatrix} \quad \ldots (44)$$

i.e., for a crack,

$$\varepsilon_0 I^c_{0,0} \begin{pmatrix} [M_2\psi^C_0(\xi) - M_1\theta^C_0(\xi)] \\ [N_2\psi^C_0(\xi) - N_1\theta^C_0(\xi)] \end{pmatrix} = \pi Hb \begin{pmatrix} -p \\ q \end{pmatrix} \xi, \quad \ldots (45)$$

and, for a punch,

$$\varepsilon_0 J^c_{0,0} \begin{pmatrix} [M_3\psi^P_0(\xi) - M_4\theta^P_0(\xi)] \\ [N_3\psi^P_0(\xi) - N_4\theta^P_0(\xi)] \end{pmatrix} = \frac{2B}{b} \begin{pmatrix} p' \\ -q' \end{pmatrix}. \quad \ldots (46)$$

Solving (45) for crack, we get

$$\psi^C_0(\xi) = \frac{\pi Hb\xi}{I^c_{0,0}} \frac{qM_1 + pN_1}{M_1N_2 - M_2N_1}; \quad \theta^C_0(\xi) = \frac{\pi Hb\xi}{I^c_{0,0}} \frac{qM_2 + pN_2}{M_1N_2 - M_2N_1} \quad \ldots (47)$$

as solution to crack face mechanical displacement and electric potential function.

Similarly solving (46) for punch, we get

$$\psi_0^P(\xi) = \frac{2B}{bJ^c_{0,0}}\frac{q'M_4+p'N_4}{M_3N_4-M_4N_3}; \quad \theta_0^P(\xi) = \frac{2B}{bJ^c_{0,0}}\frac{q'M_3+p'N_3}{M_3N_4-M_4N_3} \qquad \ldots (48)$$

as solution to mechanical stress and electric displacement function of the punch area.

Following the definition of stress-intensity factor in Roy and Chatterjee (1992) and utilizing its expression obtained there, the non-dimensional stress intensity factor $F_I^M(\varphi)$ (= $K_I^M(\varphi)/p\sqrt{b}$) and electric displacement intensity factor $F_I^E(\varphi)$ (= $K_I^E(\varphi)/q\sqrt{b}$) at the representative crack-edge point with eccentric angle φ are obtained as

$$F_I^M(\varphi) = F_I^E(\varphi) = [1-k_0^2\cos^2(\varphi)]^{1/4}/E(k_0), \qquad \ldots (49)$$

where, $k_0^2 = 1 - b^2/a^2$ and $E(k_0)$ is the complete elliptic integral of the second kind. Also, $K_I^M(\varphi)$ and $K_I^E(\varphi)$ are the mechanical stress and electric displacement intensity factors respectively. From expressions (47) and (49) it is obvious that although the mechanical displacement and electric potential components are dependent on the material parameters, the intensity factors are independent of these parameters. Another interesting feature is that, although the mechanical displacement and electric potential components depend on the coupling effect of the two loadings, both the non-dimensional intensity factors are not only independent of such coupling effect but are also equal. Figure 1 below shows the variation of the dimensionless intensity factors along the crack front for different ratios of b/a.

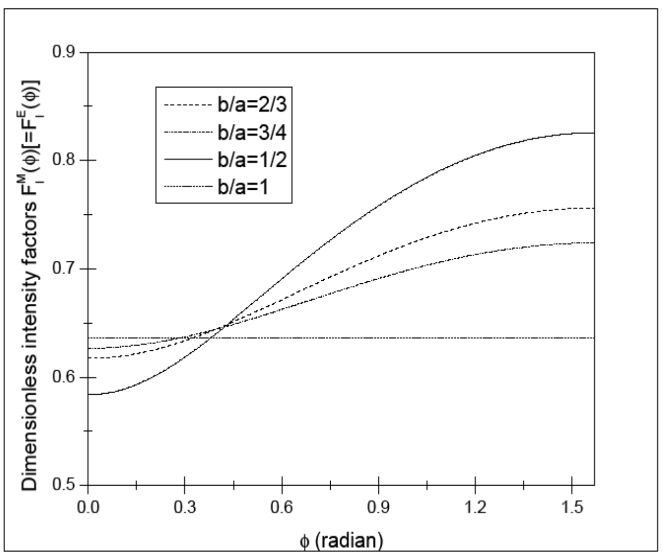

Figure 1: Dimensionless field intensity factors $F_I^M(\varphi)$ and $F_I^E(\varphi)$ along elliptical crack front of different aspect ratios.

From the figure it may be observed that the mechanical stress intensity and electric displacement intensity factors are both minimum near the edge of the major axis ($\phi = 0^\circ$) but gradually reaches to a maximum near the edge of the minor axis ($\phi = 90^\circ$) for an elliptic crack, although they are uniform for a penny-shaped crack (b/a=1). Another interesting observation is that, with the decrease in aspect ratio (b/a) of the crack, the minimum value decreases whereas the maximum value increases. Thus, we reach to two conclusions from the above figure:

(1) If fracture initiates, it will initiate from the crack-edge at the end of the minor axis;
(2) Narrower the elliptic crack i.e. smaller the aspect ratio (b/a) of the crack, higher is its tendency to fracture.

4. Conclusion

The problem of an elliptical crack embedded in an infinite transversely isotropic piezoelectric medium with its plane of isotropy parallel to the plane of the crack and the anti-crack problem of an elliptic punch indenting a transversely isotropic piezoelectric half-space surface with the plane of isotropy again parallel to the surface have been solved simultaneously. The potential function representation of the field quantities of a transversely isotropic piezoelectric medium given by Wang and Huang (1995) has been used to reduce each of the two problems considered, to pair of coupled integral equations. The coupled integral equations for the two problems could be expressed in a unified form given by equations (32) and (33). Cartesian coordinates are used in the reduction process, which has the advantage of dealing with more complicated problems that can take into consideration additional boundaries in the form of free-surface or neighboring crack/punch. Equations (32) and (33) are then solved by the integral equation method of Roy and his coworkers (Mukhopadhyay 1990, Roy and Chatterjee 1992, Roy and Saha 2000). Thus, the method developed for solving problems of pure elasticity could be extended to solve problems of piezoelectricity. In the case of crack problem, for particular case of prescribed constant normal mechanical and electric loading, exact solution for the transformed components of crack face mechanical displacement and electric potential has been given by (47). Utilizing these results, expression for dimensionless mechanical stress and electric displacement intensity factor has been given by (49). Variation of both the non-dimensional intensity factors with variation in eccentric angle and aspect ratio has been represented graphically in figure 1 and interesting observations have been made. In the case of punch problem, for particular case of prescribed constant normal indentation and constant electric potential, exact solution for the transformed components of mechanical stress and electric displacement of the punch region has been given by (48). The main point to be noted is that, a single method can deal with the crack and punch problems considered here. The solutions are obtained directly in Cartesian coordinates and not as limit of other results like inclusion/cavity, as followed by other researchers. The method may be applied to other complicated fracture mechanical problems of piezoelectric solids with elliptical crack/punch interacting with (a) incident time-harmonic waves, (b) neighboring cracks/punches or (c) with additional boundaries. Applications of the method to different fracture mechanical problems of piezoelectric media are under consideration.

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