

## Reaction of undamped systematically driven vibrators via Gupta Transform

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### Abstract

This paper deals with the dictation of the reaction of undamped, systematically driven electrical and mechanical vibrators by the Gupta transform (GT) and tenders an alternate style for the dictation of the reaction of undamped, systematically driven electrical and mechanical vibrators. It reveals that the GT is an effectual method for the dictation of undamped, systematically driven electrical and mechanical vibrators and proves the materiality of the GT for the dictation of reaction of undamped, systematically driven electrical and mechanical vibrators.

**Keywords:** Reaction, Gupta transform (GT), undamped, systematically driven electrical and mechanical vibrators

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### 1. Introduction

The integral GT has been alleged by the authors Rahul Gupta and Rohit Gupta in recent years (Gupta et al, 2022). This paper deals with the dictation of the reaction of undamped, systematically driven electrical and mechanical vibrators by the Gupta transform (GT). There are a number of styles like the Laplace transform, matrix approach, Rohit transform, Shehu transform, SEE transform, Aboodh transform, Mohand transform, etc. This paper tenders an alternate style for the dictation of the reaction of undamped, systematically driven electrical and mechanical vibrators. It reveals that the GT is an effectual method for the dictation of undamped, systematically driven electrical and mechanical vibrators and proves the materiality of the GT for the dictation of reaction of undamped, systematically driven electrical and mechanical vibrators. The GT (Gupta et al, 2022) is defined for a function of exponential order as follows: Considering functions in the set  $C$  defined as:

$$C = \{g(t): \exists R, q_1, q_2 > 0, \quad |g(t)| < Re^{q_1 t(t)}, \text{ if } t \in (-1)^i X[0, \infty)\}$$

For a given function in set C, the constant R must be a finite number  $q_1$  and  $q_2$ , may be finite or infinite.

The GT of a function  $g(t)$  is defined by the integral equations as

$$\mathring{R}\{g(t)\} = G(q) = \frac{1}{q^3} \int_0^\infty e^{-qt} g(t) dt, t \geq 0, q_1 \leq q \leq q_2.$$

The variable  $q$  in this transform is used to factor the variable  $t$  in the argument of the function  $g$ . The key stimulation for appealing GT for the dictation of the reaction of undamped, systematically driven electrical and mechanical vibrators is that the procedure of solving a ruling ordinary differential equation for such issue is clarified to an algebraic problem (Gupta et al, 2022). This procedure of metamorphosing the issues of calculus to algebraic issues is well known as operational calculus. The GT procedure has two chief edges over the calculus procedure:

- i. issues entailing differential equations are reckoned up more directly i.e. initial (or boundary) value issues are reckoned up without first ascertaining a common solution.
- ii. A Non-homogenous differential equation is reckoned up without first reckoning up the correlating homogeneous differential equation.

The GT, when applied to a function, metamorphose that function into a novel function by using a procedure that entails integration.

The GT of some basic functions is as follows

- o  $\mathring{R}\{t^n\} = \frac{n!}{q^{n+4}}, \text{ where } n = 0,1,2,3 \dots \dots$
- o  $\mathring{R}\{e^{at}\} = \frac{1}{q^3(q-a)}, q > a$
- o  $\mathring{R}\{\sin at\} = \frac{a}{q^3(q^2+a^2)}, q > 0$
- o  $\mathring{R}\{\cos at\} = \frac{1}{q^2(q^2+a^2)}, q > 0$

$$\mathring{R}\{\delta(t - a)\} = \frac{1}{q^4} e^{-aq}$$

The GT of some derivatives is given by

$$\mathring{R}\{g'(t)\} = \frac{1}{q^3} \int_0^\infty e^{-qt} g'(t) dt$$

$$\begin{aligned} \mathring{R}\{g'(t)\} &= \frac{1}{q^3} \{-g(0) - \int_0^\infty -qe^{-qt} g(t) dt\} \\ &= \frac{1}{q^3} \{-g(0) + q \int_0^\infty e^{-qt} g(t) dt\} \\ &= qG(q) - \frac{1}{q^3} g(0) \end{aligned}$$

Now cutting out  $g(t)$  by  $g'(t)$  and  $g'(t)$  by  $g''(t)$ , we have

$$\begin{aligned} \mathring{R}\{g''(t)\} &= q\mathring{R}\{g'(t)\} - \frac{1}{q^3} g'(0) \\ &= q \left\{ q\mathring{R}\{g(t)\} - \frac{1}{q^3} g(0) \right\} - \frac{1}{q^3} g'(0) \\ &= q^2 \mathring{R}\{g(t)\} - \frac{1}{q^2} g(0) - \frac{1}{q^3} g'(0) \\ &= q^2 G(q) - \frac{1}{q^2} g(0) - \frac{1}{q^3} g'(0) \end{aligned}$$

Hence  $\mathring{R}\{g'(t)\} = q\mathring{R}\{g(t)\} - \frac{1}{q^3} g(0)$ ,

$$\mathring{R}\{g''(t)\} = q^2 G(q) - \frac{1}{q^2} g(0) - \frac{1}{q^3} g'(0)$$

and so on.

## 2. Methodology

### I. Undamped Mechanical Systematically Driven Vibrator

The systematically driven mechanical vibrator (Gupta et al, 2022 and Gupta et al, 2023) is specified by

$$m\ddot{y}(t) + b\dot{y}(t) + ky(t) = F \cos \omega t \tag{1}$$

For an **Undamped Mechanically Systematically Driven Vibrator**, draining constant (Olivar et al, 2017),  $b = 0$ , therefore, (1) transforms to

$$m\ddot{y}(t) + ky(t) = F \cos \omega t$$

Or

$$\ddot{y}(t) + \omega_0^2 y(t) = \frac{F}{m} \cos \omega t \tag{2}$$

$$\text{where } \omega_0 = \sqrt{\frac{k}{m}}$$

Here,

(i)  $y(0) = 0$ .

(ii)  $\dot{y}(0) = 0$ .

The GT of (2) furnishes

$$q^2 \bar{y}(q) - \frac{1}{q^2} y(0) - \frac{1}{q^3} \dot{y}(0) + \omega_0^2 \bar{y}(q) = \frac{F}{m} \frac{1}{q^2(q^2 + \omega^2)} \tag{3}$$

Here  $\bar{y}(q)$  denotes the GT of  $y(t)$ .

Put  $y(0) = 0$  and  $\dot{y}(0) = 0$ , we get

$$q^2 \bar{y}(q) + \omega_0^2 \bar{y}(q) = \frac{F}{m} \frac{1}{q^2(q^2 + \omega^2)}$$

$$\bar{y}(q) = \frac{F}{m} \frac{1}{q^2(q^2 + \omega^2)(q^2 + \omega_0^2)}$$

$$\bar{y}(q) = \frac{F}{m} \left\{ \frac{1}{q^2(-\omega_0^2 + \omega^2)(q^2 + \omega^2)} + \frac{1}{q^2(-\omega^2 + \omega_0^2)(q^2 + \omega_0^2)} \right\}$$

Relating the inverse GT, we have

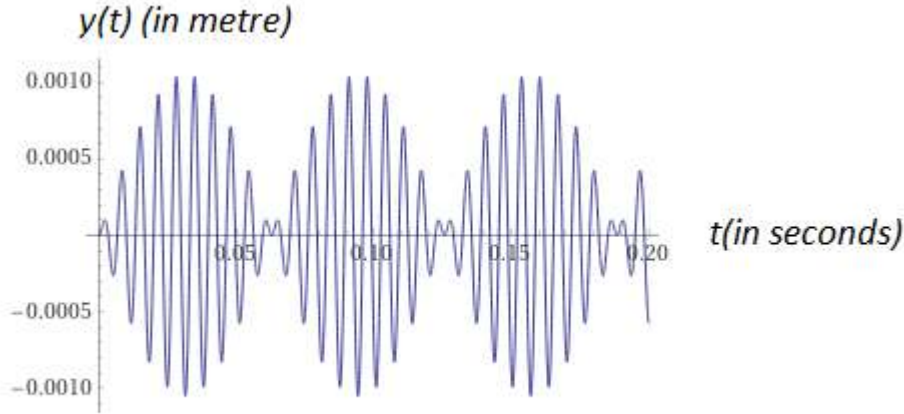
$$y(t) = \frac{F}{m} \left\{ \frac{1}{(-\omega_0^2 + \omega^2)} \cos \omega t + \frac{1}{(-\omega^2 + \omega_0^2)} \cos \omega_0 t \right\}$$

$$y(t) = \frac{F}{m} \left\{ \frac{1}{(\omega^2 - \omega_0^2)} \cos \omega t - \frac{1}{(\omega^2 - \omega_0^2)} \cos \omega_0 t \right\}$$

$$y(t) = \frac{F}{m} \frac{1}{(\omega^2 - \omega_0^2)} (\cos \omega t - \cos \omega_0 t)$$

$$y(t) = \frac{2F}{m} \frac{1}{(\omega_0^2 - \omega^2)} \sin \frac{(\omega_0 - \omega)t}{2} \sin \frac{(\omega_0 + \omega)t}{2} \tag{4}$$

If  $\omega > \omega_0$ , then (4) is supposed to be the product of two terms:  $\frac{2F}{m} \frac{1}{(\omega_0^2 - \omega^2)} \sin \frac{(\omega_0 - \omega)t}{2}$  and  $\sin \frac{(\omega_0 + \omega)t}{2}$ . Since  $\omega$  is nearly more than  $\omega_0$ , therefore,  $|\omega - \omega_0|$  is small, and the angular frequency of the term  $\sin \frac{(\omega_0 + \omega)t}{2}$  is very high than that of the term  $\frac{2F}{m} \frac{1}{(\omega_0^2 - \omega^2)} \sin \frac{(\omega_0 - \omega)t}{2}$ . In this case, (4) specifies a vibration of a high frequency whose amplitude is regulated by a vibration of a low frequency as shown in figure 1. Taking  $F = 10^4$  N,  $m = 10^2$  Kg,  $\omega = 9 \times 10^2$  rad/sec and  $\omega_0 = 10^3$  rad/sec, the graph of equation (4), between  $y$  and  $t$  is shown in Figure 1.



**Figure 1**

**II. Undamped Electrical Systematically Driven Vibrator**

The systematically driven electrical vibrator (Gupta et al, 2022 and Gupta et al, 2023) is specified by

$$L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C} q(t) = V \cos \omega t$$

$$\text{Or } \ddot{q}(t) + \frac{R}{L} \dot{q}(t) + \omega_0^2 q(t) = \frac{V}{L} \cos \omega t \tag{5}$$

For an **Undamped Electrical Systematically Driven Vibrator** (Deshpande et al, 2014), the resistance,  $R = 0$ , therefore, equation (5) becomes

$$\ddot{q}(t) + \omega_0^2 q(t) = \frac{V}{L} \cos \omega t \tag{6}$$

where  $\omega_0 = \sqrt{\frac{1}{LC}}$  and  $q(t)$  is the instantaneous charge.

Here (Berman et al, 2018),

- (i)  $q(0) = 0$ .
- (ii)  $\dot{q}(0) = 0$ .

The GT of (6) furnishes

$$p^2 \bar{q}(p) - \frac{1}{p^2} q(0) - \frac{1}{p^3} \dot{q}(0) + \omega_0^2 \bar{q}(p) = \frac{V}{L} \frac{1}{p^2(p^2 + \omega^2)} \tag{7}$$

Here  $\bar{q}(p)$  denotes the GT of  $q(t)$ .

Put  $q(0) = 0$  and  $\dot{q}(0) = 0$ , we have

$$p^2 \bar{q}(p) + \omega_0^2 \bar{q}(p) = \frac{V}{L} \frac{1}{p^2(p^2 + \omega^2)}$$

$$\bar{q}(p) = \frac{V}{L} \frac{1}{p^2(p^2 + \omega^2)(p^2 + \omega_0^2)}$$

$$\bar{q}(p) = \frac{V}{L} \left\{ \frac{1}{p^2(-\omega_0^2 + \omega^2)(p^2 + \omega^2)} + \frac{1}{p^2(-\omega^2 + \omega_0^2)(p^2 + \omega_0^2)} \right\}$$

Relating the inverse GT, we have

$$q(t) = \frac{V}{L} \left\{ \frac{1}{(-\omega_0^2 + \omega^2)} \cos \omega t + \frac{1}{(-\omega^2 + \omega_0^2)} \cos \omega_0 t \right\}$$

$$q(t) = \frac{V}{L} \left\{ \frac{1}{(\omega^2 - \omega_0^2)} \cos \omega t - \frac{1}{(\omega^2 - \omega_0^2)} \cos \omega_0 t \right\}$$

$$q(t) = \frac{V}{L} \frac{1}{(\omega^2 - \omega_0^2)} (\cos \omega t - \cos \omega_0 t)$$

$$q(t) = \frac{2V}{L} \frac{1}{(\omega_0^2 - \omega^2)} \sin \frac{(\omega_0 - \omega)t}{2} \sin \frac{(\omega_0 + \omega)t}{2} \tag{8}$$

If  $\omega$  is nearly more than  $\omega_0$ , then (8) is supposed to be the product of two terms:  $\frac{2V}{L} \frac{1}{(\omega_0^2 - \omega^2)} \sin \frac{(\omega_0 - \omega)t}{2}$  and  $\sin \frac{(\omega_0 + \omega)t}{2}$ . Since  $\omega$  is nearly more than  $\omega_0$ , therefore,  $|\omega - \omega_0|$  is small and the angular frequency of the term  $\sin \frac{(\omega_0 + \omega)t}{2}$  is much higher than that of the term  $\frac{2V}{L} \frac{1}{(\omega_0^2 - \omega^2)} \sin \frac{(\omega_0 - \omega)t}{2}$ . In this case, (8) specifies a vibration of a high frequency whose amplitude is regulated by a vibration of a low frequency as shown in figure 2. Taking  $V = 3 \times 10^2$  volt,  $L = 10^{-1}$  H,  $\omega = 9 \times 10^2$  rad/sec and  $\omega_0 = 10^3$  rad/sec, the graph of equation (8), between  $q$  and  $t$  is shown in Figure 2.

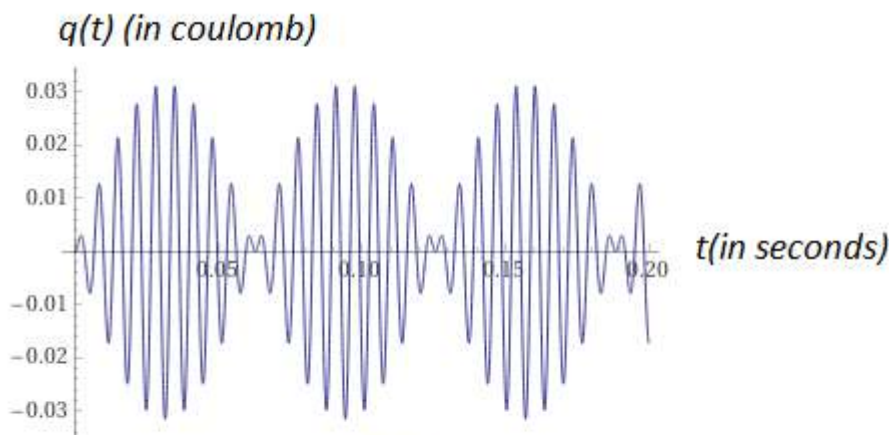


Figure 2

### 3. Discussion

The reaction of undamped, systematically driven electrical and mechanical vibrators has been fortuitously dictated by the integral GT. This paper has tendered an alternate style for the dictation of the reaction of undamped, systematically driven electrical and mechanical vibrators. In an undamped, systematically driven mechanical vibrator, if  $\omega$  is nearly more than  $\omega_0$ , then  $|\omega - \omega_0|$  is small and the angular frequency of the term  $\sin \frac{(\omega_0 + \omega)t}{2}$  is much higher than that of the term  $\frac{2F}{m} \frac{1}{(\omega_0^2 - \omega^2)} \sin \frac{(\omega_0 - \omega)t}{2}$ . In this case, the reaction of mechanical oscillator specifies a vibration of a high frequency and whose amplitude is regulated by a vibration of a low frequency as shown in figure 1. In an undamped, systematically driven electrical vibrator, if  $\omega$  is nearly more than  $\omega_0$ , then  $|\omega - \omega_0|$  is small and the angular frequency of the term  $\sin \frac{(\omega_0 + \omega)t}{2}$  is much higher than that of the term  $\frac{2V}{L} \frac{1}{(\omega_0^2 - \omega^2)} \sin \frac{(\omega_0 - \omega)t}{2}$ . In this case, the reaction of electrical oscillator specifies a vibration of a high frequency whose amplitude is regulated by a vibration of a low frequency as shown in figure 2.

### 4. Conclusions

The paper exemplified the GT for dictating the reaction of undamped, systematically driven electrical and mechanical vibrators. It reveals that the GT is an effectual method for the dictation of undamped, systematically driven electrical and mechanical vibrators and proves the materiality of the GT for the dictation of reaction of undamped, systematically driven electrical and mechanical vibrators.

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