

Mean value first order second moment analysis of buckling of axially loaded thin plates with random geometrical imperfections

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Abstract

Buckling strength of thin plate structures under axial compression is more dominantly affected by the initial geometric imperfections than the other types of imperfections present in them. Since these initial geometric imperfections are random in nature, the collapse strength distribution will also be random. Hence a probabilistic approach is required for reliable design of these thin plate structures. In this paper, by keeping the variance of imperfections of all the models at assumed manufacturing tolerance of 1.71 mm and maintaining the maximum amplitude of imperfections within ± 8 mm, 1024 random geometrical imperfect plate models are generated by the linear combination of first 10 eigen affine mode shapes using 2^k factorial design. These imperfect models are analysed using ANSYS non-linear FE buckling analysis including both geometrical and material non-linearities. From these FE analysis results, the strength distribution of the plate is obtained and reliability analysis is carried out using Mean Value First Order Second Moment (MVFOSM) method.

Keywords: Buckling strength, Thin plates, Geometrical imperfections, Random modeling, Reliability based design, MVFOSM.

1. Introduction

Thin plate structures are widely used in many fields like mechanical, marine, aerospace and in civil engineering structures. The manufacturing process involved in making perfect thin plate is difficult, because there will be some geometrical imperfections like local indentations, swelling, non-uniform thickness etc., and material imperfections like inhomogenities, cracks, vacancies, impurities etc., and also other imperfections like residual stresses and strains induced during manufacturing. These imperfections generally affect the buckling behavior of plates and to study this, complete information about the imperfections are required. Out of all these imperfections, geometrical imperfections are more dominant in determining the load carrying capacity of thin shell structures.

Reliable prediction of buckling strength of these structures are important, because buckling failure is catastrophic in nature and also geometrical imperfections present in these structures are highly random in nature which require probabilistic approach to determine the safe load for the structure. The structural reliability analysis is classified into 3 groups (Ranganathan, 2000) namely level1, level2 and level3 methods. Since the buckling strength of thin plates are widely scattered and has large deviation from the theoretical value, it will be appropriate to use level2 method. In this paper the level2 First Order Second Moment (FOSM) method is adopted to determine the safe critical load of the structure.

2. Literature Review

The modeling of imperfections can be classified into deterministic and random geometrical imperfection modeling. In case of the deterministic approach, imperfections are either obtained from actual measurement (for example Arbocz and Hol, 1991; Scheneider, 1996; Singer, 1999; Athiannan and Palaninathan, 2004 and Sadvovsky *et al.*, 2005 & 2006) or from assumed

imperfection pattern. The assumed imperfection pattern may be sinusoidal pattern (Pircher *et al.*, 2001 & Khamlichi *et al.*, 2004; Ikeda *et al.*, 2007) or first eigen mode shape pattern (Teng and Song, 2001; Kim and Kim, 2002; Khelil, 2002; Featherston, 2003; and Visweswaran *et al.*, 2006). There are two ways by which random modeling of imperfections can be achieved. The first method is by varying the nodal locations of the structural model randomly and the second method is the stochastic FE approach.

Each manufacturing process has its own characteristic imperfection shapes that can be represented by double Fourier series. In the earlier studies, these Fourier coefficients were made as random variables to get different random geometrical imperfection models (for example Athiannan and Palaninathan, 2004; Chryssanthopoulos, 1998). Elishakoff (1979) gave a reliability method based on Monte Carlo simulation technique and applied to the problem of buckling of finite column with initial geometrical imperfections, which is assumed as Gaussian random fields. Elishaoff *et al.* (1987) explained about the MVFOSM to predict the reliability of cylindrical shell possessing axisymmetric and asymmetric random geometrical imperfections using the second order statistical properties obtained from measured initial geometric imperfections. Results of reliability calculations were verified with results from Monte Carlo simulation. Chryssanthopoulos *et al.* (1991) presented Response Surface Methodology (RSM) to determine the reliability of stiffened cylindrical and plate shells subjected to axial compression, considering the manufacturing variabilities such as initial geometrical imperfections and welding residual stresses. The paper by Guedes Soares and Kmiecik (1993) addresses the collapse strength of rectangular steel plates under uniaxial compressive stress. A set of typical patterns of initial distortions were simulated so as to represent a random sample of typical distortions in ship plating. The strength of the set of initially distorted plates was calculated using a non-linear finite element code. The variability of the resulting ultimate plate strength was observed to depend on plate slenderness and simulation results obtained were compared with previous results.

Náprstek (1991) explained about stochastic finite element methodology taking large displacement as source of nonlinearities and studied about the response of the structures with random imperfection of Gaussian type. Sadvovsky and Bulaz (1996) discussed about FORM – based inverse reliability method. One of the conclusions was that realistic treatment of effects of imperfections on strength of the structure will lead to marginally conservative design which in turn relaxes the fabrication tolerances when adopting such a probabilistic approach. This idea was adopted to determine the reliability for unstiffened thin plates and girders of double symmetric I- cross section in compression and bending. Warren (1997) generated random geometrical imperfections by linear combinations of eigen buckling affine mode shapes using 2^k factorial design of Design of Experiments (DoE) and the variance of the models were maintained within the tolerance of manufacturing and adopted RSM to determine reliability of framed structures. Featherston (2001) discussed about the imperfection sensitivity of flat plates under combined compression and shear loading. It was concluded that an increase in the amplitude of imperfections reduce both the pre buckling stiffness and the collapse load of the plate. Further, it was stated that modification of shape of imperfections also changes the pre buckling stiffness and the collapse load.

Bielewicz and Gorski (2002) developed a simulation method to generate random geometrical imperfections using non-homogeneous two dimensional random fields on regular nets. Schenk and Schueller (2003) in their work, using imperfection databank at Delft University of Technology, generated geometrical imperfection models utilizing Karhunen-Loève expansion method. From the deterministic analysis of random models, buckling strength distribution was obtained and from which the reliability of the structure was determined using Monte Carlo Technique. Papadopoulos and Papadrakakis (2004) developed a nonlinear triangular composites element to carry out structural stability analysis of thin shell structures with random geometrical initial imperfections, which can be described as a two-dimensional uni-variate (2D-1V) homogeneous stochastic field. In the work of Sadvovsky *et al.* (2005 & 2006), the strength of rectangular/square simply supported plates of different aspect ratios and slenderness ratios subjected to longitudinal in-plane compression was obtained by finite element code assuming elasto-plastic material properties and large deflection capabilities. Initial deflections were taken from the database given in the reference Kmiecik *et al.* (1995). Also effect of the shapes of buckling modes, compound and localized modes, on collapse load was studied. One of the important conclusions was that single buckling mode shape does not yield the lowest capacity on the studied interval of imperfections. Visweswaran *et al.* (2006) studied about the effect of imperfection sensitivity on the collapse load of thin plates under axial compression taking eigen mode shapes as imperfection pattern. One of the major conclusions was that the collapse load of the imperfect plate is more than two times the buckling load of the perfect plate. Ikeda *et al.* (2007) studied about the imperfection sensitivity of the ultimate buckling strength of elastic-plastic square plates under compression. Finite displacement elastic-plastic analysis was conducted on the simply supported square plates under compression by varying the plate thickness and initial deflection as sinusoidal form. From the numerical results, extended power law was proposed to describe the ultimate buckling strength of the elastic-plastic square plates.

Craig and Roux (2007) also used the Karhunen-Loève expansion as a method to incorporate random geometrical imperfections into the FE buckling analysis and verified the numerical results with other numerical results and experimental results. In the work of Sadvovsky *et al.* (2007), reliability calculations were calculated based on the lower strength and strength values calculated for measured initial imperfections. The resistance to failure was identified as a function of two random variables, one is integral energy measure and the other one is shape factor that describes the effects of uncertainty of imperfection shapes on the plate surface. This approach was explained on a rectangular plate under longitudinal compression considering the influence of random field of imperfections on plate strength, and this approach may lead to significantly less conservative design. In the work of Papadopoulos *et al.* (2009), the effect of material and thickness spatial variation on the buckling load of isotropic shells with random initial geometric imperfections was investigated. The main novelty of this work is that a non-Gaussian assumption is

made for the distribution of the modulus of elasticity and the shell thickness which were described by 2D-1V homogeneous non-Gaussian stochastic fields. The initial geometric imperfections were described as a 2D-1V Gaussian non-homogeneous stochastic field with properties derived from corresponding experimental measurements. From this study, it was shown that the choice of the probability distribution for the description of the material and thickness variability is crucial since it affects significantly the buckling load of imperfection sensitive shell-type structures.

In the present work, random geometrical imperfections are generated using first 10 eigen affine mode shapes of perfect plate taken for study as suggested by Arbocz and Hol (1991), Chryssanthopoulos and Poggi (1995) and combine linearly according to linear multimode combinations following 2^k factorial design of Design of Experiments (DoE) and the variance of the models were maintained within the tolerance of manufacturing as suggested by Warren (1997). From the deterministic FE analysis, strength distribution is obtained and using which the reliability of structure is determined using MVFOSM method

3. FE Modeling

An eight noded quadrilateral shell element, SHELL93 of ANSYS is used for modeling the thin plates. This element can handle membrane, bending and transverse shear effect besides forming curvilinear surface satisfactorily. This element also has plasticity, stress stiffening, large deflection and large strain capabilities.

3.1. Thin Plate Shell Model

Harada and Fujikubo (2002) considered rectangular plates with cutout having the shorter side length b of 800 mm and aspect ratio a/b of 1, 2, 3 and 4 for elastic buckling eigen value analysis. For elastoplastic large deflection analysis, they considered rectangular plates with circular cutout with $a = 2000$ mm, $b = 1000$ mm and thickness ranging from 8 to 30 mm. But in the present work, a square plate (without any cutout) is taken for the study, the dimension and material properties of the plate are as given below.

Length (L)	= 1 m	Young's modulus (E)	= 205.8 GPa
Width (W)	= 1 m	Yield stress (σ_y)	= 313.6 Mpa
Thickness (t)	= 8 mm	Mass density (ρ)	= 7800 kg/m ³
Poisson's ratio (γ)	= 0.3	Zero strain hardening effect is assumed.	

3.2 Boundary Conditions

Simply supported boundary conditions as shown in Figure 1, are applied on all the edges of the thin plate and uniform displacement loading is applied on one side of the plate model and corresponding opposite side is restrained from moving along load direction (Harada and Fujikubo 2002).

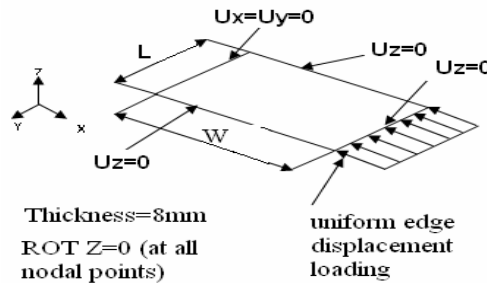


Figure 1. Geometry, boundary conditions, and loading conditions used in buckling analysis of a thin plate (not to scale)

3.3 Model validation and determination of eigen affine mode shapes

The mesh convergence study is done to choose the optimum number of elements for the analysis and it is found that 40 elements along both directions gives accurate solution and hence same number of elements are used for all analysis. The analytical solution (Timoshenko and Gere, 1965) of the perfect thin plates is compared with the FE eigen buckling analysis result at different modes as shown in Table 1 and thus FE model validation is ensured.

3.4 Modeling of imperfect plates

To achieve the aim of randomness, i.e., amplitude of imperfections at any nodal point of FE model (except the nodes at boundary edges of thin plate model should be random) and the first ten eigen affine mode shapes of linear buckling mode shapes of thin plate should be combined linearly using 2^k factorial design of Design of Experiments (DoE).

Table 1. Comparison of analytical solution with FE eigen buckling analysis result.

Mode No	Number of transverse half lobes (m)	Number of longitudinal half lobes (n)	Buckling Strength (N)		% Error
			Analytical Solution	FE Solution	
1	1	1	380936	378115.3	0.7404
2	2	1	595213	591989.6	0.5415
3	3	1	1058160	1053573.8	0.4334
4	2	2	1523750	1510774.2	0.8515
5	4	1	1720170	1712254.2	0.4601
6	3	2	1788280	1773286.6	0.8384
7	4	2	2385590	2361591.1	1.0059
8	1	2	2385590	2367998.9	0.7373
9	5	1	2580260	2560567.4	0.7632
10	5	2	3210060	3176441.5	1.0472

The modeling of the initial random geometrical imperfections is accomplished using the following assumptions/conditions.

- ❖ Δ - imperfection amplitudes at all nodes except the nodes at the boundary edges should follow independent normal distribution
- ❖ Mean value of imperfection amplitude of a node from all random models should be made equal to zero.
- ❖ Equal importance should be given for the all eigen affine mode shapes considered for random modeling.
- ❖ The random imperfection shapes generated should be linear combinations of the eigen affine mode shapes considered.

Based on the above assumptions, the nodal amplitude of imperfection vector for the entire structure (except the edge nodes, where the displacements are constrained) is given by

$$\Delta_{ix1} = \Phi_{ixj} \times M_{jx1} \tag{1}$$

where, Δ - Nodal imperfection amplitude vector

Φ - The matrix of eigen vectors containing the modal imperfection amplitudes at all nodal points of selected eigen affine mode shapes with equal maximum amplitude of imperfections

M - Modal imperfection magnitude vector

i - number of nodes

j - number of eigen affine mode shapes

If the nodal amplitudes of imperfections are known, the modal imperfection magnitudes can be obtained using the relation

$$M_{jx1} = \Phi^*_{jxi} \times \Delta_{ix1} \tag{2}$$

where, the matrix Φ^* is the pseudo-inverse of the matrix Φ . The pseudo-inverse is calculated using the following equation based on method of least squares

$$\Phi^* = (\Phi^T \Phi)^{-1} \times \Phi^T \tag{3}$$

If the nodal imperfections Δ_i are independent normally distributed random variables then the mean value and variance of each modal magnitude is given by

$$\mu_{Mj} = \sum_{i=1}^j \Phi^*_{ji} \mu_{\Delta i} \tag{4}$$

$$\sigma^2_{mj} = \sum_{i=1}^j (\Phi^*_{ji})^2 \sigma^2_{\Delta i} \tag{5}$$

where, $\mu_{\Delta i}$ and σ^2_{Δ} - mean and variance of the nodal imperfection amplitude respectively
 μ_M and σ^2_M - mean and variance of the modal imperfection magnitude respectively.

Similarly, mean and variance of each nodal amplitude is given by

$$\mu_{\Delta i} = \sum_{j=1}^j \Phi_{ji} \mu_{Mj} \tag{6}$$

$$\sigma^2_{\Delta i} = \sum_{j=1}^j (\Phi_{ji})^2 \sigma^2_{mj} \tag{7}$$

Since it is required to have nodal amplitude Δ_i of any node i of the structure to follow normal distribution with $\mu_{\Delta} = 0$ and as per Eqn. 4, μ_M also becomes zero. Hence, to get amplitude of imperfections of all nodes for each model, the modal magnitude of each model has to be obtained by using Eqn. 5. Using the modal magnitudes obtained from previous step the nodal amplitudes of imperfections can be obtained by using the Eqn. 1. Thus by varying the modal magnitudes of imperfections randomly using 2^k factorial design matrix of Design of Experiments, random geometrical imperfection models can be generated.

3.5 Steps Followed in Random Geometrical Imperfections Modeling

Step –I : Initially, substitute variance of modal imperfection magnitude vector as

$$\sigma^2_M = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{8}$$

Step-II : Using Eqn.(7) the variance of nodal imperfection amplitude vector σ^2_{Δ} is determined.

Step –III : Each element of the resulting σ^2_{Δ} vector from Step-I is normalized with the maximum value of element in that vector and multiplied σ^2_{tol} with value so as to limit the maximum amplitude of imperfections.

Step –IV : Using the σ^2_{Δ} vector obtained from the Step-III, new σ^2_M vector is found using Eqn.5.

Step –V : Since, $\mu_{\Delta} = 0, \mu_M = 0$, using σ^2_M new vector determine the modal imperfection magnitude vector M such that $M = \pm \sigma_M$.

Step –VI : Using 2^k factorial design, design matrix is generated and each column of design matrix is selected and is multiplied with corresponding element in the M vector obtained from previous step. This new design matrix is used to generate 2^k (for $k=10, 2^{10} = 1024$) random geometrical imperfection models.

i.e., $\Delta = \Phi \times \text{new design matrix} \tag{9}$

With the value of modal imperfection magnitude vector M , Δ nodal imperfection vector is determined using the Eqn.1. But the \pm value of the modal imperfection magnitude is decided by +1 or -1 of design matrix obtained from DoE. The Δ matrix, thus formed has 1024 rows, and each row corresponds to nodal displacements of all nodes of one random imperfect plate model. By adopting the procedure explained above, 512 pairs of mirror image random imperfect plate models can be generated.

Here, in the present work, 1024 random geometrical imperfect plate models are generated keeping RMS value of imperfections = 1.711 mm and the maximum amplitude of imperfection is maintained within ± 8 mm (Featherston, 2003). The maximum amplitude of imperfections in all 1024 models are shown in Figure.2

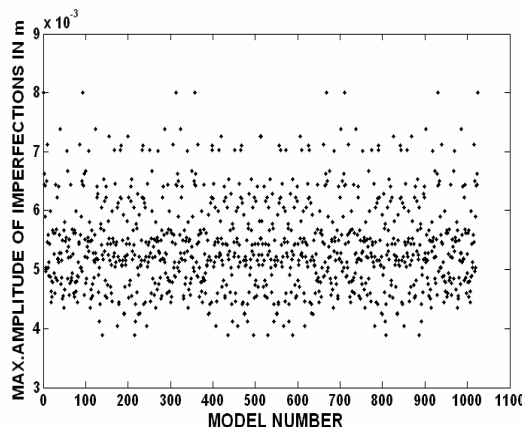


Figure 2. Scatter of maximum amplitude of imperfections from 1024 models

From the Figure 2, it can be noted that maximum amplitude of imperfections from model number 1 to 512 are exactly mirrored between model numbers 1024 to 513. A sample of a pair of thin plate models with mirror image random imperfections are shown in Figure 3.



Figure 3. A pair of mirror image random imperfections plate models (amplitude enlarged by 50 times)

To verify the assumptions made that imperfection amplitude of a node except boundary nodes are randomly distributed, the distribution of out of plane displacements of a particular node from all 1024 random plate models is plotted as shown in Figure.4(a) & (b).

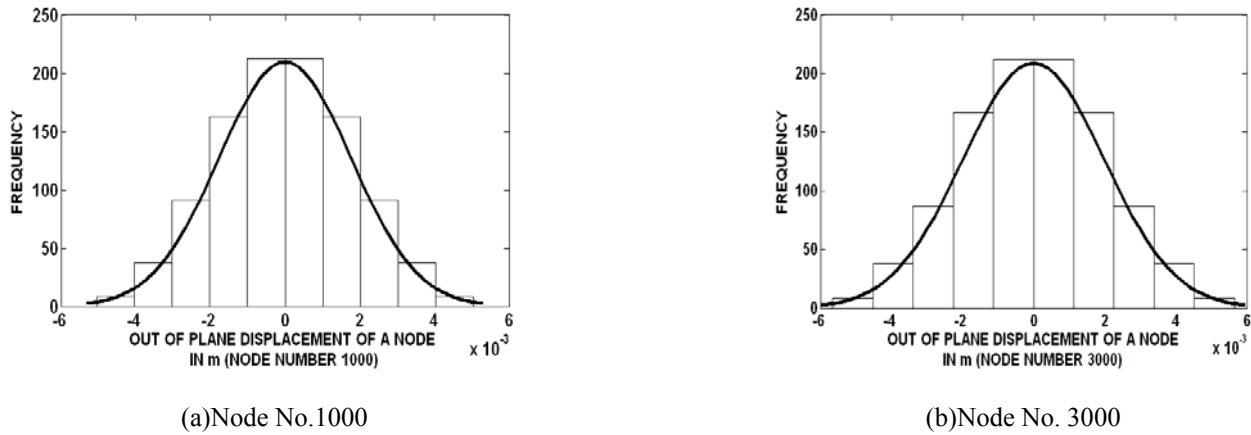


Figure 4. Normal distribution of out of plane displacements of a particular node from all 1024 random plate models

From the Figure.4, it can be seen that the out of plane displacement of nodal point distribution follows normal distribution with mean $(\mu_{\Delta}) = 0$.

4. Reliability Analysis

For any structure, the strength and load are highly probabilistic and their distribution are non-Gaussian in nature (Papadopoulos et al 2009) and the normal distribution is a good approximation. Hence, assuming that the strength (S) and load (L) are normally distributed as shown in Figure 5, the failure function is defined as,

$$G = S - L \tag{10}$$

Then, the distribution of failure function $f_G(g)$ is shown in Figure 6.

The probability of failure of the structure is,

$$P_f = P(G < 0) \tag{11}$$

The reliability of the structure is given as,

$$R = 1 - P_f \tag{12}$$

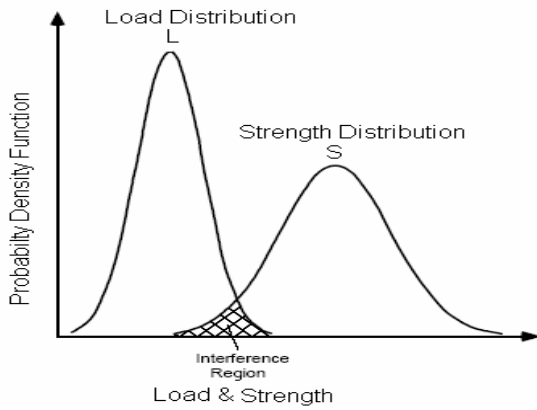


Figure 5. Load and Strength distribution

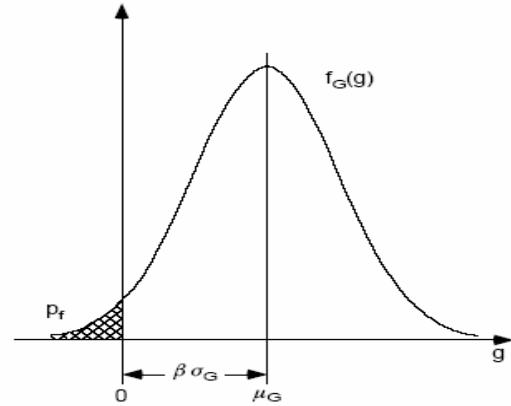


Figure 6. Normal distribution of failure function

In the First Order Second Moment (FOSM) method of determining the reliability of the structure, the mean and variance of the random variables (in this case, the strength and load) are considered. The first order approximation of failure function $f_G(g)$ is used for finding the mean and variance of the failure function. Thus, the mean and variance of the strength and load variables are required in order to carry out the reliability analysis.

5. Results and Discussion

Using non-linear FE analysis, buckling strength of first 512 models is determined including both material and geometrical nonlinearities. Determining the buckling strength of the next 512 models is nothing but inverting the first 512 models and obtaining the buckling strength. For reliability calculation buckling strength of 1024 models or first 512 models can be considered because it will not affect the reliability calculations.

By considering 1024 models, only the frequency of buckling strength values occurrence will be doubled. But here for reliability calculation, buckling strength ratio (BSR) of first 512 models is considered. For calculation purpose, first eigen mode buckling strength of perfect plate (to be called as eigen strength of the perfect plate here onwards) is taken as reference. Buckling strength ratio (BSR) can be defined as ratio between ultimate collapse strength of imperfect plate to the eigen strength of perfect plate. Since thin plates are having positive post buckling behavior, its BSR values are greater than 1 (Featherston, 2003).

Figure 7 shows the stiffness curve obtained for model No.1. From this figure it can be seen that at limit load condition at which the plate structure fails, as the slope of the stiffness curve becomes zero. Figure 8 shows the von Mises stress contour obtained for model No.1 at limit load condition.

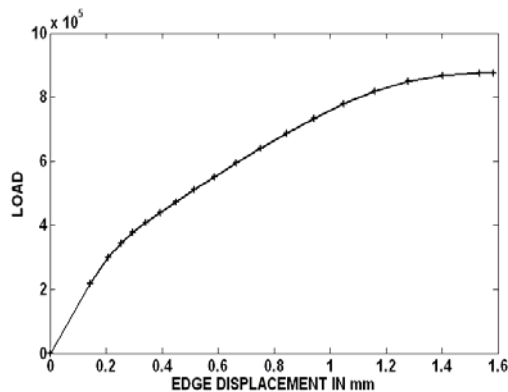


Figure 7. Stiffness curves of pair of mirror image random imperfection of model No. 1

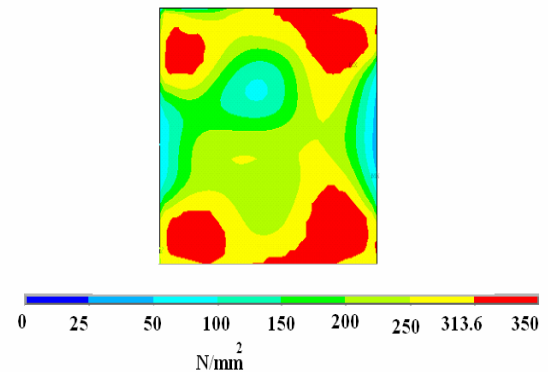


Figure 8. The von Mises stress contour of random imperfection of model No.1 at its limit load condition

Table 2 shows the BSR values obtained for few of the first 512 models and is shown as frequency graph in Figure 9. Usually the ultimate collapse strength of the imperfect plate will be more than 2 times the buckling strength of perfect plates as mentioned by Featherston (2001, 2003) and Visweswaran et al (2006). From the BSR values given in Table 2, here also it can be noted that the ultimate collapse strength of the imperfect plate is more than 2 times the eigen strength of the perfect plate. From the Figure 9, it can be seen that the distribution does not follow normal distribution exactly, but it is a skewed distribution.

Table 2. BSR values obtained from select few of the first 512 FE thin plate models with random geometrical imperfections

Model No.	BSR	Model No.	BSR	Model No.	BSR	Model No.	BSR	Model No.	BSR	Model No.	BSR
1	2.2988	86	2.2792	171	2.4609	256	2.4421	341	2.3857	493	2.4782
2	2.3786	87	2.4527	172	2.5912	257	2.301	342	2.2807	494	2.4836
3	2.334	88	2.324	173	2.5613	258	2.3759	343	2.4619	495	2.4902
4	2.4262	89	2.3621	174	2.6014	259	2.3438	344	2.3279	496	2.5234
5	2.3581	90	2.358	175	2.5372	260	2.4356	345	2.374	497	2.5666
6	2.3621	91	2.4039	176	2.5982	261	2.3593	346	2.359	498	2.5045
7	2.3895	92	2.3896	177	2.4951	262	2.3741	347	2.4311	499	2.5774
8	2.4038	93	2.3786	178	2.4787	263	2.3935	348	2.3935	500	2.5104
9	2.2793	94	2.2987	179	2.5495	264	2.4311	349	2.3758	501	2.4992
10	2.3822	95	2.4262	180	2.4935	265	2.2808	350	2.301	502	2.4073
11	2.3241	96	2.334	181	2.4919	266	2.386	351	2.4356	503	2.5657
12	2.4525	97	2.2808	182	2.4193	267	2.3281	352	2.3437	504	2.433
13	2.346	98	2.386	183	2.5435	268	2.462	353	2.2793	505	2.4825
14	2.4305	99	2.3281	184	2.4436	269	2.349	354	2.3822	506	2.4527
15	2.3858	100	2.462	185	2.5683	270	2.443	355	2.3241	507	2.5442
16	2.4481	101	2.349	186	2.4848	271	2.3892	356	2.4525	508	2.4723
17	2.3689	102	2.4439	187	2.6058	272	2.4767	357	2.346	509	2.4629
18	2.3291	103	2.3892	188	2.5027	273	2.3876	358	2.432	510	2.3898
19	2.4352	104	2.4748	189	2.5099	274	2.3493	359	2.3858	511	2.5125
20	2.367	105	2.301	190	2.4012	275	2.4401	360	2.4429	512	2.4264

Table 3. Mean BSR values obtained for 100 samples with sample size of 200 each

Sample No.	Mean BSR	Sample No.	Mean BSR	Sample No.	Mean BSR	Sample No.	Mean BSR	Sample No.	Mean BSR	Sample No.	Mean BSR
1	2.5594	18	2.4825	35	2.4617	52	2.4289	69	2.4992	86	2.5126
2	2.4848	19	2.4359	36	2.4356	53	2.3848	70	2.4724	87	2.5442
3	2.4356	20	2.3692	37	2.4742	54	2.5073	71	2.4084	88	2.4932
4	2.4522	21	2.4117	38	2.3888	55	2.5983	72	2.4357	89	2.5234
5	2.3787	22	2.373	39	2.4084	56	2.4749	73	2.4357	90	2.2887
6	2.2988	23	2.5049	40	2.5737	57	2.4908	74	2.4311	91	2.3671
7	2.4056	24	2.3821	41	2.4995	58	2.4522	75	2.2887	92	2.444
8	2.5392	25	2.3473	42	2.347	59	2.4507	76	2.3281	93	2.5373
9	2.3935	26	2.5442	43	2.4752	60	2.2987	77	2.4763	94	2.4908
10	2.4788	27	2.4357	44	2.3265	61	2.3264	78	2.463	95	2.5683
11	2.349	28	2.4522	45	2.5373	62	2.3264	79	2.4303	96	2.4522
12	2.3698	29	2.3594	46	2.2887	63	2.349	80	2.4193	97	2.5373
13	2.2792	30	2.5104	47	2.3692	64	2.4191	81	2.373	98	2.3622
14	2.4012	31	2.4357	48	2.4919	65	2.3341	82	2.4992	99	2.5844
15	2.4943	32	2.3936	49	2.5373	66	2.4359	83	2.514	100	2.4359
16	2.4823	33	2.4056	50	2.4352	67	2.3877	84	2.5594		
17	2.5772	34	2.3499	51	2.3291	68	2.5073	85	2.3858		

Since the normal distribution shape is the simplest, best developed, most known and expedient (Verderaime, 1994), the skewed strength distribution is converted into an equivalent normal distribution using the method suggested by Verderaime (1994). According to this method, the mode of the strength distribution is taken as the mean of the equivalent normal distribution. The left side of the skewed distribution is alone considered for the equivalent normal distribution. To obtain the right side of the distribution, the left side distribution is mirrored about the mode. Thus, the equivalent normal distribution of strength is obtained and it is shown in Figure 10.

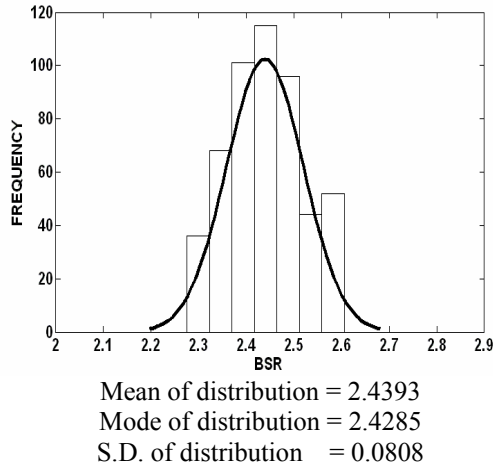


Figure 9. Actual Strength distribution obtained using BSR values

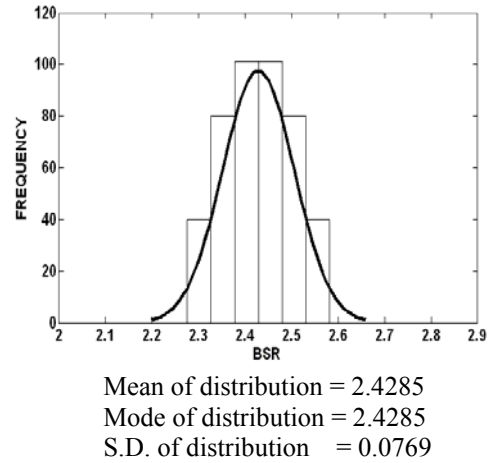


Figure 10. Equivalent normal strength distribution required for reliability calculations.

Another method adopted is based on Central Limit Theorem. According to Central limit theorem, if a random sample of n observations is selected from any population and when the sample size is sufficiently large ($n \geq 30$), the sampling distribution of the mean tends to approximate to the normal distribution. The larger the sample size, the better will be the approximation to normal distribution.

Hence, 100 samples were taken with each set containing 200 observations (Smith and Wells, 2006) drawn randomly from BSR of 512 models. The mean of 200 observations taken randomly in each sample was calculated and the means of all 100 samples are shown in Table 3 and also plotted in Figure 11. Figure 12 shows the equivalent normal strength distribution obtained from means of 100 samples (shown in Figure 11) and it was found that it deviates slightly from the normal distribution at 5% level of significance. The mean of the distribution obtained using Central Limit Theorem differs from the mean of the actual distribution by only -0.098%. Moreover, the skewness of the distribution is approximately = 0 (i.e., -0.05), which also confirms that the distribution is Gaussian.

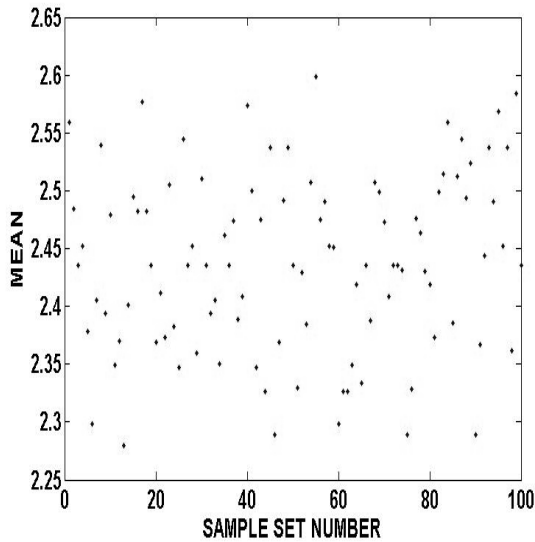


Figure 11. Sample set number and their corresponding mean

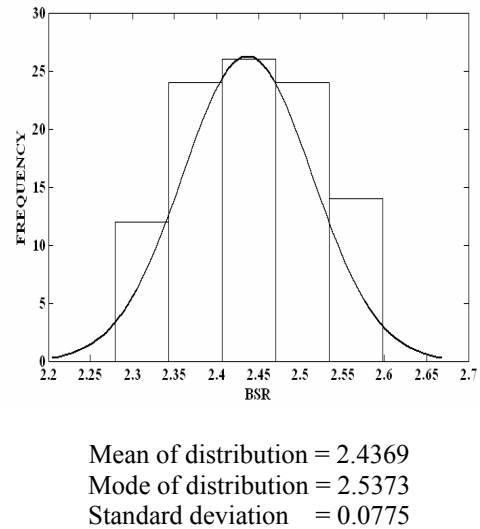


Figure 12. Equivalent strength distribution obtained by Central Limit Theorem for reliability calculations

According to the Mean Value First Order Second Moment (MVFOSM) method, the reliability index is defined as

$$\beta = \frac{\mu_S - \mu_L}{\sqrt{\sigma_S^2 + \sigma_L^2}} \quad (13)$$

where μ_S = Mean of strength distribution
 μ_L = Mean of load distribution

σ_S = S.D. of strength distribution
 σ_L = S.D. of load distribution

The probability of failure is given by,

$$P_f = \Phi(-\beta) \tag{14}$$

where, Φ = cumulative normal distribution function.

Then, reliability of the structure is given as,

$$R = 1 - P_f \tag{15}$$

In this case, the load applied is assumed as a deterministic single value. Hence, $\sigma_L = 0$ and now β is defined as,

$$\beta = (\mu_S - \text{Load applied in terms of BSR}) / \sigma_S \tag{16}$$

By varying the load applied, the reliability of the structure at each load is obtained. The failure probabilities at different loads are shown in Figure 13.

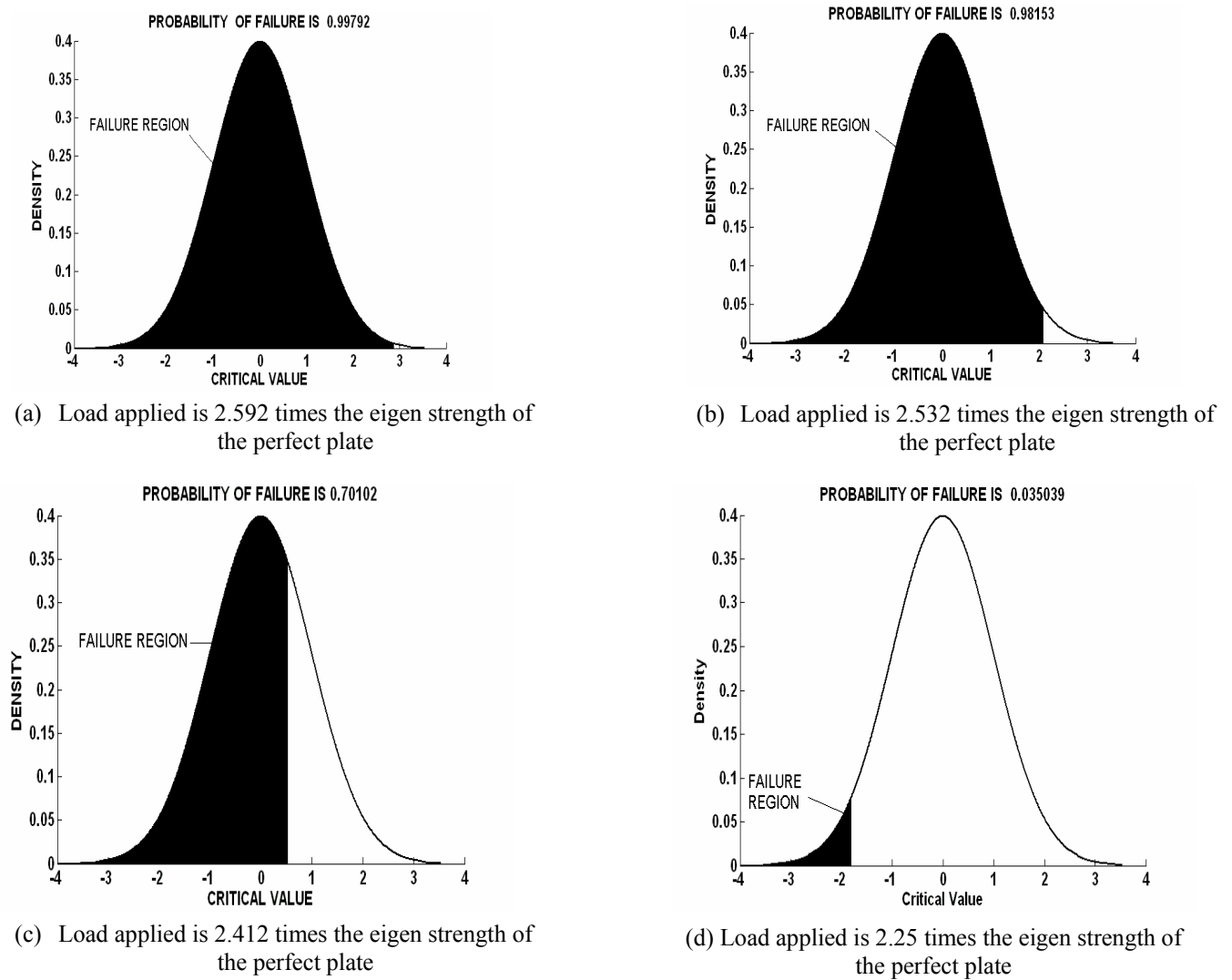


Figure 13. Failure probability at different loads

The variation of reliability found using MVFOSM method with respect to the applied load (BSR) is shown in Figure 14 & 15. From the reliability curves, it can be noted that for the plate taken for study, upto 2.1 times of eigen strength of the perfect plate, the reliability is 100%, when the load applied is more than 2.65 times the eigen strength of the perfect plate, the reliability is zero.

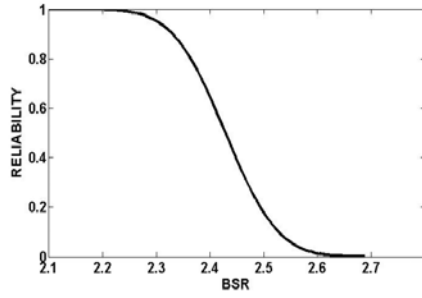


Figure 14. Reliability Vs BSR (Verderaime, 1994)

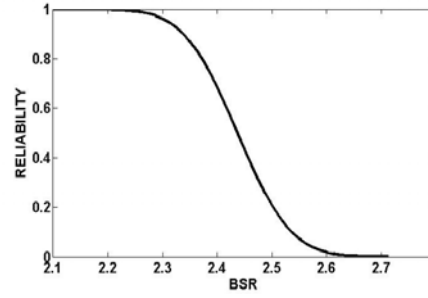


Figure 15. Reliability Vs BSR (Smith and Wells, 2006)

Table 4 shows the comparison of the reliability values obtained from the method adopted by Verderaime (method-I) and Smith and Wells (method-II). From the table, it was found that both the methods gave approximately same values.

Table 4. Comparison of reliability obtained using two methods

BSR	Reliability by Method-I (Verderaime, 1994)	Reliability by Method-II (Smith and Wells, 2006)	Difference
2	1	1	0
2.1	0.999993	0.999990	0.000003
2.2	0.998880	0.998511	0.000369
2.3	0.961303	0.952611	0.008692
2.4	0.682826	0.644836	0.037990
2.5	0.207600	0.176733	0.030867
2.6	0.017638	0.012967	0.004671
2.7	0.000342	0.000211	0.000131

5. Conclusions

The following conclusions are derived from the analysis carried out for the thin plate structure taken for study.

1. The slope of stiffness curve decreases gradually as the load applied increases and becomes zero at limit load condition and thereby imperfect thin plate shell structures collapse.
2. To increase reliable prediction of safe load of the structure further, more number of eigen affine mode shapes can be considered.
3. Ultimate crushing strength of the imperfect thin plate is more than two times the eigen strength of the perfect plate.
4. Using the adopted MVFOSM method of reliability, it is found that the reliability of thin plate taken for study under axial compression is 100% upto 2.1 times the eigen strength of the perfect plate and the reliability becomes zero when the load applied is more than 2.65 times the eigen strength of the perfect plate.
5. The methods adopted by Verderaime and Smith and Wells for reliability calculations are in good agreement with each other and hence recommended for determining reliability of imperfect plate structures.

Nomenclature

E	Young's modulus	P_f	Probability of failure
$f_G(g)$	Distribution of failure function	R	Reliability of structure
G	Failure function	ROTZ	Rotation about z-direction
i	Number of nodes	S	Strength distribution
j	Number of eigen affine mode shapes	t	Thickness of the plate
l	Length of the plate	U_x	Displacement along x-direction
L	Load distribution	U_y	Displacement along y-direction
m	Number of transverse half lobes	U_z	Displacement along z-direction
M	Modal imperfection magnitude vector	w	Width of the plate
n	Number of longitudinal half lobes		

Greek Letters

γ	Poisson's ratio	Φ	Eigen vector matrix
σ_y	Yield strength	Φ^*	Pseudo inverse of Φ matrix
ρ	Mass density	Φ^T	Transpose of Φ matrix
Δ	Nodal imperfection amplitude vectors	μ_S	Mean of strength distribution
σ_{Δ}^2	Variance of Nodal imperfection amplitude vector	μ_L	Mean of load distribution
σ_M^2	Variance of Modal imperfection amplitude vector	σ_S	Standard deviation of strength distribution
σ_{tol}^2	Variance of tolerance	σ_L	Standard deviation of load distribution
μ_{Δ}	Mean of Nodal imperfection amplitude vector	\square	Cumulative normal distribution function
μ_M	Mean of Modal imperfection amplitude vector	β	Safety index

Abbreviations

2D-1V	Two-dimensional Uni-variate	MVFOSM	Mean Value First Order Second Moment
BSR	Buckling Strength Ratio	RSM	Response Surface Methodology

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