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Multivariate control chart for controlling variability – A comparative study

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Abstract

Control chart is an important technique for statistical process control. In most of the practical situations data are collected on more than one characteristic under study. Hence multivariate control chart has drawn the attention of quality control practitioners. For successful implementation of multivariate control chart, two different charts are to be constructed. One is for controlling mean and another is for controlling variance-covariance matrix or ∑ matrix. For effective implementation of control chart for mean, one need to establish that variability is under control. Hence control chart for controlling variance-covariance matrix is playing an important role in multivariate control chart. There are several control charts available in the literature which can be used for controlling ∑ matrix. In this article, we have discussed different such control charts and studied their performance with respect to their ability to detect the shift in the components of ∑ matrix. Out of control average run length (ARL) is considered as a measure for this purpose and the best method to detect the shift have been identified under different conditions.

Keywords: Multivariate normal distribution, average run length, simulation, variance covariance matrix

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1. Introduction

Variation is inevitable in any process (manufacturing or service) which results in nonconformities or dissatisfaction of customers. Hence reduction of variability is the main tasks of the quality control department of any industry. Control chartis a useful technique formonitoring the process output with the aim of reduction of variability. Control charts are two fold; control chart for mean and control chart for variance. The implementation of a control chart involves in taking samples of fixed or variable size at regular intervals, computing the statistic (mean or variance), plotting it in the prescribed control limits and searching for an assignable cause when a point falls outside the control limits. Univariate control chart is applicable when only one variable is to be controlled. In many real-life situation, it is required to monitor several correlated variables. Multivariate control charts are useful under these situations. The aim of these control charts has typically been to monitor the vector of the process mean or the covariance matrix. The basic multivariate control chart is described elaborately in the book by Montgomery (2019).

Let us illustrate some of the examples of multivariate control charts in real life situations. In the recent era, we often see that the quality of a product can be attributed to several correlated quality characteristics, all of which are to be monitored simultaneously. Many such successful applications are available in literature, especially in last few decades. Niaki and Abbasi (2005) developed a model based on artificial neural network to identify aberrant variables. They applied their technique to control the multivariate characteristics (color, free oil percentage, acidity percent and acidity number) of the solfunation process of an Iranian detergent making company.Flores et. al. (1995) exhibited an application of multivariate control chart in the microlithography process in a semiconductor industry. In the case of stepper overlay, there are typically six variables related to wafergrid-staging errors with additional variables pertaining to intra-field effects. To achieve an optimal micro lithographic performance, it is necessary to monitor these variables simultaneously. Hotelling T^2 control chart was applied successfully which resulted in reducing nonconformities.

Mason et al. (2001) applied multivariate control chart in batch process producing a spatiality plastic polymer. A thorough chemical analysis is conducted on each batch to assure that the composition of seven measured components adhere to a rigid chemical formulation. The rigid chemical formulation is required for mold release when the plastic is transformed to a usable product. They have successfully implemented multivariate control chart to control the chemical compositions. Parra and Loziza (2003)applied multivariate T^2 control chart technique in a drug manufacturing industry. A crystalline drug substance has an impurity profile consisting of seven major organic impurities. The impurity profile constitutes an identifier of a particular drug substance and its associated method of manufacture. Any modification to the method of manufacture of a drug substance carries some risk of causing adverse impact in the impurity profile. Hence maintaining consistency in impurity profile is very important for a drug substance. In this study, Hotelling T^2 control chart and decomposition procedure proposed by Mason et. al. (1995) was applied to achieve consistency in impurity profile of the drug substance. Bersimis et al.(2005) described a noble application of multivariate control chart in a chemical process. They used the method described by Maravelakis et al. (2002) to identify the out of control variable.

In the last two decades, there has been an increasing research interest in multivariate quality control, which is evidenced by the large number of papers published in statistical and quality journals. The recent development is certainly welcoming since in many industrial applications the quality of a product can be attributed to several correlated quality characteristics, all of which need to be controlled and monitored simultaneously. The majority of the research in the last 20 years focuses on developing multivariate control charts for monitoring shifts in process mean. Hotelling (1947) proposed a Shewhart type multivariate control chart that monitors mean vector of two or more correlated characteristics. Pignatiello Jr and Runger (1990) and Crosier (1988) suggested memory based multivariate control charts. They developed Multivariate CUSUM (MCUSUM) control charts that monitor the mean vector. Lowry et al. (1992) suggested a Multivariate EWMA (MEWMA) control chart which is an extension of univariate EWMA control chart. Thesememory based control charts are proved to be more efficient to detect the small shifts in the mean vector than Shewhart type chart. All these multivariate control charts for controlling mean vector assumes that the process dispersion $\sum_0$ remains constant. This assumption should be validated by using control chart for monitoring $\sum$ matrix. Hence, control chart for controlling $\sum$ matrix draws the attention of researcher in the quality control field.

Many researchers proposed different methods to control $\sum$ matrix of correlated variables. Some of them are listed below. Alt and Smith (1988) present several schemes for monitoring process dispersion. One scheme is a direct extension of the univariate S^2 control chart, and is equivalent to repeated tests of significance of the form H_0: $\sum = \sum_0$ vs H_1: $\sum \neq \sum_0$. The test is based on a modification of the likelihood ratio test. Guerrero-Cusumano (1995) suggested a Control Chart Based on Conditional Entropy. Tang and Barnett (1996 a,b) suggested a Control Chart Based on the Decomposition of S_t.Levinson, et al. (2002) proposed the G chart. This scheme is based on the comparison of the sample variance-covariance matrix of each subgroup with an overall estimate of $\sum_0$. Khoo and Quah (2003) proposed a multivariate control chart for process dispersion based on individual observations. Surtihadi et al (2004) developed a methodology to detect the shift in $\sum$ matrix based on Shewhart and CUSUM principle. Yeh et al. (2004, 2005) proposed a likelihood ratio based EWMA control chart for monitoring variability. Vargos and Lagos (2007) proposed a modified G chart using a robust estimator of the variance covariance matrix. Hawkins (1991, 1993) proposed a multivariate control chart for monitoring the process mean based on regression adjusted variables.

The same idea coupled with his earlier work in Hawkins (1981) can be extended to constructing multivariate control charts for monitoring process variability. This idea is expanded and discussed in more detail in Huwang et al. (2007). Costa and Machado (2008) proposed a control chart based on maximum value of sample variances of two quality characteristics. Yen and Shiau (2010) proposed a technique to detect the increase in dispersion matrix based on one-sided likelihood ratio test for phase-II control chart. Jeong and Cho (2012) proposed a technique based on maximum likelihood estimator. Hamed (2014) illustrated an application of generalized variance control chart with multivariate data in High Pressure Stage, Low Pressure Stage and Evaporation and Prilling Stage in Urea production process of Delta Fertilizers and Chemical Industries in Egypt. Abujiya et. al. (2015) presented a new CUSUM control chart for controlling multivariate dispersion based on well-structured sampling techniques - extreme ranked set sampling, extreme double ranked set sampling and double extreme ranked set sampling. They applied their technique on a real life data set from a real dataset based on the problem of filling bottles on a Pepsi-Cola production line.

Alfaro and Ortega (2019) suggested a new chart based on exponentially weighted covariance matrix combination.Riaz et. al (2019) proposed a new control chart combining EWMA and CUSUM for detecting a shift in multivariate dispersion matrix. Haq and Khoo (2020) proposed 4 new control charts for monitoring multivariate dispersion matrix. First one is based on CUSUM control chart proposed by Crosier (1986) which is a slight modification of original CUSUM, second one is based on EWMA chart proposed by Robert (1959), third and fourth one are adaptive version of first and second one. They have shown the run length performance of the proposed chart and exhibited a real life application on the data of bimetal thermostat. Ajadi et. al (2021) established an EWMA control chart by taking logarithm to the diagonal elements of the estimated covariance matrix. This chart is

also robust to non-normality. Yang and Liu (2022) proposed a new multivariate dispersion control chart which is not dependent on whether process mean is under control or not. They have used their methodology in the multivariate data from a semiconductor manufacturing industry. Adegoke et. al. (2022) proposed a nonparametric control chart for detecting the shift in var-covariance matrix based on Alt's likelihood ratio test. This chart is efficient in detecting shift for both normal and non-normal distributions.
Since a large number of control charts for controlling $\sum$ matrix are available in the literature, a comparison study of their performance is called for. Hence the aim of this paper is to discuss different methods for controlling $\sum$ matrix and compare their performance to detect the shift in covariance matrix.

2. Different multivariate control chart procedure for controlling variability under study

In this section, we will describe the methods for monitoring multivariate dispersion matrix considered for this study in a nut shell. Due to time constraint it was not possible to include each and every technique in a simulation study for comparison purpose. Hence we have selected some techniques developed in twenty first century viz. 2000 onwards. A brief description of the methodologies under study is given below. Interested readers can go through the original papers.

2.1 Control Chart Based On G Statistic (G-Chart)

Levenson et al. (2002) suggested G chart based on G statistic proposed by Kramer and Jensen (1969). Kramer and Jensen developed G statistic to test the equality of variance covariance matrix ($\sum$) of two populations combining the estimates of $\sum$ based on full data set (S_1) and Mean square successive difference or MSSD (S_2) from n_1 and n_2 data. Weighted average of S_1 and S_2 is following:

$$S = \frac{(n_1-1)*S_1 + (n_2-1)*S_2}{n_1+n_2-2} \quad (1)$$

$$M = \ln \frac{|S|^{n_1+n_2-2}}{|S_1|^{n_1-1}|S_2|^{n_2-1}} \quad (2)$$

$$m = 1- \left[\frac{1}{n_1-1} + \frac{1}{n_2-1} - \frac{1}{n_1+n_2-2}\right] * \left[\frac{2p^2+3p-1}{6(p+1)}\right] \quad (3)$$

where p = number of variables.

$$G = M*m \quad (4)$$

Where, $G \sim \chi^2_{p(p+1)/2}$

Levenson et al. proposed to plot G statistic in a chart with following control limits:

$$UCL = \chi^2_{\frac{p(p+1)}{2},\ \alpha/2} \quad (5)$$

$$LCL = \chi^2_{\frac{p(p+1)}{2},\ 1-\alpha/2} \quad (6)$$

The advantage of this chart is its operational simplicity. Calculating G statistic is very easy and control limits are also simple since G follows χ^2 distribution. The performance of this chart is evaluated by the author through a simulation study but no performance measure like ARL (average run length) is given in the paper. We have included it in the present study for its operational simplicity.

2.2 Control Charts Based On Generalized Likelihood Ratio Statistic (L- chart)

Surtihadi et al. (2004) suggested a control chart based on generalized likelihood ratio statistic. They considered both Shewhart type and CUSUM type control chart. They considered three types of shifts. The first is the change by known scalar in one of the components. The second accounts for the change in one of the components by unknown scalar. The third takes care of the proportional scaling of the covariance. For these three different cases they proposed three different statistics L_1, L_2 , L_3 based on generalized likelihood ratiotest. The proposed statistics do not follow any standard distribution. Hence they used simulation to get 100(1-α) percentile points as upper control limit or UCL.

Where, $L_1 = \max_{j=1,\dots,p}\left\{-\sum_{i=1}^{n} X_i^T B_j X_i\right\}, B_j = \sum_j^{-1} - \sum_0^{-1}$ (7)

$$L_2 = \max_{j=1,\dots,p}\left\{-n \ln c_j^* - \frac{1}{2}\left(\frac{1}{\left(c_j^*\right)^2} - 1\right)\sigma^{jj}\sum_{i=1}^{n} X_{ij}^2 - \left(\frac{1}{c_j^*} - 1\right)\sum_{i=1}^{n}\left(X_{ij}\left(\sum_{k=1,k\neq j}^{p} X_{ik}\,\sigma^{jk}\right)\right)\right\} \quad (8)$$

$$c_j^* = \frac{1}{2n}\sum_{i=1}^{n} X_{ij}\left(\sum_{k=1,k\neq j}^{p} X_{ik}\sigma^{jk}\right) + \frac{1}{2}\left\{\left[\frac{1}{n}\sum_{i=1}^{n}\left(X_{ij}\left(\sum_{k=1,k\neq j}^{p} X_{ij}\sigma^{jk}\right)\right)\right]^2 + \frac{4}{n}\sigma^{jj}\sum_{i=1}^{n} X_{ij}^2\right\}^2 \quad (9)$$

$$L_3 = \sum_{i=1}^{n} X_i^T \textstyle\sum_0^{-1} X_i - \frac{np}{2}\ln\left[\sum_{i=1}^{n} X_i^T \sum_0^{-1} X_i\right] \quad (10)$$

They have proposed two types of control chart viz. Shewhart type and CUSUM type for three types of shift. The distribution of the statistics do not have any closed form. Hence they calculated the control limits for different values of n and p. They have evaluated the performance of their proposed methods with respect to out of control ARL and demonstrated that their technique is showing better result than existing multivariate S^2 chart proposed by Alt (1973). But the limitation of this chart is its computational complexity. It is not only difficult to calculate the statistics but the calculation of control limits is involving simulation. One has to write simulation program to construct the control limits. Another restriction is that the chart can be applied for the three shifts considered in this work. So it is not a complete work. There exist some other types of shifts when number of variables is more than 2 which is not covered in the paper. Since its ARL performance is better than multivariate S^2 chart we include it in our study.

2.3 Control Chart Based On Max Statistic (Max chart)

Costa et al. (2008) proposed a chart based on maximum of sample variance for bivariate population for monitoring their ∑ matrix. Let X and Y be two quality characteristics.

$$S_x^2 = \frac{\sum_{i=1}^{n}(X_i-\mu_{0x})^2}{n} \quad (11)$$

$$S_y^2 = \frac{\sum_{i=1}^{n}(Y_i-\mu_{0y})^2}{n} \quad (12)$$

Chart statistic is Max (S_x^2, S_y^2)

The MAX chart combines two univariate S^2 charts into one chart.

This chart is applicable for bivariate case only. It is an alternative to the use of two S^2 charts. It has both computational simplicity and better diagnostic feature. If an out of control signal is pinned, it can be easily found out which variable is responsible for that. This chart is also applicable if sample size is not constant. They developed the expression for ARL of the MAX chart. They have shown that out of control ARL performance of this chart is slightly better than generalized $|S|$ chart.

2.4 Control Chart Based On MLE Statistic (MLE chart)

Jeong and Cho (2012) proposed the following procedure.

Let X_{ijk}denotes the jth observation of ith variable of kth sample, where i = 1,…,p, j= 1,….,n, k= 1,2,…

$$Z_{kij} = \frac{X_{kij}-\mu_{0i}}{\sigma_{0i}}, \quad (13)$$

Where, μ_{0i} is the ith component in control μ and σ_{0i} is the ith component in control σ.

$Z_{kj} = (Z_{k1j}, Z_{k2j}, \ldots\ldots, Z_{kpj})$ be the standardized vector.

$\sum_Z$ *be the covariance matrix of* Z_{kj}

Let $\overline{X_{ki}} = \frac{\sum_{j=1}^{n} X_{kij}}{n}$ be the sample mean for ith variable and kth sample

$Z_{ki} = \sqrt{n}\,(\bar{X}_{ki} - \mu_{0i})/\sigma_{0i}$ be the standardized sample mean for i=1,2,…p

For kth sample $\widehat{\sum}_{zk}$ is the maximum likelihood estimator of $\sum_Z$, where (i, i') th element of $\widehat{\sum}_{zk}$ is $\sum_{j=1}^{n} z_{kij}\, z_{ki'j}$/n

Hotelling (1947) suggested a control statistic to monitor ∑ as follows:

$$\sum_{j=1}^{n}(Z_{k1j}, Z_{k2j}, \ldots, Z_{kpj}) \textstyle\sum_{z0}^{-1} (Z_{k1j}, Z_{k1j}, \ldots, Z_{k1j})' = n\mathrm{tr}\,(\widehat{\sum}_{zk}\sum_{z0}^{-1}) = Y_k \quad (14)$$

Since Y_k is not following a standard distribution when the process is out of control, Control limits are obtained through simulation. The calculation of the statistic is quite easy but it is difficult to obtain the control limits since the statistic is not following any standard distribution. Hence for effective implementation of this control chart a simulation study is needed. They have calculated the ARL for the proposed chart for different shifts in variance and covariance component with different values of n (sample size), p (number of variables) and ρ (correlation coefficient) but the results are not compared with any existing method.

2.5 Control Chart Based On EWMA Covariance Matrix Combination(MEWCM chart)

Alfaro and Ortega (2019) developed a control chart to test change in ∑ using multivariate exponentially weighted covariance matrix combinations. It can detect the change in ∑ matrix in both the situations, i) when mean vector changes as well as ii) when mean does not change.

They have used the following statistic:

$$F_t = wV_t + (1-w)\, F_{t-1} \quad (15)$$

Where 0 <w< 1 $F_1 = V_1$ is the sample variance-covariance matrix that can be estimated from phase-1 analysis.
For t >1, V_t the sample variance-covariance matrix that can be estimated from phase-2 analysis.
Hence F_t can be written as

$$F_t = \sum_{i=1}^{n} c_{it} V_i \tag{16}$$

Using the concept given by Huwang et. al (2007) Alfaro and Ortega suggested to use the trace of F_t to construct the MEWCM chart.

$$\mathrm{Tr}\,(F_t) = TF_t = \sum_{i=1}^{t} c_{it} tr(V_t) \tag{17}$$

Since The distribution of TF_t is not following any standard distribution the authors used simulation to get the control limit. But they showed that for t > 20, the distribution of TF_t follows $\frac{1}{t+1}\chi^2_{p(t+1)}$. They provided the values of control limits for t ≤ 20 in their paper for p=2 and 3 and for t > 20 one can use the percentile point of χ^2 *distribution.* Hence there should not be any computational complexity for this chart. They have computed the ARL for the proposed chart and compared them with those of MEWMS proposed by Huwang et al (2007) for p=2 and 3. They showed that their chart performs better when the shift in the variance is high and the shift in mean does not affect the performance.

2.6 Nonparametric Control Chart Based On Gower-Based Likelihood Ratio Statistic(GLR chart)

Adegoke et al. (2022) proposed a multivariate nonparametric control chart for controlling shifts in the covariance matrix of multivariate set up by projecting the multivariate dataset onto the Euclidean space less than or equal to the dimension of the monitoring features. They have used the concept given Gower (1985). Then uses these sets of coordinates to monitor shifts in the dispersion parameter. The least absolute shrinkage and selection operator (LASSO)-based Alt's likelihood ratio statistic used to monitor the ∑ matrix. A bootstrap method was used to obtain the control-chart limit for a suitable distance-based model through time. The matrix q gives an ordination from which the dispersion statistic is calculated in Phase-I. Here, they define q = $(q1, q2, \ldots, qr)^T$ as a random vector representing the first r axes of the q.
The Shewhart-type statistic based on the approximate Alt's likelihood ratio is given as:

$$w_i = tr(S_i) - \log |S_i| - p, \quad i=1,\ldots n \tag{18}$$

where $S_i = q_i^T q_i$
For phase-II, the approximate Alt's likelihood ratio for monitoring the ith sample is

$$w_i^* = tr(H_i) - \log |H_i| - p, \quad i=1,\ldots n \tag{19}$$

H_i is the inverse of the LASSO-based penalized normal likelihood estimate of S_i^*

A bootstrap method was used to obtain the control-chart limit for a suitable distance-based model through time.
The uniqueness of the proposed method is that it can be applied for discrete or a mixture of discrete and continuous multivariate random variables. There exists some operational complexity for constructing control limits. They have given a step by step procedure for implementation of the proposed chart. The out of control ARL values are computed considering different distributions. This comparison shows that the chart performs better for multivariate normal distribution. They applied the proposed chart using lapping process data in wafer semiconductor manufacturing.

3. Performance Comparison

All the above methods described in section 2.1-2.6 are having their own merits and demerits. Details are available in the respective paper. Their performance to detect the shift is given in some of the original papers. But different author considers different types of shifts. Hence, in this paper, we are providing their performance to detect the shift in the component of ∑ matrix considering the same pattern of shifts bringing them in the same platform, so that one can identify their relative position. For comparison purpose Average run length (ARL) for different shifts is taken as yard stick. Run length is defined as the number of samples required to get an out of control signal when the process is actually out of control. Run length is having its own distribution. Average run length is the mean value of run length for a particular shift. It is worthwhile to note here that lower the value of ARL better the performance. Without loss of generality, we consider bivariate normal distribution as the parent distribution. We further assume that under in control state the joint distribution of the two characteristics under study (say X and Y) follows bivariate normal with mean vector $\mu_0 = [0,0]^T$ and covariance matrix $\sum_0$.

Where, $\sum_0 = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$

0 ≤ ρ ≤ 1, ρ being the correlation coefficient between X and Y.
For this study purpose the value of ρ is taken as 0.5.

To investigate the performance of the control chart following cases are considered.
Case 1: Change in standard deviation of one characteristic by δ multiplier i.e. $\sigma_{0,x} \rightarrow \delta\sigma_{0,x}$ or $\sigma_{0,y} \rightarrow \delta\sigma_{0,y}$
Case 2: Change in standard deviation of both the characteristic by δ multiplier i.e. $\sigma_{0,x} \rightarrow \delta\sigma_{0,x}$ and $\sigma_{0,y} \rightarrow \delta\sigma_{0,y}$
For 6 methods under study control limits are computed assuming false alarm probability as 0.05 following the procedure described in the concerned papers. Then simulating observation from bivariate normal with mean vector μ_0 and covariance matrix $\sum_1$, where $\sum_1$ is the changed covariance matrix for different values of δ run lengths to detect the shift are computed by writing a computer program in MATLAB. Then average run length is calculated by simulating 5000 run lengths for each value of δ. The values of δ are taken as 0.2, 0.4, 0.6, 0.8, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0. The results are summarized in the following tables.

Table 1: Performance of different control charts with respect to ARL for various δ (Case 1)

Values of δ		G-Chart	L- chart	Max chart	MLE chart	MEWCM chart	GLR chart
δ < 1	0.2	8.33	1.4	3.5	1.1	2.9	3.2
	0.4	33.3	4.2	12.7	10.8	13.4	11.7
	0.6	58.8	7.8	23.9	16.3	26.8	25.2
	0.8	84.1	9.1	35.4	24.6	40.1	34.6
δ > 1	1.5	59.1	6.2	29.6	21.2	37.2	31.2
	2.0	34.7	2.2	9.8	2.2	18.1	14.8
	2.5	14.7	1.6	8.3	1.6	15.8	3.7
	3.0	8.2	1.2	3.4	1.0	8.8	2.8
	3.5	5.1	1.1	2.8	1.0	6.1	1.5
	4.0	3.7	1.0	2.3	1.0	3.9	1.0
	4.5	2.8	1.0	1.7	1.0	3.1	1.0
	5.0	2.1	1.0	1.5	1.0	2.3	1.0

Table 2: Performance of different control charts with respect to ARL for various δ (Case 2)

Values of δ		G-Chart	L- chart	Max chart	MLE chart	MEWCM chart	GLR chart
δ < 1	0.2	2.22	1.6	3.1	1.2	2.5	3.3
	0.4	14.2	4.1	12.1	9.6	11.3	9.8
	0.6	40	6.8	22.2	15.4	27.9	16.3
	0.8	69.4	5.2	34.7	21.6	41.2	29.4
δ > 1	1.5	59.2	13.2	49.6	20.1	36.8	28.5
	2.0	24.2	4.3	22.4	2.02	18.1	12.1
	2.5	8.65	2.7	16.7	1.5	13.9	2.6
	3.0	4.04	1.8	9.1	1.0	8.2	1.9
	3.5	2.4	1.5	7.9	1.0	5.1	1.2
	4.0	1.9	1.2	5.2	1.0	3.6	1.0
	4.5	1.4	1.0	4.1	1.0	2.7	1.0
	5.0	1.2	1.0	3.2	1.0	2.1	1.0

4. Conclusions

This paper has discussed the various control charts and studied their performance with respect to their ability to detect the shift in the components of $\sum$ matrix. Furthermore, the following conclusions can be drawn from the entire study.

1. G chart is the simplest chart to operate but its performance is not as good as the other charts. There is some operational complexity in implementing L chart since the computation of its control limits needs simulation. But its performance for

detecting shift is noteworthy. Max chart is very easy to operate and its performance is better than G chart. MLE chart is suffering from computational complexity but its performance is better than the other charts except L chart. MEWCM chartis easy to operate for t > 20, and its performance is commendable for detecting high shift. GLR chart is involving some operational complexity but its performance is good for detecting large shift and this is the only chart among the six charts considered in this study which can be applied for discrete or a mixture of discrete and continuous multivariate random variables.

2. All the methods are showing better result in case-2 (shift in the variability of both the characteristics) than in case-1 (shift in the variability of one characteristic).
3. For $\delta > 1$, when shift occurs in variability of one characteristic, L chart is performing better than the other charts but for case-2, i.e. when shift occurs in variability of both the characteristics, L chart and MLE chart are at par and showing better result than the other charts
4. For $\delta < 1$, L chart and MLE chart are at par and showing better result than the other charts
5. GLR chart is also performing well for detecting large shift i.e. $\delta > 2.5$

This study can be extended for the other methods mentioned in section 2. We have restricted our work in bivariate case which also can be extended for more than 2 variables. This involves in rigorous computation which may not be feasible without higher order computational facility. The performance analysis also can be done by simulating observations from different non normal distributions also can be explored. This also requires high level simulation software. The effect of changing the value of correlation coefficient (ρ) can also be studied. The different types of shifts in the component of $\sum$ matrix (change in one direction or change in different direction) can also be studies.

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