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Sum of defects (SOD) chart for joint monitoring of the two parameters of a zero-inflated Poisson process

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Abstract

Rapid technological advancements and automation in today's world have led to many high quality manufacturing processes, where most of the products are free from defects. The defects data from high quality processes are commonly modeled by two-parameter zero-inflated Poisson (ZIP) distribution. Most of the existing control charting procedures for ZIP processes are developed assuming that all the manufactured units are inspected one by one, and most often separate control charts are proposed for monitoring the two parameters of a ZIP process. However, joint monitoring of the two parameters in a single chart can offer significant operational advantages because one needs to focus on a sole chart and a single charting statistic. Again, in many manufacturing set ups 100% inspection may not always be feasible. Recently, a few one-chart schemes with group inspection of small samples are reported in literature, and all these schemes require estimation of the two parameters based on accumulated samples till the current sampling stage for computation of the monitoring statistics. This accumulation of samples introduces several limitations in these schemes. In this paper, probability mass function and the cumulative distribution function (cdf) of the sum of defects (SOD) in a sample of size n are defined, and a control chart for SOD is proposed. Both the parameters of a ZIP process can be jointly monitored using the SOD chart. Again, since no accumulation of the samples is required for application of the SOD chart, it is free from all limitations of the existing one-chart schemes with group inspection. Two case studies based on past data are presented. The performance of the SOD chart is found to be very encouraging.

Keywords: Zero-inflated Poisson (ZIP), On-chart scheme, Group inspection, Sum of defects (SOD), Probability mass function, Cumulative distribution function, SOD chart.

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1. Introduction

Attribute c- and u-charts are widely used to monitor the number of non-conformities in products under the assumption that number of nonconformities follow a Poisson distribution with a single parameter $\lambda(\lambda>0)$. An increase in the Poisson parameter is often of more interest because it implies process deterioration, and therefore, both c- and u-charts are often one sided. However, when an unusually large number of zeros are present in the data, the use of the Poisson distribution tends to result in under-estimated mean and variance, which lead to tighter control limits, and subsequently higher than expected false alarm rates (Sim and Lim, 2008). Thus, these charts are not appropriate at all for monitoring high-quality zero-inflated processes. To account for this problem in a manufacturing application, Lambert (1992) and Xie et al. (2001) have developed zero-inflated Poisson (ZIP) model for count data with excess zeros. According to Lambert (1992) zero-inflated data essentially arise from a mixture of two distributions - a Bernoulli distribution that account for the inflated zero values and the standard Poisson (λ) distribution. Thus, it consists of two parameters, the first one is zero inflation parameter ϕ ($\phi \in [0,1]$) and the second one is the Poisson parameter λ (λ>0). On the other hand, Xie et al. (2001) have considered that zero-inflated data arise because of occurrences of random shocks in a near zero-defect process, where the number of non-conformities caused by random shocks follows a Poisson distribution with parameter λ and the shocks occur independently with probability p. However, although the inflation model (Lambert, 1992) and shock model (Xie et al., 2001) for zero-inflated data are conceptually little different, in their mathematical forms they are essentially the same because $p = 1 - \phi$. So, both the models fit well to the zero-inflated data and yield the same results.

In modern days, advanced technologies are integrated into the manufacturing processes to guarantee high quality products. So many researchers/practitioners have been motivated to investigate the issue of monitoring zero-inflated processes. Xie and Goh (1993) have proposed to use probability limits as the control limits instead of the standard 3-sigma limits for monitoring number of nonconformities per sample unit. Xie et al. (1995) have proposed to use cumulative count of conforming (CCC)-chart for monitoring the parameter ϕ and c-chart for monitoring the parameter λ. Chen et al. (2008) have proposed control charts based on the generalized ZIP (GZIP) distribution, which is an extension of the ZIP distribution. Sim and Lim (2008) constructed np-chart and c-chart based on the Jeffreys intervals for monitoring the parameters ϕ and λ respectively. The effect of estimation errors on Shewhart-type chart for ZIP processes are studied by Rakitzis and Castagliola. Zhang et al. (2019) have proposed to monitor the cumulative count of items until the r^{th}nonconforming one is met, and accordingly they have proposed CCC- r chart, which is an extension of the CCC-chart. He et al. (2012) have developed twp separate CUSUM control charts, one for detecting changes in λ and the other for detecting changes in ϕ. Leong et al. (2015) have developed a EWMA charting scheme representing a combination of two individual EWMA charts for monitoring two ZIP parameters. A control chart for detecting the scale parameter of the zero-inflated Poisson model is proposed by Zhao et al. (2024). Mamzeridou and Rakitzis (2023) have studied the performance of the upper one-sided Shewhart-type control charts for monitoring a zero-inflated Poisson process, in the case of estimated process parameters. Recently, many other non Shewhart-type control charts for monitoring ZIP processes have been proposed in literature, e.g. generally weighted moving average (GWMA) control chart (Alevizakos and Koukouvinos, 2020), generalized likelihood ratio based control charts (Lai et al., 2023; Rizzo and Bucchianico, 2023), unbiased function-based Poisson adaptive EWMA control chart (Abbas et al., 2023), etc.

It is noted that most of the existing control charting procedures for ZIP processes are developed assuming that all the manufactured units are inspected one by one, and most often separate control charts are proposed for monitoring the two parameters of a ZIP process. So implementation of these prevailing control charting schemes require that 100% products are inspected for the presence of nonconformities and it is feasible only if inspections are sensor-based (which happen in many manufacturing set ups now-a-days). However, in many other manufacturing set ups 100% inspection may not be feasible. Monitoring such processes using group inspection of size n after every certain interval should be a cost-effective approach. On the other hand, one-chart scheme for joint monitoring of the two parameters allows the practitioners to focus on a sole chart (and, a single charting statistic) and thus, it offers significant operational advantages. Therefore, a plausible strategy for monitoring a ZIP process may be to use a one-chart scheme with group inspection of size n. It may be noted that if inspections are sensor-based and online, then every n number of manufactured products may be considered as the group sample of size n.

Recently, Mukherjee and Rakitzis (2019) have suggested three one-chart schemes (DS chart, Max chart and Likelihood ratio based LR chart) for joint monitoring of the two parameters of a ZIP(ϕ, λ) process using group samples of small size. They make use of the current maximum likelihood estimates (MLEs) of ZIP parameters for computation of the monitoring statistics of these charts. Generally, a large number of samples are needed for estimating the parameters of a ZIP(ϕ, λ) process. Therefore, Mukherjee and Rakitzis's (2019) schemes require accumulation of previously collected samples till the current sampling stage and then obtaining MLEs of the parameters based on the accumulated samples. Since all three schemes of Mukherjee and Rakitzis (2019) require accumulation of samples progressively, they are called as progressive monitoring schemes.

It is observed that the accumulation of samples introduces several limitations in these schemes. For example, the issue of correlation in the successive values of the monitoring statistic arises, which results in interpretation problems. Also, because of accumulation of samples, these charts are more likely to give higher type I as well as type II errors. For example, number of defects in a sample may increase suddenly due to occurrence of an assignable cause, but it may not result in high values for the estimated ZIP parameters because of accumulation effect of zero or very less number of defects observed in the previously collected samples. Consequently, there may not be enough change in the value of monitoring statistic so that the chart indicates an out-of-control situation. Similarly, after detecting a shift in the ZIP parameters and taking necessary corrective measures, the monitoring statistics in the subsequent samples may continue to indicate out-of-control situation because of the accumulation effects. Further, no closed form equation for determination of the control limits of the monitoring statistics is given by Mukherjee and Rakitzis (2019). Therefore, simulation studies are needed for designing the DS chart or Max chart or LR chart, which may become a difficult task for many practitioners.

If a ZIP process can be monitored using small samples of size n but without accumulation of the samples, then limitations of the existing progressive monitoring schemes will be resolved. This motivates us for the current work. It is observed that the probability mass function (pmf) and the cumulative distribution function (cdf) of the sum of defects (SOD) in a sample of size n can be defined easily. Since SOD is a smaller-the-better (STB) type random variable, a control chart for monitoring SOD will require determination of the upper control limit (UCL) only. If the parameters are known or the parameters can be estimated quite accurately, it is always possible to compute the cumulative probability for different values of SOD, which is a discrete variable. Then, the value of SOD for which cdf is greater than equal to $1-\alpha$, where α is pre-defined acceptable type I error, may be considered as the UCL of the SOD chart for sample size n.

The rest of the article is organized as follows. The ZIP distribution and estimation of its parameters are presented in section 2. The distribution of total number of defects, i.e. SOD in a sample of size n is presented in Section 3. Some numerical illustrations about computation of pmf, cdf, expectation and variance of SOD are presented in Section 4. The designing of the SOD chart and its properties are described in Section 5. Application of the proposed SOD chart is presented in section 6. Section 7 concludes the paper.

2. ZIP Distribution and Estimation of its Parameters

Let the random variable Y, representing the number of defects in a product, follows a ZIP(ϕ,λ) distribution where $\phi \in [0,1]$ is the zero inflation parameter and λ (>0) is the mean of the standard Poisson distribution. Lambert (1992) defined the probability mass function of the ZIP distribution as

$$f(Y=y)=\begin{cases}\phi+(1-\phi)e^{-\lambda} & \text{for } y=0\\ (1-\phi)\frac{\lambda^{y}e^{-\lambda}}{y!} & \text{for } y>0.\end{cases} \tag{1}$$

Thus, the distribution of Y is a convex combination of a distribution degenerate at zero and a Poisson distribution with mean λ. The mean and variance of this ZIP distribution are given by the following two expressions:

$$E(Y)=\lambda(1-\phi) \tag{2}$$

$$Var(Y)=\lambda(1-\phi)(1+\phi\lambda) \tag{3}$$

Parameters of the ZIP distribution can be estimated from a sample data using the maximum likelihood method or by using the method of moments. The sample size n has to be sufficiently large enough (preferably $n \geq 200$), in order to obtain reliable estimates of the parameters.

Suppose, number (frequency) of sample units each having exactly 'd' number of defects is denoted by n_d (d = 0, 1, 2,..., m). Then, the log-likelihood function of ϕ and λ for an observed dataset of size n can be written as

$$\begin{aligned} &lnL(\phi,\lambda)\\ &= ln[\{P(Y=0)\}^{n_0}] + ln\,[\textstyle\prod_{d=1}^{m}\{P(Y=d)\}^{n_d}]\\ &= n_0\, ln\big[\phi+(1-\phi)e^{-\lambda}\big] + \textstyle\sum_{d=1}^{m} n_d ln\,\{(1-\phi)\frac{e^{-\lambda}\lambda^{d}}{d!}\}\\ &= n_0\, ln\big[\phi+(1-\phi)e^{-\lambda}\big] + (n-n_0)[ln\,\{(1-\phi)-\lambda\}] + D\, ln\,\lambda - \textstyle\sum_{d=1}^{m} n_d\, ln(d!) \end{aligned} \tag{4}$$

where $D=\sum_{d=1}^{m} dn_d$ is the total number of defects in the sample of size n. The partial derivatives of the log-likelihood function with respect to ϕ and λ result in the following two score functions (Xie and Goh, 1993):

$$\frac{\partial(\ln L)}{\partial\phi}=\frac{n_0(1-e^{-\lambda})}{\phi+(1-\phi)e^{-\lambda}}-\frac{n-n_0}{1-\phi} \tag{5}$$

$$\frac{\partial(\ln L)}{\partial\lambda}=\frac{-n_0(1-\phi)e^{-\lambda}}{\phi+(1-\phi)e^{-\lambda}}+\frac{D}{\lambda}-(n-n_0) \tag{6}$$

The MLEs of ϕ and λ can be obtained by equating the above two score functions to zero and then solving numerically. The optimal values of ϕ and λ can easily be determined by performing enumerative search using the 'Solver' tool of Microsoft Excel (Pal and Gauri, 2021). As an alternative, the maximum likelihood estimates $\hat{\phi}$ and $\hat{\lambda}$ can be obtained numerically by solving the following system of nonlinear equations (Mukherjee and Raktizis, 2019):

$$\hat{\phi}=1-\frac{\bar{y}}{\hat{\lambda}} \tag{7}$$

$$\hat{\lambda}=\bar{y}^{+}(1-e^{-\hat{\lambda}}) \tag{8}$$

where $\bar{y}$ is the sample mean of overall y_i $(i=1,2,3,\dots,n)$ values and $\bar{y}^{+}$ is the mean of non-zero y_i values. Clearly, there is no closed-form expression for the $\hat{\phi}$ or $\hat{\lambda}$, and they must be determined iteratively. Since the ZIP distribution belongs to two parameter exponential family, the MLEs of the ZIP parameters can be assumed to be asymptotically normal for reasonably large sample sizes (Nanjundan and Naika, 2012).

3. Distribution of Sum of IID ZIP Random Variables

Let us now assume that Y_1 and Y_2 are two independent and identically distributed (iid) ZIP random variables with parameters (ϕ,λ). Let the random variable S is defined as the sum of Y_1 and Y_2. Then, the probabilities $P[S=s]$ for $s=0,1,2,\dots$ of S can be computed as follows:

For $s = 0$,

$$P(S=0) = P(Y_1+Y_2=0) = P(Y_1=0)\times P(Y_2=0) = [\phi+(1-\phi)e^{-\lambda}]^2 \quad (9)$$

For $s \geq 1$,

$$\begin{aligned} P(S=s) &= P(Y_1+Y_2=s) = \textstyle\sum_{i=0}^{s} P[Y_1=i]\times P[Y_2=(s-i)] \\ &= P[Y_1=0]\times P[Y_2=s] + P[Y_1=s]\times P[Y_2=0] + \textstyle\sum_{i=1}^{s-1} P[Y_1=i]\times P[Y_2=(s-i)] \\ &= 2[\phi+(1-\phi)e^{-\lambda}]\times\left[(1-\phi)e^{-\lambda}\frac{\lambda^s}{s!}\right] + (1-\phi)^2 e^{-2\lambda}\textstyle\sum_{i=1}^{s-1}\left[\frac{\lambda^i}{i!}\times\frac{\lambda^{s-i}}{(s-i)!}\right] \\ &= 2\phi\times\left[(1-\phi)e^{-\lambda}\frac{\lambda^s}{s!}\right] + (1-\phi)^2 e^{-2\lambda}\textstyle\sum_{i=0}^{s}\left[\frac{\lambda^i}{i!}\times\frac{\lambda^{s-i}}{(s-i)!}\right] \\ &= 2\phi\times\left[(1-\phi)e^{-\lambda}\frac{\lambda^s}{s!}\right] + (1-\phi)^2 e^{-2\lambda}\frac{(2\lambda)^s}{s!} \\ &= 2\phi\times P[Y_1=s] + (1-\phi)^2\times Poisson[U=s \mid rate\ (\lambda')=2\lambda] \end{aligned} \quad (10)$$

Therefore, the probability mass function of S can be written as

$$f(S=s) = \begin{cases} [\phi+(1-\phi)e^{-\lambda}]^2 & \text{for } s=0 \\ 2\phi(1-\phi)\lambda e^{-\lambda}\frac{\lambda^x}{x!} + (1-\phi)^2\times Poisson\ [U=s \mid \lambda'=2\lambda] & \text{for } s>0 \end{cases} \quad (11)$$

Since, the random variable S is the sum of two independent and identically distributed ZIP random variables Y_1 and Y_2, the mean and variance of X can be obtained as

$$E(S) = E(Y_1)+E(Y_2) = 2(1-\phi_y)\lambda_y \quad (12)$$

and $Var(S) = Var(Y_1)+Var(Y_2) = 2\lambda_y(1-\phi_y)\{1+\phi_y\lambda_y\}$ (13)

Let us now assume that $Y_1, Y_2, \dots, Y_n$ are n iid ZIP random variables with parameters (ϕ,λ). Let the probability of Y_i $(i = 1,2,\dots,n) = j$ $(j = 0,1,2,\dots)$ is denoted by p_j, which can be computed using the pmf given in Eqn. 1. Suppose the random variable S is defined as the sum of $Y_1, Y_2, \dots, Y_n$, i.e. $S = \sum_{i=1}^{n} Y_i$ Then, the probabilities $P[S=s]$ for $s = 0,1,2,\dots$ of S can be computed as follows:

$$\begin{aligned} P(S=0) &= P(Y_1+Y_2+\dots+Y_n=0) \\ &= P(Y_1=0)\times P(Y_2=0)\times\dots\times P(Y_n=0) \\ &= [\phi+(1-\phi)e^{-\lambda}]^n \end{aligned} \quad (14)$$

For $s \geq 1$,

$$\begin{aligned} P(S=s) &= P(Y_1+Y_2+\dots+Y_n=s) \\ &= \textstyle\sum_{k_1=0}^{s} p_{k_1}\times\left\{\sum_{k_2=0}^{s-k_1} p_{k_2}\times\left[\sum_{k_3=0}^{s-k_1-k_2} p_{k_3}\times\dots\times\left[\sum_{k_{n-1}=0}^{s-k_1-k_2-\dots-k_{n-2}} p_{k_{n-1}}\times p_{s-k_1-k_2-\dots-k_{n-1}}\right]\right]\right\} \end{aligned} \quad (15)$$

The above equation can be simplified and written in the following form:

$$P(S=s) = \textstyle\sum_{k=1}^{n}\binom{n}{k}\phi^{n-k}(1-\phi)^k e^{-k\lambda}\frac{(k\lambda)^s}{s!} \text{ for } s\geq 1 \quad (16)$$

Therefore, the probability mass function of X can be written as

$$f(S=s) = \begin{cases} [\phi+(1-\phi)e^{-\lambda}]^n & \text{for } s=0 \\ \sum_{k=1}^{n}\binom{n}{k}\phi^{n-k}(1-\phi)^k\times Poisson\ [U=s \mid \lambda'=k\lambda] & \text{for } s>0 \end{cases} \quad (17)$$

3.1 Cumulative distribution function (cdf) of sum of n IID ZIP random variables

Let us denote the pmf of Poisson random variable U with parameter $\lambda' = k\lambda$ as $g_{k\lambda}(u)$. Then the pmf of $S = \sum_{i=1}^{n} Y_i$ can be written as

$$f(S=s) = \begin{cases} [\phi+(1-\phi)e^{-\lambda}]^n & \text{for } s=0 \\ \sum_{k=1}^{n}\binom{n}{k}\phi^{n-k}(1-\phi)^k\times g_{k\lambda}(s) & \text{for } s>0 \end{cases} \quad (18)$$

The cumulative distribution function $F_S(s)$ of S is

$$\begin{aligned} F_S(s) &= P[S\leq s] = \textstyle\sum_{t=0}^{s} f_S(t) \\ &= [\phi+(1-\phi)e^{-\lambda}]^n + \textstyle\sum_{t=1}^{s}\sum_{k=1}^{n}\binom{n}{k}\phi^{n-k}(1-\phi)^k g_{k\lambda}(t) \\ &= \phi^n + \textstyle\sum_{k=1}^{n}\binom{n}{k}\phi^{n-k}(1-\phi)^k g_{k\lambda}(0) + \sum_{t=1}^{x}\sum_{k=1}^{n}\binom{n}{k}\phi^{n-k}(1-\phi)^k g_{k\lambda}(t) \\ &= \phi^n + \textstyle\sum_{t=0}^{s}\sum_{k=1}^{n}\binom{n}{k}\phi^{n-k}(1-\phi)^k g_{k\lambda}(t) \\ &= \phi^n + \textstyle\sum_{k=1}^{n}\binom{n}{k}\phi^{n-k}(1-\phi)^k\sum_{t=0}^{s} g_{k\lambda}(t) \\ &= \phi^n + \textstyle\sum_{k=1}^{n}\binom{n}{k}\phi^{n-k}(1-\phi)^k G_{k\lambda}(s) \end{aligned} \quad (19)$$

where $G_{k\lambda}(s)$ denotes the cumulative distribution function of Poisson distribution with parameter $k\lambda$. It can be easily observed that $f_S(s) > 0$ for $\phi \in [0,1]$ and λ (>0).

Now, $\sum_{S=0}^{\infty} f_S(s) = \phi^n + \sum_{x=0}^{\infty}\sum_{k=1}^{n}\binom{n}{k}\phi^{n-k}\left(1-\phi_y\right)^k g_{k\lambda}(s)$

$$= \phi^n + \sum_{k=1}^{n}\binom{n}{k}\phi^{n-k}(1-\phi)^k \sum_{x=0}^{\infty} g_{k\lambda}(s)$$

$$= \phi^n + \sum_{k=1}^{n}\binom{n}{k}\phi^{n-k}(1-\phi)^k \times 1$$

$$= [\phi + (1-\phi)]^n = 1 \quad (20)$$

and

$$E(S) = E(Y_1) + E(Y_2) + \cdots + E(Y_n) = n\lambda(1-\phi) \quad (21)$$

$$Var(S) = Var(Y_1) + Var(Y_2) + \cdots + Var(Y_n) = n\lambda(1-\phi)(1+\phi\lambda) \quad (22)$$

4. Numerical illustrations of computation of pmf, cdf, expectation and variance of S

4.1 Illustration 1

Let the random variable Y represents the observed numbers of nonconformities/defects in the products produced in a manufacturing process, and it follows ZIP distribution with parameters $\phi = 0.90$ and $\lambda = 3$. Suppose, a sample of four products are collected from the manufacturing process and the number of defects present in these products are counted. Then, $Y_i(i = 1,2,3,4)$ are identically and independently distributed (iid) random variables, where $Y_i \sim$ ZIP(0.90, 3). So the probability mass function of the new variable $S = \sum_{i=1}^{4} Y_i$ can be written (according to Eqn. 17) as follows:

$$f(S = s) = \begin{cases} [0.90 + (1-0.90)e^{-3}]^4 & \text{for } s = 0 \\ \sum_{k=1}^{4}\binom{4}{k}(0.90)^{4-k}(1-0.90)^k \times e^{-3k}\frac{(3k)^s}{s!} & \text{for } s > 0 \end{cases}$$

and the cumulative distribution function can be written (according to Eqn. 19) as

$$F_S(s) = (0.90)^4 + \sum_{k=1}^{4}\binom{4}{k}(0.90)^{4-k}(1-0.90)^k \sum_{k=0}^{s} e^{-3k}\frac{(3k)^s}{s!}$$

Table 1, given below, shows the computed probability masses and cumulative probability masses for different values for S i.e. sum of defects in a sample of size 4 obtained from the ZIP (0.90, 3) process.

Table 1. The probability and cumulative probability masses for different values of S, i.e. sum of defects in a sample of size 4 obtained from ZIP(0.90, 3) process

s	0	1	2	3	4	5	6	7	8	9	10	11
$f_S(s)$	0.6707	0.0443	0.0675	0.0697	0.0556	0.0374	0.0228	0.0134	0.0079	0.0046	0.0027	0.0015
$F_S(s)$	0.6707	0.7150	0.7825	0.8523	0.9079	0.9453	0.9681	0.9816	0.9894	0.9940	0.9967	0.9982

The expectation and variance of S, i.e. sum of defects in a sample of size 4 can be obtained as

$$E(S) = n\lambda(1-\phi) = 4 \times 3 \times (1-0.90) = 1.20$$

and $$Var(S) = n\lambda(1-\phi)(1+\phi\lambda) = 4 \times 3 \times (1-0.90)(1+0.90 \times 3) = 4.44$$

4.2 Illustration 2

Let us now consider another manufacturing process where the number of defects (Y) in the output products follows a ZIP distribution with parameters $\phi = 0.95$ and $\lambda = 2$, i.e. $Y \sim$ZIP(0.95, 2) distribution. Suppose, a sample of five products are collected from the manufacturing process and the number of defects present in these products are counted. Then, $Y_i(i = 1,2,\ldots,5)$ are iid random variables, where $Y_i \sim$ZIP(0.95,2). So the probability mass function and the cumulative probability mass function of the new variable $S = \sum_{i=1}^{5} Y_i$ can be written (according to Eqns. 17 and 19 respectively) as follows:

$$f(S = s) = \begin{cases} [0.95 + (1-0.95)e^{-2}]^5 & \text{for } s = 0 \\ \sum_{k=1}^{5}\binom{5}{k}(0.95)^{5-k}(1-0.95)^k \times e^{-2k}\frac{(2k)^s}{s!} & \text{for } s > 0 \end{cases}$$

$$F_S(s) = (0.95)^5 + \sum_{k=1}^{5}\binom{5}{k}(0.95)^{5-k}(1-0.95)^k \sum_{k=0}^{s} e^{-2k}\frac{(2k)^s}{s!}$$

The computed probability masses and cumulative probability masses for different values for S, i.e. sum of defects in randomly collected five products from the ZIP (0.95, 2) process are shown in Table 2.

Table 2. The probability and cumulative probability masses for different values of S, i.e. sum of defects in a sample of size 5 obtained from ZIP(0.95, 2) process

s	0	1	2	3	4	5	6	7	8	9	10
$f_S(s)$	0.8017	0.0567	0.0583	0.0410	0.0227	0.0109	0.0049	0.0021	0.0009	0.0004	0.0002
$F_S(s)$	0.8017	0.8584	0.9167	0.9578	0.9805	0.9914	0.9963	0.9984	0.9993	0.9997	0.9999

The expectation and variance of the sum of four ZIP variables, i.e. S can be obtained as

$$E(S) = n\lambda(1-\phi) = 5 \times 2 \times (1-0.95) = 0.50$$

and

$$Var(S) = n\lambda(1-\phi)(1+\phi\lambda) = 5 \times 2 \times (1-0.95)(1+0.95 \times 2) = 1.45$$

The results in Table 1 reveals that there is only 0.15% chance that total number of defects, i.e. SOD in the sample of four products will be 11 and there is 99.82% chance that the SOD in the sample will be less than equal to 11. In other words, there is only 0.18% chance that the SOD in the sample will be 12 or more. Therefore, if the SOD in a sample of four products is found to be 12 or more, it may be concluded (with type I error $\alpha = 0.0018$) that the values of the parameters ϕ and/or λ must have become different from their known values, i.e. 0.90 and 3 respectively. Similarly, the results in Table 2 reveals that there is only 0.02% chance that SOD in the sample of five products will be 10 and there is 99.99% chance that the SOD in the sample will be less than equal to 10. In other words, there is only 0.01% chance that the SOD in the sample will be 11 or more. Therefore, if the SOD in a sample of five products is found to be 11 or more, it may be concluded (with type I error $\alpha = 0.0001$) that the values of the parameters ϕ and/or λ must have become different from their known values, i.e. 0.95 and 2 respectively.

Thus, the numerical results of the two illustrative examples clearly indicate that the two parameters of a ZIP(ϕ, λ) process can jointly be monitored effectively by implementing an appropriately designed control chart for monitoring the observed SOD in a sample of small size.

5. Designing Sum of Defects (SOD) Control Chart

It may be noted that $Y \sim$ ZIP(ϕ, λ), which is highly positively skewed and therefore, the distribution of sum of defects in a sample of size n, i.e. $S = \sum_{i=1}^{n} Y_i$ is also skewed. No known skewed probability distribution could be mapped to the distribution of sum of defects S. Therefore, it is recommended to make use of cdf of S for designing an appropriate control chart for the SOD. Since SOD is a STB type random variable, the control chart for monitoring SOD will require determination of the UCL only. If the parameters are known or the parameters can be estimated quite accurately, it is always possible to compute the cumulative probability for different values of S, which is a discrete variable. Then, the value of S for which cdf is greater than equal to $1-\alpha$, where α is pre-defined acceptable type I error, may be considered as the UCL of the SOD in a sample of size n. Thus, the procedure for constructing the control chart for SOD can be described as follows:

Case 1: When the parameters of the ZIP distribution are known

1) Note down the known values of the parameters ϕ and λ of the ZIP distribution.
2) Decide how many products will be collected at each sampling point, i.e. choose the value of sample size (n).
3) Let the observed number of defects in the i^{th} product of the sample is represented by Y_i, then consider sum of defects, $S = \sum_{i=1}^{n} Y_i$ as the monitoring statistic.
4) Define the cumulative distribution of S as
$$F_S(s) = P[S \le s] = \phi^n + \sum_{k=1}^{n} \binom{n}{k} \phi^{n-k}(1-\phi)^k \sum_{t=0}^{s} g_{k\lambda}(t)$$
5) Choose an acceptable value of type I error α
6) Determine the value of s such that $P[S \le s] \ge 1-\alpha$, and consider this value of s as the UCL of the control chart for SOD.
7) Compute $E(S) = n\lambda(1-\phi)$, and consider the expected value of S as the CL of the control chart for SOD.

For example, let us consider that we wish to monitor occurrences of number of defects in the manufactured product, and it is known that the occurrences of number of defects in the products follows ZIP distribution with parameters $\phi = 0.9$ and $\lambda = 3.0$. It is decided to collect random sample of size 4 after every certain intervals and monitor the SOD in the sampled products, i.e. it is chosen that $n = 4$ and $S = \sum_{i=1}^{4} Y_i$ is the monitoring statistic. The cdf of S is defined and it is chosen that the acceptable type I error $\alpha = 0.005$. Then, determine the value of s such that $P[S \le s] \ge 1 - 0.005 = 0.995$. The cdf values in Table 1 reveal that $P(S \le 9) = 0.9940$ and $P(S \le 10) = 0.9967$. Therefore, the UCL of the SOD will be 10. On the other hand, since $E(S) = 1.20$, the CL of the SOD chart will be 1.20. Similarly, the UCL and CL of the SOD chart for ZIP($\phi = 0.95, \lambda = 2.0$) process for sample size 5 can be determined from cdf values in Table 2 as follows: $UCL = 6$ and $CL = 0.50$.

Case 2: When the parameters of the ZIP distribution are unknown

1. Collect about m samples of size n each while the process is at in-control state. Preferably $m = 20$ and $n = 4$ or 5. Count the number of defects present in each of the mn products. Obtain MLEs of ϕ and λ using the methods described earlier.
2. Let the observed number of defects in the i^{th} product of the sample is represented by Y_i, then consider sum of defects, S=$\sum_{i=1}^{n} Y_i$ as the monitoring statistic
3. Define the cumulative distribution of S as
4. $F_S(s) = P[S \le s] = \hat{\phi}^n + \sum_{k=1}^{n} \binom{n}{k} \hat{\phi}^{n-k} (1-\phi)^k \sum_{t=0}^{s} g_{k\hat{\lambda}}(t)$
5. Choose an acceptable value of type I error α
6. Determine the value of s such that $P[S \le s] \ge 1-\alpha$, and consider this value of s as the UCL of the control chart for SOD.
7. Compute $E(S) = n\hat{\lambda}(1-\hat{\phi})$, and consider the expected value of S as the CL of the control chart for SOD.

Properties of the chart

Type I error (α) occurs when there is no shift in any of the two parameters (ϕ, λ), i.e. the ZIP process is at in-control state, but the chart plots a point above UCL. On the other hand, type II error (β) occurs when mean of any one (or both) of parameters (ϕ, λ) in the ZIP process shifts from its in-control value to any other undesirable value, but the chart fails to detect the same on the first subsequent sample, i.e. plotted point falls below UCL. For better understanding, let us consider that the in-control values of the parameters are ϕ_0 and λ_0 respectively, and the shifted values of the parameters ϕ and λ are ϕ_1 and λ_1 respectively. Then, the type I error (α) and type II error (β) of the SOD can be computed as follows:

$$\alpha = P(S > UCL|\phi = \phi_0, \lambda = \lambda_0)$$
$$= 1 - P(S \le UCL|\phi = \phi_0, \lambda = \lambda_0)$$
$$= 1 - \left[\phi_0^n + \sum_{k=1}^{n} \binom{n}{k} \phi_0^{n-k} (1-\phi_0^n)^k \sum_{t=0}^{UCL} g_{k\lambda_0}(t)\right]$$
$$\beta = P(S \le \text{UCL}|\phi = \phi_1, \lambda = \lambda_1)$$
$$= \phi_1^n + \sum_{k=1}^{n} \binom{n}{k} \phi_1^{n-k} (1-\phi_1^n)^k \sum_{t=0}^{UCL} g_{k\lambda_1}(t)$$

Then, in-control average run length (ARL_0) and out-of-control average run length (ARL_1) can be estimated as follows:

$$ARL_0 = \frac{1}{\alpha} \quad \text{and} \quad ARL_1 = \frac{1}{1-\beta}$$

Here, ARL_0 value signifies the average number of points that will be plotted on or inside the UCL of SOD chart before a point falls outside the UCL when there is no shift in the two process parameters, and ARL_1 value signifies the average number of points that will be plotted on or inside the UCL before a point falls outside the UCL when there is a shift in one or more process parameters.

For illustration purpose, let us consider the case of monitoring of ZIP(*0.9*, *3*) process described earlier. It is decided that the process will be monitored by plotting observed SOD in a sample of size 4 on the SOD chart. The UCL of the SOD chart is determined as 10 implying that the expected type I error $\alpha = 0.0033$ (see Table 1). Therefore, the in-control ARL is found to be $ARL_0 = \frac{1}{0.0033} = 300$. On the other hand, suppose that the concerned ZIP(*0.9*, *3*) has become out-of-control because the values of ϕ and λ have become different from their in-control values. Say, the values of ϕ and λ have become 0.80 and 5.0 respectively, i.e. $\phi_1 = 0.80$, $\lambda_1 = 5.0$. Then, the type II error β can be computed as

$$\beta = P(S \le 10|\phi_1 = 0.80, \lambda_1 = 5.0)$$
$$= 0.80^4 + \sum_{k=1}^{4} \binom{4}{k} \phi^{4-k} (1-0.80)^k \sum_{t=0}^{10} g_{k\lambda}(t) \quad = 0.9062$$

Therefore, the out-of-control ARL is found to be $ARL_1 = \frac{1}{1-\beta} = \frac{1}{1-0.9062} = 10.70$.

6. Applications

He et al. (2012) have proposed a CUSUM control charting procedure for monitoring a zero-inflated Poisson process and they have presented a case study of a LED packaging industry. On the other hand, Fatahi et al. (2012) have proposed a ZIP-EWMA scheme for monitoring rare health-related events. They have applied the ZIP-EWMA scheme in a high technology hospital in Tehran for monitoring and detecting an increase in the counts of the number of unintentional needle-stick occurrences per day. The published data of these two case studies are taken into consideration for demonstration of application of the proposed SOD chart.

6.1 Case study data of LED packaging industry (He et al., 2012)

In the LED packaging process, products are produced batch by batch. Within each batch, there is a large number of LEDs soldered into printed circuit boards (PCB) and then cut into components. In the case study application, the LEDs within a batch were 100%

inspected after the batch was manufactured, and the defective LEDs were counted and recorded. He et al. (2012) presented defect data observed in 200 batches of LED packaging. They assumed that the first 100 data points as phase I observations for determining the control limits of their proposed t-CUSUM chart for simultaneous monitoring of the two parameters of a ZIP process and the second 100 data points as phase II observations for monitoring purpose. They removed four data points from 100 numbers of phase I sample data assuming them as possible outliers. With the remaining 96 observations they fitted an appropriate ZIP model for describing the defect data in LED packaging process. They obtained the maximum likelihood estimates (MLEs) of the parameters of ZIP distribution as $\hat{\phi} = 0.8745$ and $\hat{\lambda} = 5.5619$. Thus, the in-control values of the parameters are assumed to be known as $\phi_0 = 0.8745$ and $\lambda_0 = 5.5619$.

It may be noted that t-CUSUM chart simultaneously monitors the two parameters based on individual observations. However, our proposed SOD chart is designed for simultaneously monitoring the two parameters based on group sample observations Suppose we decide to implement the SOD chart for monitoring the same LED packaging process using samples of subgroup size $n = 3$. This implies that the preliminary samples of 96 observations (phase I data) of He et al. (2012) may be viewed as the 96/3=32 subgroups of size 3 each. Let the sample observations in a subgroup are denoted by the random variables Y_1, Y_2 and Y_3. Then, the monitoring statistic of the SOD chart will be $S = \sum_{i=1}^{3} Y_i$, where Y_i is independently and identically distributed as $ZIP(\phi_0 = 0.8745, \lambda_0 = 5.5619)$. The probability mass $[f_S(s)]$ and cumulative probability mass $[F_S(s)]$ for a given value of $S = s$ can be obtained using Eqns. (17) and (19) respectively. Table 3 shows the computed probability masses and cumulative probability masses for different values of the random variable S.

Table 3. The probability and cumulative probability masses for different values of S for subgroup size 3

s	0	1	2	3	4	5	6	7	8	9
$f_S(s)$	0.6699	0.0062	0.0172	0.0319	0.0445	0.0499	0.0471	0.0387	0.0287	0.0199
$F_S(s)$	0.6699	0.6761	0.6932	0.7250	0.7695	0.8195	0.8665	0.9052	0.9339	0.9539
s	10	11	12	13	14	15	16	17	18	19
$f_S(s)$	0.0136	0.0094	0.0067	0.0049	0.0036	0.0026	0.0018	0.0013	0.0008	0.0005
$F_S(s)$	0.9674	0.9768	0.9835	0.9884	0.9920	0.9946	0.9965	0.9977	0.9986	0.9991

Suppose, the acceptable type I error as $\alpha = 0.0027$, which is commonly chosen for the Shewhart type control chart. So $1 - \alpha =$ 0.9973. It can be found from Table 3 that if $s = 17$, then $P[S \leq s] = P[S \leq 17] = 0.9977 \geq 0.9973$. Therefore, the UCL of the SOD chart can be considered as 17, i.e.

$$UCL_{SOD} = 17$$

Again, the expectation of S can be computed using Eqn. (19) as $E(S) = n\lambda_0(1 - \phi_0) = 3 \times 5.5619 \times (1 - 0.8745) = 2.094$. Therefore, the centre line (CL) of the SOD chart can be considered as 2.094, i.e.

$$CL_{SOD} = 2.094$$

The sum of defects (S) are computed for each of the 32 subgroups and these are plotted in the SOD chart with UCL=17 and CL=2.094 (see Figure 1). The chart shows that all the plotted points are well below the UCL and hence it can be concluded that the LED process was in statistical control during phase I data collection period. This finding is in agreement with He et al. (2012).

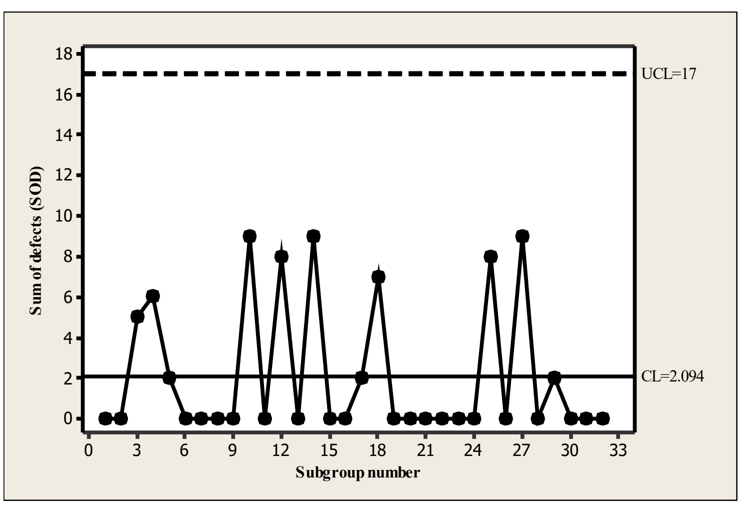

Figure 1. SOD chart with $n = 3$ for phase I data obtained from LED process by He et al. (2012)

He et al. (2012) collected 100 observations in phase II and applied their proposed *t*-CUSUM chart on these data. We take into consideration the first 99 observations for application of the SOD chart. Since we considered subgroup of size 3, there will be 99/3=33 sample points on the SOD chart. Figure 2 shows the plots of the phase II data on the SOD chart.

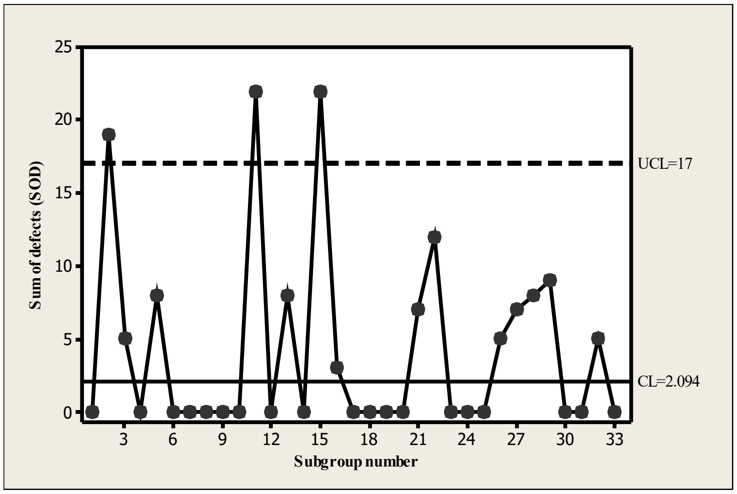

Figure 2. SOD chart with subgroup size 3 for phase II sample data of LED process

Figure 2 shows that there are three points (values of s for subgroup numbers 2, 11 and 15), which fall above the UCL of the SOD chart. This implies that some assignable cause has occurred during the period of phase II data collection. Detailed examinations of the defects data of these subgroups reveal that the point corresponding to subgroup number 2 falls above UCL because one of the sample of it contained a very large number of defects although there is zero defects in other two samples. This is indicative that there is a hike in the rate parameter λ. On the other hand, only one sample contains zero defects and other two samples contain moderate number of defects in both subgroup numbers 11 and 15. This is indicative that possibly there are moderate shifts in both the parameters in undesirable directions, i.e. inflation parameter ϕ has decreased and rate parameter λ has increased. Figure 2 further reveals that all the plotted points after subgroup number 15 fall below the UCL of SOD chart. Possibly some corrective action was taken after detection of the out-of-control process condition but it is not reported by He et al. (2012).

It may be worth to mention here that He et al. (2012) also identified the same out-of-control process at similar time point using their *t*-CUSUM chart. That implies that both the *t*-CUSUM chart and SOD chart result in equivalent performance. However, SOD chart for joint monitoring using group inspection has several advantages over the *t*-CUSUM chart for joint monitoring using individual observations, e.g. 1) Computation of monitoring statistic of SOD chart, its interpretation and practical significance are much simpler than the monitoring statistic of *t*-CUSUM chart, 2) Once the value of the monitoring statistic falls on or above the UCL in *t*-CUSUM chart, the value of the monitoring statistic is required to be reset to its ideal value; otherwise, subsequent values of the monitoring statistic will tend to fall beyond the control limits because of the cumulative effect. No such issue arises in case of SOD chart. 3) The process history is lost due to resetting of the monitoring statistic in case of t-CUSUM chart. The process history always remains available in case of SOD chart.

6.2 Case study data of Shariati hospital in Tehran (Fatahi et al., 2012)

In hospitals and infirmaries, the subject of needle-stick occurrence for the personnel or patients is becoming more and more important due to the resulting incurable diseases like HIV and Hepatitis. It would be infeasible to record every time a needle was handled by any personnel whether the needle-stick would have occurred or not. But the number of needle-stick occurred for all personnel, can be counted at the end of a day. It is very important to monitor and detect an increase in the counts of the number of unintentional needle-stick occurrences per day in a hospital and infirmaries. Fatahi et al. (2012) have observed that the number of unintentional needle-stick occurrences per day in a hospital and infirmaries follows ZIP distribution, and they have proposed a ZIP-EWMA control scheme for monitoring the ZIP distribution describing the occurrences of unintentional needle-stick. They demonstrated application of their proposed method through a case study carried in Shariati hospital (a high technology hospital) in Tehran,

Fatahi et al. (2012) collected data on the number of unintentional needle-stick occurrences for consecutive 90 days in the hospital. The data set is available in Fatahi et al. (2012). Aly et al. (2022) used the same data to illustrate application of adaptive EWMA chart for joint monitoring of the parameters of a ZIP process. Aly et al. (2022) considered the first 40 observations as the phase I data and the remaining 50 observations as the phase II data. The first 40 observations (i.e. phase I data) are used for estimating in-control parameters of the ZIP process and designing the control chart. The designed control chart is applied on the remaining 50 observations (i.e. phase II data). It may be noted that Aly et al. (2022) proposed adaptive EWMA chart simultaneously monitors the two parameters based on individual observations.

Let us consider that we wish to monitor the occurrences of numbers of needle-stick using SOD chart with subgroup size 5. In this case, it effectively means that we want to monitor the number of needle-stick per 5 days. Likewise Aly et al. (2022), here also the first 40 observations can be used for estimating the parameters of the ZIP process. Aly et al. (2022) obtained the maximum likelihood estimates of the ZIP parameters as $\hat{\phi} = 0.5593$ and $\hat{\lambda} = 2.3827$. Thus, the in-control values of the parameters are assumed to be known as $\phi_0 = 0.5593$ and $\lambda_0 = 2.3827$.

The preliminary samples of 40 observations (phase I data), considered by Aly et et al. (2012), may be viewed as the 40/5=8 subgroups of size 5 each. Let the sample observations in a subgroup are denoted by the random variables Y_1, Y_2, Y_3, Y_4 and Y_5. Then, the monitoring statistic of the SOD chart will be $S = \sum_{i=1}^{5} Y_i$, where Y_i is independently and identically distributed as $ZIP(\phi_0 = 0.5593, \lambda_0 = 2.3827)$. The probability mass $[f_S(s)]$ and cumulative probability mass $[F_S(s)]$ for a given value of $S = s$ can be obtained using Eqns. (15) and (17) respectively. Table 4 shows the computed probability masses and cumulative probability masses for different values of the random variable S.

Table 4. The probability and cumulative probability masses for different values of S for subgroup size 5

s	0	1	2	3	4	5	6	7	8	9
$f_S(s)$	0.0778	0.0628	0.0951	0.1110	0.1145	0.1100	0.0996	0.0852	0.0692	0.0537
$F_S(s)$	0.0778	0.1406	0.2357	0.3467	0.4613	0.5713	0.6708	0.7560	0.8253	0.8790
s	10	11	12	13	14	15	16	17	18	19
$f_S(s)$	0.0399	0.0284	0.0195	0.0129	0.0083	0.0051	0.0030	0.0018	0.0010	0.0005
$F_S(s)$	0.9188	0.9473	0.9668	0.9797	0.9880	0.9931	0.9961	0.9979	0.9989	0.9994

Suppose, the acceptable type I error as $\alpha = 0.0027$. So $1 - \alpha$ = 0.9973. It can be found from Table 4 that if s=17, then $P[S \leq s] = P[S \leq 17] = 0.9979 > 0.9973$. Therefore, the UCL of the SOD chart can be considered as 17, i.e.

UCL_{SOD} =17

Again, the expectation of S can be computed using Eqn. (19) as $E(S) = n\lambda_0(1 - \phi_0) = 3 \times 2.2837 \times (1 - 0.5593) = 5.25$. Therefore, the centre line (CL) of the SOD chart can be considered as 5.25, i.e.

CL_{SOD} =2.094

The sum of defects (S) are computed for each of the 8 subgroups and these are plotted in the SOD chart with UCL=17 and CL=5.25 (see Figure 3). The chart shows that all the plotted points are well below the UCL and hence it can be concluded that the numbers of needle-stick per 5 days were in statistical control during phase I data collection period. This finding is in agreement with Aly et al. (2022).

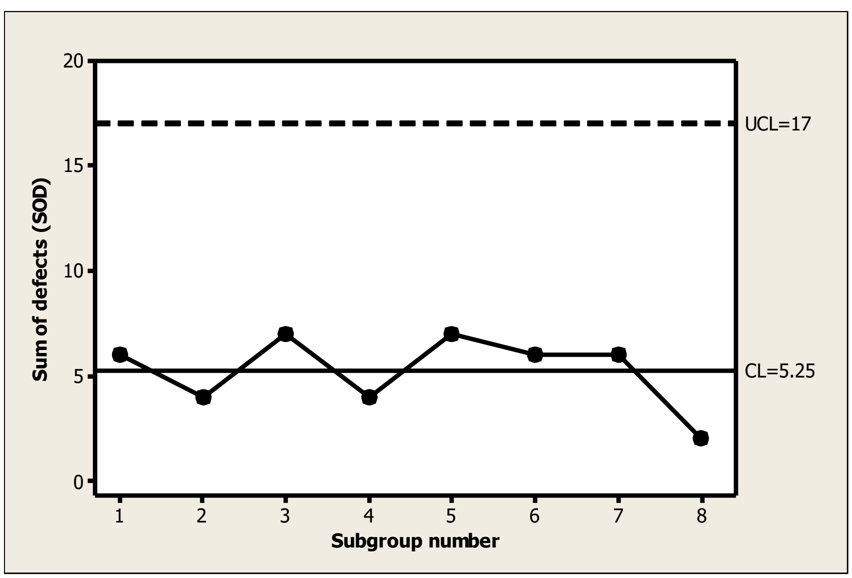

Figure 3. SOD chart with subgroup size 5 for phase I sample data on needle-stick occurrences

Now the SOD chart is applied on the remaining 50 points for monitoring purposes. Since we considered subgroup of size 5, there will be 50/5=10 sample points on the SOD chart. Figure 4 shows the plots of the phase II data on the SOD chart.

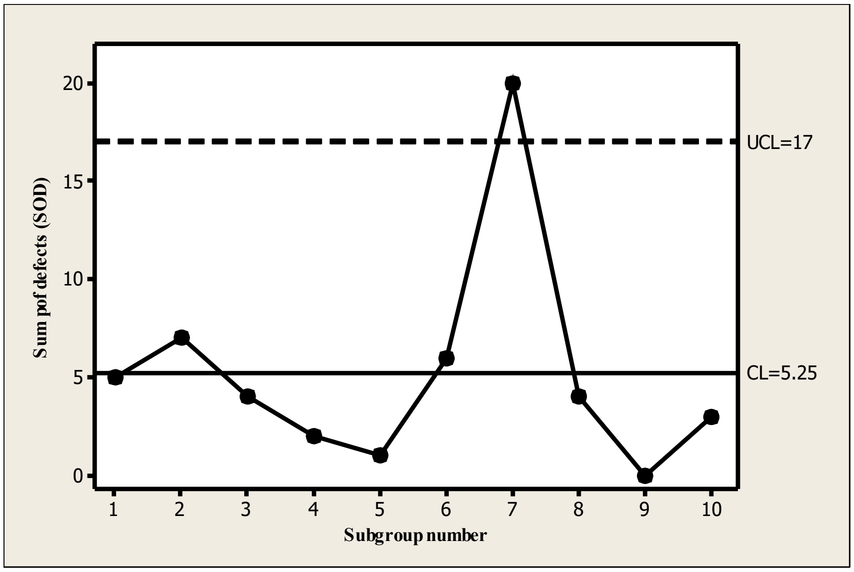

Figure 4. SOD chart with subgroup size 5 for phase II sample data on needle-stick occurrences

Figure 4 shows that one point (subgroup number 7) falls outside the UCL of the SOD chart. This is indicative that the process has become out-of-control. Examination of the five values in the subgroup number 7 reveals that there are four non-zero observations in the subgroup of size five, which implies that there is a shift in zero inflation parameter ϕ of the ZIP process towards undesirable direction.

It is important to note that the adaptive EWMA chart, proposed by Aly et al. (2022), also could detect the out-of-control process condition at the same time, which implies that SOD chart and adaptive EWMA chart results in equivalent performance. However, SOD chart for joint monitoring using group inspection has several advantages over the adaptive EWMA chart for joint monitoring using individual observations, e.g. 1) Computation of monitoring statistic of SOD chart, its interpretation and practical significance are much simpler than the monitoring statistic of adaptive EWMA chart, 2) Once the value of the monitoring statistic falls outside the control limits in adaptive EWMA chart, the value of the monitoring statistic is required to be reset to its ideal value; otherwise, subsequent values of the monitoring statistic will tend to fall beyond the control limits because of the cumulative effect. No such issue arises in case of SOD chart. 3) The process history is lost due to resetting of the monitoring statistic in case of adaptive EWMA chart. The process history always remains available in case of SOD chart.

7. Conclusions

High quality manufacturing processes are quite common in today's high-tech world. Two-parameter zero-inflated Poisson (ZIP) distribution is commonly used to model the defects data arising in high quality manufacturing processes. It is necessary to monitor both the parameters to ensure stability of the ZIP process. Most often separate control charts are used for monitoring the two parameters and these charts are developed assuming that all the manufactured units are inspected one by one. Since in many manufacturing set ups 100% inspection may not be feasible, and one-chart scheme offers significant advantages over simultaneously using of two charts, some researchers have been motivated to develop one-chart schemes with group inspection. Recently, a few one-chart schemes with group inspection are reported in literature, and all these schemes require estimation of the two parameters based on accumulated samples till the current sampling stage for computation of the monitoring statistics. This accumulation of samples introduces several limitations in these schemes. In this paper, probability mass function and the cumulative distribution function (cdf) of the sum of defects (SOD) in a sample of size n are defined. Although the distribution of SOD could not be mapped to any known probability distribution, it is found that a control chart for SOD can be designed easily based on the cdf of SOD. The proposed SOD chart can jointly monitor the two parameters of a ZIP process. Again, since no accumulation of the samples is required for application of the SOD chart, it is free from all the limitations of the existing one-chart schemes with group inspection. Two case studies, where t-CUSUM and adaptive EWMA charts for joint monitoring using individual observations are applied, are taken into consideration. The SOD chart is applied on the data of those two case studies. It is found that SOD chart results in equivalent performance to t-CUSUM and adaptive EWMA charts. However, SOD chart offers several advantages over the t-CUSUM and adaptive EWMA charts. For example, 1) Computation of monitoring statistic of SOD chart, its interpretation and practical significance are much simpler, 2) Need for resetting the value of the monitoring statistic after detection of out-of-control signal does not arise, 3) The process history always remains available since the monitoring statistic is never reset. Further study is necessary for characterizing the distribution of SOD in terms of defining the appropriate parameters of the distribution. This will make designing of the SOD chart simpler.

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