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An index of capability for bivariate zero-inflated processes

Surajit Pal [1], Susanta Kumar Gauri [2]*

[1] *SQC & OR Unit, Indian Statistical Institute, 110, N. Manickam Road, Chennai-600029, INDIA*
[2] *SQC & OR Unit, Indian Statistical Institute, 203, B. T. Road, Kolkata-700108, INDIA*
[*]Corresponding Author: e-mail: susantagauri@hotmail.com, Tel. No.: 091-033-2575-3350; Fax No.: 091-033-2577-6042
ORCID iDs: https://orcid.org/ 0000-0001-6066-5782 (Pal), https://orcid.org/ 0000-0003-4654-7696 (Gauri)

Abstract

Rapid technological advancement and implementation of automation and computerization in today's manufacturing set up resulted in many high quality processes, where defects are rarely observed. There are many high quality manufacturing processes where two or more types of defects may be generated from different types of equipment/process problems. The zero-inflated defects data containing two types of defects are commonly modeled by bivariate zero-inflated (BZI) Poisson distribution. Pal and Gauri (2022[a]) proposed a methodology for measuring capability of a BZI Poisson process. However, they ignored the count of zero defect (ZD) products produced in a BZI process. Because of that, Pal and Gauri (2022[a]) proposed approach fails to discriminate the BZI processes which produces different proportions of ZD units but having almost the same proportion of nonconforming items with respect to the USL of combined number of defects or USLs of individual defect types. In this paper, a new measure of process capability for BZI processes is proposed that can truly discriminate different BZI processes taking into account the USL of combined number of defects (or USLs of individual defect types) as well as the proportion of ZD units produced in these processes. The proposed methodology is illustrated using two case studies. The results of the case studies show that the proposed index well represents the true capability of BZI processes.

Keywords: Process capability index; Bivariate zero-inflated process; Proportion of zero defect units; Bivariate zero-inflated Poisson process.

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1. Introduction

The occurrence of count data (e.g. defect, error etc.) is very common in manufacturing environment and is characterized by the number of times an event occurs. The most basic statistical model used to analyse count data is the Poisson model. When there is evidence of overdispersion, other models (e.g. negative binomial (NB) model and generalized Poisson model) may replace the Poisson model. Rapid technological advancement and implementation of automation and computerization in today's manufacturing set up resulted in many high quality processes, particularly in electronics industry, where defects are rarely observed. For example, thermo sonic wire bonding process of integrated circuit assembly (Chang and Gan, 2001), computer hard disk manufacturing process (Xie and Goh, 1992), etc. In these processes, the count variable (e.g., d =number of defects or errors) contains excess zeroes above what is to be expected from the Poisson model. A possible explanation for such excess zeros, given by Lambert (1992), is that slight, unobserved changes in the environment cause the process to move randomly back and forth between a perfect stage in which the defects are extremely rare and an imperfect state in which defects are possible but not inevitable. Xie and Goh (1993) referred to such processes as zero-defect process with random shocks, and in the industrial terminology such a near zero-defect process is referred to a high yield process or high quality process.

A common approach for analyzing such data is zero-inflated models such as zero-inflated Poisson (ZIP) (Lambert, 1992), zero-inflated negative binomial (ZINB) (Martin and Hall, 2017), zero-inflated generalized Poisson (ZIGP) (Wagh and Kamalja, 2018), zero-inflated geometric (ZIG) (Chang and Gan, 1999). The zero-inflated distribution can be regarded as a mixture of two distributions, one of which is degenerate at zero and other is the standard probability distribution for count data, e.g. Poisson, negative binomial, geometric distributions etc. Researchers have carried out extensive research works on the control and monitoring of the zero-inflated processes (Xie and Goh, 1993, Chang and Gan, 1999; Sim and Lim, 2008; Rakitzis et al., 2016; Alevizakos and Koukouvinos, 2021: Heuvel et al., 2022). However, an obvious requirement for a high quality process is that the input materials/components to the process should be of high quality (i.e. those should come from high quality processes). Therefore, substantial works have also been done on acceptance sampling plans (Loganathan and Shalini, 2014; Rao and Aslam, 2017) and process capability analysis (Patil and Shirke, 2012; Pal and Gauri, 2021; Pal and Gauri, 2022[b]) based on zero-inflated distributions. One important point to note that in all these works, the zero-inflated modelling of the count data have been done assuming that only a single type of defect may occur in the process.

However, there are many high quality manufacturing processes where multiple types of defects may be generated from different types of equipment/process problems and they can cause different types of product failure. For example, let us consider the LED packaging process (He et al., 2012). This process has two important steps - one is placement of LEDs onto a printed circuit board (PCB) using a robotic machine and other is the soldering process that connects the LEDs and PCB via golden wire. There can be two types of defects in the LEDs on a manufactured PCB - LED mounting errors (generated from the problems in the LED mounting equipment) and soldering error (generated from the problems in soldering equipment). The special grinding process, studied by Pal and Gauri (2022[a]), is another example. One critical component of motor cycles, are processed in a special grinding machine for a smooth outer surface finish of the component. The defects are detected only after removal of a fixed amount of material from component's outer surface. There are mainly two types of defects, namely, holes (defect type 1) and unclean surface (defect type 2). Quality of environmental air of some sterilization processes in a pharmaceutical factory (Fatahi et al., 2012) may be considered as another example. The air is desirable to be free from particles as well as microorganisms. The air quality is checked by counting number of particles (defect type 1) and number of microorganisms (defect type 2). When two types of defects are present in a process, it is essential to utilize bivariate distributions for modelling the count data, and when more than two types of defects are present in a process, it is essential to utilize multivariate distributions for modelling the count data, particularly if different types of defects are correlated with each other. The literatures on the modelling of bivariate and multivariate count data are quite rich (Marshall and Olkin, 1985; Zhang et al., 2017; Weems et al., 2021).

But the modelling becomes a issue when there are zero inflation in the multivariate count data, which is the direct fallout of the rapid technological advancement and implementation of automation in today's manufacturing set up. Several researchers have taken interest in modelling bivariate and/or multivariate zero-inflated count data (Li et al., 1999; Fatahi et al., 2012; Liu and Tian, 2015; Zhang et al., 2015; Faroughi and Ismail, 2017; Ermawati et al., 2022). Substantial works have also been carried out, in the recent past, on developing appropriate schemes for control and monitoring of the bivariate zero-inflated (BZI) and/or multivariate zero-inflated (MZI) processes. Aebtarm and Bouguila (2010) proposed an optimal bivariate Poisson field chart to monitor two correlated characteristics of count data for both manufacturing and non-manufacturing processes. Fatahi et al. (2012) applied copula function approach to achieve the joint distribution of two correlated ZIP distributions for developing a bivariate control chart which can be used for monitoring rare events. He et al. (2012) proposed a CUSUM based procedure to monitor bivariate zero-inflated Poisson processes with an application in LED packaging Industry. The methodology uses the combination of two CUSUM based control procedures for detecting shifts in the two sets of parameters in a BZIP process. But the other aspects of quality control in BZI processes, e.g. evaluation of process capability, developing sampling plan etc. have drawn very little attention of the researchers.

Only Pal and Gauri (2022[a]) have attempted to measure the capability of a BZI process. They evaluate the capability of the BZI process assuming that only USL of combined number of defects (or USLs of individual defect types) is specified. However, in reality, the manufacturer often specify an additional requirement for a BZI process in terms of a lower specification limit (LSL) for proportion of zero-defect (ZD) product. This is important because different BZI processes can almost equally satisfy the same USLs for individual defect types (or USL for the combined number of defects) but they can produce different counts of ZD products. In this article, a new approach for assessment of capability of a BZI process is presented, which takes into account both the USLs for the individual defect types (or USL for the combined number of defects) as well as LSL for the proportion of ZD product. The article is organized as follows. The literature review on process capability assessment is presented briefly in section

2. The proposed approach for evaluation of process capability of a BZI process is described in section 3. Applications of the proposed method are presented as two case studies in section 4. The article is concluded in section 5.

2. Literature Review

Traditionally C_p, C_{pk}, C_{pm}, and C_{pmk} (Kane, 1986; Kotz and Johnson, 2002; Chen et al., 2017) most commonly known process capability indices for the univariate case. Traditionally, all these indices are developed assuming that the quality characteristic is a continuous variable. However, in reality, many quality characteristics are discrete in nature e.g. defect, error, defective items etc., and these are called as attribute data. These data usually follow Poisson or binomial distribution. Therefore, standard formulas cannot be used for computation of capability indices of a process involving such characteristics. To alleviate the problem, some generalized indices for univariate case are proposed in literature, e.g. C-index (Borges and Ho, 2001), C_f index (Yeh and Bhattacharya, 1998), C_{pc} index (Perakis and Xekalaki, 2005) and C_{py} index (Maiti et al., 2010). These indices are applicable to any process regardless of whether the quality characteristic is discrete or continuous and irrespective of its underlying probability distribution. Pal and Gauri (2020[a], 2020[b]) have shown that Borges and Ho (2001) proposed C-index gives the most appropriate measure of the process capability. Therefore, the capability of a Poisson process can easily be evaluated using Borges and Ho (2001) proposed C-index.

The quality revolution caused by the rapid advancement of technologies and automation in today's world has led to tremendous improvement in the quality of manufactured products. This led to high-quality processes which have more count of zeros than are expected under chance variation of Poisson distribution (Sim and Lim, 2008). These processes are usually referred to as zero-inflated (ZI) Poisson processes (Lambert, 1992; Sim and Lim, 2008), and these processes are modelled as a mixture of a degenerate distribution at zero and a Poisson or negative binomial distribution. Assessing capabilities of such ZI processes is an important issue. Only Patil and Shirke (2012) and Pal and Gauri (2021, 2022[b]) have attempted to measure the capability of a ZI process. Patil and Shirke (2012) have modified the Perakis and Xekalaki (2005) proposed C_{pcu} index by incorporating the inflation of zero parameter into C_{pcu} index. But it fails to represent the true capabilities of ZI processes consistently. Particularly, for small value (≤ 0.5) of inflation of zero parameter, the value of their process capability index becomes unusually high, which gives a wrong impression about the capability of the concerned process. On the other hand, Pal and Gauri (2021) have applied the concept of Borges and Ho (2001) for measuring the capability of a ZI Poisson process. Pal and Gauri's (2021) approach ensure that the computed capability index never be unreasonably high. But their proposed approach fails to discriminate the ZI Poisson processes which produces different proportions of ZD units but having the same proportion of nonconforming items with respect to the USLs of individual defect types (or USL of combined number of defects). This is because Pal and Gauri (2021) ignored count of ZD products produced in a ZI process. Pal and Gauri (2022[b]) developed a new measure of process capability for ZI processes that overcome the limitation of their previously proposed method.

However, there are many zero-inflated manufacturing processes (i.e. high quality manufacturing processes) where multiple types of defects may be generated from different types of equipment/process problems and they can cause different types of product failure. The zero-inflated defects data containing two types of defects and more than two types of defects are commonly modelled by bivariate zero-inflated (BZI) Poisson distribution and multivariate zero-inflated (MZI) Poisson distribution respectively. Evaluating capability of such BZI and MZI processes is an important issue. Our extensive review of literature reveals that there is no work that aims to evaluate capability of a MZI process. Only Pal and Gauri (2022[a]) have attempted to measure the capability of a BZI process. However, their proposed method suffers from a limitation. It does not take into consideration the proportion of ZD product produced in a BZI process. However, in reality, a manufacturer often specify not only the USLs for individual defect types (or USL for combined number of defects) but also specify the LSL for proportion of ZD product produced in a BZI process.

3. Proposed Approach for Evaluation of Capability of a Bivariate Zero-inflated (BZI) Process

Suppose, the requirements for a BZI process are specified by the manufacturer as follows: the USL for the combined number of defects is c^{usl} and the LSL for proportion of zero-defect (ZD) units is p_{ZD}^{lsl}. However, sometimes a manufacturer may specify its requirements in terms of individuals USLs (say, c_1^{usl} and c_2^{usl}) for each defect type instead of specifying the USL for the combined number of defects. It is important to note that the specification for the number of defects (combined defects, or individual defect type) and the specification for the proportion of ZD units are of different significance. If a product contains number of defects more than the USL of number of defects (combined or individual defect type), it becomes nonconforming (NC) and is unacceptable. On the other hand, p_{ZD}^{lsl} (the LSL for proportion of ZD units) is essentially specifies the desirable properties of the process. If this specification is violated, no product becomes nonconforming although it is indicative that the process performance is deteriorating. Two or more BZI processes producing almost equal proportion of NC items but producing different proportions of

ZD items should be discriminated appropriately. In this context, the proportion of ZD products produced in a BZI process should be an integral part of the process capability index of a BZI process.

The basic purpose of process capability index is to assess the ability of a process in producing outputs according to the specified requirements or predicting the expected NC outputs with respect to the given specifications. It is well known that given the Z-value of a standard normal distribution, one can easily find out the expected proportion of NC outputs. So it will be good if an index can have one-to-one correspondence (mapping) between the proportion of NC items produced in a process and the Z-value of the standard normal distribution. This idea is originally introduced by Borges and Ho (2001). According to this idea, when only USL is specified the expected proportion of NC outputs in a process with respect to USL is mapped to the Z-score in the right side of standard normal distribution, and 1/3rd of this Z-score is considered as the measure of the process capability with respect to the USL. Obviously, in this approach, the measure of process capability responds to changes in the NC region and not to changes in the distribution of the observed quality characteristic(s). Therefore, the same approach can be used for computation of process capability index with respect to USL of combined number of defects or individual USLs of different defect types in a BZI process.

Keeping this idea in mind, it is proposed to obtain first a measure of process capability with respect to the USL of combined number of defects (i.e. c^{usl}) or individual USLs for each defect type (i.e. c_1^{usl} and c_2^{usl}), and then to apply an appropriate multiplying factor taking into consideration the proportion of ZD units with respect to its specified lower limit. The multiplying factor will ensure that different BZI processes are discriminated properly even when they equally satisfy the given USL on combined number of defects or individual USLs for each defect type.

Suppose, the proportion of NC units in the process is p_{NC}. Then, according to the above said idea, we need to find out the Z-value in the right side of the standard normal distribution that results in probability area equal to p_{NC}. Let Z_U is the value of Z that results in probability area p_{NC} above it. The Z_U value can be obtained by using inverse cumulative probability of the standard normal distribution function as follows: $Z_U = \Phi^{-1}(1 - p_{NC})$. Then the process capability index of the BZI process with respect to USL will be obtained as $C_{pu}^{BZI} = \frac{1}{3} \times Z_U = \frac{1}{3} \times \Phi^{-1}(1 - p_{NC})$.

Now as discussed earlier, a BZI process which produces more proportion of ZD products is more desirable to a BZI process which produces less proportion of ZD products. This aspect of a BZI process should be reflected in the overall measure of process capability of a BZI process. If we consider a multiplying factor, $m = \frac{p_{ZD}}{p_{ZD}^{lsl}}$ (where, p_{ZD} is proportion of ZD output produced in the BZI process and p_{ZD}^{lsl} is the LSL on the proportion of ZD outputs) to C_{pu}^{BZI}, then the above requirement will be satisfied. For example, suppose three BZI processes equally satisfy the USL of combined number of defects, but proportions of ZD products produced in these processes are less than p_{ZD}^{lsl}, equal to p_{ZD}^{lsl} and more than p_{ZD}^{lsl}, respectively. Then the values of the multiplying factors for these processes will be less than one, equal to one and more than one. This will imply that the overall process capability index will be the minimum for the first process and the maximum for the third process, which is expected under consideration of proportion of ZD products produced in these processes. Thus, the overall process capability index of a BZI process (C_p^{BZI}) can be obtained as

$$\begin{aligned} C_p^{BZI} &= m \times C_{pu}^{BZI} \\ &= \frac{p_{ZD}}{p_{ZD}^{lsl}} \times \frac{1}{3}\Phi^{-1}(1 - p_{NC}) \end{aligned} \tag{1}$$

If $C_p^{BZI} \geq 1$, the concerned BZI process will be considered capable of producing products satisfying the USL of combined number of defects (or USLs of individual defect type) as well as LSL of proportion of ZD units. Otherwise, the BZI process will be considered not capable of satisfying the USL of combined number of defects (or USLs of individual defect type) and/or the LSL of proportion of ZD units. It is worth to mention that if the value of $(1 - p_{NC})$ is less than 0.5, then the value of C_p^{BZI} will be negative. The value of $1 - p_{NC} < 0.5$ implies $p_{NC} > 0.5$. It gives sufficient indication that the process is producing plenty of nonconforming units with respect to the USL of combined number of defects (or USLs of individual defect type), and thus the process is not capable at all. So it is recommended to consider $C_p^{BZI} = 0$ if $1 - p_{NC} < 0.5$. This will ensure that the process capability index C_p^{BZI} is always greater than or equal to zero.

Again, when the value of proportion of NC units (p_{NC}) is zero, the value of $C_{pu}^{BZI} = \frac{1}{3}\Phi^{-1}(1) \cong \frac{4}{3} = 1.33$. In that case, depending on the proportion of ZD units (p_{ZD}) produced in the process and the LSL of proportion of ZD units (p_{ZD}^{lsl}), the maximum value of C_p^{BZI} can be $\hat{C}_p^{BZI} = 1.33 \times \frac{\hat{p}_{ZD}}{p_{ZD}^{L}}$. For example, in a high quality BZI process with zero NC unit, if the LSL of the proportion of ZD units, i.e. p_{ZD}^{lsl} is specified as 0.80, the maximum value of C_p^{BZI} will not exceed $\hat{C}_p^{BZI} = 1.33 \times \frac{1.0}{0.8} = 1.6625$.

3.1 Procedure for obtaining estimate of C_p^{BZI}

An estimate of the overall process capability index ($\hat{C}_p^{BZI}$) of a BZI process can be obtained using the following steps:

1) ***Collect a sample of n units from the concerned bivariate zero-inflated (BZI) process and observe the numbers of two different types of defects in each of the sample item.***

Let there are $n$pairs of values of (y_1, y_2), where y_1 gives the number of defects of type 1 and y_2 gives the number of defects of type 2 in a single item. For a high quality BZIP process, most of the values will be $(0,0)$. Let number of such $(0,0)$ pairs in the sample of size n is n_{00}. Let us denote the maximum numbers of defects of type 1 and type 2 in an item by u and v respectively. So, there can be a maximum of $(u+1)(v+1)$ number of (i,j) pairs, where $i = 0,1,2,\dots,u$ and $i = 0,1,2,\dots,v$. Let the frequency of occurrence of $(i,j)^{th}$ pair is denoted by n_{ij}. Therefore, $\sum_{i=0}^{u}\sum_{j=0}^{v} n_{ij} = n$. In the sample data, there can be many (i,j) pairs whose frequencies may be equal to zero.

Let the random variables (Y_1, Y_2) represent the number of two types of defects present in an item.

2) ***Select an appropriate bivariate zero-inflated (BZI) probability distribution for describing the sample data.***

The most commonly used models for BZI count data are bivariate zero-inflated Poisson (BZIP) distribution. Researchers have proposed different possible ways for developing BZIP distributions. Li et al. (1999) proposed a BZIP model which is a mixture of a bivariate Poisson, two univariate Poisson and a point mass at (0,0). Fatahi et al. (2012) generated joint distribution of two correlated ZIP distribution, i.e. BZIP distribution applying the copula function approach. Motivated from the stochastic representation of the univariate ZIP random variable, Liu and Tian (2015) proposed a multivariate ZIP distribution, called as Type I multivariate ZIP distribution, to model correlated multivariate count data with extra zeros. Faroughi and Ismail (2017) derived joint probability mass function of the BZIP distribution from the Sarmanov (Lee, 1996) family of bivariate distributions. Among all these BZIP models, Li et al. (1999) proposed BZIP model is the most flexible and appealing one. While fitting an appropriate BZIP probability distribution model to sample data, it was observed that Li et al. (1999) proposed BZIP model has the maximum flexibility to fit different types of sample data. This can be observed from the two case studies described later. In the first case study, sample data is well modelled by BZIP distributions proposed by Li et al. (1999), Liu and Tian (2015) and Faroughi and Ismail (2017), but the copula-based BZIP distribution proposed by Fatahi et al. (2012) does not fit the sample data. In the second case study, BZIP distribution proposed by Li et al. (1999) and copula-based BZIP model fits the sample data well, whereas other two BZIP models does not fit the sample data. Because of this flexible nature of Li et al. (1999) proposed BZIP distribution for fitting bivariate zero-inflated count data, it is decided to utilize that model. Hence, herein this article, the procedure for obtaining the estimate of C_p^{BZI} are described considering that the concerned process data is well modelled by Li et al. (1999) proposed BZIP distribution.

As mentioned earlier, the random variables (Y_1, Y_2) represent the number of two types of defects present in an item. According to Li et al. (1999) proposed BZIP model, the distribution of (Y_1, Y_2) can be written as follows:

$$
\begin{aligned}
(Y_1, Y_2) \quad & \sim (0,0) \text{with probability } p_{00} \\
& \sim [Poisson(\lambda_1), 0] \text{with probability } p_{10} \\
& \sim [0, Poisson(\lambda_2)] \text{with probability } p_{01} \\
& \sim bivariate\ Poisson(\lambda_{10}, \lambda_{20}, \lambda_{00}) \text{with probability } p_{11}
\end{aligned}
$$

where each $p_{00}, p_{10}, p_{01}, p_{11} > 0$ and $p_{00} + p_{10} + p_{01} + p_{11} = 1$. A bivariate Poisson distribution (X_1, X_2) with parameters $(\lambda_{10}, \lambda_{20}, \lambda_{00})$ is represented as follows (Marshall and Olkin, 1995):

$$X_1 = U_1 + Z \text{ and } X_2 = U_2 + Z$$

where U_1, U_2 and Z are independent and have univariate Poisson distributions with respective means λ_{10}, λ_{20}, and λ_{00}, (each > 0).

The probability mass function of the BZIP of Li et al. (1999) is given by:

$$P(Y_1 = 0, Y_2 = 0) = p_{00} + p_{10}\exp(-\lambda_1) + p_{01}\exp(-\lambda_2) + p_{11}\exp(-\lambda) \tag{2}$$

$$P(Y_1 = y_1, Y_2 = 0) = \frac{1}{y_1!}[p_{10}{\lambda_1}^{y_1}\exp(-\lambda_1) + p_{11}{\lambda_{10}}^{y_1}\exp(-\lambda)] \tag{3}$$

$$P(Y_1 = 0, Y_2 = y_2) = \frac{1}{y_2!}[p_{01}{\lambda_2}^{y_2}\exp(-\lambda_2) + p_{11}{\lambda_{20}}^{y_2}\exp(-\lambda)] \tag{4}$$

$$P(Y_1 = y_1, Y_2 = y_2) = p_{11}\left[\sum_{j=0}^{\min(y_1,y_2)}\left\{\frac{\lambda_{10}^{y_1-j}\lambda_{20}^{y_2-j}\lambda_{00}^{j}\exp(-\lambda)}{(y_1-j)!\,(y_2-j)!\,j!}\right\}\right] \tag{5}$$

for $y_1, y_2 = 1,2,\ldots,$ and $\lambda = \lambda_{10} + \lambda_{20} + \lambda_{00}$. Li et al. (1999) also assumed that $\lambda_1 = \lambda_{10} + \lambda_{00}$ and $\lambda_2 = \lambda_{20} + \lambda_{00}$ for simplification of the BZIP model and to let the marginal distributions become univariate ZI Poisson distributions.

3) ***Estimate the parameters of the selected BZI distribution from the observed count data.***

It can be observed from the above BZIP model that there are total 10 unknown parameters, namely p_{00}, p_{10}, p_{01}, p_{11}, λ, λ_1,λ_2, λ_{10}, λ_{20} and λ_{00}. Among them, three parameters, λ, λ_1, and λ_2 can be obtained from λ_{10}, λ_{20}, and λ_{00} values, and parameter p_{11} can be automatically obtained since the constraint $p_{00} + p_{10} + p_{01} + p_{11} = 1$ must be satisfied. Therefore, we need to estimate only six unknown parameters, namely p_{00}, p_{10}, p_{01}, λ_{10}, λ_{20} and λ_{00}. The unknown six parameters can be estimated from a random sample of size n using the maximum likelihood method (Li et al., 1999). The log likelihood function can be written as

$$lnL = ln\prod_{i=0}^{u}\prod_{j=0}^{v} P(Y_1 = i, Y_2 = j)^{n_{ij}} = \sum_{i=0}^{u}\sum_{j=0}^{v} n_{ij} \times lnP(Y_1 = i, Y_2 = j) \tag{6}$$

This log likelihood function is to be maximized by changing the six unknown parameters (p_{00}, p_{10}, p_{01}, λ_{10}, λ_{20} and λ_{00}) subjecting to two constraints. These constraints are: i) all parameters are positive, and ii) $p_{00} + p_{10} + p_{01} + p_{11} = 1$. Using enumerative search procedures, the estimates of the unknown parameters can be obtained. While performing this search procedure, one has to check about the expected proportion value computed from the fitted BZIP model. The expected probability $P(Y_1 = i, Y_2 = j)$ value for $i = 0, 1, 2, \ldots u$ and $j = 0, 1, 2, \ldots v$ should be close to (n_{ij}/n) value obtained from sample data. The analyst must perform Chi-square goodness-of-fit test for checking the adequacy of the fitted model.

Suppose, the estimated parameters of the BZIP distributions are $\hat{p}_{00}$, $\hat{p}_{10}$, $\hat{p}_{01}$, $\hat{p}_{11}$, $\hat{\lambda}$, $\hat{\lambda}_1$, $\hat{\lambda}_2$, $\hat{\lambda}_{10}$, $\hat{\lambda}_{20}$ and $\hat{\lambda}_{00}$.

4) ***Estimate the expected proportion of nonconforming units ($\hat{p}_{NC}$) in the concerned BZI process.***

Case 1: USL for the combined number of defects in a unit (say, c^{usl}) is specified

A unit will be considered nonconforming if the total number of defects in it is more than the USL for the combined number of defects, i.e. c^{usl}. Then p_{NC} in the concerned BZIP process can be estimated as follows:

$$\begin{aligned}\hat{p}_{NC} &= P[(Y_1 + Y_2) > c^{usl}] = 1 - P[(Y_1 + Y_2) \le c^{usl}] \\ &= 1 - \sum_{i=0}^{c^{usl}-j}\sum_{j=0}^{c^{usl})} P(Y_1 = i, Y_2 = j)\end{aligned} \tag{7}$$

Case 2: Individuals USLs for each defect type (say, c_1^{usl} and c_2^{usl}) are specified

A unit will be considered nonconforming if the number of first type of defects in it is more than c_1^{usl} or the number of second type of defects is more than c_2^{usl} or both. Then p_{NC} in the concerned BZIP process can be estimated as follows:

$$\begin{aligned}\hat{p}_{NC} &= 1 - P(Y_1 \le c_1^{usl}, Y_2 \le c_2^{usl}) \\ &= 1 - \sum_{i=0}^{c_1^{usl}}\sum_{j=0}^{c_2^{usl}} P(Y_1 = i, Y_2 = j)\end{aligned} \tag{8}$$

5) ***Estimate the expected proportion of zero-defect units ($\hat{p}_{ZD}$) in the concerned BZI process.***

The expected proportion of zero-defect units ($\hat{p}_{ZD}$) of the BZI process can be computed using Equation (2) of the probability mass function of BZIP distribution as

$$\hat{p}_{ZD} = P(Y_1 = 0, Y_2 = 0) = \hat{p}_{00} + \hat{p}_{10}\exp\left(-\hat{\lambda}_1\right) + \hat{p}_{01}\exp\left(-\hat{\lambda}_2\right) + \hat{p}_{11}\exp\left(-\hat{\lambda}\right) \tag{9}$$

6) ***Finally, obtain the estimate of the overall process capability index ($\hat{C}_p^{BZI}$)***

The estimate of the overall process capability index of the concerned BZI process can be obtained as follows:

$$\hat{C}_p^{BZI} = \frac{\hat{p}_{ZD}}{p_{ZD}^{lsl}} \times \frac{1}{3}\Phi^{-1}(1 - \hat{p}_{NC}) \tag{10}$$

7) Obtain the confidence interval of the estimated overall process capability index ($\hat{C}_p^{BZI}$)

Since $\hat{C}_P^{BZI}$ is a point estimate obtained from sample data, it is necessary to construct confidence interval (CI) of the capability index C_P^{BZI} for inference purpose, especially when the sample size is relatively small. Deriving the CI of C_P^{BZI} taking into account the CIs of ZD and NC units is quite difficult. Hence, it is suggested to use Nagata and Nagahata (1994) proposed generalized approximation formula for construction of two-sided CI of C_P^{BZI}. According to Nagata and Nagahata (1994),

$$(1-\alpha)\% \text{ CI of } C_P^{BZI} = \left[\hat{C}_P^{BZI} - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{1}{9n}+\frac{\hat{C}_P^{BZI}}{2(n-1)}}\ ,\ \hat{C}_P^{BZI} + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{1}{9n}+\frac{\hat{C}_P^{BZI}}{2(n-1)}}\right] \tag{11}$$

where, α is the level of significance and (1-α) is the confidence coefficient.

4. Applications

The capability indices of two BZI processes are evaluated using the proposed approach and these applications are presented in this section as two case studies.

4.1. Case Study 1

The following study was carried out in an automobile industry for one critical component used in motor cycles. After receiving the input components from an authorized vendor, these components are processed for a smooth outer surface finish using a special purpose grinding machine. The defects are detected only after removal of a fixed amount of material from component outer surface. There are mainly two types of defects, namely, holes and unclean surface, either one or both occurring in one or more locations. The past history of the incoming components reveals that, in general, although around 90% components are of zero defects, there are around 10% components having either one or both type of defects in one or more locations along the component surface.

It is desirable not to have any kind of defects in any component. However, looking at the complex structure of the component, the management of the company decided that a maximum of total 4 defects in any component will be treated as acceptable and any component containing more than 4 defects will be rejected and will be sent back to the supplier at his expenses. This implies that the USL for combined number of defects was decided as 4, i.e. $c^{usl} = 4$. However, a reduction of proportion of nonconforming with respect to this specification may not necessarily signify better quality. For example, if no incoming component is nonconforming with respect to the USL of combined number of defects but the proportion of ZD reduces drastically, it may be an indication of process deterioration. Therefore, the management of the company decided also to define a LSL for the proportion of ZD components, which is equal to 0.90, i.e. $p_{ZD}^{lsl} = 0.90$. The management of the company was interested to assess the capability of the component's manufacturing process with respect to these two specifications, i.e. USL of combined number of defects (c^{usl}) and LSL of proportion of ZD components (p_{ZD}^{lsl}).

For this purpose, a sample of 1000 items is collected randomly from a lot and all those items are inspected after the grinding process. The observed frequency distribution of the two types of defects in the sampled components is given in Table 1.

Table 1. Frequency distribution of two types of defects in components after grinding

(y_1, y_2)	Frequency	(y_1, y_2)	Frequency	(y_1, y_2)	Frequency
(0, 0)	906	(0, 4)	2	(2, 3)	3
(1, 0)	12	(0, 5)	1	(2, 4)	1
(2, 0)	7	(1, 1)	10	(3, 1)	3
(3, 0)	4	(1, 2)	7	(3, 2)	3
(4, 0)	1	(1, 3)	3	(3, 3)	1
(0, 1)	8	(1, 4)	2	(4, 1)	2
(0, 2)	6	(2, 1)	8	(4, 2)	1
(0, 3)	3	(2, 2)	5	(4, 3)	1

Following the notations described earlier, it can be written that:

$$n_{00} = 906, \quad \sum_{i=1}^{4} n_{i0} = 24, \quad \sum_{j=1}^{5} n_{0j} = 20, \quad \sum_{i=1}^{4}\sum_{j=1}^{5} n_{ij} = 50, \quad n = 1000$$

It can be computed from the sample data that the total numbers of defects of type 1 and type 2 are 135 and 133 respectively, and the combined number of defects is 268. It is further observed that there are 15 items where combined number of defects is more than 4, the specified USL (c^{usl}) for the combined number of defects. This implies that the proportion of NC items in the collected samples is 1.5%.

Now the maximum likelihood procedure is applied to estimate the parameters of the BZIP distributions of Li et al. (1999). The estimates of the model parameters are obtained as follows:

$\hat{p}_{00} = 0.8969$ $\hat{p}_{10} = 0.0119$ $\hat{p}_{01} = 0.0033$ $\hat{p}_{11} = 0.0879$ $\hat{\lambda}_{00} = 0$

$\hat{\lambda}_{10} = 1.3525$ $\hat{\lambda}_{1} = 1.3525$ $\hat{\lambda}_{01} = 1.4578$ $\hat{\lambda}_{2} = 1.4578$ $\hat{\lambda} = 2.81$

The log-likelihood value is computed as *Ln L* = -585.397. For the purpose of model adequacy checking chi-square goodness-of-fit statistic is computed. The chi-square goodness-of-fit statistic is computed as $\chi_7^2 = 1.02$ having a *p*-value of 0.994, which implies that the BZIP distribution fits quite well to the sample data.

The USL for the combined number of defects in an item is specified as $c^{usl} = 4$. Thus, the expected proportion of NC unit ($\hat{p}_{NC}$) and expected proportion of ZD units ($\hat{p}_{NC}$) in the concerned BZI process are estimated using equations (7) and (9) respectively as follows:

$$\hat{p}_{NC} = 1 - \sum_{i=0}^{4-j} \sum_{j=0}^{4} P(Y_1 = i, Y_2 = j) = 0.0137$$

$$\hat{p}_{ZD} = \hat{p}_{00} + \hat{p}_{10} \exp(-\hat{\lambda}_1) + \hat{p}_{01} \exp(-\hat{\lambda}_2) + \hat{p}_{11} \exp(-\hat{\lambda}) = 0.906$$

The estimate of process capability index of the BZI process is then obtained using equation (10) as

$$\hat{C}_P^{BZI} = \frac{\hat{p}_{ZD}}{p_{ZD}^{lsl}} \times \frac{1}{3} \Phi^{-1}(1 - \hat{p}_{NC}) = \frac{0.906}{0.90} \times \frac{1}{3} \Phi^{-1}(1 - 0.0137) = \frac{0.906}{0.90} \times 0.735 = 0.740$$

Since the estimated process capability index $\hat{C}_P^{BZI}$ is less than 1.0, the concerned BZI process is considered as not capable to satisfy the given specifications for the components. The 95% confidence intervals of C_P^{BZI} is then computed using equation (11) as

$$\left[\hat{C}_P^{BZI} - Z_{1-\frac{\alpha}{2}} \sqrt{\frac{1}{9n} + \frac{\hat{C}_P^{BZI}}{2(n-1)}}, \quad \hat{C}_P^{BZI} + Z_{1-\frac{\alpha}{2}} \sqrt{\frac{1}{9n} + \frac{\hat{C}_P^{BZI}}{2(n-1)}} \right]$$

$$\Rightarrow \left[0.740 - 1.96 \sqrt{\frac{1}{9000} + \frac{0.740}{2(999)}}, \ 0.740 + 1.96 \sqrt{\frac{1}{9000} + \frac{0.740}{2(999)}} \right] \Rightarrow [0.7015, 0.7785]$$

It may be noted that there is about 9.4% items that contains one or more defects of one or both types, out of which around 1.3% items are nonconforming with respect to the USL for the combined number of defects. Hence, there is a scope for improvement in this process by further bringing reduction in the production of NC items as well as by bringing increment in the proportion of ZD items.

4.2 Case Study 2

Fatahi et al. (2012) presented a case study of a pharmaceutical factory where quality of environmental air of a sterilization process was an important issue. The air is desirable to be free from particles as well as microorganisms. The air quality is checked by counting number of particles (defect type 1) and number of microorganisms (defect type 2). In this case study, they demonstrated that the air quality could be monitored effectively using bivariate ZIP chart.

They samples environmental air three times a day during 100 consecutive days and recorded the numbers of particles and microorganisms regarding each sample of environmental air. Thus a total 300 samples of environmental air were drawn. We use here the same data for computation of the overall PCI of the sterilization process. However, for computation of the overall PCI we require to know the USL for the combined number of defects (or USLs for individual defect types) and the LSL for the proportion of ZD cases. Fatahi et al. (2012) computed the upper control limit for their bivariate ZIP control chart (for the combined number of defects) as 15. We consider the same value as the specified USL for the combined number of defects, i.e., $c^{usl} = 15$. Again we assume that there is a minimum requirement that at least 75% cases a sample should be free of particles and microorganisms, i.e. $p_{ZD}^{lsl} = 0.75$. The frequency distribution of the number of particles and microorganisms in the sampled environmental air, observed by Fatahi et al (2012), is reproduced in Table 2

Table 2. Frequency of number of particles and microorganisms in sterilization process air

(y_1, y_2)	Frequency	(y_1, y_2)	Frequency	(y_1, y_2)	Frequency	(y_1, y_2)	Frequency
(0, 0)	221	(7, 0)	2	(2, 3)	2	(5, 5)	1
(1, 0)	6	(8, 0)	4	(2, 6)	1	(6, 2)	1
(2, 0)	5	(9, 0)	1	(3, 2)	2	(7, 2)	1
(3, 0)	12	(0, 1)	5	(3, 3)	1	(7, 6)	1
(4, 0)	10	(0, 2)	2	(4, 1)	1	(8, 1)	1
(5, 0)	6	(0, 3)	4	(4, 2)	2	(10, 4)	1
(6, 0)	7	-	-	-	-	-	-

Following the notations described earlier, it can be written that:

$$n_{00} = 221, \qquad \sum_{i=1}^{10} n_{i0} = 53, \quad \sum_{j=1}^{6} n_{0j} = 11, \quad \sum_{i=1}^{10}\sum_{j=1}^{6} n_{ij} = 50, \qquad n = 300$$

Also, it can be computed from the sample data that the total number of type 1 defects is 289, total number of type 2 defects is 65 and combined number of defects is 354. There is not a single item that has combined number of defects more than the specified USL ($c^{usl} = 15$).

Now the maximum likelihood procedure is applied to estimate the parameters of the BZIP distribution of Li et al. (1999). The estimates of the model parameters are obtained as follows:

$$\hat{p}_{00} = 0.7295 \quad \hat{p}_{10} = 0.1731 \quad \hat{p}_{01} = 0.0403 \quad \hat{p}_{11} = 0.0571 \quad \hat{\lambda}_{00} = 0.1629$$
$$\hat{\lambda}_{10} = 4.0138 \quad \hat{\lambda}_1 = 4.1768 \quad \hat{\lambda}_{01} = 2.0551 \quad \hat{\lambda}_2 = 2.2180 \quad \hat{\lambda} = 6.2319$$

The log-likelihood value is computed as *Ln L* = - 427.485. For the purpose of model adequacy checking the chi-square goodness-of-fit statistic is computed. The chi-square goodness-of-fit statistic is computed as $\chi_6^2 = 5.00$ having a *p*-value of 0.54, which implies that the BZIP distribution fits reasonably well to the sample data.

The USL for the combined number of defects in an item is specified as $c^{usl} = 15$. Thus, the expected proportion of NC unit ($\hat{p}_{NC}$) and expected proportion of ZD units ($\hat{p}_{NC}$) in the concerned BZI process are estimated using equations (7) and (9) respectively as follows:

$$\hat{p}_{NC} = 1 - \sum_{i=0}^{15-j}\sum_{j=0}^{15} P(Y_1 = i, Y_2 = j) = 0.00076$$

$$\hat{p}_{ZD} = \hat{p}_{00} + \hat{p}_{10}\exp(-\hat{\lambda}_1) + \hat{p}_{01}\exp(-\hat{\lambda}_2) + \hat{p}_{11}\exp(-\hat{\lambda}) = 0.7367$$

The estimate of process capability index of the BZI process is then obtained using equation (10) as

$$\hat{C}_P^{BZI} = \frac{\hat{p}_{ZD}}{p_{ZD}^{lsl}} \times \frac{1}{3}\Phi^{-1}(1 - \hat{p}_{NC}) = \frac{0.7367}{0.75} \times \frac{1}{3}\Phi^{-1}(1 - 0.00076) = 1.038$$

Since the estimated process capability index $\hat{C}_P^{BZI}$ is less than 1.0, the concerned BZI process is considered as not capable to satisfy the given specifications for the components. The 95% confidence intervals of C_P^{BZI} is then computed using equation (11) as

$$\left[\hat{C}_P^{BZI} - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{1}{9n} + \frac{\hat{C}_P^{BZI}}{2(n-1)}}, \qquad \hat{C}_P^{BZI} + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{1}{9n} + \frac{\hat{C}_P^{BZI}}{2(n-1)}}\right]$$
$$\Rightarrow \left[1.038 - 1.96\sqrt{\frac{1}{2700} + \frac{1.038}{2(299)}}, \qquad 1.038 + 1.96\sqrt{\frac{1}{2700} + \frac{1.038}{2(299)}}\right]$$
$$\Rightarrow [0.948, 1.128]$$

It may be noted that there is about 26.33% cases where one or more number of particles and microorganisms are present in environmental air. However, the combined number of particles and microorganisms has never exceeded the respective USL for the combined number of particles. There is ample scope for improvement in environmental air of this sterilization process by increasing the proportion of ZD cases.

5. Conclusions

Rapid technological advancement and implementation of automation and computerization in today's manufacturing set up resulted in many high quality processes, where defects are rarely observed. There are many high quality manufacturing processes where two or more types of defects may be generated from different types of equipment/process problems. When only two types of defects are generated, the process data is commonly modelled using bivariate zero-inflated (BZI) Poisson distribution. Existing method for evaluation of process capability of BZI processes assume that only USL for combined number of defects (or USLs for individual defect types) are specified. However, proportion of ZD products produced is an integral part of a BZI process. In this article, a new approach for assessment of capability of a BZI process is presented, which takes into account both the USL for the combined number of defects (or USLs for the individual defect types) as well as LSL for the proportion of zero-defect (ZD) product. In the proposed approach, at first a measure of process capability with respect to the USL of combined number of defects (or USLs of individual defect types) is computed, and then the overall process capability index is obtained by multiplying a factor defined based on the actual value of proportion of ZD units and the LSL of the proportion of ZD units. Two case studies are presented which demonstrate that the proposed index well represents the true capability of BZI processes.

In this article, Li et al. (1999) proposed BZIP model is used for modelling the defect data. However, several other BZIP models are proposed in literature. Further, in many situations, especially when there is overdispersion in the BZI process, bivariate zero-inflated generalized Poisson (BZIGP) distributions or bivariate zero-inflated negative binomial (BZINB) distributions are utilized for modelling the defect data. The proposed approach for computing the overall PCI can be used for those cases also. The proposed approach can also be used for assessing capability of a multivariate zero-inflated (MZI) Poisson process.

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Biographical notes

Dr. Surajit Pal is a Faculty Member in the Statistical Quality Control and Operations Research Unit of the Indian Statistical Institute, Chennai Centre, India. His fields of interest are quality engineering, process optimization, statistical quality control and multiple response optimization. He has published about forty five papers in different National and International journals.

Dr. Susanta Kumar Gauri is a Faculty Member in the Statistical Quality Control and Operations Research Unit of the Indian Statistical Institute, Kolkata, India. His fields of interest are quality engineering, process optimization, statistical quality control and multiple response optimization. He has published about sixty papers in different National and International journals.