Mixed-mode (I/II) crack initiation direction for elastic-plastic materials based on crack-tip plastic zone

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Abstract

In this paper, crack-initiation direction for an elastic-plastic material (interstitial free steel) under mixed-mode (I/II) load is studied based on crack-tip plastic zone under monotonic load. The feasibility of using the minimum plastic zone radius (MPZR) criterion for elastic-plastic materials is also examined. The shape and size of crack-tip plastic zones have been estimated by elastic-plastic finite element analysis in a plane stress single edge notch tensile (SENT) specimen under mixed-mode (I/II) loading according to von Mises yield criteria. The results show that the minimum plastic zone sizes were not clearly identified; indicating MPZR criterion is not suitable for the analysis of elastic-plastic materials. It is shown that the crack initiation direction for elastic-plastic materials depends on the path of the maximum plastic zone size. The variation of crack initiation angle is also studied with respect to mode mixity factor and the results are found to be in good agreement with the experimental results available in the literature.

Keywords: plastic zone size, *J*- integral, mixed mode (I/II), finite element analysis, mode mixity, SENT specimen

1. Introduction

The problem of crack initiation under monotonic mixed-mode loading is of great interest in the structural integrity assessment because of complex nature of actual loading condition at the crack-tip. Prediction of crack initiation and orientation with its propagation path under mixed-mode loading is much desirable for life prediction of engineering materials. The problem involving mixed mode fracture has been extensively investigated during the past decade, Nobile (2000), Sonsino (2001), Li and Chandra (2003), Ayatollahi *et al.* (2004), Borrego *et al.* (2006), Johnson and Qu (2007), Smith *et al.* (2008), Guo *et al.* (2008), Shahani and Tabatabaei (2009) and Kudari and Sharanaprabhu (2010) have analyzed the mixed mode crack initiation and orientation problems. Several criteria have been proposed for predicting crack initiation angles under Linear Elastic Fracture Mechanics (LEFM) regime. Some important criteria are maximum tangential stress criterion proposed by Erdogan and Sih (1963), and minimum strain energy density criterion proposed by Sih (1974).

It is well known that, the ductile engineering materials develop a plastically deformed region ahead of a crack-tip referred as plastic zone. This zone contains damage that leads to either a total or progressive fracture depending on the properties of the material. The growth of the crack is linked to the existence of this plastic zone at the crack-tip, whose formation and development are accompanied by energy dissipation. The fracture analysis of a material requires a clear understanding of the plastic zone morphology, deformation and stress fields ahead of the crack-tip. Determination of crack initiation angle based on crack-tip plastic zone emerged recently. Golos and Wasiluk (2000) suggested a criterion for crack initiation angles based on the minimum radius of the plastic zone. Kahn and Khraisheh (2004) and Bian and Kim (2004) proposed a minimum plastic zone radius (MPZR) criterion for crack initiation angle based on the analytical estimations of crack-tip plastic zone size considering von-Mises yield criteria. In mixed mode fracture, Kahn and Khraisheh (2004) and Bian and Kim (2004) have shown that the crack initiation angle depends on the loading angle. Benrahou *et al.* (2007) have identified that the type of loading alters the shape and size of the crack-

tip plastic zone. In the earlier works (Kahn and Khraisheh, 2004 and Bian and Kim, 2004) the estimates of plastic zone size were done by choosing only LEFM parameter, stress intensity factor, which is valid only in small-scale yielding (SSY) conditions.

However, the study of the plastic zones for elastic-plastic materials demands examination using an elastic-plastic fracture mechanics (EPFM) parameter, *J*-integral. Some reports on the estimation on plastic zone sizes (PZS) together with the *J*- integral are available in the work of Kuang and Chen (1996), Yuan and Broks (1998), and Benguediab *et al.* (2001) for mode-I type of specimens. Kudari *et al.* (2007) have clearly shown the variation of PZS with respect to the *J*-integral in detail for various geometries of mode-I fracture specimens. Arun Roy and Narasimhan (1997) have computed the *J*-dominance in mixed mode ductile fracture specimens such as Compact Tension Shear (CTS), four point bend (4PB) and Modified Compact Tension specimen. Pirondi and Dalle (2001) have proposed the relationship between crack-tip displacement vector and *J* for CTS specimen. Li *et al.* (2004) originated the J-M_p based criteria for bifurcation assessment of a crack in elastic-plastic materials under mixed mode (I/II) loading. In this paper, an effort is made to extend the work of Bian and Kim (2004) on the mixed mode crack initiation angle, for elastic-plastic materials. Finite element method is used to determine the crack-tip plastic zone under elastic-plastic analysis. The objectives of this work are: (i) to evaluate the shape and size of elastic-plastic crack-tip plastic zones (mixed mode-I/II) with respect to the *J* integral, (ii) to examine the suitability of MPZR criterion for elastic-plastic materials, and (iii) to propose a possible criterion for estimating the mixed mode crack initiation angle for elastic-plastic materials.
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2. Finite element analysis

The general-purpose finite element (FE) code ABAQUS (2004) is used in this study. The Single Edge Notch Tensile (SENT) specimen geometry (Mohanty *et al.* 2008) is used in this analysis and is shown in Fig.1 (a). The load is applied at various angles (β), 0° (pure mode-I), 18°, 36°, 54°, 72° and 90° (pure mode-II) to study the plastic deformation ahead of the crack-tip. In the experimental analyses (Mohanty *et al.* 2008, Borroego *et al.* 2006) the specimens were given mixed mode load with the help of a loading Jig. In the present FE analysis, the loading on the specimen, carried out with the help of a jig in experiments is simulated by applying uni-axial forces at the loading holes (Fig.1b). The force and displacement boundary condition as used in the work of Borroego *et al.* (2006) for compact tensile and shear (CTS) specimen is extended for the SENT specimen in this FE analysis. The specimen loading at various angles (β) is ascertained by applying uniaxial loads F_1 to F_6 as shown in the Fig.1(b), estimated by equations (1)-(3) as given below. The displacement boundary condition as shown in Fig.1(b) is provided to avoid rigid body movement without affecting the global rigidity of the specimen (Borroego *et al.*, 2006).
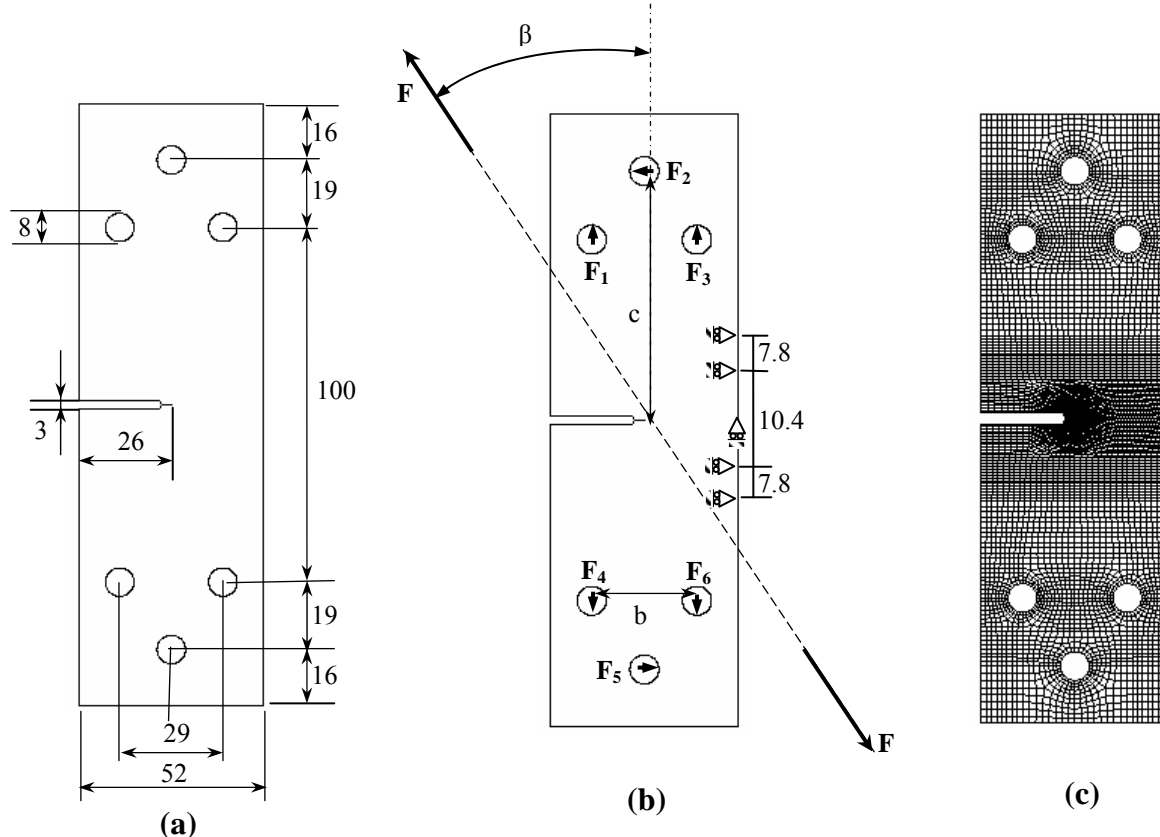

Figure 1.(a) Specimen configuration used in the analysis (all dimensions in mm), thickness = 3 mm. **(b)** Loading and boundary conditions, **(c)** FE mesh used in the analysis.

$$F_1 = F_6 = F\left(\frac{1}{2}\cos\beta + \frac{c}{b}\sin\beta\right) \tag{1}$$

$$F_2 = F_5 = F\sin\beta \tag{2}$$

$$F_3 = F_4 = F\left(\frac{1}{2}\cos\beta - \frac{c}{b}\sin\beta\right) \tag{3}$$

Where, F is the total applied load, β is the loading angle, b and c are the dimensions shown in the Fig.1(b).

A series of elastic-plastic finite element computations have been made on the SENT specimen (Fig.1a) having a/W=0.5 considering the full specimen geometry due to the unsymmetrical loading. A typical two-dimensional FE mesh used in the analysis is shown in Fig.1(c). The analysis domain is descritized using 7636 number of 8-noded isoparametric 2D plane stress reduced integration solid elements (CPS8R in ABAQUS). This kind of elements is used in the work of Yun-Jae Kim *et al.* (2004) for computation of J integral. In this FE analysis, interstitial-free (IF) steel possessing yield strength, σ_y=155MPa, ultimate strength, σ_u=275MPa, poisons ratio, υ=0.3, Ramberg-Osgood strain hardening parameter, N=3.358 and elastic modulus, E=197 GPa (Kudari *et al.* 2007), has been considered. The material response in the plastic region was modeled as multi-linear by taking twenty divisions of plastic portion of true stress-strain curve of the material as shown in Fig.2. The magnitude of J has been evaluated for a path at a specific loading condition using the expression (Rice 1968):

$$J = \int_\Gamma \left(Wdy - T_i\frac{\partial u_i}{\partial x}ds\right) \tag{4}$$

$$W = \int_0^\varepsilon \sigma_{ij}d\varepsilon_{ij}, \qquad T_i = \sigma_{ij}n_j$$

Where W is the strain energy density, T_i is the surface traction vector, u_i is the displacement vector and s is the element of arc length along the contour Γ. The magnitudes of J integrals were extracted using a contour integral with the general purpose FE code ABAQUS (2004). In this analysis, J is computed using Eq.(4) based on the domain integral along five contours. In this Finite element analysis it is observed that the outer four contours gave almost path independent values of J. Hence, in this study, the average value of J is obtained using the results from the outer four contours, which is used for further analysis.
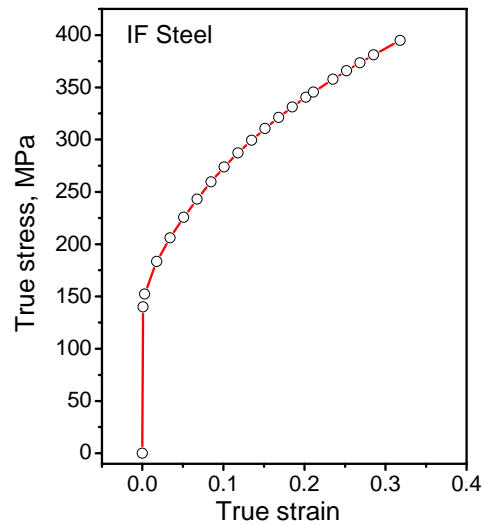

Figure 2. Multilinear stress-strain curve used in the elastic-plastic FE analysis.

3. Results and Discussions

The load was applied on the specimen to estimate the J integral and to study the plastic zone shape and size ahead of the crack-tip. A typical variation of J-integral (J) with the applied load (F) for various loading angles is shown in Fig.3. The nature of variation of J *vs*. F (Fig.3a) is in good agreement with similar earlier results by Arun Roy and Narasimhan (1997). It is observed from Fig.3(a) that, J appears to be independent of loading angle (β) for F <4.3 kN but, for F >4.3 kN J increases exponentially with respect to applied load (F). To find the clarity in this observation, variation J *vs*. F has been plotted for F<5kN in Fig. 3(b). This plot indicates that: (i) only for F=4.3 kN J is independent of F for various β and (ii) for any magnitude of F<4.3 kN, J

depends on the loading angle, though the dependency is insignificantly small. It is observed from Fig.3(a) that specimen withstands maximum load and develops higher magnitude of J for $\beta=0^o$ (mode-I) than other loading angles.
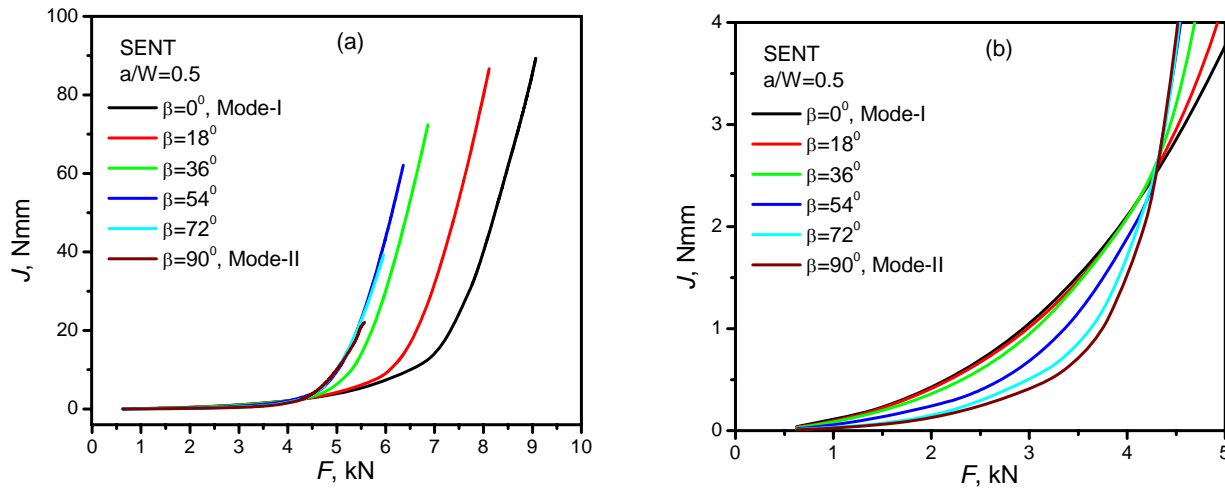

Figure 3. A typical variation of J with F (a) for full load (b) for load <5kN

Next, the shape of the plastic zone ahead of the crack-tip has been ascertained by plotting iso-contours of the effective stress, which causes yielding according to von Mises criterion (Gdoutos and Papkatalis, 1987). The sequential development of crack-tip plastic zone in the SENT specimen has been obtained by superimposing the plastic zone contours with increasing J values. For simplicity the displacement scaling in these specimens are set to zero. Typical shapes of plastic zones for $\beta=0^o$ to 90° are shown in Fig.4 (a) to Fig.4 (f). The plastic zone shapes have been also obtained for higher magnitudes of J, but it is observed that for higher magnitudes of J the crack-tip plastic zones merge with plastic zones produced due to support boundary conditions, which are not shown in Fig.4. The Fig.4 indicates that the shape of plastic zone is affected by the loading angle and the plastic zone is found to be maximum ahead of the crack-tip due to the effect of plasticity. It is observed from Fig.4 that for $\beta=0^o$ (Mode-I) the shape of plastic zone is elliptical and is in good agreement with the results of Kudari *et al.* (2007). For $\beta=90^o$ (Mode-II) the plastic zone stretches along the ligament because of shear loading. The Fig.4 show that the minimum plastic zone size is not clearly identified as in the case of elastic analysis shown by Bin and Kim (2004) and Sharanaprabhu and Kudari (2009). Hence, the results obtained in this analysis infer that the minimum plastic zone radius (MPZR) criterion to estimate the crack initiation angle is not appropriate for the analysis of elastic-plastic materials. It is well known that the crack in elastic-plastic materials propagates due to nucleation, growth and coalescence of voids in the crack-tip process zone within the plastic zone. The nucleation, growth and coalescence are controlled by plastic strain in the process zone, indicating that the failure direction or crack initiation direction for elastic-plastic materials should be controlled by plastic strain in the process zone. In this FE analysis it is noted that, the plastic strain is found to be highest for the direction in which the plastic zone size is maximum. Based on this hypothesis, it is considered that crack propagation initiates along the direction of maximum plastic zone size for elastic-plastic materials. Further, the size of maximum plastic zone (r_o) and the angle at which maximum plastic zone occur (θ_o) for various plastic zone contours obtained in this analysis are measured. The plot showing the variation of maximum plastic zone size, r_o *vs.* J is shown in Fig.5. This figure shows that r_o *vs.* J is apparently independent of loading angle. It is known that for small scale yielding (SSY) condition the allowable plastic zone size is one sixteenth of the ligament (Tracey, 1976), which is marked in the Fig.5 to demarcate SSY from LSY. This shows that for SSY condition, r_o<1.3 mm and J<1.3 Nmm. From Fig.5 one can find the magnitude of r_o for a particular J or vice versa. To obtain such a relation the data in Fig.5 is fit by a third degree polynomial. The resulting analytical expression is as given below.

$$r_o = 0.8121 + 0.8733(J) - 0.0211(J)^2 - 2.023E - 4(J)^3 \tag{5}$$

The polynomial fit (Eq.5) is shown in Fig.5 by a red line, which shows an excellent fit to the present FE results for various β. The proposed Eq.(6) can be useful to the researchers and industrial scientists to predict the maximum plastic zone size (r_o) for SENT specimens (a/W=0.5) by only knowing the magnitude of J or vice versa for the steel considered in this analysis or any elastic-plastic materials having N approximately between 3 and 4.
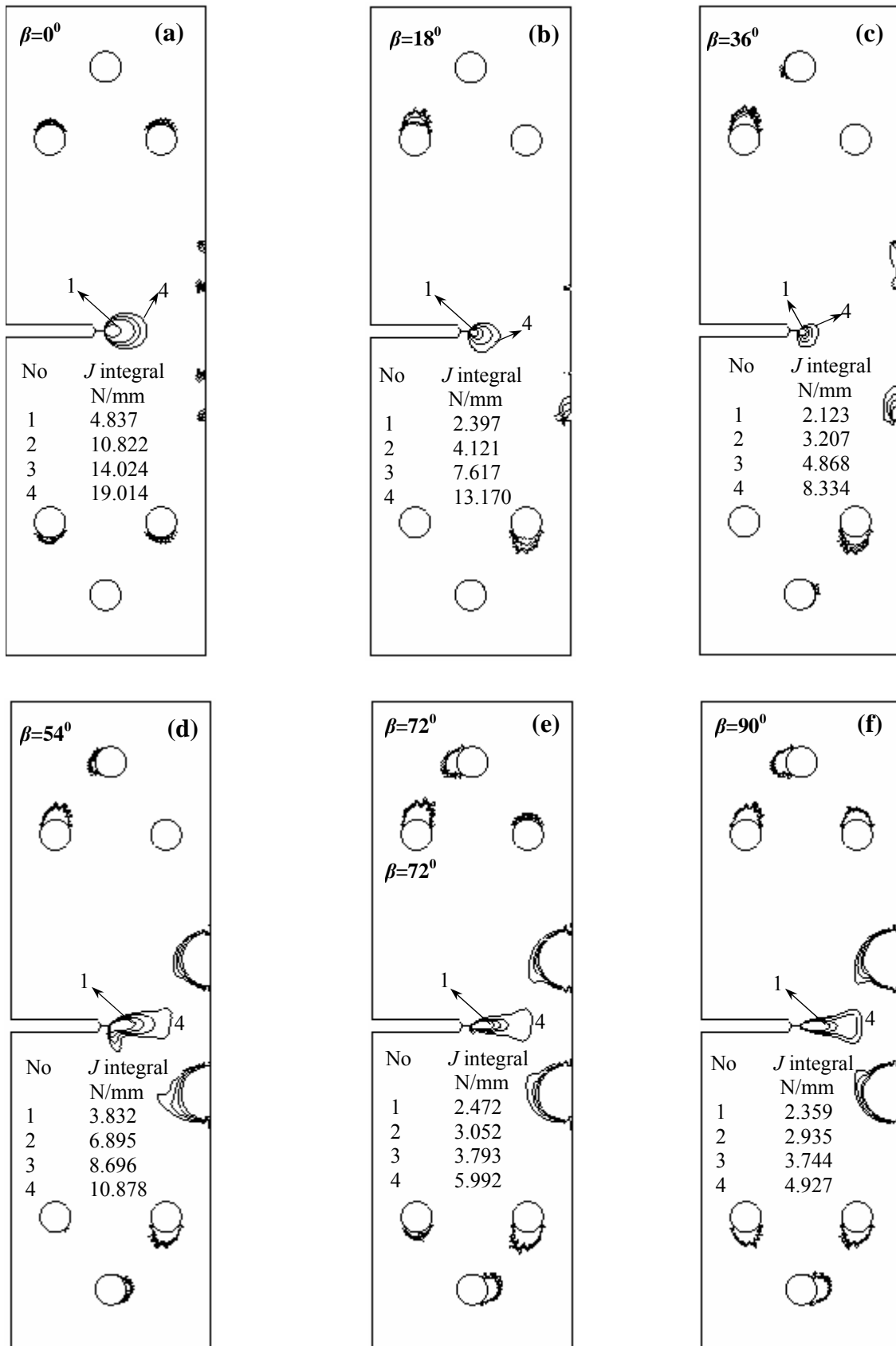
$\beta=0^0$ **(a)**

No	*J* integral N/mm
1	4.837
2	10.822
3	14.024
4	19.014

$\beta=18^0$ **(b)**

No	*J* integral N/mm
1	2.397
2	4.121
3	7.617
4	13.170

$\beta=36^0$ **(c)**

No	*J* integral N/mm
1	2.123
2	3.207
3	4.868
4	8.334

$\beta=54^0$ **(d)**

No	*J* integral N/mm
1	3.832
2	6.895
3	8.696
4	10.878

$\beta=72^0$ **(e)**

$\beta=72^0$

No	*J* integral N/mm
1	2.472
2	3.052
3	3.793
4	5.992

$\beta=90^0$ **(f)**

No	*J* integral N/mm
1	2.359
2	2.935
3	3.744
4	4.927

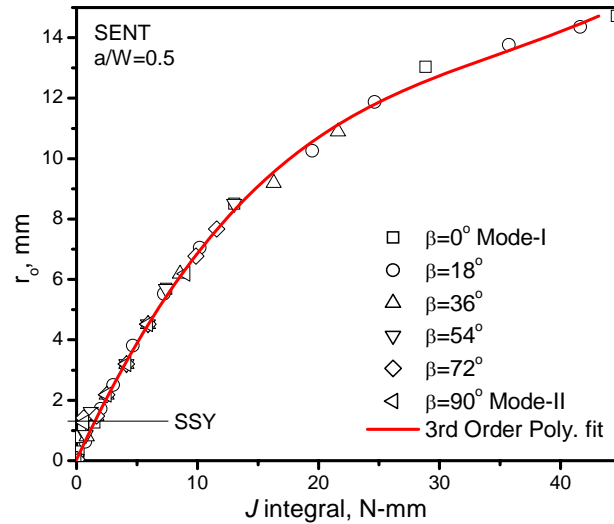**Figure 4.** Typical shapes of sequential plastic zone shapes for various loading

Figure 5. A typical variation of r_0 *vs.* J for various loading angles

Based on the hypothesis that the crack initiates at the direction of maximum plastic zone size the crack initiation/deflection angles have been estimated using the plastic zones for various loading angles. The variation of crack-deflection angle *vs.* loading angle is shown in the Fig.6 for various applied loads. This figure indicates that crack-deflection angle is independent of applied load. It increases from 0° to 26° for $\beta \leq 36^o$, then the θ_o shifts from 26° to 0° for $54^o > \beta < 90^o$. This nature of variation of θ_o shown in Fig.6 is inconsistent with the similar results obtained based on MPZR criterion (Bian and Kim, 2004, Kahn and Khraisheh, 2004), which may be attributed to the consideration of elastic-plastic analysis in this work and elastic analysis in the work of Bian and Kim (2004) and Kahn and Khraisheh (2004).
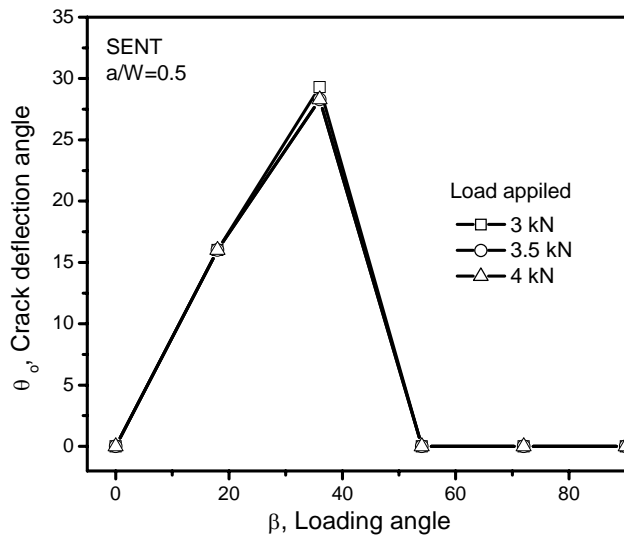

Figure 6. A typical variation of θ_0 *vs.* β for various load steps

To validate the finite element analysis results presented in this study (Fig.6), the FE results are verified with the available experimental results of Pirondi and Dalle (2001). Pirondi and Dalle (2001) have shown the variation of crack deflection angle estimated experimentally using 4 mm thick CTS specimens for the ductile material (fine grained structural steel StE 550), with respect to elastic mode mixity factor (M_e). In mixed mode (I/II) loading, the elastic mode mixity factor is defined as (Shih, 1974):

$$M_e = (2/\pi) \arctan (K_I/K_{II}) \tag{6}$$

M_e ranges from 0 to 1, with $M_e = 0$ for pure mode-II, $M_e = 1$ for pure mode-I, which can be used instead of loading angle β for elastic materials. For analysis of elastic plastic materials one can use plastic mixity factor, M_p (Shih, 1974) as:

$$M_p = \lim_{r \to 0} \frac{2}{\pi} \tan^{-1}\left\{\frac{\sigma_{\theta\theta}(r,\theta=0)}{\sigma_{r\theta}(r,\theta=0)}\right\} = \lim_{r \to 0} \frac{2}{\pi} \arctan\left\{\frac{\sigma_{\theta\theta}(r,\theta=0)}{\sigma_{r\theta}(r,\theta=0)}\right\} \qquad (7)$$

M_p also ranges from 0 to 1, with $M_p = 0$ for pure mode-II, $M_p = 1$ for pure mode-I, which can be used instead of loading angle β for elastic-plastic materials. In the work of Pirondi and Dalle (2001) the elastic mode mixity ratio M_e is adopted instead of plastic mixity factor M_p because of its simplicity. It is also known that M_p practically does not depend on N generally for ductile steels (Pirondi and Dalle, 2001). In this analysis M_p is computed by elastic-plastic analysis and M_e is computed by a separate elastic FE analysis using in equations (7) and (6) respectively. The computed M_e and M_p for the SENT specimen geometry are plotted in Fig.7 along with the results of Shih, (1974). This figure indicates that the relation between M_e and M_p are in excellent agreement with the results of Shih (1974) for $N=3$. Hence, Fig.7 gives an excellent validation of the FE results on M_e and M_p computed in this work.
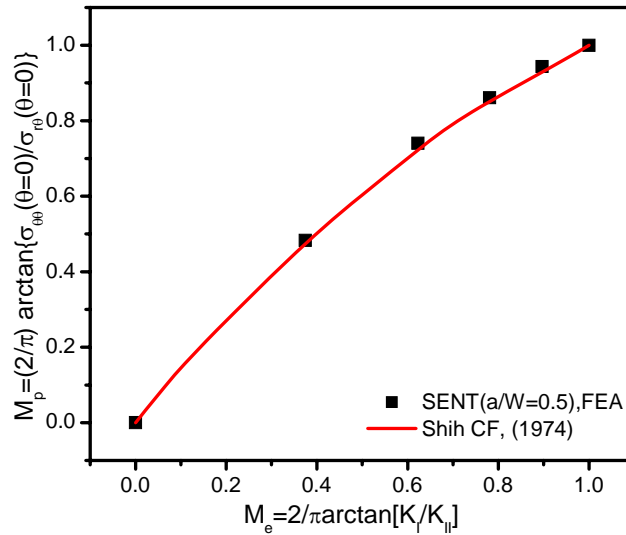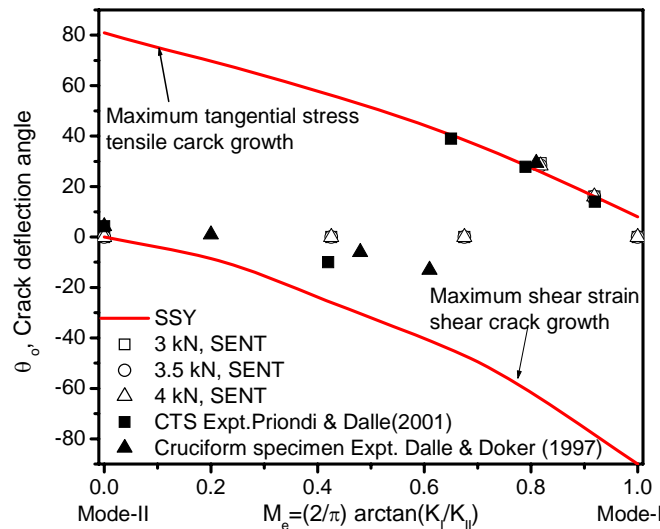

Figure 7. Load mixity M_p *vs.* M_e

Figure 8. A typical variation of θ_0 *vs.* M_e for various load steps

The magnitudes of M_e and the crack deflection angle estimated in the present work are superimposed on the similar plot containing experimental results and maximum tangential stress and maximum shear strain criterion from the work of Priondi and Dalle (2001) in Fig.8. This figure indicates that the results on crack deflection angle on SENT specimen are in good agreement with the experimental results of Priondi and Dalle (2001) for CTS specimen and, Dalle and Doker (1997) for Cruciform specimen. The small deviation in present results and results presented by Priondi and Dalle (2001) may be attributed to the geometry of the

specimen used in the analysis. Figure 8 shows that for mode M_e<0.42 the crack deflection angle (θ_o) better fits to the maximum shear strain criterion, where the shear load on the specimen is maximum. For mode M_e>0.68 the crack deflection angle better fits the maximum tangential stress criterion, where the opening load dominate. 0.42>M_e<0.68 may be a case of transition from maximum shear strain criterion to maximum tangential stress criterion. The results presented in this study, show that for dominating tensile loads ($\beta\cong0$) crack initiation follows maximum tangential stress criterion and for dominating shear loads ($\beta\cong90$) it follows maximum shear strain criterion. The authors consider the results presented in this paper are premature to explain the crack initiation criterion for elastic-plastic materials. Further, this kind of study has to conducted for various mixed mode specimen geometries and various elastic-plastic materials having varied strain hardening parameter (N) to clearly understand the crack initiation criterion based on maximum plastic zone size for elastic-plastic materials. But, from this work it is very much clear that the MPZR criterion for crack initiation direction is only suitable for materials under elastic analysis (LEFM), and it is not appropriate for the use of elastic-plastic materials.

4. Conclusions

The following conclusions are drawn from the present study:
- o The MPZR criterion is not suitable for the analysis crack initiation direction for elastic-plastic materials
- o Possibly, the crack initiates along the path of maximum plastic zone size for elastic-plastic materials
- o The relationship between maximum plastic zone along the crack plane (r_o) and J integral is proposed for a SENT specimen
- o The crack deflection angle (θ_o), better fits for maximum shear strain criterion for M_e<0.42 and it fits maximum tangential stress criterion for M_e>0.68

Nomenclature
a crack length
K_I mode-I stress intensity factor
F uniaxial applied load
F_1-F_6 Force applied on specimen loading holes (Fig.1)
K_{II} mode-II stress intensity factor
J J-integral
M_e elastic mode mixity
M_p plastic mode mixity
N Ramberg Osgood strain hardening constant
r_0 plastic zone size
W width of the specimen
β loading angle
θ_o crack initiation/deflection angle

Abbreviations
CTS   compact tension shear
EPFM   elastic plastic fracture mechanics
FE   Finite element
FEA   finite element analysis
LEFM   linear elastic fracture mechanics
MPZR   minimum plastic zone radius
SENT   single edge notch tension
4PB   four point bend

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