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Multi-Stage optimization under uncertainty in reverse logistics operations: An industrial scenario

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Abstract

The major concern of reverse logistics operations involves reducing logistics costs, improving after sales service, waste disposal and providing good quality service in today's green environment. Process optimization involving uncertainty under reverse logistics set-up is a major challenge. There is a need to consider the dynamics of the real situation because of the variation of product/process characteristics to make the system more robust. A comparative analysis and the need for multi-stage optimization are illustrated here based on a single stage optimization model after performing its sensitivity analysis. Accordingly, a multi-stage optimization problem is formulated here to solve the cost optimization problem of charge mix under a reverse logistics set-up of a foundry business and to deal with the uncertainties involved in a practical business scenario. The proposed framework can be effectively implemented in a similar reverse logistics environment considering uncertainties of the process parameters/product characteristics.

Keywords: Reverse logistics, uncertainty, sensitivity analysis, multi-stage optimization, charge mix

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1. Introduction

Forward logistics is a process of managing the forward movement of goods right from raw materials to finished products and finally to the end users. Reverse logistics is a process that deals with the return of a product to the distributor or manufacturer for performing different operations like repairing, recycling, remanufacturing, refurbishment or redistribution, including its disposal (Carter and Lisa, 1998). It generally starts from the end-user to the manufacturer and sometimes directly to the raw materials through recycling. In other words, reverse logistics stands for all operations related to the movement of products and materials from their typical final destination to at least one step backwards in the supply chain for re-utilisation or waste disposal.

The management of reverse logistics requires robust infrastructure accompanied by accurate information systems to focus on their core activities in an optimal manner. Reducing logistics costs (Mathiyazhagan *et al.*, 2021), improving customer satisfaction, and providing good quality service are the main concerns in the reverse logistics path of business. Since this path involves plenty of uncertainty in terms of operational cycle time and decision-making, it requires accurate prediction about the impact of possible control actions.

Though the academic and industrial interest in reverse logistics has increased significantly in recent years, the main challenges (Autry *et al.*, 2001) involve the complexity, reliability and applicability of the processes and times of completion. There are some difficulties which need to be considered while implementing the reverse logistics process, for example, determining the value per unit of material under process, tracking the warranty period, and dealing with the internal suppliers, in particular. The major concern with the returned products lies with the decision-making in the form of reuse, remanufacturing, repairing, recycling, reselling etc.

To understand the scope of cost reduction, the breakup of the cost components of a typical foundry needs to be understood first. There are some methods which can be adopted to reduce the cost like bulk purchasing of the materials, hedging of raw materials cost, and reuse of the returns/ rejects in the charge mix. Ziółkowski (2013) shows that the effort of optimising the cost of manufacturing the casting alloys generates different processes to determine the charging burden required for the casting depending on the properties of the casting alloys. The application of linear programming for the optimization of charge mixing in foundry operation is common. Woubante (2017) presents a technologically adaptive and cost-efficient system using linear programming techniques. There are different procedures for finding the judicious balance of product mix to obtain the desired/optimal chemistry in the cast, and the linear programming approach (Rabani, 2007) is the best fit to do this job. A logistics network can also be designed with the support of a linear programming optimization model (Farkas *et al*., 1993). Cost optimization of charge burden can be done with the help of a linear programming model by using a dynamic pricing approach for returned products in a forward/reverse logistics network (Keyvanshokooh *et al.*, 2013). With the help of continuous variables to define a portion of every charge material and a traditional linear optimization method, the charge burden calculation can also be done (Źiółkowski, 2017). Moreover, the multi-stage optimization gives an improved result than the single-stage optimization as given in (Ziółkowski and Schmalenberg, 2019). The multi-stage approach is very important in cost optimization and investment management problems (Lau and Womersley, 2001). This approach is used to provide an optimal or near-optimal solution in a multi-stage supply chain network (Manimaran and Selladurai, 2014). The solution of the optimization task gives more realistic results and can be used in real foundry operations. Reverse logistics is a very important and sustainable method, particularly, in today's green world scenario, and quite popular in different industries as well (Dowlatshahi, 2000). Turrisi *et al.* (2013) showed that a closed-loop structure using reverse logistics is economically profitable as the collection of the products obtained from the reverse logistics avoids the variability of orders to the supplier. Bazan *et al*. (2016) pointed out that showed that use of products, materials or components at the end of their useful life is environment friendly, cost saving and sustainable. A reverse logistics profile for social sustainability can be made by connecting various sustainability indicators to different reverse logistics processes according to (Das *et al*., 2020). Kaynak *et al*. (2014) present the different roles of reverse logistics for logistics centres. The firms have to deal with higher levels of uncertainties and also, the flow of returns due to product recalls, warranty and service has increased leading to concern about the cost of the returning materials. Recently, with the increase in environmental concerns, legal requirements as well as the need for improving core competencies many companies are focusing on the implementation of reverse logistics as demonstrated in Tarin *et al.* (2019). According to Babazadeh *et al*. (2016), environmental and business factors are the main reasons behind the development of reverse logistics frameworks. Lambert *et al.* (2011) show a conceptual framework using reverse logistics decisions which will be flexible to cover many practical situations. *Morais*, *et al.* (2018) introduced the effect of reverse logistics on purchasing of raw materials and the method of optimizing the costs which would help the company in the competitive market. According to Johnson (1998), the importance of reverse logistics is being increased for the firms due to various reasons such as public concern with sustainable development, green products etc.

This paper introduces an innovative optimization method for minimizing the cost of charge materials required in a foundry operation, specifically designed for a reverse logistics set-up. Unlike previous studies that focused on deterministic approaches to optimize this process, we introduce a stochastic approach that leverages statistical methods based on industrial data to handle the uncertainty associated with charge mix combinations. Both single-stage and multi-stage linear programming optimization techniques were employed, providing a comprehensive solution for cost optimization that can be easily adapted to different contexts. Furthermore, we proposed a novel modeling framework that integrates economic and environmental considerations to minimize the cost of the charge materials while reducing the environmental impact associated with this operation. The proposed approach offers a flexible and more effective solution for foundry operators to optimize their operations under a reverse logistics context. Finally, we demonstrated the effectiveness of our approach through numerical experiments and sensitivity analysis, providing insights for practitioners to design and implement cost-effective and environmentally sustainable operations.

2. Scope and Approach

Despite all those difficulties and problems mentioned earlier, reverse logistics has a very wide scope in the field of foundry operations if it can be implemented properly. There has been a growing need for reverse logistics in foundries and other firms for a variety of reasons. Government regulations and public concern for eco-friendly and sustainable development have played a significant role in this trend, driving firms to pursue more efficient and responsible ways of managing their operations. Also, reverse logistics can help firms achieve better planning and create a more sustainable socio-economic and ecological order. Additionally, reverse logistics can help firms reduce costs by providing a more cost-effective way to manage scrap and returns compared to the cost of producing new alloys. Overall, the adoption of reverse logistics in foundries and other firms can have a positive impact on the environment, society, and the economy. (Johnson, 1998). Also, due to the changes in the supply chain, the uncertainty has increased along with returns and scrap of the foundry.

In contrast to previous research, this study places greater emphasis on the materials received through the reverse logistics process. While prior studies have largely focused on the forward supply chain, neglecting the potential value of returns and end-of-life products, our research recognizes the importance of these materials and their potential to create value for firms. By taking a more holistic approach to supply chain management that considers both forward and reverse logistics, our study contributes to a more comprehensive understanding of the factors that influence the success and sustainability of firms in today's complex and dynamic business environment. The path of the whole foundry system is described here, the suppliers supply all the charged materials to the foundry and in the reverse logistics, scraps are returned from the internal customers to the foundry. Finally, in the foundry, all the charged materials along with the scrap are used to make the molten (hot) metal for the cast. Then the cast product is supplied to the customers and the scrap generated from these or the previous cast materials are being returned to the foundry for further use. Thus, the cycle continues. The schematic diagram of the foundry system involving its reverse logistics path is shown in **Fig.1** below.

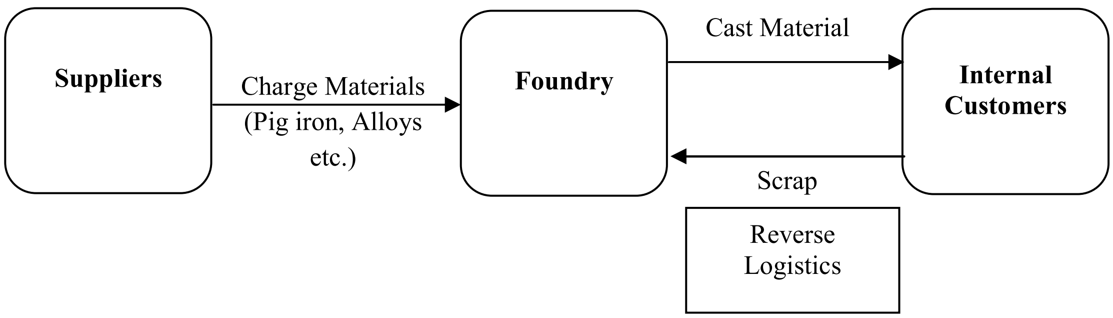

Figure 1. Foundry in-house reverse logistics – A schematic diagram

Foundry operations are critical to many industries, including aerospace, automotive, and construction, among others. The cost of performing these operations can be substantial, with charge material representing a significant portion of the overall expenses. In order to optimize the cost of foundry operations, it is essential to minimize the cost of the charge material required per day. The optimization of the charge material composition is a complex task that requires consideration of various factors, including the composition of the charge materials, their cost components, and the desired cast metal (Ziółkowski and Schmalenberg, 2019). Earlier research has highlighted the effectiveness of linear programming as a tool for optimizing the charge material mix. However, sensitivity analysis is essential to account for variations in chemical composition that can occur in practice. To this end, this study not only applies linear programming but also utilizes a multi-stage approach to gain a deeper understanding of the real-world constraints faced by foundries. The Excel solver and Lindo software are used for the entire analysis, providing a user-friendly and efficient solution that can be easily adopted by practitioners in the industry. By optimizing the cost of foundry operations, this study contributes to a more sustainable and competitive manufacturing industry that can meet the needs of a rapidly changing world.

3. Mathematical models

3.1 Linear programming problem (LPP)

In this study, the optimization problem is defined by the cost minimization of the charge materials and a system of constraints which includes the balance of the chemistry of the charge materials and the desired cast materials, the limitations in the mass fraction of the different elements, the amount of scrap to be used (through reverse logistics) and the total cast material (in kg) to be produced in a single day. Now, making a batch of molten metal with a specific chemical composition requires a perfect selection of charge materials. The operational objective is to minimize the total cost involved per day of production for the production of cast components (finished products) by calculating the mass fraction of the charged materials and also by utilizing the scrap as much as possible without hampering the quality of the cast components. Under the normal scenario, the weight percentage of the scrap lies within 5% to 30% of the total weight of the cast.

Objective function:

$$\min \sum_{j=1}^{n} c_j x_j \tag{1}$$

Constraints:

$$\left.\begin{aligned}
&\sum_{j=1}^{N} A_{ij}x_j \geq A_i^{\min} m, \quad i = 1,2,\ldots,M \\
&\sum_{j=1}^{N} A_{ij}x_j \leq A_i^{\max} m, \; i = 1,2,\ldots,M \\
&x_j \geq x_j^{\min} \geq 0 \\
&x_j \leq x_j^{\max} \leq m \\
&\sum_{j=1}^{N} x_j = m
\end{aligned}\right\} \tag{2}$$

N = Number of charge materials
c_j = Unit price of the j^{th} charge material (Rs. per kg.)
x_j = Mass fraction of the j^{th} charge material (in kg.)
A_{ij} = Content of i^{th} element in the j^{th} charge material (%)
$A_i^{\min}$ = Minimum value of the i^{th} element required in the cast material (%)
$A_i^{\max}$ = Maximum value of the i^{th} element required in the cast material (%)
m = Total amount of cast material to be produced per day (in kg.)
$x_j^{\min}$ = Minimum value of j^{th} material value in the charge (in kg.)
$x_j^{\max}$ = Maximum value of j^{th} material value in the charge (in kg.)
M = Number of chemical elements

After solving (1) and (2), the obtained solution will be a vector $\boldsymbol{x^{opt}} = [x_1^{opt} x_2^{opt} \ldots . x_N^{opt}]$

3.2 Multi-stage problem

The charge materials, required in the casting industry, are characterized not only by their chemical composition and cost but by their dispersion properties too. This is very important in terms of the deviation of the test proportion of one material (j) from their optimized value (x_j^{opt}) obtained from the solution of linear programming problem. The estimations of the deviation are very important which may sometimes results in mistakes, delay and inefficiency. To address this problem, the multi-stage optimization algorithm has been followed.

Steps of the algorithm:

Step-1: Solve the linear programming problem using the objective function (1) and the system of constraints (2) and obtain the solution vector $\boldsymbol{x^{opt}}$.

Step-2: Depending on the standard deviation of each of the charge materials, based on the previous records of the number of charge materials required for some heats, a list of charge materials (y) is prepared in such a way that the materials listed first are the one with the highest standard deviation values and the list ends with the components with the least standard deviation values. A parameter T is defined as the number of charge materials for which, in subsequent stages, the limits of the mass ranges of the given components will be defined.

Step-3: Considering t = 1, the optimization problem is formulated as follows.

Objective functions:

$$\min \; y_1^L \tag{3}$$

$$\max \; y_1^R \tag{4}$$

Constraints:

$$\left.\begin{aligned}
&\sum_{j=1}^{N} A_{ij} y_j \geq A_i^{\min} m \text{ , } i = 1,2,\ldots,M \\
&\sum_{j=1}^{N} A_{ij} y_j \leq A_i^{\max} m \text{ , } i = 1,2,\ldots,M \\
&y_j \geq y_j^{\min} \geq 0 \\
&y_j \leq y_j^{\max} \leq m \\
&\sum_{j=1}^{N} y_j = m
\end{aligned}\right\} \tag{5}$$

After solving the two pairs of equations (3), (5) and (4), (5) respectively, a range of values of y_1 is obtained as $y_1 \in [y_1^L : y_1^R]$. The charge material is then weighted to assign a value of y_1^* and, if it satisfies $y_1 \in [y_1^L : y_1^R]$, then go to Step-4.

Step-4: Considering that the t^{th} variable is to be optimized at each stage, continue solving the following updated optimization problem where $t = t+1$ with $t \leq T$, as follows.

Objective functions:

$$\min \; y_t^L \tag{6}$$

$$\max \; y_t^R \tag{7}$$

Constraints:

$$\left.\begin{aligned}
&\sum_{j=1}^{N} A_{ij} y_j \geq A_i^{\min} m \text{ , } i = 1,2,\ldots,M \\
&\sum_{j=1}^{N} A_{ij} y_j \leq A_i^{\max} m \text{ , } i = 1,2,\ldots,M \\
&y_j \geq y_j^{\min} \geq 0 \\
&y_1 = y_1^* \\
&\ldots\ldots\ldots \\
&y_{t-1} = y_{t-1}^* \\
&\sum_{j=1}^{N} y_j = m
\end{aligned}\right\} \tag{8}$$

Solving again the two pairs of equations (6), (8) and (7), (8) respectively, and the final part of the stage, a range of values is obtained as $y_t \in [y_t^L : y_t^R]$. The charge material is then weighted to give a value of y_t^* and, if it satisfies $y_t \in [y_t^L : y_t^R]$, then go to Step 5.

Step-5: Solve the optimization problem for each different set of constraints obtained recursively as in *Step 4*, considering both online and offline costs components together, to get the number of charge materials (kg.) to be required in both cases. Also, calculate the optimized costs for each of the cases.

4. An illustration: foundry reverse logistics operations

The process considered here is the manufacturing of Austenite Manganese Steel-based products in a foundry. Manganese Steel has a wide scope of its usage for various purposes, one of them is used as a cement mixer in cement manufacturing plants. In this case, the mother company has two manufacturing set-ups, one is a cement manufacturing plant (internal customer) and the other is a foundry, its sister company. The equipment which is needed to prepare the cement mix is known as a cement mixer. Because of the self-hardening properties of the Manganese Steel, it has been used to make the cement mixers and the foundry unit of the company supplies/sells the cement mixers to the cement manufacturing plant at Rs. 80/kg (material cost only) of the Mn Steel. So, the cement plant is the internal customer of the foundry. After using it for a certain period, the cement mixers become unable to further use. Then the foundry of the company buys the used Mn Steel from the cement plant as returns (Rs. 72/kg.) which is re-used in the charge through the path of reverse logistics. As usual, the forward logistics is the transfer of new Mn Steel-based product to the cement plant from the foundry and the reverse logistics path is the path in which the used Mn Steel comes back to the foundry from the cement plant as a raw material for the charge mix. So, the reverse logistics path is very useful for the mother company and important too from a cost aspect for the entire foundry operations.

The data on heat-wise charge details of 32 heats have been collected from the foundry for three months. Six different charge materials have been used to produce the Mn Steel. These materials are Mild Steel Scrap (MS Scrap), Manganese Scrap (Mn Scrap), High Carbon Ferromanganese (FeMnHC), Medium Carbon Ferromanganese (FeMnMC), Ferrosilicon (FeSi). The descriptive statistics of the data are given in **Table 1**.

Table 1. Descriptive statistics of charge mix components

	MS Scrap	Mn Scrap	FeMnHC	FeMnMC	FeSi	Mn Returns
N	32	32	32	32	32	32
Mean	319.50	810.75	56.91	29.09	3.50	385.19
Min	200.00	400.00	35.00	15.00	2.00	100.00
Max	495.00	1093.00	90.00	70.00	7.00	919.00
Std. Dev	82.7495	180.912	14.8874	12.3219	1.24434	201.856

The data for the chemistry of individual charge mix components have also been collected (*ref.* **Table 2)**. These components are Carbon (*C*), Silicon (*Si*), Manganese (*Mn*), Phosphorus (*P*), Sulphur (*S*), Chromium (*Cr*), Molybdenum (*Mo*) and Nickel (*Ni*). So, the charge materials are chosen in such a manner that they can fulfil the requirement of the chemical components of the cast Mn Steel. **Table 3** represents the required chemical composition of Mn Steel. The costs of the charge materials were obtained both from the online (*ref.* https://www.indiamart.com/) and offline market study. Subsequently, the optimization problem is formulated and thus illustrated in the next section.

Table 2. Chemistry of charge mix components

Sr. No.	Chemical Composition										
	Charge Mix	Matl. Id.	UOM	*C*	*Si*	*Mn*	*P*	*S*	*Cr*	*Mo*	*Ni*
1	MS Scrap	x_1	wt%	0.16	0.23	0.81	0.02	0.02	0.17	0.02	0.02
2	Mn Scrap	x_2	wt%	1.15	0.48	11.74	0.07	0.01	1.88	0.07	0.24
3	FeMnHC	x_3	wt%	6.00	0.00	60.00	0.09	0.04	0.00	0.00	0.00
4	FeMnMC	x_4	wt%	0.65	0.00	72.00	0.20	0.00	0.00	0.00	0.00
5	FeSi	x_5	wt%	1.39	67.00	0.00	0.05	0.01	0.00	0.00	0.00
6	Mn Returns	x_6	wt%	1.151	0.386	12.012	0.062	0.008	1.866	0.080	0.379

Table 3. Chemical composition of Manganese Steel

Chemical Element		Content (wt.%)
Name	Symbol	
Carbon	*C*	1.10 - 1.17
Silicon	*Si*	0.3 - 0.9
Manganese	*Mn*	11.9 - 13.0
Phosphorus	*P*	0.08 max
Sulfur	*S*	0.02 max
Chromium	*Cr*	1.7 - 2.0
Molybdenum	*Mo*	.01 - .15
Nickel	*Ni*	.2 - .6

4.1 Problem formulation

The optimized mass fraction of the charge materials at the lowest cost is calculated by solving the linear programming problem. From the previous subsection, it is clear that, in this problem, two different sets of costs are considered. So, there will be two different objective functions. The objective function (9.1) will be based on the online cost figures and the objective function (9.2) will be based on the offline cost figures as shown below.

Objective functions (using (1)):

$$\text{Min } Z_1 = 25.5x_1 + 32x_2 + 85x_3 + 110x_4 + 95x_5 + 72\,x_6 \quad (9.1)$$

$$\text{Min } Z_2 = 35x_1 + 40x_2 + 90x_3 + 100x_4 + 80x_5 + 72\,x_6 \quad (9.2)$$

Regarding formulation of the constraints, as formulated below, the first fourteen constraints (10.1 to 10.14) are based on the chemistry of the raw materials and the required cast metal (Mn Steel) chemistry (*ref.* **Table 2** & **3)**. Constraints (10.15) - (10.18) and (10.20) are the upper limit constraints for MS Scrap, Mn Scrap, FeMnHC, FeMnMC and Mn Returns respectively, which are obtained from the maximum limits of the charge materials in the heats (*ref.* **Table 1**). From constraint (10.19), it can be seen that it is the lower bound constraint of FeSi (*ref.* **Table 1**) which is available from the heat data and also specified by the industry. Constraint (10.21) represents the melting loss constraint. The melting loss is considered to be maximum 5% and the average requirement of the hot metal in every heat is 1520 kg, as specified by the foundry. This may, of course, vary over the availability of raw materials (charge mix) from the outside vendors and own plants through reverse logistics, days of production, the number of heats per shift/day/month, etc. depending on the customer requirements.

Constraints:

$$0.16x_1 + 1.15x_2 + 6x_3 + 0.65x_4 + 1.39x_5 + 1.151\,x_6 \geq 1672 \quad (10.1)$$

$$0.16x_1 + 1.15x_2 + 6x_3 + 0.65x_4 + 1.39x_5 + 1.151\,x_6 \leq 1778.2 \quad (10.2)$$

$$0.23x_1 + 0.48x_2 + 67x_5 + 0.386\,x_6 \geq 456 \quad (10.3)$$

$$0.23x_1 + 0.48x_2 + 67x_5 + 0.386\,x_6 \leq 1368 \quad (10.4)$$

$$0.81x_1 + 11.74x_2 + 60x_3 + 72x_4 + 12.012\,x_6 \geq 18088 \quad (10.5)$$

$$0.81x_1 + 11.74x_2 + 60x_3 + 72x_4 + 12.012\,x_6 \leq 19760 \quad (10.6)$$

$$0.02x_1 + 0.07x_2 + 0.09x_3 + 0.20x_4 + 0.05x_5 + 0.062\,x_6 \leq 121.6 \quad (10.7)$$

$$0.02x_1 + 0.01x_2 + 0.04x_3 + 0.01x_5 + 0.008\,x_6 \leq 30.4 \quad (10.8)$$

$$0.17x_1 + 1.88x_2 + 1.866\,x_6 \geq 2584 \quad (10.9)$$

$$0.17x_1 + 1.88x_2 + 1.866\,x_6 \leq 3040 \quad (10.10)$$

$$0.02x_1 + 0.07x_2 + 0.08\,x_6 \geq 15.2 \quad (10.11)$$

$$0.02x_1 + 0.07x_2 + 0.08\,x_6 \leq 228 \quad (10.12)$$

$$0.02x_1 + 0.24x_2 + 0.379\,x_6 \geq 304 \quad (10.13)$$

$$0.02x_1 + 0.24x_2 + 0.379\,x_6 \leq 912 \quad (10.14)$$

$$x_1 \leq 495 \quad (10.15)$$

$$x_2 \leq 1093 \quad (10.16)$$

$$x_3 \leq 90 \quad (10.17)$$

$$x_4 \leq 70 \quad (10.18)$$

$$x_5 \geq 2 \quad (10.19)$$

$$x_6 \leq 919 \quad (10.20)$$

$$0.95(x_1 + x_2 + x_3 + x_4 + x_5 + x_6) = 1520 \quad (10.21)$$

non-negativity restrictions: $x_i \geq 0$, for $i = 1,2,3,4,5,6$

4.2 Computational results of LPP and Sensitivity analysis

After solving the above LPP using Lindo software, the following two sets of results were obtained (*ref.* **Table 4**). These are the optimal weights (in kg.) of the charge material for this problem.

Table 4. Results of LPP

Charge Components	Variable name	Optimal weights of the charge materials (in kg.)	
		Considering Online Cost	Considering Offline Cost
MS Scrap	x_1^{opt}	208.95	212.51
Mn Scrap	x_2^{opt}	1093.00	1093.00
FeMnHC	x_3^{opt}	29.92	10.43
FeMnMC	x_4^{opt}	1.58	17.84
FeSi	x_5^{opt}	2.00	2.00
Mn Returns	x_6^{opt}	264.54	264.22
Total Wt. (Charge Mix)		1599.99994	1600.00008
COST (Obj. Fn.) in Rs.		62258.84	73064.32

The weight% of each charge material concerning the total charge material weight has been calculated by dividing the optimal weights (x_i^{opt}) of each charge material by the total charge material required.

Sensitivity Analysis:

The sensitivity analysis is performed next on the optimum results achieved by solving the LPP (*ref.* **Table-4**). The objective of performing this analysis is to find out the sensitivity of the optimum solution by changing the objective function coefficients for a decision variable and changing the right-hand side of a constraint. **Table 5** shows the sensitivity report for ranges of the objective function coefficients considering both online and offline cost components. Similarly, **Table 6** shows the sensitivity report for the effect of change of the right-hand side of a constraint considering both online and offline cost components.

Table 5. Objective Coefficient Ranges

	considering online costs			considering offline costs		
Charge Components	Current Coefficient (Rs. /kg.)	Allowable Increase (Rs. /kg.)	Allowable Decrease (Rs. /kg.)	Current Coefficient (Rs. /kg.)	Allowable Increase (Rs. /kg.)	Allowable Decrease (Rs. /kg.)
MS Scrap (x_1)	25.5	42.99834	19.44541	35	7.678376	INFINITY
Mn Scrap (x_2)	32	39.94609	INFINITY	40	31.95752	INFINITY
FeMnHC (x_3)	85	10.20413	INFINITY	90	158.1878	1.401913
FeMnMC (x_4)	110	2002.428	12.23437	100	1.680841	62.88332
FeSi (x_5)	95	INFINITY	76.2604	80	INFINITY	48.07005
Mn Returns (x_6)	72	213.4419	35.01312	72	INFINITY	26.6111

From the sensitivity analysis of the objective function coefficients (*ref.* **Table 5**), the allowable increase/decrease columns tell us that, for ex, provided the coefficient of MS Scrap (x_1) in the objective function lies between (25.5+42.99 ≅ 68) and (25.5-19.44 ≅ 6.1), the values of the variables in the optimal LP solution will remain unchanged (insensitive). However, the actual optimal solution value will change as the objective function coefficient of MS Scrap (x_1) is changing. Using this logic, it appears that for both types of costs, MS Scrap (x_1), FeMnHC (x_3) and FeMnMC (x_4) are sensitive either towards a small increase or decrease of their optimal values, i.e., outside the range of coefficients. The reduced cost components are found as zero everywhere.

Table 6. Right-Hand Side Constraints Ranges

		Considering online costs			Considering offline costs		
Constraint no.	Current RHS of the Constraint (kg.)	Allowable Increase (kg.)	Allowable Decrease (kg.)	Shadow/ Dual price	Allowable Increase (kg.)	Allowable Decrease (kg.)	Shadow/ Dual price
(10.1)	1672	106.2	INFINITY	0	106.2	56.79671	0.257335
(10.2)	1778.2	10.34622	106.2	1.873074	INFINITY	106.2	0
(10.3)	456	352.8129	INFINITY	0	353.5064	INFINITY	0
(10.4)	1368	INFINITY	559.1871	0	INFINITY	558.4936	0
(10.5)	18088	1672	105.0639	-1.24826	1672	1183.505	-0.94806
(10.6)	19760	INFINITY	1672	0	INFINITY	1672	0
(10.7)	121.6	INFINITY	21.39967	0	INFINITY	19.85123	0
(10.8)	30.4	INFINITY	11.95781	0	INFINITY	12.66898	0
(10.9)	2584	156.5009	298.9243	-20.2673	107.0652	298.678	-15.4037
(10.10)	3040	INFINITY	456	0	INFINITY	456	0
(10.11)	15.2	86.65253	INFINITY	0	86.69777	INFINITY	0
(10.12)	228	INFINITY	126.1475	0	INFINITY	126.1022	0
(10.13)	304	62.76101	INFINITY	0	62.7093	INFINITY	0
(10.14)	912	INFINITY	545.239	0	INFINITY	545.2907	0
(10.15)	495	INFINITY	286.0472	0	INFINITY	282.4879	0
(10.16)	1093	262.514	387.3413	39.94609	262.1923	649.7579	31.95752
(10.17)	90	INFINITY	60.08025	0	INFINITY	79.57437	0
(10.18)	70	INFINITY	68.416	0	INFINITY	52.15685	0
(10.19)	2	8.371662	2	-76.2604	8.36128	2	-48.0700
(10.20)	919	INFINITY	654.4566	0	INFINITY	654.7808	0
(10.21)	1600	260.6128	113.1061	-21.3432	257.37	193.6162	-31.5722

From the sensitivity analysis of the right-hand side of constraints, it is seen that the objective function values will remain unchanged (insensitive) within the limits given in the allowable increase/decrease columns corresponding to each constraint (*ref.* **Table 6**). The direction of the change in the objective function (down) depends upon the direction of the change on the right-hand side of the constraint for the minimization problem. If the constraint is more (or, less) restrictive after the change in the right-hand side, then for a minimization problem, the objective function value will be increased (or, decreased) for a worse (or, better) situation. The shadow/dual price reflects the amount of change in the objective function value if the right-hand side of the corresponding constraint is changed within the limits given in the allowable increase/decrease quantity. In other words, the shadow price will be zero if a small change in the right-hand side of the constraint cannot alter the optimal solution.

It is observed from **Table 6** that ranges for RHS of constraints (no. 10.3-10.4, 10.6-10.8, 10.10-10.15, 10.20) are almost the same for both online and offline cost cases. The constraints (no. 10.1, 10.3, 10.5, 10.9, 10.11, 10.13) represent lower bounds for *C*, *Si, Mn, Cr, Mo* and *Ni* weights respectively whereas constraints (no. 10.2, 10.4, 10.6-10.8, 10.10, 10.12, 10.14) represents lower bounds for *C*, *Si, Mn, P, S, Cr, Mo* and *Ni* weights respectively. The constraints 10.15-10.18, 10.20 present the upper bounds for MS scrap, Mn scrap, FeMnHC, FeMnMC and Mn returns whereas the constraint (10.19) presents the lower bound for FeSi. Overall, it appears that constraints (no. 10.1-10.2, 10.5-10.7, 10.9-10.10, 10.19) are sensitive to the change of right-hand side bounds considering both the costs components and allowable increase/decrease quantities (small %changes in terms of existing RHS bounds) with respect to *C*, *Mn*, *P*, *Cr* and *FeSi* respectively. So, appropriate care should be taken in controlling their variability/permissible limits in the process.

4.3 Implementation of multi-stage optimization

Step-1: Solving (9) and (10) we get a vector $\boldsymbol{x^{opt}} = [\, x_1^{opt} x_2^{opt} x_3^{opt} \,]$

Step-2: Now depending on the standard deviation of each of the charge materials from **Table 2**,
we get that the standard deviation of the charge materials in a decreasing manner;
Mn Returns (x_6) > Mn Scrap (x_2) > MS Scrap (x_1) > FeMnHC (x_3) > FeMnMC (x_4) >FeSi (x_5).

Now, among the charge materials, three of them have a significant amount of standard deviation. So, taking $T = 3$, the vector y becomes $y = [x_1 x_2 x_3]$. Next, following Step 3 and Step 4, as explained in Section 3.2, results of multi-stage optimization are generated and displayed stage wise in **Table 7**.

Table 7. Results of multi-stage optimization

Stages (T)	Objective Function	Constraint set	Results of optimization	Remarks
t=1	$\min\ x_6 = y_1^L$	(10.1) – (10.21)	$y_1^L = 264.2192$, x_1 = 212.5121, x_2 = 1093, x_3 = 10.4256, x_4 = 17.4831, x_5 = 2	$x_6 = y_1^* = 320$ is chosen, which lies in the range [264.2192, 919] and is the *median* value of Mn returns (in Kg.) of the *skewed* distribution of Mn returns (*ref*. Table 1).
	$\max\ x_6 = y_1^R$		$y_1^R = 919$, x_1 = 63.8374, x_2 = 615.1626, x_3 = 0, x_4 = 0, x_5 = 2	
t=2	$\min\ x_2 = y_2^L$	(10.1) – (10.19), (10.21) x_6 = 320;	$y_1^L = 320$, $y_2^L = 1037.647$, x_1 = 212.3736, x_3 = 10.3625, x_4 = 17.6168, x_5 = 2	$x_2 = y_2^* =$ 1065.3235 is chosen, which lies in the range [1037.647, 1093] and is the *average* value of Mn scrap (in Kg.) of the *symmetric* distribution of Mn scrap (*ref*. Table 1).
	$\max\ x_2 = y_2^R$		$y_1^R = 320$, $y_2^R = 1093$, x_1 = 132.6005, x_3 = 16.5870, x_4 = 27.5238, x_5 = 10.2887	
t=3	$\min\ x_1 = y_3^L$	(10.1) – (10.19), (10.21) x_6 = 320; x_2 = 1065.3235;	$y_1^L = 320$, $y_2^L = 1065.3235$, $y_3^L = 155.1245$, x_3 = 21.2198, x_4 = 27.9225, x_5 = 10.4096	$x_1 = y_3^* =$ 172.4281 is chosen, which lies in the range [155.1245, 189.7320] and is the *average* value of MS scrap (in Kg.) of the *symmetric* distribution of MS scrap (*ref*. Table 1).
	$\max\ x_1 = y_3^R$		$y_1^R = 320$, $y_2^R = 1065.3235$, $y_3^R = 189.7320$, x_3 = 5.7022, x_4 = 17.2423, x_5 = 2	

Now, after all the steps carried out from *Step 1* to *Step 4*, the final updated list of constraints will remain the same from (10.1) to (10.21), except constraint no. (10.15) will now be replaced as $\boldsymbol{x_1} = \mathbf{172.4281}$; constraint no. (10.16) will be considered as $\boldsymbol{x_2} = \mathbf{1065.3235}$ and constraint no. (10.20) will be considered as $\boldsymbol{x_6} = \mathbf{320}$ along with non-negativity restrictions: $x_i \geq 0$, for i = 1,2,3,4,5,6.

For each different set of constraints stage wise, the total optimized costs have been calculated considering both online and offline cost components and are presented in **Table 8**.

Table 8. Solution set considering Online and Offline costs

separately weighted variables in optimization	Charge Material	Amount of charge material (kg.)		Optimum Cost		The total amount of charge material (kg.)
		Online	Offline	Online	Offline	
(All, LPP)	MS Scrap	208.95	212.5121	62258.84	73064.32	1600
	Mn Scrap	1093.00	1093.00			
	FeMnHC	29.92	10.4256			
	FeMnMC	1.58	17.8432			
	FeSi	2.00	2.00			
	Mn Returns	264.55	264.22			

Table 8 (cont'd). Solution set considering Online and Offline costs

separately weighted variables in optimization	Charge Material	Amount of charge material (kg.)		Optimum Cost		The total amount of charge material (kg.)
		Online	Offline	Online	Offline	
x_6	MS Scrap	163.795	167.09	64200.55	74548.71	1600
	Mn Scrap	1093.00	1093.00			
	FeMnHC	20.59	1.042			
	FeMnMC	0.614	16.868			
	FeSi	2.00	2.00			
	Mn Returns	320	320			
X_6, x_2	MS Scrap	186.437	189.732	64329.59	74690.99	1600
	Mn Scrap	1065.323	1065.323			
	FeMnHC	25.251	5.702			
	FeMnMC	0.989	17.242			
	FeSi	2.00	2.00			
	Mn Returns	320	320			
x_6, x_2, x_1	MS Scrap	172.4281	172.4281	65449.02	75477.63	1600
	Mn Scrap	1065.323	1065.323			
	FeMnHC	22.813	22.813			
	FeMnMC	9.085	9.085			
	FeSi	10.35	10.35			
	Mn Returns	320	320			

4.3.1 Observations on the multi-stage optimization results:

1) By comparing the amounts required for each of the charged materials, it is clear that for the High Carbon Ferromanganese (FeMnHC) and the Medium Carbon Ferromanganese (FeMnMC), the amounts required in both the online and offline costs are very different.
2) For both online and offline costs, as the number of variables gets optimized over stages, the being optimized at each successive stage gets increased. The optimized cost takes the least value when the result is obtained through single stage optimization using the LPP approach but without considering the uncertainty of charge mix components in descending order of magnitude. This inherent uncertainty or variation is tackled using a multi stage optimization approach with a negligible increase in the overall cost of production in a reverse logistics set-up.
3) After optimizing the values of the three decision variables (with the higher uncertainty involved) in the multi-stage approach, it is observed that the amount required for each of the charge materials (kg.) is the same for both the online and the offline costs.

5. Discussions

In this paper, multi-stage optimization methodology is used to solve the cost optimization problem of charge mix under a reverse logistics set-up of a foundry business. The deployment of an environment friendly process through reverse logistics is very relevant nowadays. One of the problems associated with single stage optimization using linear programming is that, sometimes, it is unable to solve problems considering the real-world constraints prevailing in the industry. Hence, a multi stage approach is necessary to adopt. Along with the formulation of the methodology, a case study has also been performed in this work.

From the derivations, it is clear that after the first stage of the optimization for the High carbon Ferromanganese (FeMnHC) and the Medium Carbon Ferromanganese (FeMnMC), the amount to be required for both the online and offline costs cases is very different. In the case of online cost, the required amount is non-uniformly distributed (FeMnHC - 29.92 kg. and FeMnMC - 1.58 kg.); whereas for the offline cost, the amount is relatively uniformly distributed (FeMnHC - 10.43 kg. and FeMnMC - 17.84 kg.). But, after completing all the stages of the multi stage optimization, we observe that their required amount reached the mean values which are equal for both online and offline costs. This has happened because, in multi stage optimization, we have considered some other constraints to make the problem more realistic.

For both the online and offline costs, increasing the number of stage variables increases the cost because by fixing the realistic optimal values of more variables, we are limiting the flexibility of the variables to take any unrealistic value under uncertainty, which finally gives us a feasible realistic set of values of these variables. This is the reason why the cost for single stage linear programming is the least. The solution for single stage optimization produces the most optimal values of the variables, but it does not check whether the charge mix variables values are realistic or not. But after fixing the most realistic values of the three variables considering their statistical uncertainties, namely, Mn scrap, MS scrap and Mn returns, using the multi-stage optimization, we obtain the actual feasible solution. Even after fixing the values of these three decision variables (with the standard deviation values in descending order) in the multi-stage approach, we can see that for both the online and the offline costs, the amount required for each of the charge materials (kg.) is the same. Thus, the solution of the multi-stage approach becomes completely independent of the cost of the individual charge materials.

From the results, it can also be said that in the case of both costs, the amount of Mn returns, which is obtained via the reverse logistics path, is the same. It signifies that the reverse logistics path is dependable and consistent. Also, the price per kg. of the Mn returns is one of the lowest among all of the charged materials. So, it is very clear that the use of the returns as a charge material using a reverse logistics path in the foundry has certainly reduced the total charge material cost satisfying its chemistry in the charge mix. This also shows the usefulness of applying the reverse logistics concept in the foundry operations.

For this type of charge mix problem under uncertainty, the main objective is to find the optimal combination of the values of the charge mix variables considering the realistic constraints. In this paper, it is evident from the final solution that in the last stage of the multi stage approach, the amount to be required for each of the charge materials is the same for both online and offline costs. This proves that the solution gives the best possible mixture of all the charge materials to get the desired result independent of the cost values. Overall, this study throws light on the domain of multistage optimization under an internal reverse logistics set-up of an organization.

6. Conclusions

Previously, the number of charge materials to be used is determined manually by the experienced workers, which could increase the uncertainty of the entire foundry operations, including its reverse logistics path and the cost involved therein. However, if the mathematical model is appropriately used, then the exact amount of the charge materials to be required can be calculated, even after suitably handling the uncertainty involved. Thus, the proper composition of the molten metal cast can also be maintained.

In this study, with the use of optimization techniques (both single and multi-stage linear programming problems), a realistic cost estimate towards hot metal production can be obtained under the uncertainty of charge mix components satisfying the required amount of the specified chemistry. From the multi-stage optimization results, it is seen that for both online and offline costs, as the number of stage-optimized variables increases, the optimized cost value also increases. This cost takes the least value when the result is obtained through the single stage LPP and takes the highest value when the three variables have been weighted through multi-stage optimization. This happens due to tackling the uncertainty of charge mix components through multi stages of optimization under reverse logistics set-up. By taking the multi-stage approach, more realistic values for the weights of the charged materials are getting fixed considering their uncertainties statistically. So, the cost becomes more practical and realistic than the ideal condition (LPP).

In general, in every real-world scenario, there will be always some amount of variation of product/process characteristics due to assignable causes present in the system. Considering this variation, these characteristics are treated as stochastic parameters of the system which are difficult to control in a deterministic manner. Hence, a fixed deterministic model may not work always in a real field but obviously can provide a guideline to control the system. Accordingly, there is a need to consider the dynamics of the real situation to make the system more robust. Two-stage stochastic programming considers the fact that the probability distributions governing the data are known or can be estimated. So, this kind of optimization is usually called for to deal with the uncertainties involved in any real life business scenario. The proposed optimization framework can be effectively implemented in a similar industrial environment under the presence of uncertainties in the entire closed loop supply chain.

References

Autry, C.W., Daugherty P.J. and Glenn R.R, 2001. The challenge of reverse logistics in catalog retailing, *International Journal of Physical Distribution & Logistics Management*, Vol.31, No.1, pp 26-37. https://doi.org/10.1108/09600030110366384

Babazadeh, R., Jolai, F., and Razmi, J., 2015. Developing scenario-based robust optimisation approaches for the reverse logistics network design problem under uncertain environments, *International Journal of Services and Operations Management*, Vol.20, pp 418–440. https://doi.org/10.1504/IJSOM.2015.068526

Bazan, E., Jaber, M.Y. and Zanoni, S., 2016. A review of mathematical inventory models for reverse logistics and the future of its modeling: An environmental perspective, *Applied Mathematical Modelling,* Vol.40, pp 4151–4178. https://doi.org/10.1016/j.apm.2015.11.027

Carter, Craig R. and Ellram Lisa M., 1998. Reverse logistics: a review of the literature and framework for future investigation, *Journal of Business Logistics*, Vol.19, No.1, pp 85-102.

Das, D., Kumar, R. and Rajak, M.K., 2020. Designing a reverse logistics network for an e-commerce firm: A case study, *Operations and Supply Chain Management: An International Journal,* Vol.13, pp 48–63. http://doi.org/10.31387/oscm0400252

Dowlatshahi, S., 2000. Developing a theory of reverse logistics, *Interfaces (Providence),* Vol.30, pp 143–155. https://www.jstor.org/stable/25062603

Farkas, A., Koltai, T. and Szendrovits, A., 1993. Linear programming optimization of a network for an Aluminum plant: A case study, *International Journal of Production Economics*, Vol.32, No.2, pp 155- 168. https://doi.org/10.1016/0925-5273(93)90065-S

Johnson, P.F., 1998. Managing value in reverse logistics systems, *Transportation Research Part E: Logistics and Transportation Review*, Vol.34E, pp 217–227. https://doi.org/10.1016/S1366-5545(98)00016-7

Kaynak, R., Koçoğlu, İ., and Akgün, A.E., 2014. The role of reverse logistics in the concept of logistics centers, *Procedia - Social and Behavioral Sciences*, Vol.109, pp 438–442. https://doi.org/10.1016/j.sbspro.2013.12.487

Keyvanshokooh, E., Fattahi, M., Seyed-Hosseini, S.M., and Tavakkoli-Moghaddam, R., 2013. A dynamic pricing approach for returned products in integrated forward/reverse logistics network design, *Applied Mathematical Modelling*. Vol.37, pp 10182–10202. https://doi.org/10.1016/j.apm.2013.05.042

Lambert, S., Riopel, D., and Abdul-Kader, W., 2011. A reverse logistics decisions conceptual framework, *Computers & Industrial Engineering*, Vol.61, pp 561–581. https://doi.org/10.1016/j.cie.2011.04.012

Lau, K.K. and Womersley, R.S., 2001. Multistage quadratic stochastic programming, *Journal of Computational and Applied Mathematics*, Vol.129, pp 105–138. https://doi.org/10.1016/S0377-0427(00)00545-8

Manimaran, P. and Selladurai, V., 2014. Efficient heuristics for cost optimisation in multi stage supply chain network with a fixed transportation charge, *International Journal of Services and Operations Management*, Vol. 18, pp 115–141. https://doi.org/10.1504/IJSOM.2014.061997

Mathiyazhagan, K., Rajak, S., Sampurna Panigrahi, S., Agarwal, V. and Manani, D., 2021. Reverse supply chain management in manufacturing industry: a systematic review, *International Journal of Productivity and Performance Management*, Vol.70, No. 4, pp 859-892. https://doi.org/10.1108/IJPPM-06-2019-0293

Morais, M. de O., Brejão, A.S., Araújo, M.B. and Neto, P.L. de O.C., 2018. The reverse logistics helping to reduce costs of raw material in a pressure aluminum casting, *Environmental Quality Management*, Vol.28, pp 39–46. https://doi.org/10.1002/tqem.21596

Rabani, Y., 2007. Linear programming, in *Handbook of Approximation Algorithms and Metaheuristics*, Chapman and Hall, CRC 2007, ISBN 978-1-58488-550-4.

Tarin, N., Azar, A., and Ebrahimi, S.A., 2019. Two-phase approach to an integrated reverse logistics network considering different qualities of returned products, *Journal of Advances in Management Research*, Vol.17, pp 369–395. https://doi.org/10.1108/JAMR-07-2019-0137

Turrisi, M., Bruccoleri, M., and Cannella, S., 2013. Impact of reverse logistics on supply chain performance, *International Journal of Physical Distribution & Logistics Management*, Vol.43, pp 564–585. https://doi.org/10.1108/IJPDLM-04-2012-0132

Woubante, G., 2017. The Optimization Problem of Product Mix and Linear Programming Applications: Case Study in the Apparel Industry, *Open Science Journal*, Vol.2, No.2.

Ziółkowski, E. and Schmalenberg, K., 2019. Determination of the charge materials range in multistage charge burden optimization for foundry furnaces, *Journal of Materials Engineering and Performance*, Vol.28, pp 4012–4017. https://doi.org/10.1007/s11665-019-04144-6

Ziółkowski, E., 2013. Algorithm for burden calculation for foundry furnaces using charge materials with an uncertain composition, *Archives of Metallurgy and Materials*. Vol.58, pp 887–889. https://doi.org/10.2478/amm-2013-0093

Źiółkowski, E., 2017. Burden optimisation of lump charge materials for foundry furnaces, *Archives of Metallurgy and Materials*, Vol.62, pp 2217–2221. https://doi.org/10.1515/amm-2017-0327

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