Propagation of shear waves in an irregular magnetoelastic monoclinic layer sandwiched between two isotropic half-spaces

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Abstract

  In the present paper we study the propagation of horizontally polarised shear waves (SH waves) in an internal magnetoelastic monoclinic stratum with rectangular irregularity in lower interface and is sandwiched between two semi-infinite isotropic elastic media. The dispersion equation has been obtained. It is observed that the dispersion equation is in assertion with the classical Love-type wave equation, for isotropic layer sandwiched between two isotropic half spaces, in the absence of magnetic field and irregularity. The effects of depth of irregularity and monoclinic-magnetoelastic coupling parameters on dispersion curves are depicted by means of graphs. This study shows the remarkable effect of wave number, size of irregularity and magnetic field on the phase velocity.

Keywords: SH wave, magnetoelastic monoclinic, irregularity, dispersion equation, perturbation

1. Introduction

  Many results of theoretical and experimental studies revealed that a real earth is considerably more complicated than the models presented earlier. This has led to a need for more realistic representation as a medium through which seismic waves propagate. The study of wave propagation in elastic medium with different boundaries is of great importance to seismologists as well as to geophysicists to understand and predict the seismic behaviour at the different margins of earth.
  The propagation of Love waves has been studied by many authors with assuming different forms of irregularities at the interface. Bhattacharya (1962) discussed the dispersion curves for Love wave propagation in a transversely isotropic crustal layer with an irregularity in the interface. Chattopadhyay (1975) studied the Love wave propagation due to irregularity in the thickness of non-homogeneous crustal layer. The wave propagation in crystalline media plays a very interesting role in geophysics and also in ultrasonic and signal processing. Chattopadhyay and Bandyopadhyay (1986) studied the propagation of shear waves in an infinite monoclinic crystal plate as well as in infinite anisotropic non-homogeneous monoclinic plate. The study of wave propagation in irregular boundaries is also of equal importance to seismologists and geophysicists. Ge and Chen (2007) proposed a direct boundary element method for calculating elastic wave propagation in two-dimensional irregularly stratified models. Further a more efficient approach was developed by same authors (Ge and Chen, 2008) for seismic-wave simulation in a stratified medium with irregular interfaces. In particular different authors have done the study of SH-wave propagation in different irregular mediums. Chattopadhyay *et al*. (1983) studied the propagation of SH guided wave in an internal stratum with parabolic irregularity in the lower interface. Campillo (2006) studied the case of a dome in a stratified medium for modelling of SH-wave propagation in an irregularly layered medium. Ding and Dravinski (1998) studied the scattering of SH-waves in irregular interfaces. Reflection and transmission of SH-wave at a corrugated interface has been studied before. Kaur *et al*. (2005) obtained the Reflection and transmission coefficients due to incident plane SH-waves at a corrugated interface between two isotropic, laterally and vertically heterogeneous visco-elastic solid half spaces. Tomar and Kaur (2007) studied the reflection and transmission of SH-waves at a corrugated interface between anisotropic elastic and viscoelastic solid half-spaces.
  Keeping in the mind the characteristic and structure of earth, the medium of monoclinic is of core interest for seismologists to study the seismic wave propagation. Singh and Tomar (2007) studied the propagation of quasi-P wave at the interface between two monoclinic half-spaces. Kalyani *et al*. (2008) studied the propagation of SH waves in the plane of mirror symmetry of a

monoclinic multilayered medium with displacement normal to the plane. Recently, the propagation of SH waves in an irregular monoclinic crustal layer has been studied by Chattopadhyay *et al*. (2008). In this paper we have discussed the propagation of SH waves in an irregular magnetoelastic monoclinic layer lying in between two isotropic half-spaces. The irregularity is in the form of rectangle. The perturbation technique indicated by Eringen and Samuels (1959) has been used. It is observed that phase velocity is strongly influenced by the depth of irregularity and magnetic field. These important peculiarities are highlighted by means of graphs.

2. Formulation of the problem
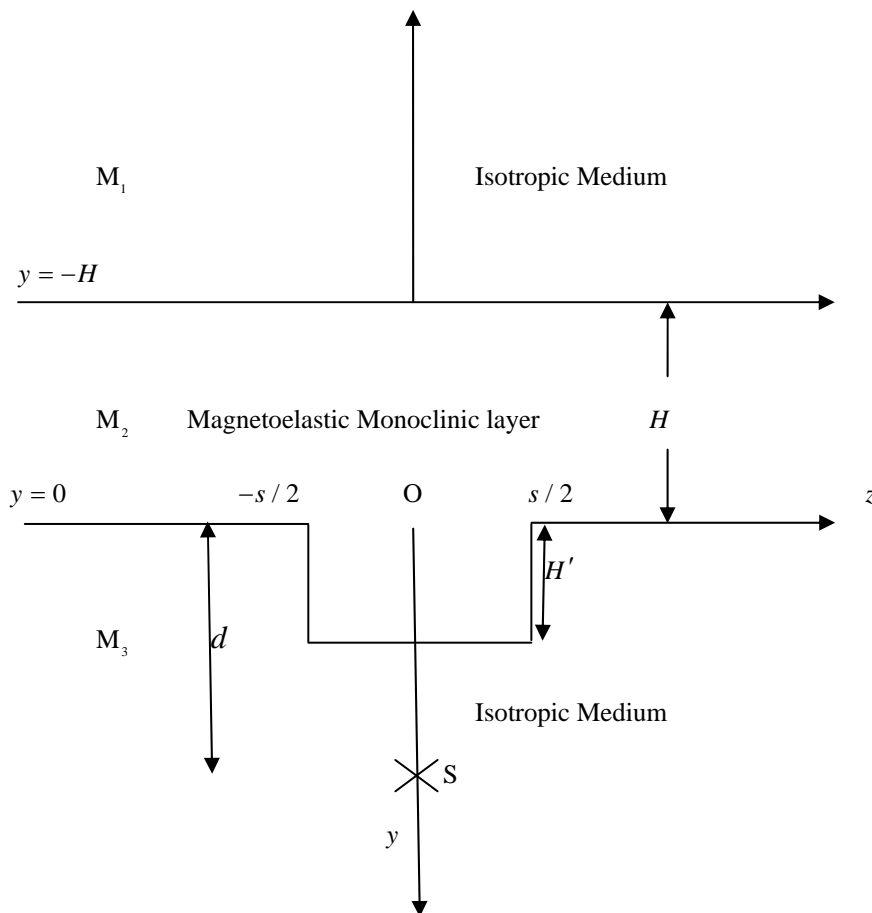
Figure 1: Geometry of the problem

We consider the model, which is comprised of perfectly conducting magnetoelastic monoclinic irregular internal stratum, M_2 sandwiched between two isotropic elastic semi-infinite media M_1 (upper) and M_3 (lower) as shown in Figure 1. The irregularity is considered in the form of rectangle on the lower interface of the internal stratum. We assume the rectangular irregularity with the length s and depth H'. Let us choose the rectangular coordinate system with the origin O at the middle point of the span, s, of the irregularity. We have taken y-axis vertically downwards and z axis along the regular interface between the lower semi-infinite medium and the monoclinic magnetoelastic layer. Let H be the thickness of the internal stratum except for the irregular region and S be the position of the source on the y-axis at a depth d below the origin ($d > H'$).
The equation of the interface between layer and lower half space is defined as

$$y = \varepsilon h(z) \tag{1}$$

where

$$h(z) = \begin{cases} 0 & \text{for } z \leq \dfrac{-s}{2}, \ z \geq \dfrac{s}{2} \\ f(z) & \text{for } \dfrac{-s}{2} \leq z \leq \dfrac{s}{2} \end{cases}$$

where $\varepsilon = \dfrac{H'}{s}$ and $\varepsilon \ll 1$.

Let us consider μ_r, ρ_r, u_r $(r = 1, 2, 3)$ as the rigidities, densities and displacements of the upper half space, sandwiched layer, lower half space respectively.

For upper (M_1) and lower (M_3) semi-infinite medium,

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} = \frac{1}{\beta_1^2} \frac{\partial^2 u_1}{\partial t^2} \tag{2}$$

and

$$\frac{\partial^2 u_3}{\partial y^2} + \frac{\partial^2 u_3}{\partial z^2} = \frac{1}{\beta_3^2} \frac{\partial^2 u_3}{\partial t^2} \tag{3}$$

where $\beta_1 = \sqrt{\dfrac{\mu_1}{\rho_1}}$ and $\beta_3 = \sqrt{\dfrac{\mu_3}{\rho_3}}$ are the shear wave speed in the semi-infinite medium M_1 and M_3.

For the monoclinic magnetoelastic layer (M_2), the stain-displacement relations for monoclinic layer are

$$S_1 = \frac{\partial u}{\partial x}, \; S_2 = \frac{\partial v}{\partial y}, \; S_3 = \frac{\partial w}{\partial z}, \; S_4 = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}, \; S_5 = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \; S_6 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \tag{4}$$

where u, v, w are displacement components in the direction x, y, z respectively, and

S_i $(i = 1, 2, ..., 6)$ are the strain components.

Also, the stress-strain relation for a rotated y-cut quartz plate, which exhibits monoclinic symmetry with x being the diagonal axis are

$$\begin{aligned}
T_1 &= C_{11}S_1 + C_{12}S_2 + C_{13}S_3 + C_{14}S_4, \\
T_2 &= C_{12}S_1 + C_{22}S_2 + C_{23}S_3 + C_{24}S_4, \\
T_3 &= C_{13}S_1 + C_{23}S_2 + C_{33}S_3 + C_{34}S_4, \\
T_4 &= C_{14}S_1 + C_{24}S_2 + C_{34}S_3 + C_{44}S_4, \\
T_5 &= C_{55}S_5 + C_{56}S_6, \\
T_6 &= C_{56}S_5 + C_{66}S_6
\end{aligned} \tag{5}$$

where T_i $(i = 1, 2, ..., 6)$ are the stress components and $C_{ij} = C_{ji}$ $(i = 1, 2, ..., 6)$ are the elastic constants.

Equations governing the propagation of small elastic disturbances in a perfectly conducting monoclinic medium having electromagnetic force $\mathbf{J} \times \mathbf{B}$ (the Lorentz force, $\mathbf{J}$ being the electric current density and $\mathbf{B}$ being the magnetic induction vector) as the only body force are

$$\frac{\partial T_1}{\partial x} + \frac{\partial T_6}{\partial y} + \frac{\partial T_5}{\partial z} + \left(J \times B \right)_x = \rho_2 \frac{\partial^2 u_2}{\partial t^2},$$

$$\frac{\partial T_6}{\partial x} + \frac{\partial T_2}{\partial y} + \frac{\partial T_4}{\partial z} + \left(J \times B \right)_y = \rho_2 \frac{\partial^2 v_2}{\partial t^2}, \tag{6}$$

$$\frac{\partial T_5}{\partial x} + \frac{\partial T_4}{\partial y} + \frac{\partial T_3}{\partial z} + \left(J \times B \right)_z = \rho_2 \frac{\partial^2 w_2}{\partial t^2}$$

where ρ_2 is the density of the layer.

For SH wave propagating in the z- direction and causing displacement in the x- direction only, we shall assume that

$$u_2 = u_2 \left(y, z, t \right), \; v_2 = w_2 = 0 \text{ and } \frac{\partial}{\partial x} \equiv 0. \tag{7}$$

Using Eqs. (4) and (7), the stress-strain relation (5) becomes

$$T_1 = T_2 = T_3 = T_4 = 0,$$

$$T_5 = C_{55} \frac{\partial u_2}{\partial z} + C_{56} \frac{\partial u_2}{\partial y}, \tag{8}$$

$$T_6 = C_{56} \frac{\partial u_2}{\partial z} + C_{66} \frac{\partial u_2}{\partial y}.$$

Using Eq. (8) in Eq. (6), we have

$$C_{66} \frac{\partial^2 u_2}{\partial y^2} + 2C_{56} \frac{\partial^2 u_2}{\partial y \partial z} + C_{55} \frac{\partial^2 u_2}{\partial z^2} + (J \times B)_x = \rho_2 \frac{\partial^2 u_2}{\partial t^2}. \tag{9}$$

The well known Maxwell's equations governing the electromagnetic field are $\nabla . B = 0, \nabla \times E = -\dfrac{\partial B}{\partial t}, \nabla \times H = J$ with

$$B = \mu_e H, J = \sigma \left(E + \frac{\partial u_i}{\partial t} \times B \right) \tag{10}$$

where E is the induced electric field, J is the current density vector and magnetic field H includes both primary and induced magnetic fields. μ_e and σ are the induced permeability and conduction coefficient respectively.

The liberalized Maxwell's stress tensor $\left(\tau_{ij}^0 \right)^{M_x}$ due to the magnetic field is given by $\left(\tau_{ij}^0 \right)^{M_x} = \mu_e \left(H_i h_j + H_j h_i - H_k h_k \delta_{ij} \right).$

Let $H = (H_1, H_2, H_3)$ and $h_i = (h_1, h_2, h_3)$ where h_i is the change in the magnetic field. In writing the above equations, we have neglected the displacement current.

From Eq. (10), we get

$$\nabla^2 H = \mu_e \sigma \left\{ \frac{\partial H}{\partial t} + \nabla \times \left(\frac{\partial u_i}{\partial t} \times H \right) \right\}. \tag{11}$$

In component form, Eq. (11) can be written as

$$\frac{\partial H_1}{\partial t} = \frac{1}{\mu_e \sigma} \nabla^2 H_1 + \frac{\partial \left(H_2 \dfrac{\partial u_2}{\partial t} \right)}{\partial y} + \frac{\partial \left(H_3 \dfrac{\partial u_2}{\partial t} \right)}{\partial z},$$

$$\frac{\partial H_2}{\partial t} = \frac{1}{\mu_e \sigma} \nabla^2 H_2, \tag{12}$$

$$\frac{\partial H_3}{\partial t} = \frac{1}{\mu_e \sigma} \nabla^2 H_3.$$

For perfectly conducting medium, $\sigma \rightarrow \infty$ the equation (12) become

$$\frac{\partial H_2}{\partial t} = \frac{\partial H_3}{\partial t} = 0, \tag{13}$$

and

$$\frac{\partial H_1}{\partial t} = \frac{\partial \left(H_2 \dfrac{\partial u_2}{\partial t} \right)}{\partial y} + \frac{\partial \left(H_3 \dfrac{\partial u_2}{\partial t} \right)}{\partial z}. \tag{14}$$

It is clear from Eq. (13) that there is no perturbation in H_2 and H_3, however from Eq. (14) there may be perturbation in H_1. Therefore, taking small perturbation, say h_1 in H_1, we have

$$H_1 = H_{01} + h_1, \; H_2 = H_{02} \text{ and } H_3 = H_{03}, \text{ where } (H_{01}, H_{02}, H_{03}) \text{ are components of the initial magnetic field } H_0.$$

We can write $H_0 = (0, H_0 \cos \phi, H_0 \sin \phi)$, where $H_0 = |H_0|$ and ϕ is the angle at which the wave crosses the magnetic field. Thus we have

$$H = (h_1, H_0 \cos \phi, H_0 \sin \phi). \tag{15}$$

We shall take initial value of h_1 as $h_1 = 0$. Using Eq. (15) in Eq. (14), we get

$$\frac{\partial h_1}{\partial t} = \frac{\partial\left(H_0 \cos\phi \dfrac{\partial u_2}{\partial t}\right)}{\partial y} + \frac{\partial\left(H_0 \sin\phi \dfrac{\partial u_2}{\partial t}\right)}{\partial z}. \tag{16}$$

Integrating with respect to t, we get

$$h_1 = H_0 \cos\phi \frac{\partial u_2}{\partial y} + H_0 \sin\phi \frac{\partial u_2}{\partial z}. \tag{17}$$

Considering $\nabla\left(\dfrac{H^2}{2}\right) = -(\nabla \times H) \times H + (H.\nabla)H$ and Eqs (10), we get

$$J \times B = \mu_e \left\{ -\nabla\left(\frac{H^2}{2}\right) + (H.\nabla)H \right\}. \tag{18}$$

In the component form Eq. (18) can be written as

$$(J \times B)_y = (J \times B)_z = 0$$

and $\tag{19}$

$$(J \times B)_x = \mu_e H_0^2 \left(\cos^2\phi \frac{\partial^2 u_2}{\partial y^2} + \sin 2\phi \frac{\partial^2 u_2}{\partial y \partial z} + \sin^2\phi \frac{\partial^2 u_2}{\partial z^2} \right).$$

Using Eqs. (9) and (19), we find the equation of motion for the magnetoelastic monoclinic layer in the form

$$M_{66} \frac{\partial^2 u_2}{\partial y^2} + 2M_{56} \frac{\partial^2 u_2}{\partial y \partial z} + M_{55} \frac{\partial^2 u_2}{\partial z^2} = \rho_2 \frac{\partial^2 u_2}{\partial t^2}. \tag{20}$$

where

$$M_{66} = C_{66}\left(1 + m_H \cos^2\phi\right),$$

$$M_{55} = C_{66}\left(\frac{C_{55}}{C_{66}} + m_H \sin^2\phi\right), \tag{21}$$

$$M_{56} = C_{66}\left(\frac{C_{56}}{C_{66}} + m_H \cos\phi \sin\phi\right).$$

Where $m_H = \dfrac{\mu_e H_0^2}{C_{66}}$ is monoclinic-magnetoelastic coupling parameter. Here in equations (2), (3) and (20) suffixes "1", "2" and

"3" are used to represent the upper semi-infinite medium (M_1), monoclinic layer (M_2) and lower semi-infinite medium (M_3) respectively.

The boundary conditions are as follows:

(i) The stresses are continuous at the interface of M_1 and M_2,

$$\mu_1 \frac{\partial u_1}{\partial y} = M_{56} \frac{\partial u_2}{\partial z} + M_{66} \frac{\partial u_2}{\partial y} \qquad at \ y = -H . \tag{22}$$

(ii) The Displacements are continuous at the interface of M_1 and M_2,

$$u_1 = u_2 \quad at \ y = -H . \tag{23}$$

(iii) The stresses are continuous at the interface of M_2 and M_3,

$$(M_{56} - M_{55}\varepsilon h') \frac{\partial u_2}{\partial z} + (M_{66} - M_{56}\varepsilon h') \frac{\partial u_2}{\partial y} = \mu_3 \left(\frac{\partial u_3}{\partial y} - \varepsilon h' \frac{\partial u_3}{\partial z} \right) \qquad at \ y = \varepsilon h(z) \tag{24}$$

where $h' = \dfrac{dh(z)}{dz}$.

(iv) The Displacements are continuous at the interface of M_2 and M_3,

$$u_2 = u_3 \quad at \ y = \varepsilon h(z). \tag{25}$$

3. Solution of the problem

Let us assume $u_i = U_i(y,z)e^{i\omega t}$ $(i = 1, 2, 3)$. Under this assumption equations (2), (3) and (20) reduce to

$$\frac{\partial^2 U_1}{\partial y^2} + \frac{\partial^2 U_1}{\partial z^2} + \frac{\omega^2}{\beta_1^2} U_1 = 0, \tag{26}$$

$$\frac{\partial^2 U_3}{\partial y^2} + \frac{\partial^2 U_3}{\partial z^2} + \frac{\omega^2}{\beta_3^2} U_3 = 0 \tag{27}$$

and

$$M_{66} \frac{\partial^2 U_2}{\partial y^2} + 2M_{56} \frac{\partial^2 U_2}{\partial y \partial z} + M_{55} \frac{\partial^2 U_2}{\partial z^2} + \rho_2 \omega^2 U_2 = 0. \tag{28}$$

where ω is the circular frequency.

Defining the Fourier transform $\overline{U}_r(y, \eta)$ of $U_r(y, z)$ $(r = 1, 2, 3)$ as

$$\overline{U}_r(y, \eta) = \int_{-\infty}^{\infty} U_r(y, z)e^{i\eta z} dz,$$

We find the inverse Fourier transform as

$$U_r(y, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{U}_r(y, \eta)e^{-i\eta z} d\eta.$$

Using Fourier transform in equations (26), (27) and (28), we have

$$\frac{d^2 \overline{U}_1}{dy^2} - p_1^2 \overline{U}_1 = 0, \tag{29}$$

$$\frac{d^2 \overline{U}_3}{dy^2} - p_3^2 \overline{U}_3 = 0 \tag{30}$$

and

$$\frac{d^2 \overline{U}_2}{dy^2} + a \frac{d\overline{U}_2}{dy} + p_2^2 \overline{U}_2 = 0 \tag{31}$$

where

$$p_1^2 = \eta^2 - \frac{\omega^2}{\beta_1^2}, \quad p_2^2 = \frac{\omega^2}{\beta_2^2} - \frac{M_{55}}{M_{66}} \eta^2, \quad p_3^2 = \eta^2 - \frac{\omega^2}{\beta_3^2} \text{ and } a = -2i\eta \frac{M_{56}}{M_{66}}.$$

The appropriate solutions of Eqs. (29) and (30) can be taken as

$$\overline{U}_1 = A e^{p_1 y} \tag{32}$$

and

$$\overline{U}_3 = E e^{-p_3 y}. \tag{33}$$

The solution of Eq. (31) is

$$\overline{U}_2 = e^{-\frac{a}{2} y} (B \cos py + D \sin py) \tag{34}$$

where

$$p = \sqrt{p_2^2 - \frac{a^2}{4}} = \left(\frac{\omega^2}{\beta_2^2} - \frac{M_{55}}{M_{66}} \eta^2 + \frac{M_{56}^2}{M_{66}^2} \eta^2 \right)^{1/2}.$$

The constants A, B, D and E are functions of η.

The displacements in M_1, M_2 and M_3 are

$$U_1(y, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A e^{p_1 y} e^{-i\eta z} d\eta, \tag{35}$$

$$U_2(y, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\frac{a}{2}y} \left(B \cos py + D \sin py \right) e^{-i\eta z} d\eta, \tag{36}$$

and

$$U_3(y, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(E e^{-p_3 y} + \frac{2}{p_3} e^{p_3 y} e^{-p_3 d} \right) e^{-i\eta z} d\eta. \tag{37}$$

where the second term in the integrand of U_3 is introduced due to the effect of source at S in M_3 (Willis, 1948). We set the following approximations due to small value of ε

$$B \cong B_0 + B_1\varepsilon, \ D \cong D_0 + D_1\varepsilon, \ E \cong E_0 + E_1\varepsilon. \tag{38}$$

Since the boundary is not uniform at the interface of M_2 and M_3 so the term B, D and E appearing in Eq.(38) are also function of ε, expanding these terms in ascending powers of ε and keeping in view that ε is small and so retaining the term up to the first order of ε, hence approximated as in Eq.(38). These assumptions are justified in the real earth model where the depth H' of the irregular boundary is too small with respect to the length of the boundary s.

Also for small ε, following approximations can be accepted

$$e^{\pm v\varepsilon h} \cong 1 \pm v\varepsilon h, \ \cos p\varepsilon h \cong 1, \ \sin p\varepsilon h \cong p\varepsilon h.$$

where v is any quantity.

Using the boundary conditions (i), (ii) and (iv), we have the following equations

$$M_{56}\left\{\left(B_0 + B_1\varepsilon\right)\cos pH - \left(D_0 + D_1\varepsilon\right)\sin pH\right\}\left(-i\eta\right)e^{\frac{aH}{2}} - M_{66}\left\{\left(B_0 + B_1\varepsilon\right)\cos pH\right.$$

$$-\left(D_0 + D_1\varepsilon\right)\sin pH\left\}\left(\frac{a}{2}\right)e^{\frac{aH}{2}} + M_{66}\left\{\left(B_0 + B_1\varepsilon\right)\sin pH + \left(D_0 + D_1\varepsilon\right)\cos pH\right\}\left(p\right)e^{\frac{aH}{2}} \tag{39}$$

$$= \mu_1 A p_1 e^{-p_1 H},$$

$$\left\{\left(B_0 + B_1\varepsilon\right)\cos pH - \left(D_0 + D_1\varepsilon\right)\sin pH\right\}e^{\frac{aH}{2}} = A e^{-p_1 H} \tag{40}$$

and

$$\varepsilon \int_{-\infty}^{\infty} \left(\frac{a}{2}B_0 - pD_0 - p_3 E_0 + 2e^{-p_3 d}\right)h(z) e^{-i\eta z} d\eta$$

$$= \int_{-\infty}^{\infty} \left\{\left(B_0 - E_0 - \frac{2}{p_3}e^{-p_3 d}\right) + \varepsilon(B_1 - E_1)\right\}e^{-i\eta z} d\eta. \tag{41}$$

Now applying the Fourier transform of $h(z)$ defined as

$$\overline{h}(\lambda) = \int_{-\infty}^{\infty} h(z)e^{i\lambda z} dz \tag{42}$$

so that

$$h(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{h}(\lambda)e^{-i\lambda z} d\lambda \tag{43}$$

and

$$h'(z) = -\frac{i}{2\pi} \int_{-\infty}^{\infty} \lambda \overline{h}(\lambda)e^{-i\lambda z} d\lambda. \tag{44}$$

Now applying the transform (43) and (44) on (41), we have

$$\frac{\varepsilon}{2\pi} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \left(\frac{a}{2}B_0 - pD_0 - p_3 E_0 + 2e^{-p_3 d}\right)\overline{h}(\lambda) e^{-i(\eta+\lambda)z} d\eta \right] d\lambda$$

$$= \int_{-\infty}^{\infty} \left\{\left(B_0 - E_0 - \frac{2}{p_3}e^{-p_3 d}\right) + \varepsilon(B_1 - E_1)\right\}e^{-i\eta z} d\eta. \tag{45}$$

Taking $\eta + \lambda = k$ for the inner integral in the left-hand side of the Eq. (45), so that λ may be treated as a constant such that $d\eta = dk$, replacing η by k in the right-hand side of the Eq.(45), and finally after taking Fourier transform as defined above we can have

$$\left(B_0 - E_0 - \frac{2}{p_3} e^{-p_3 d} \right) + \varepsilon (B_1 - E_1) = \varepsilon R_1(k) \tag{46}$$

where

$$R_1(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\frac{a}{2} B_0 - pD_0 - p_3 E_0 + 2e^{-p_3 d} \right]^{\eta = k - \lambda} \overline{h}(\lambda) d\lambda \tag{47}$$

From the boundary condition (iii), we obtain

$$M_{66} \left\{ \frac{a}{2} B_0 - pD_0 + \varepsilon(\frac{a}{2} B_1 - pD_1) \right\} + iM_{56} k \left(B_0 + B_1 \varepsilon \right) - \mu_3 (p_3 E_0 - 2e^{-p_3 d} + \varepsilon p_3 E_1)$$

$$= \varepsilon R_2(k) \tag{48}$$

where

$$R_2(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\left\{ M_{66} \left(\frac{a^2}{4} B_0 - p^2 B_0 - apD_0 \right) + M_{56} \left(\frac{a}{2} B_0 - pD_0 \right) ik - \mu_3 \left(p_3^2 E_0 + 2p_3 e^{-p_3 d} \right) \right\} \right.$$

$$\left. + \lambda \left\{ iM_{56} \left(D_0 p - \frac{a}{2} B_0 \right) + M_{55} kB_0 - k\mu_3 \left(E_0 + \frac{2}{p_3} e^{-p_3 d} \right) \right\} \right]^{\eta = k - \lambda} \overline{h}(\lambda) d\lambda. \tag{49}$$

Equating the absolute term (term not containing ε) and the coefficient of ε from Eqs. (39), (40), (46) and (48) we get

$$B_0 \left(-i\eta M_{56} \cos pH - \frac{a}{2} M_{66} \cos pH + pM_{66} \sin pH \right) e^{\frac{aH}{2}}$$

$$+ D_0 \left(i\eta M_{56} \sin pH + \frac{a}{2} M_{66} \sin pH + pM_{66} \cos pH \right) e^{\frac{aH}{2}} = Ap_1 \mu_1 e^{-p_1 H}, \tag{50}$$

$$B_1 \left(-i\eta M_{56} \cos pH - \frac{a}{2} M_{66} \cos pH + pM_{66} \sin pH \right) e^{\frac{aH}{2}}$$

$$+ D_1 \left(i\eta M_{56} \sin pH + \frac{a}{2} M_{66} \sin pH + pM_{66} \cos pH \right) e^{\frac{aH}{2}} = 0, \tag{51}$$

$$\left(B_0 \cos pH - D_0 \sin pH \right) e^{\frac{aH}{2}} = Ae^{-p_1 H}, \tag{52}$$

$$\left(B_1 \cos pH - D_1 \sin pH \right) e^{\frac{aH}{2}} = 0, \tag{53}$$

$$B_0 - E_0 - \frac{2}{p_3} e^{-p_3 d} = 0, \tag{54}$$

$$B_1 - E_1 = R_1(k), \tag{55}$$

$$B_0 \left(\frac{a}{2} M_{66} + ikM_{56} \right) - \left(pM_{66} \right) D_0 - \mu_3 \left(p_3 E_0 - 2e^{-p_3 d} \right) = 0, \tag{56}$$

and

$$B_1 \left(\frac{a}{2} M_{66} + ikM_{56} \right) - \left(pM_{66} \right) D_1 - (\mu_3 p_3) E_1 = R_2(k). \tag{57}$$

Solving the above eight equations, we get

$$B_0 = \frac{-8\mu_3 e^{-p_3 d}}{G(k)}\left(2ikM_{56}\tan pH + aM_{66}\tan pH + 2pM_{66} + 2p_1\mu_1\tan pH\right),$$

$$D_0 = \frac{-8\mu_3 e^{-p_3 d}}{G(k)}\left(2ikM_{56} + aM_{66} - 2pM_{66}\tan pH + 2p_1\mu_1\right),$$

$$E_0 = \frac{-2e^{-p_3 d}}{p_3 G(k)}\left(E_2 + E_3\right),$$

$$B_1 = \frac{-2\left(\mu_3 p_3 R_1 - R_2\right)}{G(k)}\left(2ikM_{56}\tan pH + aM_{66}\tan pH + 2pM_{66} + 2p_1\mu_1\tan pH\right),$$

$$D_1 = \frac{-2\left(\mu_3 p_3 R_1 - R_2\right)}{G(k)}\left(2ikM_{56} + aM_{66} - 2pM_{66}\tan pH + 2p_1\mu_1\right),$$

$$E_1 = \frac{\left(E_4 + E_5\right)}{G(k)},$$

$$A = \left(B_0\cos pH - D_0\sin pH\right)e^{\left(\frac{a}{2}+p_1\right)H},$$

$$E_2 = 4ikp_3\mu_3 M_{56}\tan pH + 2ap_3\mu_3 M_{66}\tan pH + 4p_3\mu_3 pM_{66} + 4p_3\mu_3 p_1\mu_1\tan pH,$$

$$E_3 = 4p^2 M_{66}^2\tan pH - 4pp_1\mu_1 M_{66} + 4ikaM_{56}M_{66}\tan pH + a^2 M_{66}^2\tan pH$$
$$+ 2ap_1\mu_1 M_{66}\tan pH - 4k^2 M_{56}^2\tan pH + 4ikp_1\mu_1 M_{56}\tan pH,$$

$$E_4 = 4ikR_2 M_{56}\tan pH + 2aR_2 M_{66}\tan pH + 4pR_2 M_{66} + 4p_1\mu_1 R_2\tan pH - 4ikaR_1 M_{56}M_{66}\tan pH$$
$$- 4ikR_1 p_1\mu_1 M_{56}\tan pH,$$

$$E_5 = -4p^2 R_1 M_{66}^2\tan pH + 4pp_1\mu_1 R_1 M_{66} - a^2 R_1 M_{66}^2\tan pH - 2ap_1\mu_1 R_1 M_{66}\tan pH + 4k^2 R_1 M_{56}^2\tan pH,$$

$$G(k) = 4p^2 M_{66}^2\tan pH - 4pp_1\mu_1 M_{66} + 4ikaM_{56}M_{66}\tan pH + a^2 M_{66}^2\tan pH + 2ap_1\mu_1 M_{66}\tan pH$$
$$- 4k^2 M_{56}^2\tan pH + 4ikp_1\mu_1 M_{56}\tan pH - 4ikp_3\mu_3 M_{56}\tan pH - 2p_3\mu_3 aM_{66}\tan pH$$
$$- 4p_3\mu_3 pM_{66} - 4p_3\mu_3 p_1\mu_1\tan pH.$$

With the help of above obtained values the displacement in the monoclinic layer is given by

$$U_2 = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{8\mu_3 e^{-p_3 d}e^{-\frac{a}{2}y}}{G(k)}\left[1 + \frac{\varepsilon(\mu_3 p_3 R_1 - R_2)}{4\mu_3 e^{-p_3 d}}\right]$$
$$\times\left\{\left(2ikM_{56}\tan pH + aM_{66}\tan pH + 2pM_{66} + 2p_1\mu_1\tan pH\right)\cos py\right.$$
$$\left. + \left(2ikM_{56} + aM_{66} - 2pM_{66}\tan pH + 2p_1\mu_1\right)\sin py\right\}e^{-ikz}dk. \tag{58}$$

From Eq. (1), the interface for the rectangular irregularity gives

$$\bar{h}(\lambda) = \frac{2s}{\lambda}\sin\left(\frac{\lambda s}{2}\right). \tag{59}$$

Further on simplification, we get

$$\mu_3 p_3 R_1 - R_2 = \frac{2s\mu_3}{\pi}\int_{-\infty}^{\infty}\{\psi(k-\lambda) + \psi(k+\lambda)\}\frac{1}{\lambda}\sin\left(\frac{\lambda s}{2}\right)d\lambda, \tag{60}$$

where

$$\psi(k-\lambda) = \left[\left(B_2 + B_3 + B_4 + B_5 + B_6 + B_7 + B_8 + B_9 + B_{10} + B_{11} + B_{12}\right)\frac{e^{-p_3 d}}{p_3 G(k)}\right]^{\eta = k-\lambda}$$

and

$$B_2 = -2a^2 p_3^2 \mu_3 M_{66} \tan pH - 2a^2 pp_3 M_{66}^2 - 8p^3 p_3 M_{66}^2 - 8p^2 p_1 p_3 \mu_1 M_{66} \tan pH + 2p_3^2 E(k),$$

$$B_3 = -4iak\mu_3 p_3^2 M_{66} \tan pH - 8p^2 p_3^2 \mu_3 M_{66} \tan pH + 4ap^2 p_3 M_{66}^2 \tan pH,$$

$$B_4 = 8ikpp_3^2 \mu_3 M_{66} + 4ika^2 p_3 M_{56} M_{66} \tan pH + a^3 p_3 M_{66}^2 \tan pH - 8iakpp_3 M_{56} M_{66},$$

$$B_5 = -4ap_1 p_3^2 \mu_1 \mu_3 \tan pH + 8pp_1 p_3^2 \mu_1 \mu_3 + 2a^2 p_1 p_3 \mu_1 M_{66} \tan pH - 8app_1 p_3 \mu_1 M_{66},$$

$$B_6 = -8\lambda kpp_3 M_{56}^2 + 4\lambda akp_3 M_{56}^2 \tan pH + 8i\lambda k^2 p_3 M_{55} M_{56} \tan pH + 8\lambda kpp_3 M_{55} M_{66},$$

$$B_7 = -4i\lambda k^2 p_3 \mu_3 M_{56} \tan pH - 2a\lambda k\mu_3 p_3 M_{66} \tan pH + 4a\lambda kp_3 M_{55} M_{66} \tan pH,$$

$$B_8 = -4\lambda kp^2 M_{66}^2 \tan pH - 4ia\lambda k^2 M_{56} M_{66} \tan pH - 4\lambda kpp_3 \mu_3 M_{66} + \lambda kE(k),$$

$$B_9 = 4\lambda k^3 M_{56}^2 \tan pH - a^2 \lambda kM_{66}^2 \tan pH - 4ak^2 p_3 M_{56}^2 \tan pH - 4\lambda kpp_1 \mu_1 M_{66},$$

$$B_{10} = -8i\lambda p^2 p_3 M_{56} M_{66} \tan pH - 2ia^2 \lambda p_3 M_{56} M_{66} \tan pH + 8k^2 pp_3 M_{56}^2,$$

$$B_{11} = 8i\lambda pp_1 p_3 \mu_1 M_{56} - 4ia\lambda p_1 p_3 \mu_1 M_{56} \tan pH + 8\lambda kp_1 p_3 \mu_1 M_{55} \tan pH,$$

$$B_{12} = 2a\lambda kp_1 \mu_1 M_{66} \tan pH + 4i\lambda k^2 p_1 \mu_1 M_{56} \tan pH - 4\lambda kp_1 p_3 \mu_1 \mu_3 \tan pH.$$

Here the argument of $\psi(k-\lambda)$ is because of $\eta + \lambda = k$. Following the asymptotic formula of Willis (1948) and neglecting the terms containing $2/s$ and higher powers of $2/s$ for large s, we get

$$\int_{-\infty}^{\infty} \{\psi(k-\lambda) + \psi(k+\lambda)\} \frac{1}{\lambda} \sin\left(\frac{\lambda s}{2}\right) d\lambda \cong \frac{\pi}{2} 2\psi(k) = \pi\psi(k), \text{ (cf. Tranter, 1966).}$$

(61)

Hence on using Eq. (61) on Eq. (60) one can have

$$\mu_3 p_3 R_1 - R_2 = 2s\mu_3 \psi(k) = 2\mu_3 \frac{H'}{\varepsilon} \psi(k).$$

(62)

Therefore, in view of Eq. (62) the displacement in the magnetoelastic monoclinic layer is

$$U_2 = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{8\mu_3 e^{-p_3 d} e^{-\frac{a}{2}y}}{G(k)\left[1 - \frac{H'}{2}\psi(k)e^{p_3 d}\right]} \left\{ (2ikM_{56} \tan pH + aM_{66} \tan pH + 2pM_{66}\right.$$

$$\left. + 2p_1 \mu_1 \tan pH) \cos py + (2ikM_{56} + aM_{66} - 2pM_{66} \tan pH + 2p_1 \mu_1) \sin py \right\} e^{-ikz} dk.$$

Since the value of this integral depends entirely on the contribution of the poles of the Integrand, the dispersion equation for SH waves is given by

$$G(k)\left[1 - \frac{H'}{2}\psi(k)e^{p_3 d}\right] = 0.$$

(63)

The poles are located at the roots of above equation which has been examined in our study of shear waves (cf. Achenbach, 1975). Eq. (63) may be rewritten as

$$D_2 + D_3 + D_4 - \frac{H'}{2}(D_5 + D_6 + D_7 + D_8) = 0$$

(64)

$$D_2 = 4p^2 M_{66}^2 \tan pH - 4pp_1 \mu_1 M_{66} + 4ikaM_{56} M_{66} \tan pH + a^2 M_{66}^2 \tan pH,$$

$$D_3 = 2ap_1 \mu_1 M_{66} \tan pH - 4k^2 M_{56}^2 \tan pH + 4ikp_1 \mu_1 M_{56} \tan pH - 4ikp_3 \mu_3 M_{56} \tan pH,$$

$$D_4 = -2p_3 \mu_3 aM_{66} \tan pH - 4p_3 \mu_3 pM_{66} - 4p_3 \mu_3 p_1 \mu_1 \tan pH,$$

$$D_5 = -2a^2 p_3^2 \mu_3 M_{66} \tan pH - 2a^2 pp_3 M_{66}^2 - 8p^2 p_1 p_3 \mu_1 M_{66} \tan pH + 2p_3^2 E(k),$$

$$D_6 = -4iak\mu_3 p_3^2 M_{66} \tan pH - 8p^2 p_3^2 \mu_3 M_{66} \tan pH + 4ap^2 p_3 M_{66}^2 \tan pH,$$

$$D_7 = 8ikpp_3^2 \mu_3 M_{66} + 4ika^2 p_3 M_{56} M_{66} \tan pH + a^3 p_3 M_{66}^2 \tan pH,$$

$$D_8 = -4ak^2 p_3 M_{56}^2 \tan pH + 8k^2 pp_3 M_{56}^2 - 8iakpp_3 M_{56} M_{66} - 8p^3 p_3 M_{66}^2.$$

Let us assume that the wave is propagating along the surface with c as the common wave velocity, and then we can set the following

$$p = kP, \ p_1 = kP_1, \ p_3 = kP_3 \ \text{and} \ a = kA$$

where $P = \left(\dfrac{c^2}{\beta_2^2} - \dfrac{M_{55}}{M_{66}} + \dfrac{M_{56}^2}{M_{66}^2}\right)^{1/2}, \ P_1 = \left(1 - \dfrac{c^2}{\beta_1^2}\right)^{1/2}, \ P_3 = \left(1 - \dfrac{c^2}{\beta_3^2}\right)^{1/2}, \ A = -2i\dfrac{M_{56}}{M_{66}}.$

In the view of above setting, Eq. (64) gives

$$\tan\left\{\sqrt{\left(\dfrac{c^2}{\beta_2^2} - \dfrac{M_{55}}{M_{66}} + \dfrac{M_{56}^2}{M_{66}^2}\right)} kH\right\} = \dfrac{J_1}{L_1} + i\dfrac{J_2}{L_2} \tag{65}$$

where

$$J_1 = \left(8PP_1\mu_1 M_{66} + 8PP_3\mu_3 M_{66} + 8H'kPP_1P_3\mu_1\mu_3 - 8H'kPP_1P_3\mu_1 M_{66} - 8H'kP^3 M_{66}^2 - 8H'kPP_3^2\mu_3 M_{66}\right)$$

$$\times\left(8P^2 M_{66}^2 - 8P_1P_3\mu_1\mu_3 + 8H'kP^2 P_3\mu_3 M_{66} - 8H'kP^2 P_3 M_{66}^2 + 8H'kP^2 P_1\mu_1 M_{66} + 8H'kP_3^2 P_1\mu_1\mu_3\right)$$

$$+\left(8H'kPP_3\mu_3 M_{56} + 8H'kPP_1\mu_1 M_{56}\right)\left(8H'kP^2 M_{56} M_{66} - 8H'kP_1P_3\mu_1\mu_3 M_{56} / M_{66}\right),$$

$$L_1 = \left(8P^2 M_{66}^2 - 8P_1P_3\mu_1\mu_3 + 8H'kP^2 P_3\mu_3 M_{66} - 8H'kP^2 P_3 M_{66}^2 + 8H'kP^2 P_1\mu_1 M_{66} + 8H'kP_3^2 P_1\mu_1\mu_3\right)^2$$

$$+\left(8H'kP^2 M_{56} M_{66} - 8H'kPP_1P_3\mu_1\mu_3 M_{56} / M_{66}\right)^2,$$

$$J_2 = \left(8H'kPP_3\mu_3 M_{56} + 8H'kPP_1\mu_1 M_{56}\right)\left(8P^2 M_{66}^2 - 8P_1P_3\mu_1\mu_3 + 8H'kP^2 P_3\mu_3 M_{66} - 8H'kP^2 P_3 M_{66}^2\right.$$

$$\left. + 8H'kP^2 P_1\mu_1 M_{66} + 8H'kP_3^2 P_1\mu_1\mu_3\right) - \left(8H'kP^2 M_{56} M_{66} - 8H'kPP_1P_3\mu_1\mu_3 M_{56} / M_{66}\right)$$

$$\times\left(8PP_1\mu_1 M_{66} + 8PP_3\mu_3 M_{66} + 8H'kPP_1P_3\mu_1\mu_3 - 8H'kPP_1P_3\mu_1 M_{66} - 8H'kP^3 M_{66}^2 - 8H'kPP_3^2\mu_3 M_{66}\right),$$

$$L_2 = \left(8P^2 M_{66}^2 - 8P_1P_3\mu_1\mu_3 + 8H'kP^2 P_3\mu_3 M_{66} - 8H'kP^2 P_3 M_{66}^2 + 8H'kP^2 P_1\mu_1 M_{66} + 8H'kP_3^2 P_1\mu_1\mu_3\right)^2$$

$$+\left(8H'kP^2 M_{56} M_{66} - 8H'kPP_1P_3\mu_1\mu_3 M_{56} / M_{66}\right)^2.$$

The real part of the Eq. (65) gives the dispersion equation of SH waves. The dispersion relation for the SH waves is

$$\tan\left\{\sqrt{\left(\dfrac{c^2}{\beta_2^2} - \dfrac{M_{55}}{M_{66}} + \dfrac{M_{56}^2}{M_{66}^2}\right)} kH\right\} = \dfrac{J_1}{L_1}. \tag{66}$$

4. Particular cases

Case 1:

When $m_H = 0$ the dispersion relation (66) reduces to

$$\tan\left\{\sqrt{\left(\dfrac{c^2}{\beta_2^2} - \dfrac{C_{55}}{C_{66}} + \dfrac{C_{56}^2}{C_{66}^2}\right)} kH\right\} = \dfrac{J_3}{L_3}$$

where

$$J_3 = \left(8PP_1\mu_1 C_{66} + 8PP_3\mu_3 C_{66} + 8H'kPP_1P_3\mu_1\mu_3 - 8H'kPP_1P_3\mu_1 C_{66} - 8H'kP^3 C_{66}^2 - 8H'kPP_3^2\mu_3 C_{66}\right)$$

$$\times\left(8P^2 C_{66}^2 - 8P_1P_3\mu_1\mu_3 + 8H'kP^2 P_3\mu_3 C_{66} - 8H'kP^2 P_3 C_{66}^2 + 8H'kP^2 P_1\mu_1 C_{66} + 8H'kP_3^2 P_1\mu_1\mu_3\right)$$

$$+\left(8H'kPP_3\mu_3 C_{56} + 8H'kPP_1\mu_1 C_{56}\right)\left(8H'kP^2 C_{56} C_{66} - 8H'kP_1P_3\mu_1\mu_3 C_{56} / C_{66}\right),$$

$$L_3 = \left(8P^2 C_{66}^2 - 8P_1P_3\mu_1\mu_3 + 8H'kP^2 P_3\mu_3 C_{66} - 8H'kP^2 P_3 C_{66}^2 + 8H'kP^2 P_1\mu_1 C_{66} + 8H'kP_3^2 P_1\mu_1\mu_3\right)^2$$

$$+\left(8H'kP^2 C_{56} C_{66} - 8H'kP_1P_3\mu_1\mu_3 C_{56} / C_{66}\right)^2.$$

which is the dispersion relation for SH waves propagating in an irregular monoclinic layer sandwiched between two isotropic half spaces.

Case 2:

When $\mu_1 = 0$ and $m_H = 0$ the dispersion relation (66) reduces to

$$\tan\left\{ \sqrt{\left(\frac{c^2}{\beta_2^2} - \frac{C_{55}}{C_{66}} + \frac{C_{56}^2}{C_{66}^2} \right)} \, kH \right\} = \frac{J_4}{L_4}$$

where

$$J_4 = 2P\left(4H'kP^2C_{66}^2 + 4H'kP_3^2\mu_3C_{66} - 4P_3\mu_3C_{66} \right)$$

$$\times \left(8H'kP^2P_3C_{66}^2 - 8P^2C_{66}^2 - 8H'kP^2P_3\mu_3C_{66} \right) + 64H'^2k^2P^3P_3\mu_3C_{56}^2C_{66},$$

$$L_4 = \left(8H'kP^2P_3C_{66}^2 - 8P^2C_{66}^2 - 8H'kP^2P_3\mu_3C_{66} \right)^2 + 64H'^2k^2P^4C_{56}^2C_{66}^2.$$

Which is the result obtained by Chattopadhyay *et al.* (2008) for SH waves propagating in an irregular monoclinic layer lying over an isotropic half space.

Case 3:

When $m_H = 0$ and $H' = 0$ the dispersion relation (66) reduces to

$$\tan\left\{ \sqrt{\left(\frac{c^2}{\beta_2^2} - \frac{C_{55}}{C_{66}} + \frac{C_{56}^2}{C_{66}^2} \right)} \, kH \right\} = \frac{PP_1\mu_1C_{66} + PP_3\mu_3C_{66}}{P^2C_{66}^2 - P_1P_3\mu_1\mu_3}$$

which is the dispersion relation for SH waves propagating in a regular monoclinic layer sandwiched between two isotropic half spaces.

Case 4:

When $C_{66} = C_{55} = \mu_2$, $C_{56} = 0$, $m_H = 0$ and $H' = 0$ the dispersion relation (66) reduces to

$$\tan\left\{ \sqrt{\left(\frac{c^2}{\beta_2^2} - 1 \right)} \, kH \right\} = \frac{P_2P_1\mu_1\mu_2 + P_2P_3\mu_3\mu_2}{P_2^2\mu_2^2 - P_1P_3\mu_1\mu_3}$$

where $P_2 = \left(\frac{c^2}{\beta_2^2} - 1 \right)^{1/2}$

This is the dispersion relation for SH waves propagating in a regular isotropic layer sandwiched between two isotropic half spaces.

Case 5:

When $C_{66} = C_{55} = \mu_2$, $C_{56} = 0$, $\mu_1 = 0$ and $m_H = 0$ the dispersion relation (66) reduces to

$$\tan\left\{ \sqrt{\left(\frac{c^2}{\beta_2^2} - 1 \right)} \, kH \right\} = \frac{J_5}{L_5}$$

where

$$P_2 = \left(\frac{c^2}{\beta_2^2} - 1 \right)^{1/2},$$

$$J_5 = 2P\left(4H'kP_2^2\mu_2^2 + 4H'kP_3^2\mu_3\mu_2 - 4P_3\mu_3\mu_2 \right)$$

$$\times \left(8H'kP_2^2P_3\mu_2^2 - 8P_2^2\mu_2^2 - 8H'kP_2^2P_3\mu_3\mu_2 \right),$$

$$L_5 = \left(8H'kP_2^2P_3\mu_2^2 - 8P_2^2\mu_2^2 - 8H'kP_2^2P_3\mu_3\mu_2 \right)^2.$$

This is the result obtained by Chattopadhyay (1975) for SH waves propagating in an irregular isotropic layer lying over an isotropic half space.

Case 6:

When $\mu_1 = 0$, $m_H = 0$ and $H' = 0$ the dispersion relation (66) reduces to

$$\tan\left\{\sqrt{\left(\frac{c^2}{\beta_2^2} - \frac{C_{55}}{C_{66}} + \frac{C_{56}^2}{C_{66}^2}\right)} \, kH\right\} = \frac{\mu_3 P_3}{C_{66} P}$$

which is the result obtained by Chattopadhyay and Pal (1983) for SH waves propagating in a regular monoclinic layer lying over an isotropic half space.

5. Numerical examples

For the case of an irregular monoclinic magnetoelastic layer between two isotropic half spaces we take the following data

(i) For upper isotropic half space, M_1 (Gubbins, 1990)

$$\mu_1 = 7.10 \times 10^{10} \, \text{N/m}^2,$$

$$\rho_1 = 3,321 \, \text{Kg/m}^3.$$

(ii) For monoclinic layer, M_2 (Tiersten, 1969)

$$C_{55} = 0.94 \times 10^{11} \, \text{N/m}^2, \qquad\qquad C_{56} = -0.11 \times 10^{11} \, \text{N/m}^2,$$

$$C_{66} = 0.93 \times 10^{11} \, \text{N/m}^2, \qquad\qquad \rho_1 = 7,450 \, \text{Kg/m}^3.$$

(iii) For lower isotropic half space, M_3 (Gubbins, 1990)

$$\mu_3 = 6.77 \times 10^{10} \, \text{N/m}^2,$$

$$\rho_1 = 3,323 \, \text{Kg/m}^3.$$

Moreover, the following data are used

$$m_H = \frac{\mu_e H_0^2}{C_{66}} = 0.0, 0.35, 0.65, 0.4, 0.8; \quad \phi = 10^\circ.$$

The effect of magnetic field and irregular boundary on the propagation of plane SH waves propagating in perfectly conducting monoclinic magnetoelastic layer with rectangular irregularity sandwiched between two isotropic elastic half spaces has been depicted by means of graphs. Figure 2 to 6 gives the variation of non-dimensional phase velocity (c/β_2) with respect to non-dimensional wave number kH for different values of monoclinic-magnetoelastic coupling parameters (m_H). The small change in the non-dimensional wave number produces substantial change in non-dimensional phase velocity. In each of these figures graphs are drawn for different sizes of irregularity. It is observed that as the size of irregularity (H'/H) increases, the phase velocity decreases. Also with the increase of parameter (m_H), the phase velocity decreases.
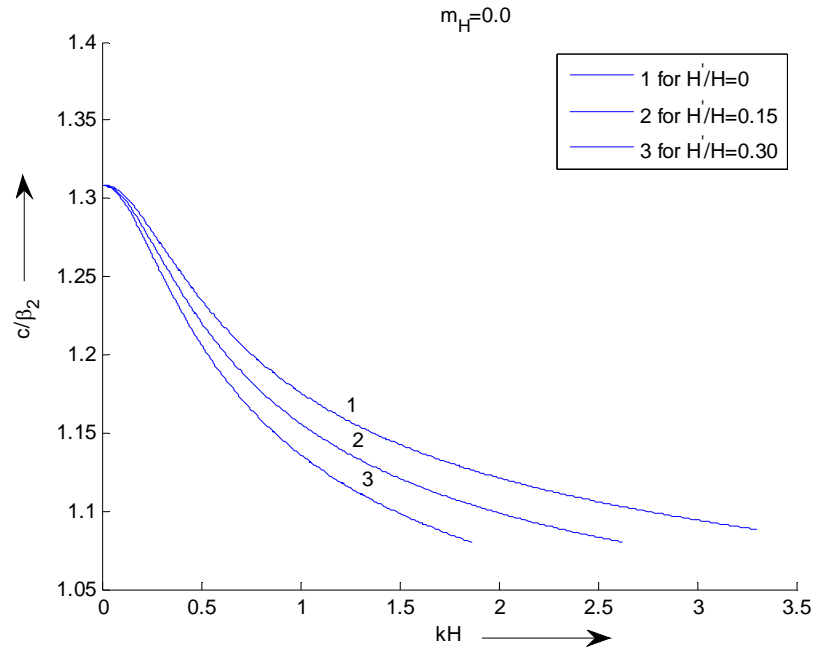
Figure 2: Dimensionless phase velocity against dimensionless wave number for $m_H = 0.0$.
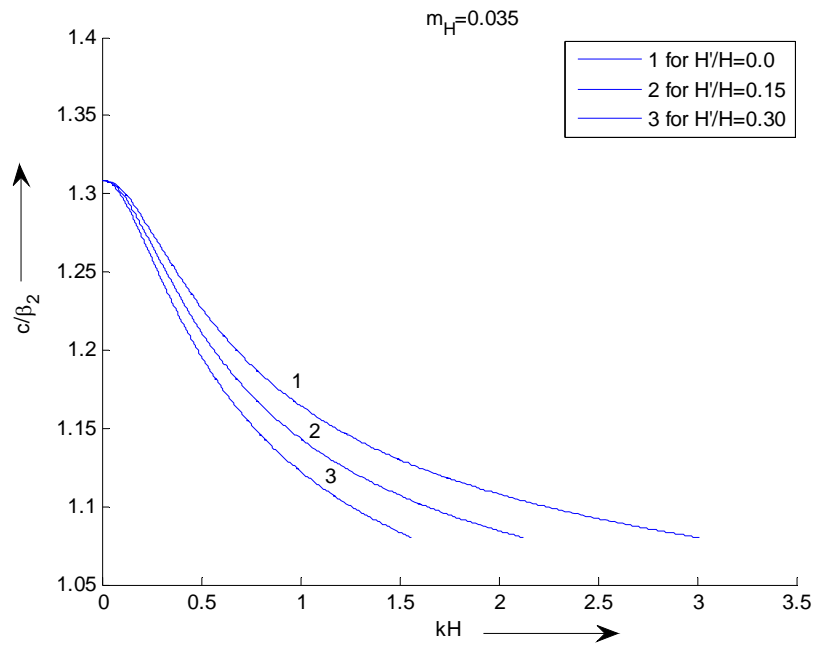
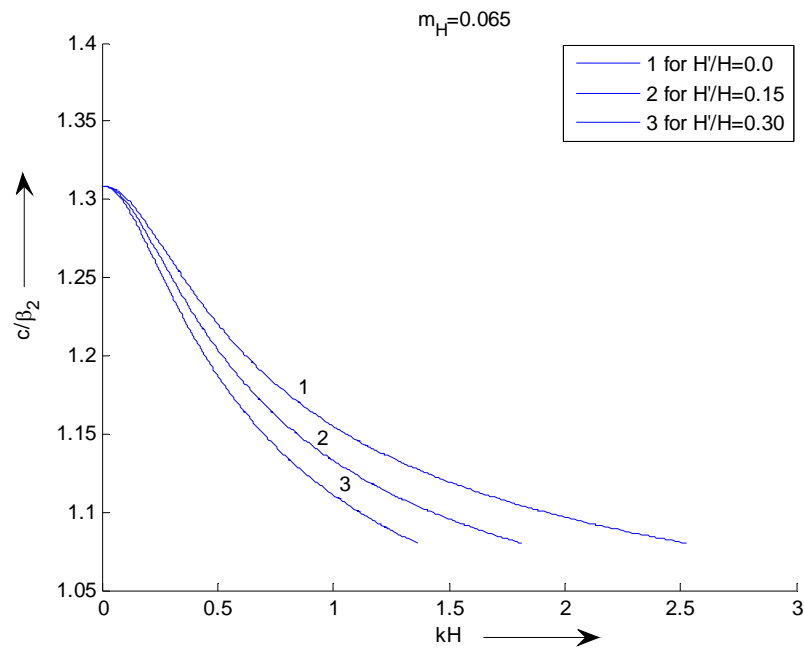**Figure 3:** Dimensionless phase velocity against dimensionless wave number for $m_H = 0.035$.

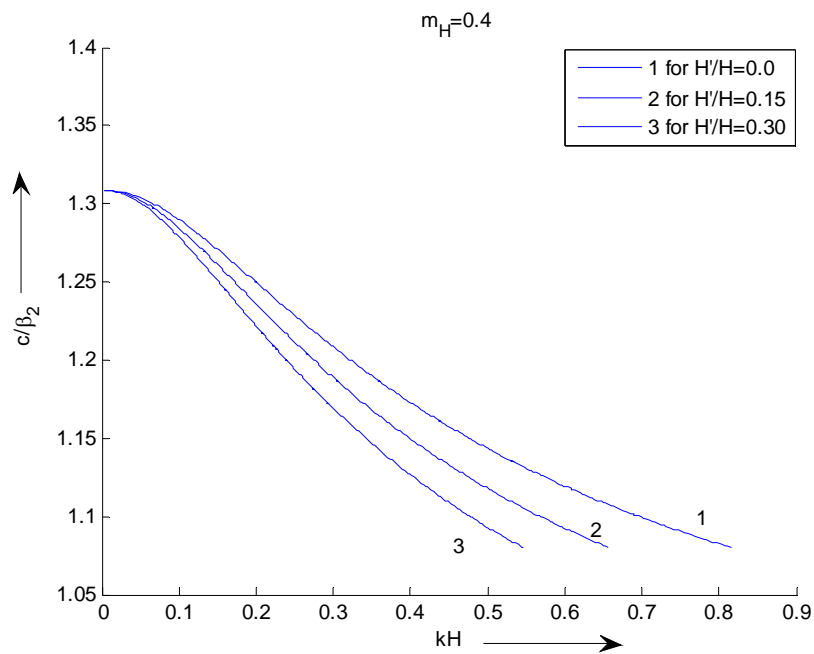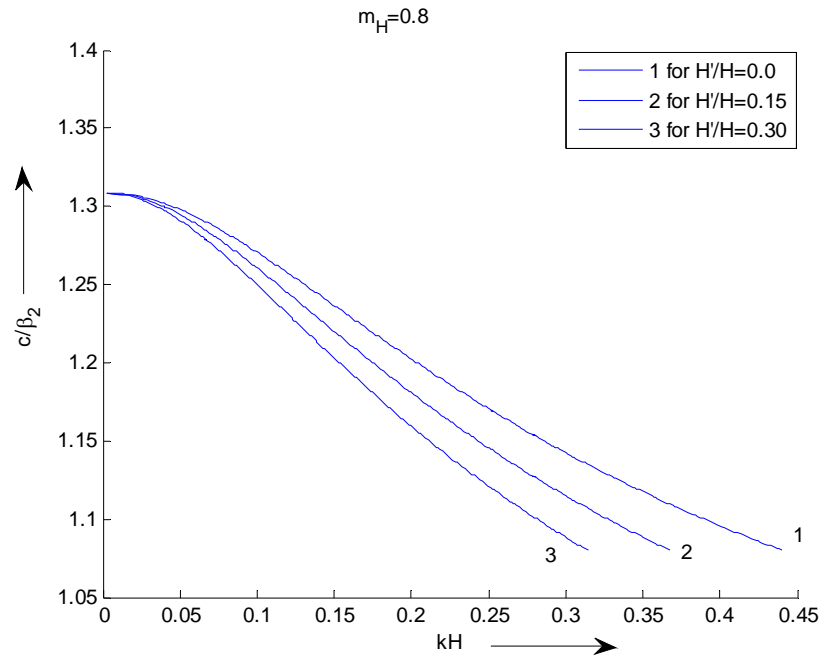**Figure 4:** Dimensionless phase velocity against dimensionless wave number for $m_H = 0.065$.

Figure 5: Dimensionless phase velocity against dimensionless wave number for $m_H = 0.4$.

Figure 6: Dimensionless phase velocity against dimensionless wave number for $m_H = 0.8$.

6. Conclusion

In this paper the dispersion relation has been obtained in the closed form. The effect of size of irregularity and magnetoelastic monoclinic parameter on the phase velocity of SH waves has been depicted and shown by means of graphs. From the above studies it can be concluded that in a monoclinic medium, with the increase in the size of irregularity the phase velocity decreases. Also, with the increase in the magnetoelastic monoclinic parameter the phase velocity retains the same nature. In the isotropic case, when irregularity and magnetic field are absent, the dispersion equation is matched with the classical SH wave equation. The present study has its application specially to the problem of waves and vibrations where the wave signals have to travel through different layers of different material properties and containing irregularities due to continental margin, mountain roots etc. These results can also be utilized in the interpretation and analysis of data of geophysical studies. The findings will be useful in forecasting formation details at greater depth through signal processing and seismic data analysis. The present study may be effectively utilized to generate initial data prior to exploitation of the formation. This study may be useful to geophysicist and metallurgist for analysis of rock and material structures through Non-Destructive Testing (NDT).

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