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Multicriteria decision-making method based on a cosine similarity measure between trapezoidal fuzzy numbers

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Abstract

The degree of similarity or dissimilarity between the objects under study plays an important role. In vector space, especially, the cosine similarity measure is often used in information retrieval, citation analysis, and automatic classification. However, it scarcely deals with trapezoidal fuzzy information and multicriteria decision-making problems. For this purpose, a cosine similarity measure between trapezoidal fuzzy numbers is proposed based on an extension of the cosine similarity between fuzzy sets and is applied to fuzzy multicriteria decision-making problems under the conditions that the criteria weights and the evaluated values in the decision matrix are expressed by the form of trapezoidal fuzzy numbers. Through the expected weight and the weighted cosine similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily identified as well. The proposed method is simple and effective. Finally, an illustrative example demonstrates the implementation process of the technique.

Keywords: cosine similarity measure, trapezoidal fuzzy number, expected weight, multicriteria decision making

1. Introduction

In many real-world situations, the decision maker cannot provide deterministic alternative values but fuzzy numbers instead. This kind of uncertainty in multicriteria decision making (MCDM) can be modeled using fuzzy set theory and is ideally suited for solving these problems. Bellman and Zadeh (1970) first proposed the fuzzy decision making model. Since then, great numbers of studies have been done on fuzzy multicriteria decision making (FMCDM) (Hwang *et al*, 1981; Chen *et al*, 1992; Xu, 2004; Wang *et al*, 2005; Wu *et al*, 2007), so that the discipline has created several methodologies so far. Chen (2000) extended one of known classical MCDM method, technique for order preference by similarity to ideal solution (TOPSIS), to develop a methodology for solving multicriteria decision-making problems in fuzzy environment. Recently, Jahanshahloo *et al* (2006) developed the TOPSIS approach to decision making with fuzzy data, where the rating of each alternative and the weight of each criterion are expressed in triangular fuzzy numbers. He *et al* (2009) proposed the extension of the expected value method for multiple attribute decision making with fuzzy data, in which the preference values take the form of triangular fuzzy numbers and attribute weights are completely unknown. Moreover, Zeng (2006) developed an expected value method for FMCDM problems, in which the criteria weights and criteria values are trapezoidal fuzzy numbers.

On the other hand, Salton and McGill (1983) proposed a cosine similarity measure between fuzzy sets and applied it to information retrieval of words. Recently, Ye (2011) proposed a cosine similarity measure between intuitionistic fuzzy sets based on the concept of the cosine similarity measure between fuzzy sets and it demonstrated a stronger discrimination among the existing similarity measures by the comparisons of a variety of similarity measures for intuitionistic fuzzy sets, and then it was applied to pattern recognition and medical diagnosis. However, the domains of fuzzy sets and intuitionistic fuzzy sets are discrete sets, Trapezoidal fuzzy numbers extend discrete sets to continuous sets and are the extension of fuzzy sets. The advantage of the continuous sets is to maintain the integrity of information; while discrete sets may be loss partial information in the information integration. Therefore, the continuous sets are superior to the discrete sets. Furthermore, the existing cosine similarity measures do not deal with trapezoidal fuzzy numbers. Therefore, this paper will propose a cosine similarity measure for trapezoidal fuzzy numbers and a FMCDM method based on the cosine similarity measure under the conditions that the criterion weights and the evaluated values in the decision matrix are expressed by means of trapezoidal fuzzy numbers. Through the expected weight and the weighted cosine similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily identified as well. The advantage is that the proposed FMCDM approach has some simple tools and concepts in the fuzzy similarity measure and aggregation approach among the existing ones. An illustrative example shows that the proposed method is simple and effective.

2. Some Preliminaries

This section introduces some definitions and basic concepts related to fuzzy sets, fuzzy numbers, and trapezoidal fuzzy numbers.

Definition 1 (Zadeh, 1965). A fuzzy set A in the universe of discourse $X = \{x_1, x_2,..., x_n\}$ is defined as follows:

$$A = \{\langle x, \mu_A(x)\rangle \mid x \in X\}\,, \tag{1}$$

which is characterized by membership function $\mu_A(x)$: $X \to [0, 1]$, where $\mu_A(x)$ indicates the membership degree of the element x to the set A.

Definition 2 (Dubois *et al*, 1983). Let A be an fuzzy number in the set of real numbers R, its membership function is defined as

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ f_A(x), & a_1 \le x \le a_2, \\ 1, & a_2 \le x \le a_3, \\ g_A(x), & a_3 \le x \le a_4, \\ 0, & a_4 < x. \end{cases}, \tag{2}$$

where $a_1, a_2, a_3, a_4 \in R$, f_A: $[a_1, a_2] \to [0, 1]$ is a increasing continuous function, $f_A(a_1) = 0$, $f_A(a_2) = 1$, which is called the left side of the fuzzy number A, and g_A: $[a_3, a_4] \to [0, 1]$ is a decreasing continuous function, $g_A(a_3) = 1$, $g_A(a_4) = 0$, which is called the right side of the fuzzy number A.

Particularly, if the increasing functions f_A and decreasing functions g_A are linear, then we have trapezoidal fuzzy numbers, which are preferred in practice. For convenience, the trapezoidal fuzzy number is usually denoted by $A = (a_1, a_2, a_3, a_4)$.

Definition 3 (Dubois *et al*, 1983). A trapezoidal fuzzy number A with four parameters $a_1 \le a_2 \le a_3 \le a_4$ is denoted as $A = (a_1, a_2, a_3, a_4)$ in the set of real numbers R. In this case, its membership function can be given as

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2, \\ 1, & a_2 \le x \le a_3, \\ \dfrac{x - a_4}{a_3 - a_4}, & a_3 \le x \le a_4, \\ 0, & a_4 < x. \end{cases}, \tag{3}$$

The trapezoidal fuzzy number degenerates to a triangular fuzzy number when $a_2 = a_3$ holds, which is considered as a special case of the trapezoidal fuzzy number.

The following properties for trapezoidal fuzzy numbers have been given by Zeng (2006).

Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers in the set of real numbers R and r be a positive scalar number. Then,

$$A+B = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4), \tag{4}$$

$$A-B = (a_1-b_1, a_2-b_2, a_3-b_3, a_4-b_4), \tag{5}$$

$$rA = (ra_1, ra_2, ra_3, ra_4), \tag{6}$$

$$AB = (a_1b_1, a_2b_2, a_3b_3, a_4b_4). \tag{7}$$

The expected value (Zeng, 2006) of a trapezoidal fuzzy number $A = (a_1, a_2, a_3, a_4)$ is

$$E(A) = \frac{1}{4}(a_1 + a_2 + a_3 + a_4)\,. \tag{8}$$

3. Cosine Similarity Measure between Trapezoidal Fuzzy Numbers

In this section we introduce a definition and some concepts of the cosine similarity between fuzzy sets (Salton *et al*, 1983) and propose a cosine similarity measure between trapezoidal fuzzy numbers, then compare the calculation results with Chen (1996) and Chen *et al* (2001, 2007).

3.1 Cosine Similarity Measure for Fuzzy Sets

A cosine similarity measure for fuzzy sets (Salton *et al*, 1983) is defined as the inner product of two vectors divided by the product of their lengths. This is nothing but the cosine of the angle between the vector representations of the two fuzzy sets.

Assume that $A = (\mu_A(x_1), \mu_A(x_2),\ldots, \mu_A(x_n))$ and $B = (\mu_B(x_1), \mu_B(x_2),\ldots, \mu_B(x_n))$ are two fuzzy sets in the universe of discourse $X = \{x_1, x_2,\ldots, x_n\}$, $x_i \in X$. A cosine similarity measure (angular coefficient) between A and B can be defined as follows (Salton *et al*, 1983):

$$C_F(A,B) = \frac{\sum_{i=1}^{n} \mu_A(x_i)\mu_B(x_i)}{\sqrt{\sum_{i=1}^{n} \mu_A^2(x_i)}\sqrt{\sum_{i=1}^{n} \mu_B^2(x_i)}}, \tag{9}$$

where $0 \le C_F(A, B) \le 1$. It is undefined if $\mu_A(x_i) = 0$ and/or $\mu_B(x_i) = 0$ ($i = 1, 2,\ldots, n$). Then, let the cosine measure value be zero when $\mu_A(x_i) = 0$ and/or $\mu_B(x_i) = 0$ ($i = 1, 2,\ldots, n$).

3.2 Cosine Similarity Measure for Trapezoidal Fuzzy Numbers

In this subsection, a cosine similarity measure between trapezoidal fuzzy numbers is proposed based on the concept of the cosine similarity measure for fuzzy sets.

Let $A = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number in the set of real numbers R, the four parameters in A can be considered as a vector representation with the four elements. Thus, a cosine similarity measure for trapezoidal fuzzy numbers is proposed in an analogous manner to the cosine similarity measure (angular coefficient) between fuzzy sets (Salton *et al*, 1983).

Assume that there are two trapezoidal fuzzy numbers $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ in the set of real numbers R. Based on the extension of the cosine similarity measure for fuzzy sets, a cosine similarity measure between A and B is proposed as follows:

$$S(A,B) = \frac{\sum_{p=1}^{4} a_p b_p}{\sqrt{\sum_{p=1}^{4} (a_p)^2}\sqrt{\sum_{p=1}^{4} (b_p)^2}}. \tag{10}$$

It is easy to check that the cosine similarity measure of two trapezoidal fuzzy numbers A and B satisfies the following properties:

P1. $0 \le S(A, B) \le 1$;

P2. $S(A, B) = S(B, A)$;

P3. $S(A, B) = 1$ if and only if $A = B$, i.e., $a_p = b_p$ for $p = 1, 2, 3, 4$.

Proof. P1. It is obvious that the property is true according to cosine value.

P2. It is obvious that the property is true.

P3. When $A = B$, there is $a_p = b_p$ for $p = 1, 2, 3, 4$. So there is $S(A, B) = 1$. When $S(A, B) = 1$, there is $a_p = b_p$ for $p = 1, 2, 3, 4$. So there is $A = B$. □

We can derive the mathematical relationship between cosine similarity and Euclidean distance when each data object has an L_2 length of 1.

Assume that there are two trapezoidal fuzzy numbers $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ in the set of real numbers R where each trapezoidal fuzzy number (considered as a vector) has an L_2 length of 1. The mathematical relationship between cosine similarity and Euclidean distance is as follows:

$$d(A,B) = \sqrt{\sum_{p=1}^{4} (a_p - b_p)^2} = \sqrt{\sum_{p=1}^{4} (a_p^2 - 2a_p b_p + b_p^2)} = \sqrt{1 - 2S(A,B) + 1} = \sqrt{2[1 - S(A,B)]}. \tag{11}$$

3.3 Comparison of Similarity Measures

To illustrate the effectiveness of the proposed measure method, we use six sets of trapezoidal fuzzy numbers to compute the results of our proposed method. Then our main task is to compare the calculation results with Chen (1996) and Chen *et al* (2001, 2007) in this subsection.

The six sets of trapezoidal fuzzy numbers are shown in Table 1 adapted from Chen (1996) and Chen *et al* (2001, 2007), and then the calculation results of different similarity measures are shown as Table 2.

By applying our proposed method, from Table 2, we can see that the results of our proposed method coincides with ones of Chen (1996) and Chen et al. (2001, 2007). Therefore, the proposed method is reasonable.

Table 1. Six sets of trapezoidal fuzzy numbers

Set numbers	Trapezoidal fuzzy numbers
Set 1	A=(0.1,0.2,0.3,0.4), B=(0.1,0.25,0.25,0.4)
Set 2	A=(0.1,0.2,0.3,0.4), B=(0.5,0.65,0.65,0.8)
Set 3	A=(0.1,0.2,0.3,0.4), B=(0.3,0.45,0.45,0.6)
Set 4	A=(0.1,0.2,0.3,0.4), B=(0.1,0.2,0.3,0.4)
Set 5	A=(0.1,0.2,0.3,0.4), B=(0.5,0.6,0.7,0.8)
Set 6	A=(0.1,0.2,0.3,0.4), B=(0.3,0.4,0.5,0.6)

Table 2. Calculation results of different similarity measures

Set numbers	Chen's method (1996)	Chen and Chen's method (2001)	Chen and Chen's method (2007)	Proposed method
Set 1	0.975	0.8357	0.9499	0.9916
Set 2	0.6	0.3086	0.5846	0.9633
Set 3	0.8	0.5486	0.7794	0.9774
Set 4	1	1	1	1
Set 5	0.6	0.36	0.6	0.9689
Set 6	0.8	0.64	0.8	0.9844

4. Cosine Similarity Measure for Multicriteria Decision-Making Problems

In this section, we present a handling method of a cosine similarity measure between trapezoidal fuzzy numbers for multicriteria decision-making problems with fuzzy weights.

Let $A = \{A_1, A_2,\ldots, A_m\}$ be a set of alternatives and let $C = \{C_1, C_2,\ldots, C_n\}$ be a set of criteria. The preference value of a criterion C_j ($j = 1, 2,\ldots, n$) on an alternative A_i ($i = 1, 2,\ldots, m$) is an trapezoidal fuzzy number $a_{ij} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4})$ ($i = 1, 2,\ldots, m$; $j = 1, 2,\ldots, n$), $a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4} \in R$, and $a_{ij1} \le a_{ij2} \le a_{ij3} \le a_{ij4}$, which indicates the degree that the alternative A_i satisfies the criterion C_j given by the decision maker or expert according to some evaluated criteria. Thus we can obtain a fuzzy decision matrix $A = (a_{ij})_{m\times n}$.

The criteria are generally incommensurate, so the criterion values need to be normalized so as to transform them into comparable values. A normalizing method has been used based on the expected value operator (Zeng, 2006). In addition, the fuzzy weight vector is normalized so that the utility value of each criterion weight belongs to [0, 1].

4.1 Normalizing Fuzzy Criterion Values

Usually, there are two categories of criteria including a benefit criterion and a cost criterion in the MCDM problem. In order to eliminate the incommensurability of the criteria in dimensionless units, each criterion value a_{ij} ($i = 1, 2,\ldots, m$; $j = 1, 2,\ldots, n$) in the matrix $A = (a_{ij})_{m\times n}$ needs to be normalized into the corresponding comparable element in the matrix $R = (r_{ij})_{m\times n}$ by using the following formulas (Zeng, 2006):

$$r_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m}\left(E(d_{ij})\right)^2}} \quad \text{for a benefit criterion,} \tag{12}$$

$$r_{ij} = \frac{p_j - d_{ij}}{\sqrt{\sum_{i=1}^{m}\left(E(p_j - d_{ij})\right)^2}} \quad \text{for a cost criterion,} \tag{13}$$

where $p_j = \max_{1\le i\le m} \sup\left\{x_{ij} \mid \mu_{ij}(x_{ij}) > 0\right\}$. Then the FMCDM addresses the problem of ranking alternatives or choosing the most desirable alternative among the finite set of alternatives based on the normalized decision matrix.

4.2 Normalizing Fuzzy Weights by Expected Weight Values

The weight vector of criteria for the different importance of each criterion is given as the fuzzy weight vector $\omega = (\omega_1, \omega_2,\ldots, \omega_n)$, in which each element is expressed by a trapezoidal fuzzy number. It is normalized into the expected weight vector $w = (w_1, w_2,\ldots, w_n)$ by using the following calculation:

$$w_j = \frac{E(\omega_j)}{\sum_{j=1}^{n} E(\omega_j)}, \tag{14}$$

where an expected weight $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$.

4.3 Decision-Making Method

In multicriteria decision-making environments, the concept of ideal point has been used to help the identification of the best alternative in the decision set. Although the ideal alternative does not exist in real world, it does provide a useful theoretical construct to evaluate alternatives. Therefore, we define an ideal trapezoidal fuzzy number for a criterion in the ideal alternative A^* as $r_j^* = (1, 1, 1, 1)$.

Thus a weighted cosine similarity measure between an alternative A_i and the ideal alternative A^* represented by the trapezoidal fuzzy numbers is defined as

$$W_i(A^*, A_i) = \sum_{j=1}^{n} w_j \frac{\sum_{p=1}^{4} r_{jp}^* r_{ijp}}{\sqrt{\sum_{p=1}^{4} (r_{jp}^*)^2} \sqrt{\sum_{p=1}^{4} (r_{ijp})^2}}, \tag{15}$$

which provides the global evaluation for each alternative regarding all the criteria. From Eq. (15), the larger the value of a weighted cosine similarity measure W_i, the better the alternative A_i (i = 1, 2,…, m). Through the weighted cosine similarity measure between each alternative and the ideal alternative, the ranking order of all the alternatives can be determined and the best alternative can be easily identified as well.

5. Illustrative Example

In this section, we consider the following numerical example discussed in Zeng (2006) to illustrate the proposed method for the FMCDM problem.

A company considers five investment engineering alternatives: A_1, A_2, A_3, A_4, and A_5. Each alterative is evaluated from four criteria: the investment amount (C_1), the expected net income (C_2), the venture profit (C_3), and the venture loss (C_4). The fuzzy ratings of the criteria and weights are listed in Table 3.

Table 3. Ratings of criteria and weights

	C_1(\$$10^6$)	C_2(\$$10^6$)	C_3(\$$10^6$)	C_4(\$$10^6$)
A_1	(0.40,0.45,0.55,60)	(0.35,0.40,0.5,0.6)	(0.32,0.42,0.5,0.6)	(0.03,0.04,0.05,0.06)
A_2	(0.9,0.95,1.05,1.1)	(0.4,0.6,0.7,0.85)	(0.4,0.5,0.65,0.7)	(0.05,0.1,0.15,0.2)
A_3	(0.4,0.45,0.55,0.6)	(0.2,0.3,0.4,0.5)	(0.15,0.3,0.4,0.55)	(0.05,0.06,0.08,0.1)
A_4	(0.85,0.9,1,1.05)	(0.35,0.4,0.5,0.6)	(0.4,0.5,0.65,0.7)	(0.04,0.08,0.15,0.2)
A_5	(0.55,0.6,0.7,0.75)	(0.25,0.3,0.4,0.45)	(0.19,0.26,0.4,0.5)	(0.02,0.07,0.1,0.15)
Weights	(0.1,0.2,0.4,0.5)	(0.1,0.2,0.3,0.4)	(0.01,0.05,0.1,0.21)	(0.21,0.31,0.36,0.47)

The decision matrix $A = (a_{ij})_{m\times n}$ is obtained from Table 3, in which each element represents the suitability of an alternative versus each criterion by using a trapezoidal fuzzy number $a_{ij} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4})$ (i = 1, 2, 3, 4, 5; j = 1, 2, 3, 4), as follows:

$$A = \begin{bmatrix} (0.40,0.45,0.55,0.60) & (0.35,0.40,0.50,0.60) & (0.32,0.42,0.50,0.60) & (0.03,0.04,0.05,0.06) \\ (0.90,0.95,1.05,1.10) & (0.40,0.60,0.70,0.85) & (0.40,0.50,0.65,0.70) & (0.05,0.10,0.15,0.20) \\ (0.40,0.45,0.55,0.60) & (0.20,0.30,0.40,0.50) & (0.15,0.30,0.40,0.55) & (0.05,0.06,0.08,0.10) \\ (0.85,0.90,1.00,1.05) & (0.35,0.40,0.50,0.60) & (0.40,0.50,0.65,0.70) & (0.04,0.08,0.15,0.20) \\ (0.55,0.60,0.70,0.75) & (0.25,0.30,0.40,0.45) & (0.19,0.26,0.40,0.50) & (0.02,0.07,0.10,0.15) \end{bmatrix}.$$

Then the fuzzy weight vector for the four criteria is ω = {(0.1,0.2,0.4,0.5), (0.1,0.2,0.3,0.4), (0.01,0.05,0.1,0.21), (0.21,0.31,0.36,0.47)}.

In the four criteria, C_1 and C_4 are cost criteria, and then C_2 and C_3 are benefit criteria. By use of Eqs. (12) and (13) the normalized matrix is given as follows:

$$R=\begin{bmatrix}(0.51,0.56,0.66,0.71) & (0.34,0.38,0.48,0.58) & (0.31,0.40,0.48,0.58) & (0.56,0.60,0.64,0.68)\\ (0.00,0.05,0.15,0.20) & (0.38,0.58,0.67,0.82) & (0.38,0.48,0.63,0.67) & (0.00,0.20,0.40,0.60)\\ (0.51,0.56,0.66,0.71) & (0.19,0.29,0.38,0.48) & (0.14,0.29,0.38,0.53) & (0.40,0.48,0.56,0.60)\\ (0.05,0.10,0.20,0.26) & (0.34,0.38,0.48,0.58) & (0.38,0.48,0.63,0.67) & (0.00,0.20,0.48,0.64)\\ (0.36,0.41,0.51,0.56) & (0.24,0.29,0.38,0.43) & (0.18,0.25,0.38,0.48) & (0.20,0.40,0.52,0.72)\end{bmatrix}.$$

By using Eq. (14), thus we can obtain the following expected weight value for a criterion C_j (j = 1, 2, 3, 4):
$w_1 = 0.3061$, $w_2 = 0.2551$, $w_3 = 0.0944$, and $w_4 = 0.3444$.
By applying Eq. (15), we can obtain the following values of weighted cosine similarity measures:
$W_1(A^*, A_1) = \mathbf{0.9889}$, $W_2(A^*, A_2) = 0.8554$, $W_3(A^*, A_3) = 0.9740$, $W_4(A^*, A_4) = 0.8870$, and $W_5(A^*, A_5) = 0.9582$.
Therefore, the alternatives can be ranked as
$A_1 > A_3 > A_5 > A_4 > A_2$,
which implies that the optimal alternative is A_1 and the same as the result in Zeng (2006). This also demonstrates that the proposed method is simple and effective by the comparison of the existing one (Zeng, 2006) in the FMCDM problem.

6. Conclusions

In this paper, we have proposed a cosine similarity measure between two trapezoidal fuzzy numbers and investigated the method of a cosine similarity measure for the FMCDM problem, in which weight values and criterion values take the form of trapezoidal fuzzy numbers. The cosine similarity measure method has been extended for ranking alternatives, and then a practical example of the developed approach has been given to select the investment alternatives. The results show that the proposed method in this paper is simple and effective. The proposed fuzzy MCDM approach is tailored of some simple tools and concepts in the fuzzy similarity measure and aggregating approach. Furthermore, the proposed cosine similarity measure is extended to trapezoidal fuzzy numbers and trapezoidal fuzzy MCDM problems. The next research work is to develop other aggregation operations for fuzzy group decision-making problems.

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