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On the one-chart schemes for joint monitoring of the two parameters of a zero-inflated Poisson (ZIP) process

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Abstract

One-chart scheme for joint monitoring of two parameters allows the practitioners to focus on a single chart and thus, it offers significant operational advantages. Recently, four one-chart schemes are reported in literature for joint monitoring of the two parameters (ϕ, λ) of a zero-inflated Poisson (ZIP) process. One of these schemes, namely - Gamma chart, is developed assuming samples of large size will be inspected and other three charts, namely DS-chart, Max-chart and LR chart, are developed assuming samples of small size will be inspected after every certain interval. The plotting statistics of these three charts are computed using the estimated ZIP parameters. However, unless the sample size (n) is sufficiently large, the excessive number of zeros in the ZIP process may offer a sample of zeros only. Therefore, these three schemes require estimation of ZIP parameters and subsequent computation of the plotting statistic based on the accumulated samples till the sampling stage. Because of the accumulation of samples, these schemes suffer from several limitations. In this paper, these one-chart schemes are modified for large sample size to make them free from their limitations. Subsequently, the performances of the four one-chart schemes are evaluated extensively using simulation studies. The results reveal that among the three modified charts likelihood ratio (LR) chart is the most preferable with respect to in-control and out-of-control average run length (ARL) performances. However, all these three modified charts give false alarms when the process parameter(s) shift towards desirable direction(s) implying process improvement. On the other hand, the Gamma chart is slightly inferior with respect to out-of-control ARL performances. But it does not give false alarm when there is a process improvement. Further, since Gamma chart directly monitors the average number of defects, it offers significant advantages in terms of implementation and interpretations.

Keywords: Zero-inflated Poisson (ZIP), On-chart scheme, Simulation experiments, Process monitoring, DS chart, Max chart, Likelihood ratio (LR) chart, Gamma chart.

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1. Introduction

In many manufacturing processes, the number of nonconformities in an item is an important quality characteristic. Traditionally *c* or *u* chart is used to monitor the number of nonconformities which is assumed to follow a Poisson distribution. However, the use of the Poisson distribution underestimates the mean and variance if the sample data contain a large number of zero observations, which is a reality in many high-quality processes. Consequently, conventional attribute control charts result in too many false alarms if used for monitoring high quality processes, and thus, are not appropriate at all for monitoring high-quality processes.

Lambert (1992) has considered that the count data with excess zeros (i.e. zero-inflated data) essentially arise from a mixture of two distributions - a Bernoulli distribution that account for the inflated zero values and the standard Poisson (λ) distribution. Thus, it consists of two parameters, the first one is zero inflation parameter ϕ ($\phi \in [0,1]$) and the second one is the Poisson parameter λ (λ>0). Xie and Goh (1993) have viewed the zero inflated data from a different angle. According to them, most often the high quality processes are in perfect state but random shocks (e.g. equipment or process problem) with probability p occur independently that causes nonconformities in the process, and the number of nonconformities follows Poisson (λ) distribution. The inflation model (Lambert, 1992) and shock model (Xie and Goh, 1993) for zero-inflated data are conceptually little different but in their mathematical forms they are essentially the same because $p = 1 - \phi$. So, both the models fit well to the zero-inflated data and yield the same results. Both inflation model and random shock model are known as zero-inflated Poisson (ZIP) model.

Rapid technological advancements and automation in today's world have led to many high quality manufacturing processes, which motivated many researchers/practitioners to investigate the issue of monitoring zero-inflated processes. Xie and Goh (1993) have proposed the random shock model of zero-inflated Poisson distribution for fitting zero-inflated process data. They have used probability limits as the control limits instead of the standard 3-sigma limits for monitoring number of nonconformities per sample unit. Xie et al. (1995) presented control charts based on the cumulative count of conforming (CCC) items until a defective one is encountered, which is generally assumed to follow a geometric distribution. They proposed to use CCC-chart for monitoring the parameter ϕ and c-chart for monitoring the parameter λ. Chen et al. (2008) proposed control charts based on the generalized ZIP (GZIP) distribution, which is an extension of the ZIP distribution. Sim and Lim (2008) constructed np-chart and c-chart based on the Jeffreys intervals for monitoring the parameters ϕ and λ respectively. Rakitzis and Castagliola (2016) studied the effect of estimation errors on Shewhart-type chart for ZIP processes. Zhang et al. (2019) proposed CCC-r chart, which is an extension of the CCC-chart. The CCC-r chart monitors the cumulative count of items until the r^{th}nonconforming one is met. He et al. (2012) considered two separate CUSUM control charts, one for detecting changes in λ and the other for detecting changes in ϕ parameter. Leong et al. (2015) developed a EWMA charting scheme representing a combination of two individual EWMA charts for monitoring two ZIP parameters. Recently, many other nonShewhart-type control charts for monitoring ZIP processes have been proposed in literature, e.g. generally weighted moving average (GWMA) control chart (Alevizakos and Koukouvinos, 2020), generalized likelihood ratio based control charts (Lai et al., 2023) etc.

Now-a-days in many manufacturing set ups inspections are sensor-based and in such cases 100% products are inspected for the presence of nonconformities. Most of the existing control charting procedures for ZIP processes are also developed assuming that all the manufactured units are inspected one by one, and most often separate control charts are proposed for monitoring the two parameters of a ZIP process. However, in many manufacturing set ups 100% inspection may not be feasible. In such cases, a cost-effective strategy will be to monitor a ZIP process using group inspection of size n after every certain interval. On the other hand, one-chart scheme for joint monitoring of the two parameters allows the practitioners to focus on a sole chart (and, a single charting statistic) and thus, it offers significant operational advantages. Therefore, a plausible strategy for monitoring a ZIP process may be to use a one-chart scheme with group inspection of size n. It may be noted that if inspections are sensor-based and online, then every n number of manufactured products may be considered as the group sample of size n.

Recently, Mukherjee and Rakitzis (2019) suggested three one-chart schemes (DS chart, Max chart and Likelihood ratio based LR chart) for joint monitoring of the two parameters of a ZIP(ϕ, λ) process using group samples of small size. They make use of the current maximum likelihood estimates (MLEs) of ZIP parameters for computation of the monitoring statistics of these charts. Generally, a large number of samples are needed for estimating the parameters of a ZIP(ϕ, λ) process. Therefore, Mukherjee and Rakitzis's (2019) schemes require accumulation of previously collected samples till the current sampling stage and then obtaining MLEs of the parameters based on the accumulated samples. Since all three schemes of Mukherjee and Rakitzis (2019) require accumulation of samples, they are called as progressive monitoring schemes. Pal and Gauri (2024) have proposed a one-chart scheme, namely Gamma chart, for monitoring a ZIP process. The plotting statistic of Gamma chart is the average number of defects observed in a large sample. Pal and Gauri (2024) observed that average number of defects in a sample of large size ($n \geq 300$) collected from ZIP(ϕ, λ) process approximately follows Gamma distribution with shape parameter $k = \frac{n\phi\lambda}{1+\lambda-\phi\lambda}$ and scale parameter $\theta = \frac{1+\lambda-\phi\lambda}{n}$. Accordingly, they have proposed a Shewhart-type Gamma chart for joint monitoring of the two parameters of a ZIP process. The gamma chart is found to be very effective in detecting out-of-control process if the shifted values of ϕ and/or λ are larger than equal to 1.4 times of their in-control values.

It is observed that the progressive monitoring schemes of Mukherjee and Raktizis (2019) suffer from several limitations. For example, accumulation of samples can introduce correlation issues resulting in interpretation problems. Also, because of accumulation of samples, these charts are more likely to give higher type I as well as type II errors. For examples, number of defects in a sample may increase suddenly due to occurrence of an assignable cause, but it may not result in high values for the estimated ZIP parameters because of accumulation effect of zero or very less number of defects observed in the previously collected samples. Consequently, there may not be enough change in thevalue of monitoring statistic so that the chart indicates an out-of-control situation. Similarly, after detecting a shift in the ZIP parameters and taking necessary corrective measures, the monitoring statistics in the subsequent samples may continue to indicate out-of-control situation because of the accumulation effects. Moreover, Mukherjee and Rakitzis (2019) have not given any closed form equations for determination of the control limits of the monitoring statistics. Therefore, simulation studies are needed for designing the DS chart or Max chart or LR chart, which may become a difficult task for many practitioners.

It may happen that a sample of small size collected from a ZIP process may not contain any defective item. In such cases, it becomes difficult to estimate the ZIP parameters.So, ideally, group samples of large size should be inspected for the purpose of monitoring the parameters of a ZIP(ϕ,λ) process. In the modern days, there have been tremendous developments in the inspection and data acquisition technologies, and inspecting group samples of large size is no longer a difficult task. This motivates us to suitably modify the three one-chart schemes (DS chart, Max chart and LR chart) of Mukherjee and Rakitzis (2019) for the scenario where samples of large size are inspected for control charting purpose, and examine if (i) the limitations of existing progressive monitoring with small sample size can be overcome, and (ii) if one or more of these scheme can be more efficient than the Pal and Gauri (2024) proposed Gamma chart for joint monitoring of the two parameters of a ZIP process.

The rest of the article is organized as follows. The ZIP distribution and properties of MLEs of its parameters are presented in section 2. Section 3 discusses about currently available three one-chart schemes (based on small sample size) for progressive joint monitoring of the two parameters of a ZIP process and the one-chart scheme (based on large sample size) for joint monitoring of the two parameters. Section 4 presents modified DS chart, Max chart and LR chart for large sample size. The performances of all the four schemes for large sample size are evaluated using simulation experiments and the related results are presented in section 5. In section 6, some issues about these one-chart schemes for large sample size are discussed. Section 7 concludes the paper.

2. Zero-inflated Poisson (ZIP) distribution

As mentioned earlier, the inflation model (Lambert, 1992) and shock model (Xie et al., 2001) for zero-inflated data are conceptually little different but their mathematical forms are essentially the same and therefore, both the models yield the same results. This work takes into consideration Lambert's inflation model which is a mixture of two distributions - a Bernoulli distribution that account for the inflated zero values and the standard Poisson distribution. Let Y be a ZIP(ϕ,λ)variable representing number of defects in a product. Lambert (1992) defines the probability mass function of a ZIP distribution as

$$f(Y=y;\phi,\lambda)=\begin{cases}\phi+(1-\phi)e^{-\lambda} & \text{for } y=0\\ (1-\phi)\frac{\lambda^{y}e^{-\lambda}}{y!} & \text{for } y>0.\end{cases} \quad (1)$$

where, $\phi\in[0,1]$is the zero inflation parameter and $\lambda\ (>0)$ is the rate (mean) of the standard Poisson distribution. The mean and variance of this ZIP distribution are given by the following two expressions:

$$E(Y)=(1-\phi)\lambda \quad (2)$$

$$Var(Y)=\lambda(1-\phi)(1+\phi\lambda) \quad (3)$$

The ZIP parameters are generally unknown and hence those are estimated from sample data of a large size collected from a ZIP process. The parameters are generally estimated using the maximum likelihood method and those estimates are called as MLEs of ZIP parameters.

Suppose, a random sample of size n (large, preferably $n\geq 200$) is collected from a ZIP process and each item is inspected for number of nonconformities in it. Let the number (frequency) of sample units each having exactly 'd' number of nonconformities is denoted by $n_d(d=0,1,2,\dots,m)$. Then, the log-likelihood function of ϕ and λ for an observed dataset of size n can be written as

$$\begin{aligned}lnL(\phi,\lambda)&\\ &=ln[\{P(Y=0)\}^{n_0}]+ln\,[\prod_{d=1}^{m}\{P(Y=d)\}^{n_d}]\\ &=n_0\,ln\big[\phi+(1-\phi)e^{-\lambda}\big]+\sum_{d=1}^{m}n_d ln\,\{(1-\phi)\frac{e^{-\lambda}\lambda^{d}}{d!}\}\\ &=n_0\,ln\big[\phi+(1-\phi)e^{-\lambda}\big]+(n-n_0)[ln\,\{(1-\phi)-\lambda\}]+D\,ln\,\lambda-\textstyle\sum_{d=1}^{m}n_d\,ln(d!)\end{aligned} \quad (4)$$

where$D=\sum_{d=1}^{m}dn_d$ is the total number of defects in the sample of size n. The partial derivatives of the log-likelihood function with respect to ϕ and λ result in the following two score functions:

$$\frac{\partial(\ln L)}{\partial\phi}=\frac{n_0(1-e^{-\lambda})}{\phi+(1-\phi)e^{-\lambda}}-\frac{n-n_0}{1-\phi} \quad (5)$$

$$\frac{\partial(\ln L)}{\partial\lambda}=\frac{-n_0(1-\phi)e^{-\lambda}}{\phi+(1-\phi)e^{-\lambda}}+\frac{D}{\lambda}-(n-n_0) \quad (6)$$

The MLEs of ϕ and λ can be obtained by equating the above two score functions to zero and then solving numerically. The optimal values of ϕ and λ can easily be determined by performing enumerative search using the 'Solver' tool of

Microsoft Excel (Pal and Gauri, 2021). As an alternative, the MLEs of ϕ and λ can be obtained numerically by solving the following system of nonlinear equations (Mukherjee and Rakitzis, 2019):

$$\hat{\phi} = 1 - \frac{\bar{y}}{\hat{\lambda}} \quad (7)$$

$$\hat{\lambda} = \bar{y}^{+}(1 - e^{-\hat{\lambda}}) \quad (8)$$

where $\bar{y}$ is the sample mean of overall y_i $(i = 1, 2, 3, \ldots, n)$ values and $\bar{y}^{+}$ is the mean of non-zero y_i values. Clearly, there is no closed-form expression for the $\hat{\phi}$ or $\hat{\lambda}$, and they must be determined iteratively.

It is interesting to note that although there are no closed form expressions for estimation of standard errors of the MLEs of ϕ and λ, approximate values of standard errors of those MLEs can be obtained from the Fisher Information matrix, derived based on sample data (Mukherjee and Rakitzis, 2019). Let $y_i (i = 1, 2, 3, .., n)$ represents the number of nonconformities in the i^{th} item in a sample of size n collected from a ZIP(ϕ, λ) process and $(\hat{\phi}, \hat{\lambda})$ are the MLEs of unknown parameters (ϕ, λ) computed from the sample data. The estimate of probability of zero defects in the ZIP process, can then be obtained from the density function (see Eqn. 4) as

$$\hat{P}_0 = P(y = 0) = \hat{\phi} + (1 - \hat{\phi})e^{-\hat{\lambda}} \quad (9)$$

The Fisher Information matrix for the ZIP model can then be computed as

$$\boldsymbol{J} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}$$

where, $J_{11} = \frac{n(1-e^{-\hat{\lambda}})}{(1-\hat{\phi})\{\hat{\phi}+(1-\hat{\phi})e^{-\hat{\lambda}}\}} = \frac{n(1-e^{-\hat{\lambda}})}{(1-\hat{\phi})\hat{P}_0}$ (10)

$$J_{12} = J_{21} = \frac{-n\, e^{-\hat{\lambda}}}{\hat{\phi}+(1-\hat{\phi})e^{-\hat{\lambda}}} = \frac{-n\, e^{-\hat{\lambda}}}{\hat{P}_0} \quad (11)$$

$$J_{22} = n\big(1 - \hat{\phi}\big)\left\{\frac{1}{\hat{\lambda}} - \frac{\hat{\phi}e^{-\hat{\lambda}}}{\hat{\phi}+(1-\hat{\phi})e^{-\hat{\lambda}}}\right\} = n\big(1 - \hat{\phi}\big)\frac{\hat{P}_0 - \hat{\lambda}\hat{\phi}e^{-\hat{\lambda}}}{\hat{\lambda}\hat{P}_0} \quad (12)$$

The inverse of Fisher Information matrix $\boldsymbol{J}$, is

$$\boldsymbol{J}^{-1} = \begin{pmatrix} J_{11}^{-1} & J_{12}^{-1} \\ J_{21}^{-1} & J_{22}^{-1} \end{pmatrix}$$

where, $J_{11}^{-1} = \frac{\hat{P}_0(1-\hat{\phi})\big(\hat{P}_0-\hat{\lambda}\hat{\phi}e^{-\hat{\lambda}}\big)}{n\{\big(1-e^{-\hat{\lambda}}\big)\big(\hat{P}_0-\hat{\lambda}\hat{\phi}e^{-\hat{\lambda}}\big)-\hat{\lambda}e^{-2\hat{\lambda}}\}}$ (13)

$$J_{12}^{-1} = J_{21}^{-1} = \frac{\hat{P}_0\hat{\lambda}e^{-\hat{\lambda}}}{n\{\big(1-e^{-\hat{\lambda}}\big)\big(\hat{P}_0-\hat{\lambda}\hat{\phi}e^{-\hat{\lambda}}\big)-\hat{\lambda}e^{-2\hat{\lambda}}\}} \quad (14)$$

$$J_{22}^{-1} = \frac{\hat{P}_0\hat{\lambda}(1-e^{-\hat{\lambda}})}{n(1-\hat{\phi})\{\big(1-e^{-\hat{\lambda}}\big)\big(\hat{P}_0-\hat{\lambda}\hat{\phi}e^{-\hat{\lambda}}\big)-\hat{\lambda}e^{-2\hat{\lambda}}\}} \quad (15)$$

The diagonal elements of $\boldsymbol{J}^{-1}$ represent the variances of $\hat{\phi}$ and $\hat{\lambda}$. Therefore, the standard errors of the MLEs of ϕ and λ can be obtained from the diagonal elements of J^{-1} as follows:

$$s_{\phi} = \sqrt{\frac{\hat{P}_0(1-\hat{\phi})\big(\hat{P}_0-\hat{\lambda}\hat{\phi}e^{-\hat{\lambda}}\big)}{n\big\{\big(1-e^{-\hat{\lambda}}\big)\big(\hat{P}_0-\hat{\lambda}\hat{\phi}e^{-\hat{\lambda}}\big)-\hat{\lambda}e^{-2\hat{\lambda}}\big\}}} \quad (16)$$

$$s_{\lambda} = \sqrt{\frac{\hat{P}_0\hat{\lambda}\big(1-e^{-\hat{\lambda}}\big)}{n(1-\hat{\phi})\big\{\big(1-e^{-\hat{\lambda}}\big)\big(\hat{P}_0-\hat{\lambda}\hat{\phi}e^{-\hat{\lambda}}\big)-\hat{\lambda}e^{-2\hat{\lambda}}\big\}}} \quad (17)$$

The correlation coefficient between the two parameters can be obtained as follows:

$$r_{\phi\lambda} = \frac{J_{12}^{-1}}{s_{\phi} \times s_{\lambda}} \quad (18)$$

3.Different One-chart Schemes for Joint Monitoring of the Parameters of a ZIP Process

Suppose, the observed number of nonconformities in the outputs of a manufacturing process, are well described by a ZIP(ϕ, λ) distribution. Let the parameters ϕ and λ at the in-control state of the ZIP(ϕ, λ) process are known to be ϕ_0 and λ_0, respectively. Let $Y_i (i = 1,2,3, .., n)$ are independently and identically distributed ZIP(ϕ, λ) variable representing number of nonconformities (defects) in the i^{th} item in a sample of size n. The estimates of the two parameters can be obtained from the observed defects data using maximum likelihood method. Let the MLEs of ϕ and λ obtained from the observed defects data in a sample of size n, collected at t^{th} time point, are $\hat{\phi}_t$ and $\hat{\lambda}_t$ respectively. The purpose of process monitoring is to detect based on the estimated $\hat{\phi}_t$ and $\hat{\lambda}_t$ values, if any one or both the parameters have shifted from their in-control values (ϕ_0, λ_0) to some other value(s). To accomplish this objective, Mukherjee and Rakitzis (2019) have proposed three different one-chart schemes and Pal and Gauri (2024) have proposed an one-chart scheme for joint monitoring of the two parameters using group inspection data. While Mukherjee and Rakitzis (2019) have considered that samples of small size will be inspected after every certain interval, Pal and Gauri (2024) have considered that samples of large size will be inspected after every certain interval. All these methods are briefly described here.

3.1 Mukherjee and Rakitzis (2019) proposed three one-chart schemes

Mukherjee and Rakitzis (2019) proposed three one-chart schemes, namely- DS chart, Max chart and LR chart for monitoring the ZIP parameters. The monitoring statistics of these charts are computed using the MLEs of ZIP parameters at any time point and the in-control values of the ZIP parameters. Generally, a large number of samples are required in order to obtain the MLEs of the two parameters of a ZIP process. However, Mukherjee and Rakitzis (2019) considered that samples of small sizes will be inspected after every certain interval for the process control purpose. Since a sample of small size collected from a ZIP process may not contain any defective item, it becomes difficult to estimate the ZIP parameters. Hence, Mukherjee and Rakitzis (2019) proposed accumulation of all samples of small size (say, m) from the beginning till a pre-specified time t_0 from a stable ZIP process for estimation of in-control ZIP parameters. The first $t_0 - 1$ samples, each of size m, are neither used for estimating the parameters nor for plotting anything on the chart. At time t_0, a sample of size m is collected from the process and it is combined into one sample of size $mt_0$with all the previously available observations. These combined observations are used for computing the MLEs of the parameters of the ZIP process. The sample of size m, collected at the next time point, again is combined into one sample with all the previously available observations, and similarly, MLEs are estimated using the combined observations. The total sample size considered at time point t is denoted by $n = m \times t$.

Mukherjee and Rakitzis (2019) defined three different plotting/monitoring statistics utilizing the in-control ZIP parameters (ϕ_0, λ_0) and MLEs ($\hat{\phi}_t$, $\hat{\lambda}_t$)of ZIP parameters estimatedat time point t. Accordingly, they have proposed three one-chart control charting schemes for progressive monitoring of the two parameters of a ZIP process. It may be noted that in their schemes, MLEs of ϕ and λ are obtained based on the overall sample size (n) at t^{th} time point and the overall sample size changes after collection of every new sample of small size. To ensure a stable run-length distribution, Mukherjee and Rakitzis (2019) have used the multiplier $\sqrt{n}$ in defining all the three monitoring statistics. Mukherjee and Rakitzis (2019) proposed three one-chart schemes, e.g. DS chart, Max chart and LR chart are described in short in the following sub-sections.

3.1.1 DS chart

In statistics, the Wald statistic (Engle, 1984; Gombay, 2002) assesses constraints on statistical parameters based on the weighted distance between the unrestricted estimate and its hypothesized value under the null hypothesis. The Wald-type statistic is popularly used for sequentially testing one or more parameters, based on maximum likelihood estimators. Mukherjee and Rakitzis (2019) have considered the Wald-type distance (DS) measure as the plotting/monitoring statistic for the DS chart.

The basic concept of DS chart is to measure the Mahalanobis distance between the in-control values and the current MLEs of the two parameters, and monitoring the same for detecting any shift in one or both the parameters. Therefore, they defined the plotting statistic at t^{th}time point as follows:

$$DS_t = \sqrt{n}(\hat{\boldsymbol{\theta}}_t - \boldsymbol{\theta}_0)\boldsymbol{J}(\boldsymbol{\theta}_t - \boldsymbol{\theta}_0)^T \quad (19)$$

where the in-control (known) values and MLEs (at t^{th} time point) of the two parameters are represented by the vectors $\boldsymbol{\theta}_0 = (\phi_0, \lambda_0)$ and $\boldsymbol{\theta}_t = (\hat{\phi}_t, \hat{\lambda}_t)$ respectively and $\boldsymbol{J}$ is Fisher Information matrix evaluated at $\phi = \phi_0$and $\lambda = \lambda_0$(see Eqns. 16-17), and $\sqrt{n}$ is the multiplier used due to the reason discussed earlier. It may be noted that here the implicit assumptions are that MLEs ofϕ and λ are normally distributed with means ϕ_0 and λ_0 respectively, and standard deviations s_{ϕ_0} and s_{λ_0} respectively. These in-control values of ZIP parameters are generally unknown and hence those are estimated from a phase I sample data of large size. During phase II monitoring, those estimated values are assumed as known in-control values of ZIP parameters. In this case, the DS_t statistic is a STB type variable and hence it can be monitored using a control chart with upper control limit (UCL) only.

The successive MLEs of ϕ and λ are estimated from successively accumulated overall samples, and thus, it is not reasonable to assume that the successive MLEs of ϕ and λ are independent. This makes derivation of the distribution of the plotting statistic DS_t quite difficult. Mukherjee and Rakitzis (2019) used Monte-Carlo simulations to estimate appropriate value of UCL for a DS chart. They also used simulation procedures for evaluation ofout-of-control average run length (ARL) performances of the DS chart for various shifts in $\phi_0$and λ_0.

3.1.2 Max chart

The concept of max chart was introduced by Chen and Chang (1998) with the aim to jointly monitor the mean and variance of a normally distributed process using a single chart. Subsequently, several authors (Mukherjee and Marozzi, 2017; Zafar et al., 2019;

Gong and Mukherjee, 2019; Sanusi et al., 2020) took interest in investigating max chart for different situations. The basic concept of max chart is as follows: If the two characteristics (variables) of interest are statistically independent and if these variables can be transformed into standard normal variables (say, $U$and V), then the two characteristics can jointly be monitored by monitoring the maximum of $|U|$and $|V|$, i.e. monitoring statistic $M = \max\{|U|, |V|\}$.

Mukherjee and Rakitzis (2019) extended the concept of max chart for joint monitoring of the two parameters of a ZIP process. They defined the plotting statistic at t^{th} time point as follows:

$$M_t = \sqrt{n}\max\left\{\frac{|\hat{\phi}_t-\phi_0|}{s_{\phi_0}}, \frac{|\hat{\lambda}_t-\lambda_0|}{s_{\lambda_0}}\right\} \tag{20}$$

wheres_{ϕ_0} and s_{λ_0} are the standard errors of the estimates ofϕ_0 and λ_0 respectively and these can be obtained from the diagonal elements of the Fisher Information matrix $\boldsymbol{J}$, evaluated at $\phi = \phi_0$ and $\lambda = \lambda_0$ (see Eqns. 16-17), and $\sqrt{n}$ is the multiplier. . It may be noted that here the implicit assumptions are that MLEs ofϕ and λ are normally distributed with means ϕ_0 and λ_0 respectively, and standard deviations s_{ϕ_0} and s_{λ_0} respectively and they are independent. These in-control values of ZIP parameters are generally unknown and hence those are estimated from a phase I sample data of large size. During phase II monitoring, those estimated values are assumed as known in-control values of ZIP parameters. Since$M_t$is a STB type variable, it can be monitored using a control chart with UCL only.

Here too the successive MLEs of ϕ and $\lambda$are not expected to be truly independent, and therefore, the derivation of the distribution of the plotting statistic M_t becomes quite difficult. Mukherjee and Rakitzis (2019) used Monte-Carlo simulations to estimate appropriate value of UCL for a Max chart as well as to evaluate the out-of-control ARL performancesof the Max chart for various shifts in ϕ_0 and λ_0.

3.1.3 LR chart

The likelihood ratio test of a parameter is based on two different maximum likelihood estimates of the parameter. One estimate, called **unrestricted estimate,** is obtained from the solution of the unconstrained maximum likelihood problem, and the other estimate, called **restricted estimate,** is obtained from the solution of the constrained maximum likelihood problem. Several authors have considered the likelihood ratio-based statistics for joint monitoring of two parameters in normal and other populations (McCracken et al., 2013;Mukherjee et al., 2015).

Mukherjee and Rakitzis (2019) have considered a similar scheme for the joint monitoring of the two parameters of a ZIP process. Let n numbers of products are inspected at t^{th} time point, and the observed numbers of defects in these products are denoted as y_i $(i = 1,2,3, \dots, n)$. Then, the LR test statistic for the ZIP(ϕ, λ) process at t^{th} time point can be defined as follows:

$$LR_t = 2ln\left(\frac{sup_{\lambda,\phi}\prod_{i=1}^{n} f_{ZIP}(y_i|\lambda,\phi)}{\prod_{i=1}^{n} f_{ZIP}(y_i|\lambda_0,\phi_0)}\right)$$

$$= 2ln\left[\frac{sup_{\lambda,\phi}\prod_{i=1}^{n}\left[\{\phi+(1-\phi)e^{-\lambda}\}^{1-a_i}\times\left\{(1-\phi)\frac{\lambda^{y_i}e^{-\lambda}}{y_i!}\right\}^{a_i}\right]}{\prod_{i=1}^{n}\left[\{\phi_0+(1-\phi_0)e^{-\lambda_0}\}^{1-a_i}\times\left\{(1-\phi_0)\frac{\lambda_0^{y_i}e^{-\lambda_0}}{y_i!}\right\}^{a_i}\right]}\right] \tag{21}$$

wherea_i = 1 if y_i>0 and a_i = 0, otherwise. Suppose, the number of products having zero defects (i.e. zero observations) is $n_0$and $\bar{y}_t^+$ is the mean of non-zero observations in the collected samples and $\bar{y}_t$ is the overall average number of defects (i.e. sample mean) at t^{th} time point. Then, the Eqn. (21) can be written as

$$LR_t = \begin{cases} 2\left\{n_0\, ln\frac{\bar{y}_t}{(\phi_0+(1-\phi_0)e^{-\lambda_0})\bar{y}_t^{+}} + (n-n_0)\, ln\frac{\bar{y}_t}{(1-\phi_0)\hat{\lambda}_t} + n\bar{y}_t\, ln\frac{\hat{\lambda}_t}{\lambda_0} - (n-n_0)(\hat{\lambda}_t-\lambda_0)\right\}, & \text{if at least one } y_i \text{ is zero} \\ 2m\left\{(\lambda_0-\bar{y}_t) + \bar{y}_t\, ln\frac{\bar{y}_t}{\lambda_0}\right\}, & \text{if all } y_i \text{ are positive} \end{cases} \tag{22}$$

Since MLEs of ϕ and λ are obtained based on the accumulated number of samples till the t^{th} time point, the MLEs are obtained from varying overall sample size (n). Mukherjee and Rakitzis (2019) have observed that usage of $\sqrt{n}$ as a multiplier results in a stable run-length distribution, and so they have defined the plotting/monitoring statistic at t^{th} time pointfor the LR chart as

$$LS_t = \sqrt{n}LR_t \tag{23}$$

Since the distribution of LS_t is unknown, Mukherjee and Rakitzis (2019) used Monte-Carlo simulations to estimate appropriate value of UCL for a LR chart as well as to evaluate the out-of-control ARL performancesof the LR chart for various shifts in ϕ_0 and λ_0.

3.1.4 The limitations of Mukherjee and Rakitzis's (2019) three one-chart schemes

Mukherjee and Rakitzis's (2019) considered that samples of small size will be collected after every certain interval for the process control purpose. Generally, a large number of samples are required for obtaining the MLEs of the two parameters of a ZIP process. A group sample of small size obtained from a ZIP process may not contain any defective item which makes it difficult to estimate the ZIP parameters. To satisfy the requirement of large number of samples, they have proposed obtaining MLEs from the progressively accumulated samples till the t^{th} sampling stage. The plotting/monitoring statistics for all the three one-chart schemes are defined based on the MLEs obtained from the progressively accumulated samples. This introduces several issues to ponder, e.g.

(i) It is quite difficult to derive the distributions of DS_t, M_t and LS_t, and therefore, no close form equations are available for determination of UCLs of DS chart, Max chart and LR chart. Therefore, simulation studies are required for determination of the UCLs of these charts. Carrying out simulation studies, call for huge computational skill of the practitioners, which may not be available always.

(ii) The UCLs of these charts are expected to be changed for every new collection of subgroup of sample because the size (n) of the progressively accumulated overall samples changes. This may cause complexity in control charting.

(iii) The successive values of DS_t, M_t and LS_t are computed based on the MLEs of ϕ and λ obtained from progressively accumulated overall samples, and thus, the plotting statistics in all these charts are expected to be auto-correlated. This may increase difficulty in interpretation of the plotted points.

(iv) Because of accumulation of samples, these charts are more likely to give higher type I as well as type II errors. For examples, number of defects in a sample may increase suddenly due to occurrence of an assignable cause, but the values of the estimated ZIP parameters may not be high enough because of the accumulation effect of many good products collected previously. Consequently, there may not be enough change in the value of monitoring statistic so that the chart indicates an out-of-control situation. This gives rise to type I error. Similarly, after detecting a shift in the ZIP parameters and taking necessary corrective measures, the monitoring statistics in the subsequent samples may continue to indicate out-of-control situation because of the accumulation effects. This gives rise to type II error (β-risk) of control charts.

(v) Simulation studies are required for determination of β-risks of DS chart, Max chart and LR chart if there are any shift in zero-inflation parameter and/or in the rate parameter of ZIP distribution.

3.2 Pal and Gauri (2024) proposed one-chart scheme: Gamma chart

Pal and Gauri (2024) attempted to develop a one-chart scheme keeping in mind that any control charting scheme developed directly on the observed quality characteristic will be intuitively appealing to the practitioners and can offer significant advantages in terms of implementation and interpretations. If $Y_i (i = 1,2,3,..,n)$ are identically and independently distributed ZIP(ϕ,λ) variable representing number of nonconformities (defects) in the i^{th} item in a sample of size n, then the average number of nonconformities in a sample (i.e. sample mean) is $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$. The expectation and variance of $\bar{Y}$ can be derived as

$$E(\bar{Y}) = \frac{1}{n}\sum_{i=1}^{n} E(Y_i) = \frac{1}{n}\sum_{i=1}^{n}(1-\phi)\lambda = (1-\phi)\lambda \quad (24)$$

$$Var(\bar{Y}) = \frac{1}{n^2}\sum_{i=1}^{n} Var(Y_i) = \frac{1}{n^2}\sum_{i=1}^{n}\lambda(1-\phi)(1+\phi\lambda) = \frac{\lambda(1-\phi)(1+\phi\lambda)}{n} \quad (25)$$

The sample mean $\bar{Y}$ can take fractional values and so, $\bar{Y}$can be treated as a continuous random variable. Through extensive simulated experiments, Pal and Gauri (2024) have observed that sample mean ($\bar{Y}$) in a ZIP(ϕ,λ) process follows approximately a Gamma(k,θ) distribution, where $k = n(1-\phi)\lambda/(1+\phi\lambda)$ and $\theta = (1+\phi\lambda)/n$. Based on this proposition, Pal and Gauri (2024) have proposed Gamma chart for joint monitoring of the two parameters of a ZIP(ϕ,λ) process. Since $E(\bar{Y}) = (1-\phi)\lambda$, the two parameters ϕ and λ of a ZIP process can be monitored simultaneously by plotting only the observed sample mean ($\bar{y}$) in a chart.

The random variable $\bar{Y}$is a STB type variable, and therefore, the out of control condition of a ZIP process can be detected by comparing it with the UCL and plotting $\bar{y}$values on the chart. When the parameters ϕ and λ are known to be ϕ_0 and λ_0 respectively, then the CL of the Gamma chart will be

$$\text{CL} = E(\bar{Y}) = (1-\phi_0)\lambda_0 \quad (26)$$

and the UCL of the Gamma chart can be computed using the inverse properties of the cumulative Gamma distribution, as

$$\text{UCL} = \Gamma^{-1}(1-\alpha) \quad (27)$$

where$\Gamma^{-1}(1-\alpha)$ is the inverse of cumulative Gamma distribution having scale parameter, $k = n(1-\phi_0)\lambda_0/(1+\phi_0\lambda_0)$ and shape parameter, $\theta = (1+\phi_0\lambda_0)/n$ with probability $1-\alpha$ (where, α is the type I error of the control chart). Knowing the values of scale and shape parameters, the value of $\Gamma^{-1}(1-\alpha)$ can easily be obtained by using "gamma.inv" function of Microsoft Excel. The syntax of the $Gamma.inv$ function will be $Gamma.inv\left(1-\alpha, \frac{n(1-\phi_0)\lambda_0}{1+\phi_0\lambda_0}, \frac{1+\phi_0\lambda_0}{n}\right)$.

If the in-control parameters ϕ_0 and λ_0 of the ZIP process shift to ϕ_1 and λ_1 respectively, then the β-risk of the Gamma control chart can be computed as follows:

$$\beta = P(\bar{Y} \le \text{UCL}|\phi = \phi_0, \quad \lambda = \lambda_1) + P(\bar{Y} \le \text{UCL}|\phi = \phi_1, \quad \lambda = \lambda_0) + P(\bar{Y} \le \text{UCL}|\phi = \phi_1, \quad \lambda = \lambda_1)$$

$\cong P(X \le \text{UCL})$, where $X \sim \text{Gamma}\left(\frac{n(1-\phi_0)\lambda_1}{1+\phi_0\lambda_1}, \frac{1+\phi_0\lambda_1}{n}\right)$

$+ P(X \le \text{UCL})$, where $X \sim \text{Gamma}\left(\frac{n(1-\phi_1)\lambda_0}{1+\phi_1\lambda_0}, \frac{1+\phi_1\lambda_0}{n}\right)$

$+ P(X \le \text{UCL})$, where $X \sim \text{Gamma}\left(\frac{n(1-\phi_1)\lambda_1}{1+\phi_1\lambda_1}, \frac{1+\phi_1\lambda_1}{n}\right)$

$$= \int_0^{\text{UCL}} \frac{x^{\frac{n\phi_0\lambda_1}{1+\lambda_1-\phi_0\lambda_1}-1} e^{-\frac{nx}{1+\lambda_1-\phi_0\lambda_1}}}{\Gamma\left(\frac{n\phi_0\lambda_1}{1+\lambda_1-\phi_0\lambda_1}\right)\left(\frac{1+\lambda_1-\phi_0\lambda_1}{n}\right)^{\frac{n\phi_0\lambda_1}{1+\lambda_1-\phi_0\lambda_1}}} dx + \int_0^{\text{UCL}} \frac{x^{\frac{n\phi_1\lambda_0}{1+\lambda_0-\phi_1\lambda_0}-1} e^{-\frac{nx}{1+\lambda_0-\phi_1\lambda_0}}}{\Gamma\left(\frac{n\phi_1\lambda_0}{1+\lambda_0-\phi_1\lambda_0}\right)\left(\frac{1+\lambda_0-\phi_1\lambda_0}{n}\right)^{\frac{n\phi_1\lambda_0}{1+\lambda_0-\phi_1\lambda_0}}} dx$$

$$+ \int_0^{\text{UCL}} \frac{x^{\frac{n\phi_1\lambda_1}{1+\lambda_1-\phi_1\lambda_1}-1} e^{-\frac{nx}{1+\lambda_1-\phi_1\lambda_1}}}{\Gamma\left(\frac{n\phi_1\lambda_1}{1+\lambda_1-\phi_1\lambda_1}\right)\left(\frac{1+\lambda_1-\phi_1\lambda_1}{n}\right)^{\frac{n\phi_1\lambda_1}{1+\lambda_1-\phi_1\lambda_1}}} dx \quad (28)$$

It is worth to mention here that Pal and Gauri (2024)have considered the shock model (Xie et al., 2001) for describing zero-inflated data, and developed the Gamma chart. Here the same Gamma chart is presented using inflation model (Lambert, 1992) for describing zero-inflated data.

4. Modified DS chart, Max chart and LR chart for large sample size

If group sampling with large sample size is considered for process control purpose, MLEs of the two ZIP parameters can be obtained independently from the observed defects data at each time point. There will be no need for accumulation of samples, and consequently, the problem of auto-correlation in the existing DS chart, Max chart and LR chart (Mukherjee and Rakitzis, 2019)will be resolved. Further, when the parameters are estimated from samples of large size the estimates of these parameters are expected to follow independently asymptotic normal distribution (Nanjundan and Naika, 2012). These may facilitate derivations of the statistical distributions of the plotting statistics DS_t, M_t and LS_t. This in turn can enable determining the close form equations for the UCL of these charts. Again, since there will be no need for accumulation of samples, the sample size will remain fixed at each time point. Consequently, control charting and interpretation of the plotted points will become simpler.

On the other hand, if Mukherjee and Rakitzis (2019) proposed three one-chart schemes are considered for large sample size, then performances of these charts will become comparable with the Pal and Gauri (2024) proposed Gamma chart. Therefore, it is decided to modify these three one-chart schemes for the process control scenario where samples of large size will be inspected after every certain interval.

It may be noted that if sample of large size is considered for the process control purpose, MLEs of the two parameters can be obtained independently from the observed defects data at each time point. Therefore, one obvious modification should be to remove the multiplier $\sqrt{n}$ from the definitions of the monitoring statistics of these schemes.

4.1 Modified DS chart for large sample size

The monitoring statistic of DS chart for large sample size can be defined as follows:

$$DS_t = (\widehat{\boldsymbol{\theta}}_t - \boldsymbol{\theta}_0)\boldsymbol{J}(\boldsymbol{\theta}_t - \boldsymbol{\theta}_0)^T$$

$$= [J_{11}(\hat{\phi}_t - \phi_0)^2 + 2J_{12}(\hat{\phi}_t - \phi_0)(\hat{\lambda}_t - \lambda_0) + J_{22}(\hat{\lambda}_t - \lambda_0)^2] \quad (29)$$

where J_{11}, J_{12} and J_{22}[defined in Eqns. (10-12)] are the elements of Fisher Information matrix $\boldsymbol{J}$ evaluated at $\phi = \phi_0$and $\lambda = \lambda_0$.

Suppose, the in-control (known) values and MLEs (at t^{th} time point) of the two parameters are represented by the vectors $\boldsymbol{\theta}_0 = (\phi_0, \lambda_0)$ and $\widehat{\boldsymbol{\theta}}_t = (\hat{\phi}_t, \hat{\lambda}_t)$ respectively, and $\boldsymbol{\Sigma}$ (=$\boldsymbol{J}^{-1}$) is the variance-covariance matrix ($\boldsymbol{J}^{-1}$ is the inverse of the Fisher Information matrix $\boldsymbol{J}$ when the process is in in-control state). Nanjundan and Naika (2012) have observed that when the parameters ($\phi$and λ) are estimated based on samples of large size, those are expected to follow independently asymptotic normal distribution. This implies that

$$\widehat{\boldsymbol{\theta}}_t \sim AN(\boldsymbol{\theta}_0, \boldsymbol{\Sigma})$$
$$\Rightarrow (\widehat{\boldsymbol{\theta}}_t - \boldsymbol{\theta}_0) \sim AN(0, \boldsymbol{\Sigma})$$
$$\Rightarrow (\widehat{\boldsymbol{\theta}}_t - \boldsymbol{\theta}_0)\boldsymbol{\Sigma}^{-1/2} \sim AN(0,1)$$
$$\Rightarrow [(\widehat{\boldsymbol{\theta}}_t - \boldsymbol{\theta}_0)\boldsymbol{\Sigma}^{-1/2}]^T[(\widehat{\boldsymbol{\theta}}_t - \boldsymbol{\theta}_0)\Sigma^{-1/2}] \sim \chi_2^2$$
$$\Rightarrow (\widehat{\boldsymbol{\theta}}_t - \boldsymbol{\theta}_0)^T\boldsymbol{\Sigma}^{-1}(\widehat{\boldsymbol{\theta}}_t - \boldsymbol{\theta}_0) \sim \chi_2^2$$
$$\Rightarrow (\widehat{\boldsymbol{\theta}}_t - \boldsymbol{\theta}_0)^T\boldsymbol{J}(\widehat{\boldsymbol{\theta}}_t - \boldsymbol{\theta}_0) \sim \chi_2^2$$

Therefore, the monitoring statistic of DS chart for large sample size follows Chi-square distribution with degrees of freedom = 2. Thus, for a given type I error (α), the UCL of the monitoring/plotting statistic can be determined as follows:

$$P(DS_t \le \text{UCL}) = 1 - \alpha \Rightarrow \text{UCL} = \left(\chi^2_{1-\alpha,2}\right)^{-1}$$

So, for type I error (α) = 0.005, the UCL of the DS chart can be determined as

$$\text{UCL} = \left(\chi^2_{1-\alpha,2}\right)^{-1} = \left(\chi^2_{0.995,2}\right)^{-1} = 10.5966$$

On the other hand, 50th percentile point can be considered as the centre line (CL) of the DS chart, i.e. the CL of the DS chart can be determined as

$$\mathrm{CL} = \left(\chi^2_{0.5,2}\right)^{-1} = 1.3863$$

It is important to note that the UCL and CL of the DS chart for large sample size do not depend on values of the sample size as well as the parameters ϕ and λ of the concerned ZIP process.

Let us now assume that the ZIP process parameters have shifted from in-control state (ϕ_0, λ_0) to (ϕ_1, λ_1). The probability that the DS chart will fail to detect this shift on the first subsequent sample is known as type II error or β-risk. It is observed that derivation of the formula for computation of β-risk is quite complicated and not easily tractable. So it is recommended to resort to simulation procedure for assessing the β-risk and out-of-control ARL performances.

4.2 Modified Max chart for large sample size

Nanjundan and Naika (2012) have observed that when the parameters ($\phi$and λ) are estimated based on samples of large size, they are expected to follow asymptotic normal distribution.Also, MLEs of ϕ and λ can be assumed to be independent if the Poisson rate parameter λ is not too small. Suppose that s_{ϕ_0} and s_{λ_0} are the standard errors of the estimates of ϕ_0 and λ_0 respectivelywhen the process is at in-control state. The values of s_{ϕ_0} and s_{λ_0} can be obtained from the diagonal elements of the Fisher Information matrix $\boldsymbol{J}$, evaluated at $\phi = \phi_0$ and $\lambda = \lambda_0$.Then, the transformed variables $U = \frac{\hat{\phi}_t - \phi_0}{s_{\phi_0}}$ and $V = \frac{\hat{\phi}_t - \lambda_0}{s_{\lambda_0}}$ are asymptotic standard normal variables and statistically independent. The monitoring statistic of Max chart for large sample size can be defined as follows:

$$M_t = \max\left\{\frac{|\hat{\phi}_t - \phi_0|}{s_{\phi_0}}, \frac{|\hat{\lambda}_t - \lambda_0|}{s_{\lambda_0}}\right\} = \max\{|U|, |V|\} \quad (30)$$

For any value $z > 0$in standard normal distribution, we know

$$P(|Z| \le z) = P(-z \le Z \le z) = \Phi(z) - \Phi(-z) = 2\Phi(z) - 1 \quad (31)$$

where$\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. Here, both the transformed variables U and V are asymptotic standard normal variables, and they are statistically independent. Thus, the cumulative distribution function of $M_t$can be obtained as

$$P(M_t \le z) = P(\max\{|U|, |V|\} \le z) = P(|U| \le z) \times P(|V| \le z) = (2\Phi(z) - 1)^2 \quad (32)$$

Thus, for a given type I error (α), the UCL of the plotting statistic $M_t$can be determined as follows:

$$P(M_t \le UCL) = 1 - \alpha$$
$$\Rightarrow \{2\Phi(UCL) - 1\}^2 = 1 - \alpha$$
$$\Rightarrow 2\Phi(UCL) - 1 = \sqrt{1 - \alpha}$$
$$\Rightarrow UCL = \Phi^{-1}\left(\frac{1 + \sqrt{1 - \alpha}}{2}\right) \quad (33)$$

For type I error (α) = 0.005, the UCL of Max chart is computed as

$$\mathrm{UCL} = \Phi^{-1}\left(\frac{1 + \sqrt{0.995}}{2}\right) = 3.0230$$

On the other hand, 50th percentile point can be considered as the CL of the Max chart, i.e. the CL of the Max chart can be determined as

$$\mathrm{CL} = \Phi^{-1}\left(\frac{1 + \sqrt{0.5}}{2}\right) = 1.0518$$

It is important to note that the UCL and CL of the Max chart for large sample size do not depend on the sample size as well as the parameters ϕ and λ of the ZIP process.

Let us now assume that the ZIP process parameters have shifted from in-control state (ϕ_0, λ_0) to (ϕ_1, λ_1). The probability that the Max chart will fail to detect this shift on the first subsequent sample is known as type II error or β-risk. Then, type II error (β) of the max chart can be computed as follows:

$\beta = P(M_t \le \mathrm{UCL}|\phi_1, \lambda_1)$ (if there is shift in one parameter, then $\phi_1 = \phi_0$ or $\lambda_1 = \lambda_0$)

$= P[\max(|U_1|, |V_1|) \le \mathrm{UCL}|\phi_1, \lambda_1]$, where $U_1 = \frac{\hat{\phi}_1 - \phi_0}{s_{\phi_0}}$ and $V_1 = \frac{\hat{\lambda}_1 - \lambda_0}{s_{\lambda_0}}$

$= P[|U_1| \le \mathrm{UCL}] \times P[|V_1| \le \mathrm{UCL}]$, [since U_1 and V_1 are independent] (34)

Let $\hat{\phi}_1$ and $\hat{\lambda}_1$ are the estimates of parameters ϕ and λ obtained based on the samples collected during out-of-control state of the process. The estimates of ϕ andλ obtained in the out-of-control state of the process, i.e. $\hat{\phi}_1$and $\hat{\lambda}_1$ will be distributed asymptotically normally (AN) as follows:

$\hat{\phi}_1 \sim AN[\phi_1, s^2_{\phi_1}]and\hat{\lambda}_1 \sim AN[\lambda_1, s^2_{\lambda_1}]$

where$s_{\phi_1}$and s_{λ_1} can be obtained from the diagonal elements of the inverse of corresponding Fisher Information matrix evaluated at $\phi = \phi_1$and $\lambda = \lambda_1$.

Now, $\hat{\phi}_1 \sim AN[\phi_1, s_{\phi_1}^2] \Rightarrow \frac{\hat{\phi}_1-\phi_0}{s_{\phi_0}} \sim AN\left[\frac{\phi_1-\phi_0}{s_{\phi_0}}, \frac{(s_{\phi_1})^2}{(s_{\phi_0})^2}\right] \Rightarrow U_1 \sim AN\left[\frac{\phi_1-\phi_0}{s_{\phi_0}}, \frac{(s_{\phi_1})^2}{(s_{\phi_0})^2}\right]$

Therefore, $P[|U_1| \leq \text{UCL}] = P[-\text{UCL} \leq U_1 \leq \text{UCL}]$

$$= P(U_1 \leq \text{UCL}) - P(U_1 \leq -\text{UCL})$$

$$= \Phi\left[\frac{UCL - \frac{\phi_1-\phi_0}{s_{\phi_0}}}{s_{\phi_1}/s_{\phi_0}}\right] - \Phi\left[\frac{-UCL - \frac{\phi_1-\phi_0}{s_{\phi_0}}}{s_{\phi_1}/s_{\phi_0}}\right]$$

$$= \Phi\left[\frac{s_{\phi_0} \times UCL - (\phi_1 - \phi_0)}{s_{\phi_1}}\right] - \Phi\left[\frac{-s_{\phi_0} \times UCL - (\phi_1 - \phi_0)}{s_{\phi_1}}\right]$$

Similarly, $P[|V_1| \leq \text{UCL}] = \Phi\left[\frac{s_{\lambda_0}\times UCL-(\lambda_1-\lambda_0)}{s_{\lambda_1}}\right] - \Phi\left[\frac{-s_{\lambda_0}\times UCL-(\lambda_1-\lambda_0)}{s_{\lambda_1}}\right]$

Therefore, the formula for estimating β-risk of the max chart can be obtained from Eqn. (34) as

$$\beta = \left\{\Phi\left[\frac{s_{\phi_0}\times UCL-(\phi_1-\phi_0)}{s_{\phi_1}}\right] - \Phi\left[\frac{-s_{\phi_0}\times UCL-(\phi_1-\phi_0)}{s_{\phi_1}}\right]\right\} \times \left\{\Phi\left[\frac{s_{\lambda_0}\times UCL-(\lambda_1-\lambda_0)}{s_{\lambda_1}}\right] - \Phi\left[\frac{-s_{\lambda_0}\times UCL-(\lambda_1-\lambda_0)}{s_{\lambda_1}}\right]\right\} \quad (35)$$

4.3 Modified LR chart

Since MLEs of ϕ and λ are obtained based on the same number of samples at each time point, there is no need to use the multiplier $\sqrt{n}$, and therefore, the plotting statistic of LR chart for large sample size can be defined as

$$LR_t = \begin{cases} 2\left\{n_0\, ln\frac{\bar{y}_t}{(\phi_0+(1-\phi_0)e^{-\lambda_0})\bar{y}_t} + (n-n_0)\, ln\frac{\bar{y}_t}{(1-\phi_0)\hat{\lambda}_t} + n\bar{y}_t\, ln\frac{\hat{\lambda}_t}{\lambda_0} - (n-n_0)(\hat{\lambda}_t - \lambda_0)\right\}, & \text{if at least one } y_i \text{ is zero} \\ 2n\left\{(\lambda_0 - \bar{y}_t) + \bar{y}_t\, ln\frac{\bar{y}_t}{\lambda_0}\right\}, & \text{if all } y_i \text{ are positive} \end{cases} \quad (36)$$

It may be may be noted that Eqn. (22) and Eqn. (36) are the same.When MLEs of the parameters ϕ and λ are estimated from group samples of large size, the estimate of each parameter is expected to follow independently asymptotic normal distribution (Nanjundan and Naika, 2012). Then, it can be shown that the monitoring statistic of LR chart is expected to follow a Chi-square distribution with 2 degrees of freedom, i.e.

$$LR_t \sim \chi_2^2$$

Therefore, for Type I error, α = 0.005, the UCL of the LR chart can be determined as

$$\text{UCL} = \left(\chi^2_{1-\alpha,2}\right)^{-1} = \left(\chi^2_{0.995,2}\right)^{-1} = 10.5966$$

On the other hand, 50th percentile point can be considered as the centre line (CL) of the LR chart, i.e. the CL of the LR chart can be determined as

$$\text{CL} = \left(\chi^2_{0.5,2}\right)^{-1} = 1.3863$$

It can be observed that the CL and UCL values of the LR chart are constant and those are independent of group sample size as well as ZIP process parameters ϕ and λ. The plotting statistic DS_t also follows χ_2^2 distribution, and therefore, CL and UCL of the LR chart and DS chart are the same.

5. Evaluation of Performances of Different One-chart Schemes with Large Sample

Arbitrarily two ZIP processes, e.g.ZIP (ϕ = 0.90, λ = 6) and ZIP (ϕ = 0.95, λ = 3) are chosen for application of the four one-chart schemes. In order to ensure that performances of all the four charts are comparable, sample size (n) and type I error (α) for all the four types of charts are chosen as 300 and 0.005 respectively. The CLs and UCLs of the four types of charts for the two ZIP processes are determined using the appropriate formulas discussed earlier and these values are shown in Table 1.

Table 1. The CLs and UCLs of the four types of charts for the two ZIP processes

ZIP process	Chart type	n = 300	
		CL	UCL
($\phi = 0.90, \lambda = 6$)	DS chart	1.3863	10.5966
	Max chart	1.0518	3.0230
	LR chart	1.3863	10.5966
	Gamma chart	0.6000	0.9312
($\phi = 0.95, \lambda = 3$)	DS chart	1.3863	10.5966
	Max chart	1.0518	3.0230
	LR chart	1.3863	10.5966
	Gamma chart	0.1500	0.2868

It can be observed from Table 1 that the CLs and UCLs of DS chart, Max chart and LR chart remain constant, irrespective of the ZIP process parameters ϕ and λ, which are due to the reasons explained in section 4. The UCL of the Gamma chart only changes depending on the values of the ZIP process parameters.

Our main point of interest here is to assess the performances of these charts in real life situation. A control chart has two important measure of performances: (1) Average number of points that will be plotted on or inside the UCL before a point falls outside the UCL, when there is no shift in the two process parameters. This measure is commonly known as in-control ARL (denoted as ARL_0). Less the value of ARL_0 for a chart, more will be the number of false alarms given by the chart; (2) Average number of points that will be plotted on or inside the UCL before a point falls outside the UCL, when there is a shift in one or more process parameters. This measure is commonly called as out-of-control ARL (denoted as ARL_1).

For a nominal-the-best type quality characteristic, both upward shift and downward shift are undesirable, and so, both in the cases the process is considered to be out-of-control. But for a STB type quality characteristic, a shift in the parameter(s) in any direction does not imply that the process has become out-of-control. A shift in a parameter in the desirable direction is indicative of process improvement, and a shift in the parameter in the undesirable direction only is indicative of out-of-control state of the process. A control chart for STB type quality characteristic should detect efficiently whenever one or more process parameters shift to undesirable direction. The detection of a shift of a parameter when it occurs in the desirable direction implies raising a false alarm. Therefore, for judging usefulness of a control chart for a STB type quality characteristic, this aspect needs to be taken into consideration besides the usual in-control and out-of-control ARL measures.

Since real life data are not available, it is decided to carry out simulation experiments for assessing relative usefulness of all the four types of control charts (mentioned in Table 1) for monitoring ZIP processes. To facilitate understanding about the in-control and out-of-control state of a ZIP(ϕ, λ) process, we use the following notations: values of ϕ and λ in in-control state are denoted as ϕ_0 and λ_0 respectively and shifted values of ϕ and λ are denoted as ϕ_1 and λ_1 respectively. For the purpose of carrying out the simulation experiments for ZIP(ϕ_0, λ_0) process, we consider the following: A shift of a fixed size may occur in each of the parameters (ϕ_0 and λ_0), and the shift may occur in undesirable direction or desirable direction. For ZIP(ϕ_0=0.90, λ_0=6) process, it is assumed that the shift size in ϕ_0 is 0.035 (which is equal to $2\times s_{\phi_0}$) and the shift size in λ_0 is 0.90 (which is equal to $2\times s_{\lambda_0}$). This implies that ϕ_1 values can be 0.935 (desirable) or 0.865 (undesirable) and λ_1 values can be 6.9 (undesirable) or 5.1 (desirable) in ZIP (0.90, 6) process. The ϕ_0 andλ_0 values, and possible different $\phi_1$and $\lambda_1$values for the simulation experiments are represented in Figure 1.

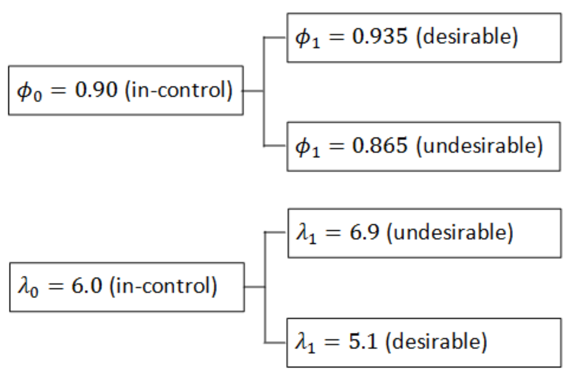

Figure 1. In-control and possible different shifted values of the parameters in ZIP(0.90,6) process

It may be determined easily from Figure 1 that there can be one combination of values of the two parameters when there is no shift in any of the parameters (i.e. in-control state), and there can be eight combinations of values of the parameters if shifts in one or more parameters are taken into consideration. These combinations of values of the two parameters and the corresponding process status are shown in Table 2.

The case numbers 8 and 9 need special attention. In these two cases, one parameter is shifted in desirable direction and another parameter is shifted in undesirable direction. The overall effect of these shifts may be nil, process deterioration or process improvement, which cannot be identified examining the shifted values of the parameters. However, the chances of occurrences of these cases are practically very rare in a manufacturing set up. It can be explained as follows: An increase in ϕimplies production of more number of zero-defect products and less number of defective products. On the other hand, an increase in λ implies an increase in the probability of occurrence of a defect in a product. Thus, simultaneously an increase in ϕ and an increase in λ is possible only when number of defective products is reduced and number of defects in the reduced number of defective products is increased, which is a unrealistic scenario in a manufacturing set up. Similarly, simultaneously a decrease in ϕ and a decrease in $\lambda$are possible when number of defective products increases and number of defects in the increased number of defective products decreases, which is again a unrealistic scenario in a manufacturing set up. Therefore, case numbers 8 and 9 are not taken into consideration for the purpose of simulation experiments.

Table 2. Different possible combinations of values of the two parameters for ZIP(0.90, 6) process

Case no.	Possible values of ϕ	Possible values of λ	Process status
1	ϕ_0=0.900	λ_0=6.0	***In-control***
2	ϕ_0=0.900	λ_1=6.9	Process deterioration
3	ϕ_1=0.865	λ_0=6.0	Process deterioration
4	ϕ_1=0.865	λ_1=6.9	Process deterioration
5	ϕ_0=0.900	λ_1=5.1	Process improvement
6	ϕ_1=0.935	λ_0=6.0	Process improvement
7	ϕ_1=0.935	λ_1=5.1	Process improvement
8	ϕ_1=0.935	λ_1=6.9	Low probability of occurrence
9	ϕ_1=0.865	λ_1=5.1	Low probability of occurrence

Similarly, for ZIP(0.95, 3) process, it is assumed that the shift size inϕ_0 is 0.026(=2s_{ϕ_0}) and the shift size in λ_0 is 0.974(=2s_{λ_0}), and then different combination of values of the two parameters are determined. These combinations of values of the two parameters are shown in Table 3. Due to the same reasons as explained above, case numbers 8 and 9 are not taken into consideration for the purpose of simulation experiments.

Table 3. Different possible combinations of values of the two parameters for ZIP(0.95, 3) process

Case no.	Possible values of ϕ	Possible values of λ	Process status
1	ϕ_0 = 0.950	λ_0 =3.0	***In-control***
2	ϕ_0 = 0.950	λ_1 = 3.974	Process deterioration
3	ϕ_1 = 0.924	λ_0 = 3.0	Process deterioration
4	ϕ_1 = 0.924	λ_1 = 3.974	Process deterioration
5	ϕ_0 = 0.950	λ_1 = 2.026	Process improvement
6	ϕ_1 = 0.976	λ_0 = 3.0	Process improvement
7	ϕ_1 = 0.976	λ_1 = 2.026	Process improvement
8	ϕ_1 = 0.976	λ_1 = 3.974	Low probability of occurrence
9	ϕ_1 = 0.924	λ_1 = 2.026	Low probability of occurrence

5.1 Simulation experiments

As mentioned earlier, the common measures of usefulness of a control chart are in-control ARL and out-of-control ARL. For STB type quality characteristic, an additional measure may be to consider the number of false alarms corresponding to a process improvement. If there is a shift implying a process improvement, the control chart should not give a false alarm indicating an out-of-control situation. The false alarm is directly related to the ARL corresponding to a process improvement. Higher is the ARL, less will be the numbers of false alarms. Therefore, it is decided to assess the ARL values for all the four types of charts for all the combination of parameter values shown in Tables 2 and 3 for ZIP(0.90,6) and ZIP(0.95,3) processes respectively.

From the ZIP process with a particular combination of values of the two parameters, 300 observations (y_1, y_2, y_3,..., y_{300}) are simulated as a sample. Then, ZIP parameters ($\hat{\phi}, \hat{\lambda}$) are estimated and using them, the plotting statistics of DSchart, Maxchart and LRchart are computed and compared with their respective UCLs. On the other hand, for Gamma chart, the plotting statistic $\bar{y}$is computed directly from simulated observations and compared with the UCL. If the plotting statistic lies below the UCL of a control chart, then again a set of 300 observations is simulated from the same ZIP process; plotting statistics is computed and compared with the UCL of the chart. In this process, samples of 300 observations are simulated until a computed plotting statistic falls above the UCL of the corresponding chart. The number of times samples of 300 observations are generated for getting an out-of-control point is taken as an estimate of run length of the control chart. In this process, run lengths are estimated 10,000 times, and then, average run lengths (ARL), standard deviation of run length (SDRL) and 95% confidence interval (CI) of run lengths are computed. The range of 97.5% percentile point and 2.5% percentile point of run length distribution is considered as the 95% CI of in-control run lengths. In the same way, ARL, SDRL, 95% CI of run lengths are obtained for all the charts. The ARL performances of all the four types of charts for the seven combinations of parameter values with respect to ZIP(0.90,6) process are shown in Table 4, and the ARL performances of all the four types of charts for the seven combinations of parameter values with respect to ZIP(0.95,3) process are shown in Table 5. Here, $\alpha = 0.005$ for all the four types of charts and so, the expected in-control ARL (denoted as ARL_0) for all the four types of charts is $1/\alpha = 1/0.005 = 200$. If the ARL value for a combination of parameter values (with shift in one or more parameters) is more than 300, it is considered very high and not mentioned in these tables.

The results in Tables 4 and 5 reveal that the in-control ARL is consistently very close to the expected value (200) for the LR chart, and consistently higher for the Gamma chart.

Table 4. ARLs for seven combinations of parameter values with respect to ZIP (0.90, 6) process

Sl. No. (process status)	ZIP process parameters		DS chart		Max chart		LR chart		Gamma chart	
			ARL	SDRL (CI)	ARL	SDRL (CI)	ARL	SDRL (CI)	ARL	SDRL (CI)
1 (IC)	ϕ_0=0.90	λ_0=6.00	175.3	27.3 (132, 238)	190.0	28.0 (142, 260)	210.0	33.0 (164, 286)	284.0	48.6 (215, 393)
2 (PD)	ϕ_0=0.90	λ_1=6.90	6.28	0.12 (6.06, 6.50)	5.88	0.12 (5.67, 6.09)	7.01	0.14 (6.75, 7.26)	27.1	1.33 (25.0, 30.0)
3 (PD)	ϕ_1=0.865	λ_0=6.00	6.3	0.13 (6.0, 6.5)	5.9	0.12 (5.6, 6.1)	7.2	0.17 (6.9, 7.6)	5.4	0.10 (5.2, 5.6)
4 (PD)	ϕ_1=0.865	λ_1=6.90	2.43	0.03 (2.39, 2.49)	3.47	0.05 (3.38, 3.57)	2.27	0.02 (2.23, 2.32)	2.0	0.02 (1.98, 2.05)
5 (PI)	ϕ_0=0.90	λ_1=5.10	7.35	0.19 (6.36, 6.94)	6.86	0.18 (6.59, 7.21)	6.65	0.17 (6.36, 6.94)	-	-
6 (PI)	ϕ_1=0.935	λ_0=6.00	6.8	0.19 (6.5, 7.2)	8.1	0.25 (7.7, 8.6)	6.2	0.17 (5.9, 6.6)	-	-
7 (PI)	ϕ_1=0.935	λ_1=5.10	2.53	0.03 (2.46, 2.58)	3.60	0.06 (3.48, 3.71)	2.75	0.03 (2.68, 2.81)	-	-

Note: IC = In-control, PD = Process deterioration, PI = Process improvement, CI=Confidence interval

Table 5. ARLs for seven combinations of parameter values with respect to ZIP (0.95, 3) process

Sl. No. (process status)	ZIP process parameters		DS chart		Max chart		LR chart		Gamma chart	
			ARL	SDRL (CI)	ARL	SDRL (CI)	ARL	SDRL (CI)	ARL	SDRL (CI)
1 (IC)	ϕ_1=0.950	λ_0=3.000	136.0	16 (108, 168)	140.0	17 (112, 179)	192.0	28 (156, 260)	276.0	56 (202, 382)
2 (PD)	ϕ_0=0.950	λ_1= 3.974	6.1	0.14 (5.9, 6.4)	5.8	0.13 (5.5, 6.1)	6.7	0.16 (6.4, 7.0)	14.4	0.50 (13.5, 15.5)
3 (PD)	ϕ_1=0.924	λ_0=3.000	6.0	0.14 (5.7, 6.3)	5.2	0.11 (5.0, 5.4)	7.7	0.21 (7.3, 8.1)	6.6	0.16 (6.3, 6.9)
4 (PD)	ϕ_1=0.924	λ_1= 3.974	2.29	0.03 (2.24, 2.34)	3.39	0.05 (3.28, 3.48)	2.01	0.02 (1.97, 2.05)	1.7	0.01 (1.69, 1.74)
5 (PI)	ϕ_0=0.950	λ_1= 2.026	6.9	0.15 (6.6, 7.2)	6.7	0.15 (6.4, 7.0)	7.1	0.16 (6.7, 7.4)	-	-
6 (PI)	ϕ_1=0.976	λ_0=3.00	7.0	0.18 (6.6, 7.4)	9.4	0.33 (8.7, 10.0)	5.4	0.13 (5.2, 5.6)	-	-
7 (PI)	ϕ_1=0.976	λ_1= 2.026	2.41	0.03 (2.36, 2.48)	3.45	0.05 (3.36, 3.54)	2.44	0.03 (2.38, 2.51)	-	-

Note: IC = In-control, PD = Process deterioration, PI = Process improvement, CI=Confidence interval

On the other hand, in-control ARL for the DS chart and Max chart are found to be inconsistent for the two ZIP processes. The out-of-control ARL performances of DS chart, Max chart and LR chart are noted to be consistently good and among these three charts, out-of-control performance of Max chart and DS-chart is better than LR-chart. On the other hand, the ARL_1 performances of the Gamma chart are observed to be poor when there is a shift in λ-value in the undesirable direction and ARL_1 is on par with other three charts when the shift is in ϕ value in the undesirable direction. But the performances of the Gamma chart are noted to be excellent when the shift in one or more parameters leads to process improvement. When the shift in one or more parameters leads to process improvement, the Gamma chart rarely detects the shift implying that the Gamma chart does not raise any false alarm. But, the other three types of charts treat the shift(s) in the parameter(s) leading to process improvement same as the out-of-control state, and so raise false alarm, which is unacceptable and a matter of serious concern.

6. Discussions

The monitoring statistics of DS chart, Max chart and LR chart charts, e.g.$\widehat{DS}_t$, $\widehat{M}_t$ and $\widehat{LR}_t$ are computed based on the MLEs of the two parameters $(\hat{\phi}_t, \hat{\lambda}_t)$ at time point t and their respective in-control (ϕ_0, λ_0) values. Therefore, it is necessary to obtain the MLEs of the two parameters $(\hat{\phi}_t, \hat{\lambda}_t)$ using the sample data of size n collected at time point t for application of these three types of control charts. Obtaining MLEs of the two parameters after collection of new sample data of size n every time is quite cumbersome and thus, implementation of these three types of charts may not be easy for many practitioners. On the other hand, there is no need for obtaining MLEs of the two parameters for application of the Gamma chart. The monitoring statistic of the Gamma chart is average number of defects $(\bar{y})$ and it can be computed directly from the observed sample data. Hence, application of the Gamma chart is much simpler and easier compared to other three charts for monitoring a ZIP process. Again, since the quality characteristic of interest is monitored directly in the Gamma chart, it is intuitively appealing to the practitioners and can offer significant advantages with respect to interpretations.

The results of simulation experiments reveal that LR chart is the most preferable one among the three types of charts that require obtaining MLEs of the two parameters for computing the monitoring statistics. The in-control ARL of the LR chart is very close to the expected values and it is consistent for different ZIP processes. The out-of-control ARLs of LR chart for different shifts in one or more parameters in undesirable direction are also found to be reasonably good. However, it also gives false alarms when there are shifts in one or more parameters in desirable direction (implying process improvement). In fact, all the three types of charts (e.g. DS chart, Max chart and LR chart) that utilize the MLEs of the two parameters for process monitoring purposes, give false alarms when actually there is a process improvement. This is because the monitoring statistics of these three schemes are defined utilizing the deviations between the MLEs of the parameters and their respective in-control values but ignoring the directions of the deviations.

On the other hand, the results of simulation experiments reveal that the in-control ARL of Gamma chart is quite higher than the expected value. A higher in-control ARL value implies lesser number of false alarms, which is desirable. However, the out-of-control ARL is also found to be quite high when there is a shift in λ-value in the undesirable direction, although the out-of-control ARL is on par with other three charts when the shift in ϕ value is in the undesirable direction. The Gamma chart is developed based on the findings that average number of defects in a ZIP(ϕ,λ) process approximately follows Gamma distribution with scale parameter, $k = n(1-\phi)\lambda/(1+\phi\lambda)$ and shape parameter, $\theta = (1+\phi\lambda)/n$, where n is the sample size. The error of approximation may be the cause behind the relatively poor out-of-control ARL performance of the Gamma chart. But the most important aspect of Gamma chart is that it does not give any false alarm when one or more parameters shift in desirable direction (implying process improvement). This is because the observed quality characteristic is directly monitored in Gamma chart.

7. Conclusions

Rapid technological advancement and automation of manufacturing processes, have resulted in many high quality processes. These high quality processes are characterized by more count of zero-defects products. A sample data on the number of defects in each item from such a high quality process will mostly consist of a large number of zero values and a few non-zero integer values. This type ofzero-inflated process data is generallymodeled by a two-parameter ZIP(ϕ, λ) distribution. In order to ensure stability of these zero-inflated processes, both the parameters (ϕ, λ) need to monitored using appropriate control charts. One-chart schemes for joint monitoring of the two parameters offer significant advantages because the practitioners need to focus on a sole chart (and, a single charting statistic) only. Recently, four one-chart schemes (Gamma chart, DS chart, Max chart and LR chart) are reported in literature for joint monitoring of the two parameters of a ZIP process. Gamma chart monitors average number of defects computed from sample of large size. The DS chart, Max chart and LR chart essentially monitor the deviations between the current MLEs of the parameters and their in-control values and these three schemes are developed assuming that sample of small size will be inspected. However, a sample of small size from a high quality ZIP process may offer a sample of zeros only, and then MLEs cannot be estimated. Therefore, under these schemes MLEs of the two parameters are estimated using the accumulated samples till the sampling stage. Because of the accumulation of samples, these schemes suffer from some limitations. In this paper, these one-chart schemes are modified for large sample size to make them free from their earlier limitations. Subsequently, the performances of the four one-chart schemes are evaluated extensively using simulation experiments. The results reveal that among the three modified charts likelihood ratio (LR) chart is slightly superior with respect to in-control and out-of-control ARL performances.

However, all these three modified charts give false alarms when one or more parameters shift towards desirable directions (implying process improvement). On the other hand, the Gamma chart is very good with respect to in-control ARL performance as well as out-of-control ARL performance with respect to shift in the inflation parameter ϕ towards lower side (i.e. undesirable direction). But the out-of-control ARL performances of the Gamma chart is relatively poor with respect to shift in the rate parameter (λ) towards higher side (i.e. undesirable direction). However, the most important aspect of the Gamma chart is that it does not give false alarm when there is a process improvement. Further, since Gamma chart directly monitors the average number of defects, it offers significant advantages in terms of implementation and interpretations.

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