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A mathematical model of shear wave propagation in the incompressible transversely isotropic thermoelastic half-spaces

T. Lalawmpuia[1], S.S. Singh[1]*, C. Zorammuana[2]

[1]Department of Mathematics and Computer Science, Mizoram University, Aizawl -796004, Mizoram, INDIA
[2]Department of Education, Assam University, Silchar - 788 011, Assam, INDIA
**Corresponding Author, Email: saratcha32@yahoo.co.uk*, Tel.: +09862626776*
ORCID iDs: http:/orcid.org/ 0000-0002-3644-2172 (Lalawmpuia); http:/orcid.org/0000-0003-4039-4019 (Singh);
http:/orcid.org/0000-0003-4222-3934 (Zorammuana)

Abstract

This article deals with the problem of reflection and transmission of shear waves at a plane interface between two dissimilar incompressible transversely isotropic thermoelastic half-spaces. Two coupled quasi-shear waves are found to propagate due to the incompressibility of such materials. Applying appropriate boundary conditions at the plane interface, amplitude ratios of the reflected and transmitted quasi-shear waves are obtained. It has been observed that these ratios are functions of the angle of incidence, elastic and thermal parameters of the materials. These ratios are computed numerically for a particular model to see the effects of specific heat and thermal expansion on quasi-shear waves in incompressible transversely isotropic thermoelastic materials. The results are also presented graphically.

Keywords: Transversely isotropic thermoelastic materials, Reflection, Transmission, Shear wave, Amplitude ratio

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1. Introduction

The theory of thermoelasticity deals with the interaction between thermal and mechanical fields in the solid bodies. It is very important in various fields such as earthquake engineering, acoustics, soil dynamics, aeronautics, astronautics, nuclear reactors, etc. Biot (1956) developed the minimum entropy production principle along with the variational form of general laws of thermoelasticity. Dhaliwal and Sherief (1980) derived the equations of generalized thermoelasticity for an anisotropic elastic medium and proved uniqueness theorem for these equations. For different types of tension and compression, Benveniste (1981) presented a detail discussion on one dimensional wave propagation in an initially deformed material. Rogerson (1991) investigated the dynamical behavior of transversely isotropic incompressible elastic materials expressing two energy flux vectors explicitly. Chadwick (1994) developed the constitutive relations for an incompressible transversely isotropic elastic material and studied the transmission of homogeneous plane waves of small amplitude in the material.

Ogden and Sotiropoulos (1997) discussed the effect of finite strain and pre-stress on the reflected waves in anisotropic incompressible elastic solid. Itskov and Aksel (2002) introduced more constraints on the elastic constants and explained the difficulty of deriving the constitutive relations for anisotropic incompressible materials. Prikazchikov and Rogerson (2004) shows that the speed of surface waves in initially stressed incompressible transversely isotropic materials depends crucially on the normal static stress. Kumar and Hundal (2005) derived the characteristic equations and the relations for discontinuities across the wave fronts in a fluid-saturated incompressible porous medium. Abd-Alla et al. (2011) obtained that the speed of shear waves depends on the direction of propagation, the anisotropy, gravity field, non-homogeneity in the initial stress anisotropic incompressible materials. Vinh and Giang (2012) and Gupta and Ahmed (2017) also explored the impact of incompressibility on propagation of surface waves in elastic medium.

The study of elastic wave propagation has been an interesting area of research since long. They are very helpful in the exploration of materials inside the Earth's crust. Singh (2007) investigated the reflection coefficients for three different materials of incompressible fibre-reinforced transversely isotropic materials. Singh and Tomar (2007a) obtained the reflection and transmission coefficients of the reflected and transmitted shear waves at a corrugated interface between two dissimilar fiber-reinforced elastic half-spaces using Rayleigh method of approximation. Singh (2011, 2013) studied the effect of thermal on the motion of coupled longitudinal and transverse waves in the thermoelastic materials with voids and obtained the amplitude and energy ratios. Singh et al. (2014) also studied the behavior of an elastic wave at the interface between two dissimilar half spaces of fibre-reinforced incompressible elastic material having transverse isotropy. Singh (2015) obtained the secular equation for Rayleigh waves in incompressible transversely isotropic thermoelastic solid using the theory of Lord and Shulman (1967). Zorammuana and Singh (2016) derived the amplitude and energy ratios of the reflected waves in thermoelastic saturated porous medium. Recently, Zorammuana et al. (2020) obtained the amplitude ratios of the reflected waves from a plane free boundary of an incompressible transversely isotropic thermoelastic material. Lalawmpuia and Singh (2020) investigated the problem of the effect of initial stresses on the elastic waves in transversely isotropic thermoelastic materials. There are many interesting problems of waves and vibrations in open literatures as Achenbach (1976), Singh (2003), Singh and Tomar (2007b), Singh and Zorammuana (2014), Singh (2015), Zorammuana and Singh (2015), Lianngenga and Singh (2019), Singh and Lalawmpuia (2019), Goyal et al. (2020) and Lalvohbika and Singh (2020).

Two coupled quasi-shear waves are found to propagate in a heat conducting incompressible transversely isotropic elastic material. The problem of reflection/transmission of quasi shear waves at a plane interface between two dissimilar half-spaces of heat conducting incompressible transversely isotropic elastic materials has been investigated. The amplitude ratios of the reflected and transmitted quasi shear waves are computed numerically. The effects of thermal coefficient and specific heat on the propagation of quasi-shear waves in the material are observed graphically.

2. Fundamental Equations

The non-deformed state of homogeneous thermal conducting incompressible elastic materials with transverse isotropy at uniform temperature, T_0 has the following set of equations (see Singh, 2015)

$$c_{11}u_{x,xx} + (c_{13}+c_{44})u_{z,xz} + c_{44}u_{x,zz} - \beta_1\, T_{,x} - P_{,x} = \rho\ddot{u}_x, \tag{1}$$

$$c_{44}u_{z,xx} + (c_{14}+c_{44})u_{x,xz} + c_{33}u_{z,zz} - \beta_3\ T\_\{,z\} - P_{,z} = \rho\ddot{u}_z, \tag{2}$$

$$K_1\, T_{,xx} + K_3\, T_{,zz} - \rho\, C_e(\dot{T} + \tau_0\,\ddot{T}) = T_0\{\beta_1(\dot{u}_{x,x} + \tau_0\,\ddot{u}_{x,x\}}) + \beta_3(\dot{u}_{z,z} + \tau_0\,\ddot{u}_{z,z}), \tag{3}$$

where c_{ij} are elastic constants, $\boldsymbol{u_x}$ and $\boldsymbol{u_z}$ are components of displacement in $\boldsymbol{x}$ and $\boldsymbol{z}$-axis respectively, P is the hydrostatic pressure, T is the increment in temperature, τ_0 and C_e are thermal relaxation times and specific heat respectively, ρ is the density, K_1 and K_3 are the coefficients of thermal conductivity. It may be noted that comma in the subscript denotes spatial derivatives, $\beta_1 = (c_{11}+c_{12})\alpha_1 + c_{13}\alpha_3$ and $\beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3$, where α_1 and α_3 are coefficients of linear expansion.
The incompressibility condition may be given as

$$u_{x,x} + u_{z,z} = 0. \tag{4}$$

Eliminating the hydrostatic pressure from Eqs. (1) and (2), we have

$$c_{11}u_{x,xxz} + (c_{13}+c_{44})u_{z,xzz} + c_{44}u_{x,zzz} - \beta_1\, T_{,xz} - \rho\ddot{u}_{x,z} = c_{44}u_{z,xxx} + (c_{13}+c_{44})u_{x,xxz} + c_{33}u_{3,133} - \beta_3\, T_{,xz} - \rho\ddot{u}_{z,x}. \tag{5}$$

Due to the incompressibility condition (4), we can find a scalar function $\varphi(x,z,t)$ such that

$$u_x = \varphi_{,z} \text{ and } u_z = -\varphi_{,x}. \tag{6}$$

3. Wave Propagation

Consider the Cartesian co-ordinates system with x and y-axes lying horizontally and z-axis along the vertical direction. We aim to study the two-dimensional problem of wave propagation in xz-plane in the half-spaces of two incompressible transversely isotropic materials $M: 0 \le z < \infty$ and $M': -\infty < z \le 0$.

The equations of motion for the half-spaces M and M' are

$$\begin{gathered} c_{44}\varphi_{,xxxx} + 2\beta\,\varphi_{,xxzz} + \beta_{13}T_{,xz} = \rho\left(\ddot{\varphi}_{,xx} + \ddot{\varphi}_{,zz}\right), \\ K_1 T_{,xx} + K_3\, T_{,zz} - \rho C_e(\dot{T} + \tau_0\,\ddot{T}) = T_0(\beta_1 - \beta_3)\left(\dot{\varphi}_{,xz} + \tau_0\,\ddot{\varphi}_{,xz}\right), \end{gathered} \tag{7}$$

$$\begin{gathered} c'_{44}\varphi'_{,xxxx} + 2\beta'\,\varphi'_{,xxzz} + \beta'_{13}T'_{xz} = \rho\left(\ddot{\varphi}'_{,xx} + \ddot{\varphi}'_{,zz}\right), \\ K'_1 T'_{,xx} + K'_3\, T'_{,zz} - \rho' C'_e\left(\dot{T} + \tau'_0\,\ddot{T}'\right) = T'_0\left(\beta'_1 - \beta'_3\right)\left(\dot{\varphi}'_{,xz} + \tau_0\,\ddot{\varphi}'_{,xz}\right), \end{gathered} \tag{8}$$

where $\beta = (c_{11}+c_{33})/2 - c_{13} - c_{44}$, $\beta' = (c'_{11}+c'_{33})/2 - c'_{13} - c'_{44}$, $\beta_{13} = \beta_3 - \beta_1$, $\beta'_{13} = \beta'_3 - \beta'_1$.

When a quasi-shear wave propagating in the half-space M be incident at the plane interface, $z = 0$ making an angle θ_0 with the normal, two quasi shear waves are reflected and transmitted in M and M' respectively. The structures of the wave field for the incident, reflected and transmitted waves may be written as

$$\langle \varphi^{(n)}, T^{(n)} \rangle = \langle A_n, a_n A_n \rangle e^{\{ik_n\{xp_1^{(n)} + zp_3^{(n)} - c_n t\}}, \quad n = 0,1,2,3,4 \tag{9}$$

where A_n is the amplitude constant, $\langle p_1^{(n)}, 0, p_3^{(n)} \rangle$ is the unit propagation vector, k_n is the wavenumber and c_n is the phase velocity. Note that $n = 0$ represents incident quasi shear wave, $n = 1, 2$ and $n = 3, 4$ represent for the reflected and transmitted quasi-shear waves respectively. The coupling constant a_n is given by

$$a_n = \begin{cases} \dfrac{k_n^2\{c_{44}\,(p_1^{(n)^4} + p_3^{(n)^4}) + 2\beta\, p_1^{(n)^2}\, p_3^{(n)^2} - \rho\, c_n^2\}}{\beta_{13}\, p_1^{(n)} p_3^{(n)}}, & n = 0, 1, 2 \\ \dfrac{k_n^2\{c'_{44}(p_1^{(n)^4} + p_3^{(n)^4}) + 2\beta' p_1^{(n)^2}\, p_3^{(n)^2} - \rho' c_n^2\}}{\beta'_{13} p_1^{(n)} p_3^{(n)}}, & n = 3, 4. \end{cases}$$

The Snell's law, in this case, is given as (Singh, 2011)

$$\frac{k_0}{k_n} = \frac{\sin\theta_n}{\sin\theta_0} \qquad \text{for} \quad n = 1, 2, 3, 4. \tag{10}$$

4. Boundary Conditions

The tractions and displacement components are continuous at $z = 0$. These conditions may be written as

(i) Continuity of normal traction:

$$\sum_{n=0}^{2}\{c_{44}\ \varphi_{,zzz}^{(n)} + c_{22}\varphi_{,xxz}^{(n)} - \rho\ddot{\varphi}_{,z}^{(n)} + \beta_{13}\, T_{,x}^{(n)}\} = \sum_{n=3}^{4}\{c'_{44}\, \varphi_{,zzz}^{(n)} + c'_{22}\varphi_{,xxz}^{(n)} - \rho'\, \ddot{\varphi}_{,z}^{(n)} + \beta'_{13}\, T_{,x}^{(n)}\} \tag{11}$$

where $c_{22} = c_{11} + c_{33} - c_{44} - 2c_{13},\ \ c'_{22} = c'_{11} + c'_{33} - c'_{44} - 2c'_{13}$.

(ii) Continuity of shear traction:

$$\sum_{n=0}^{2}\left\{c_{44}\ \varphi_{,zz}^{(n)} + \varphi_{,xx}^{(n)}\right\} = \sum_{n=3}^{4}\left\{c'_{44}\, \varphi_{,zz}^{(n)} + \varphi_{,xx}^{(n)}\right\} \tag{12}$$

(iii) Continuity of displacement components:

$$\sum_{n=0}^{2} \varphi_{,z}^{(n)} = \sum_{n=3}^{4} \varphi_{,z}^{(n)}, \quad \sum_{n=0}^{2} \varphi_{,x}^{(n)} = \sum_{n=3}^{4} \varphi_{,x}^{(n)} \tag{13}$$

Using Eqs. (9) and (10) into (11)-(13), these boundary conditions may be reduced to

$$\begin{aligned} &\sum_{n=0}^{2}\{c_{44}\ k_n^3\, p_3^{(n)^3} + c_{22}k_n^3\, p_1^{(n)^2}\ p_3^{(n)} - \rho c_n^2\, k_n^3\, p_3^{(n)} + \beta_{13}\, a_n\, p_1^{(n)}\, k_n\}A\_n - \\ &\sum_{n=3}^{4}\left\{c'_{44}\, k_n^3\, p_3^{(n)^3} + c'_{22}k_n^3\, p_1^{(n)^2} p_3^{(n)} - \rho' c_n^2\, k_n^3\, p_3^{(n)} + \beta'_{13}\, a_n p_1^{(n)} k_n\right\}A_n = 0 \end{aligned} \tag{14}$$

$$\sum_{n=0}^{2} c_{44}\, \mathrm{k_n^2}\, \left(\mathrm{p_3^{(n)^2}} - \mathrm{p_1^{(n)^2}}\right)\mathrm{A_n} - \sum_{n=3}^{4} c'_{44}\, \mathrm{k_n^2}\, \left(\mathrm{p_3^{(n)^2}} - \mathrm{p_1^{(n)^2}}\right)\mathrm{A_n} = 0 \tag{15}$$

$$\sum_{n=0}^{2} \mathrm{k_n p_3^{(\mathit{n})} A_n} - \sum_{n=3}^{4} \mathrm{k_n p_3^{(\mathit{n})} A_n} = 0, \qquad \sum_{n=0}^{2} \mathrm{k_n p_1^{(\mathit{n})} A_n} - \sum_{n=3}^{4} \mathrm{k_n p_1^{(\mathit{n})} A_n} = 0 \tag{16}$$

Equations (14)-(16) will be used for evaluation of the amplitude ratios corresponding to the reflected and transmitted waves.

5. Amplitude Ratios

Equations (14)-(16) may be rewritten in matrix notation as

$$AZ = B, \tag{17}$$

where A is a matrix of order 4×4 and $B,\ Z$ are matrices of orders 4×1 with the following entries

$$a_{1j} = \begin{cases} c_{44}k_j^3\, p_3^{(j)^3} + c_{22}k_j^3\, p_1^{(j)^2}\, p_3^{(j)} - \rho\, c_j^2\, k_j^3\, p_3^{(j)} - \beta_{13}a_j\, p_1^{(j)}\, k_j, & j = 1, 2, \\ -\{c'_{44}\, k_j^3\, p_3^{j^3} + c'_{22}k_j^3\, p_1^{(j)^2} p_3^{(j)} - \rho' c_j^2\, k_j^3\, p_3^{(j)} + \beta'_{13}\, a_j p_1^{(j)} k_j\}, & j = 3, 4, \end{cases},$$

$$a_{\{2j\}} = \begin{cases} c_{44}\, \mathrm{k_j^2}\, \left(\mathrm{p_3^{(j)^2}} - \mathrm{p_1^{(j)^2}}\right), & j = 1,2, \\ -c'_{44}\, \mathrm{k_n^2}\, \left(\mathrm{p_3^{(j)^2}} - \mathrm{p_1^{(j)^2}}\right), & j = 3,4, \end{cases}, \quad a_{3j} = \begin{cases} \mathrm{k_j p_3^{(\mathit{j})}}, & j = 1,2, \\ -\mathrm{k_j p_3^{(\mathit{j})}}, & j = 3,4, \end{cases}, \quad a_{4j} = \begin{cases} \mathrm{k_j p_1^{(\mathit{j})}}, & j = 1,2, \\ -\mathrm{k_j p_1^{(\mathit{j})}}, & j = 3,4, \end{cases},$$

$$b_{11} = -\{c_{44}k_0^3\, p_3^{(0)^3} + c_{22}k_0^3\, p_1^{(0)^2}\, p_3^{(0)} - \rho\, c_0^2\, k_0^3\, p_3^{(0)} - \beta_{13}a_0\, p_1^{(0)}\, k_0\},$$

$$b_{21} = -c_{44}\, \mathrm{k_0^2}\, \left(\mathrm{p_3^{(0)^2}} - \mathrm{p_1^{(0)^2}}\right), \quad b_{31} = -\mathrm{k_0 p_3^{(0)}}, \quad b_{41} = -\mathrm{k_0 p_1^{(0)}}$$

Eq. (17) is solved for $Z_j = \frac{A_j}{A_0}$ due to incident quasi shear wave. The amplitude ratio Z_j for $j = 1, 2$ represent for the reflected quasi-shear waves and for $j = 3, 4$ represent for the transmitted quasi shear waves.

6. Particular Cases

Case I: If we neglect the effect of thermal, the problem becomes reflection/transmission of plane waves at the interface of two dissimilar half-spaces of incompressible transversely isotropic materials. The amplitude ratios of the reflected and transmitted shear waves, in this case, are given by Eq. (17) with the following modified values

$$a_{1j} = \begin{cases} c_{44}k_j^3\, p_3^{(j)^3} + c_{22}k_j^3\, p_1^{(j)^2}\, p_3^{(j)} - \rho\, c_j^2\, k_j^3\, p_3^{(j)}, & j = 1,2, \\ -\{c_{44}^{'}\, k_j^3\, p_3^{j^3} + c_{22}^{'}k_j^3\, p_1^{(j)2} p_3^{(j)} - \rho^{'} c_j^2\, k_j^3\, p_3^{(j)}\}, & j = 3,4, \end{cases}$$

$$b_{11} = -\{c_{44}k_0^3\, p_3^{(0)^3} + (c_{11} + c_{33} - c_{44} - 2c_{13})k_0^3\, p_1^{(0)^2}\, p_3^{(0)} - \rho\, c_0^2\, k_0^3\, p_3^{(0)}\}.$$

Case II: If the half-space M' is neglected, then the problem reduces to reflection of plane waves in an incompressible transversely isotropic thermoelastic materials. The amplitude ratios are given by Eq. (17) with the modification that A is a matrix of order 2×2, B and Z are column matrices with the following entries

$$a_{1j} = c_{44}k_j^3\, p_3^{(j)^3} + c_{22}k_j^3\, p_1^{(j)^2}\, p_3^{(j)} - \rho\, c_j^2\, k_j^3\, p_3^{(j)} - \beta_{13}a_j\, p_1^{(j)}\, k_j, \qquad j = 1,2,$$

$$a_{2j} = c_{44}\, \mathrm{k}_\mathrm{j}^2\, \left(\mathrm{p}_3^{(\mathrm{j})^2} - \mathrm{p}_1^{(\mathrm{j})^2}\right), \qquad j = 1,2, \quad b_{21} = -c_{44}\, \mathrm{k}_0^2\, \left(\mathrm{p}_3^{(0)^2} - \mathrm{p}_1^{(0)^2}\right),$$

$$b_{11} = -\{c_{44}k_0^3\, p_3^{(0)^3} + c_{22}k_0^3\, p_1^{(0)^2}\, p_3^{(0)} - \rho\, c_0^2\, k_0^3\, p_3^{(0)} - \beta_{13}a_0\, p_1^{(0)}\, k_0\}.$$

The amplitude ratios of the reflected waves depend on the angle of propagation, elastic and thermal parameters of the material.

Case III: If we neglect the effect of thermal and the half-space M', the problem reduces to reflection of plane waves at the half-space of incompressible transversely isotropic material. In this case, the amplitude ratios of the reflected waves are given as in Case II with the following modified values

$$a_{1j} = c_{44}k_j^3\, p_3^{(j)^3} + c_{22}k_j^3\, p_1^{(j)^2}\, p_3^{(j)} - \rho\, c_j^2\, k_j^3\, p_3^{(j)}, \qquad j = 1,2,$$

$$b_{11} = -\{c_{44}k_0^3\, p_3^{(0)^3} + c_{22}k_0^3\, p_1^{(0)^2}\, p_3^{(0)} - \rho\, c_0^2\, k_0^3\, p_3^{(0)}\}.$$

7. Numerical Results

We have computed the amplitude ratios of reflected and transmitted shear waves due to incident quasi shear waves. The relevant value of the parameters are given in Table 1 (Chadwick and Seet, 1970).

Table 1: Value of the elastic and thermal parameters

Cobalt (M)	Value	Zinc (M')	Value	Units
ρ	8.836×10^3	P'	7.14×10^3	Kgm^{-3}
c_{11}	3.071×10^{11}	C_{11}'	1.628×10^{11}	Nm^{-2}
c_{12}	1.650×10^{11}	C_{12}'	0.362×10^{11}	Nm^{-2}
c_{13}	1.027×10^{11}	C_{13}'	0.508×10^{11}	Nm^{-2}
c_{33}	3.581×10^{11}	C_{33}'	0.627×10^{11}	Nm^{-2}
c_{44}	0.755×10^{11}	C_{44}'	0.385×10^{11}	Nm^{-2}
β_1	7.04×10^6	B_1'	5.75×10^6	$Nm^{-2}\ degree^{-1}$
β_3	6.90×10^6	B_3'	5.17×10^6	$Nm^{-2}\ degree^{-1}$
C_e	4.27×10^2	C_e'	3.9×10^2	$Jkg^{-1}degree^{-1}$
K_1	0.690×10^2	K_1'	1.24×10^2	$Wm^{-1}degree^{-1}$
K_3	0.690×10^2	K_3'	1.24×10^2	$Wm^{-1}degree^{-1}$
T_0	298	T_0'	296	K
τ_0	0.05	τ_0'	0.06	

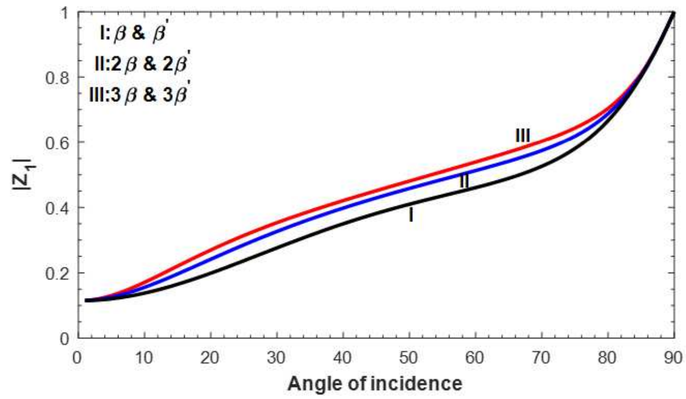

Figure 1: Variation of $|Z_1|$ with angle of incidence for different values of β and β'.

It may be noted that $(p_1^{(0)}, 0, p_3^{(0)}) = (\sin\theta_0, 0, \cos\theta_0)$ for incident quasi shear wave, $(p_1^{(1)}, 0, p_3^{(1)}) = (\sin\theta_1, 0, -\cos\theta_1)$, $(p_1^{(2)}, 0, p_3^{(2)}) = (\sin\theta_2, 0, -\cos\theta_2)$ for reflected quasi-shear wave and $(p_1^{(3)}, 0, p_3^{(3)}) = (\sin\theta_3, 0, \cos\theta_3)$, $(p_1^{(4)}, 0, p_3^{(4)}) = (\sin\theta_4, 0, \cos\theta_4)$ for transmitted quasi shear waves.

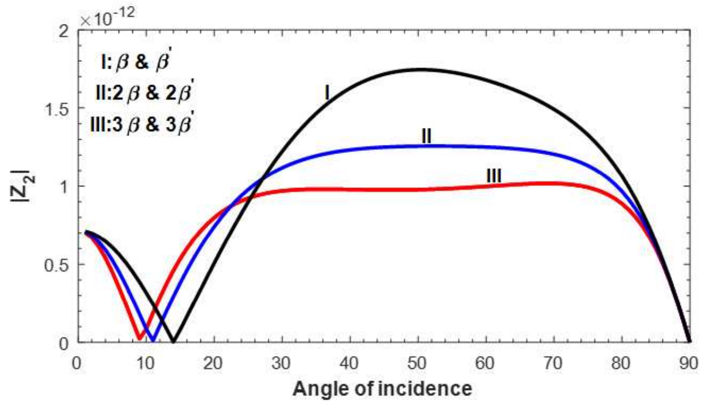

Figure 2: Variation of $|Z_2|$ with angle of incidence for different values of β and β'.

The variation of amplitude ratios with the angle of incidence, θ_0 at different values of (β, β') are depicted through **Figures 1-4**, while **Figures 5** and **6** show the variation of amplitude ratios at different values of (C_e, C_e'). The values of the amplitude ratios $|Z_1|$ and $|Z_3|$ in **Figures 1** and **3** of the reflected and transmitted shear waves increases and decreases respectively with the increase of θ_0. We have observed that the effects of (β, β') on $|Z_1|$ and $|Z_3|$ have minimum near the normal and grazing angle of incidence.

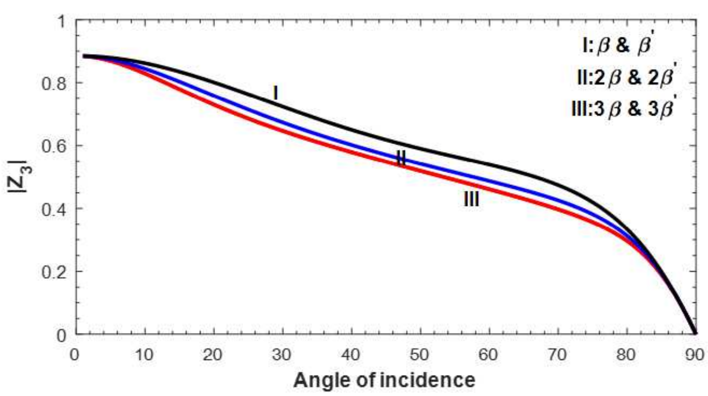

Figure 3: Variation of $|Z_3|$ with angle of incidence for different values of β and β'.

Figures 2 and **4** show that the amplitude ratios $|Z_2|$ and $|Z_4|$ have similar fashion. They started from certain values which decrease with the increase of θ_0 and increase thereafter to the maximum value followed by decreasing with the increase of θ_0. Here also the minimum effect of (β, β') on $|Z_2|$ and $|Z_4|$ is observed near normal and grazing angle of incidence.

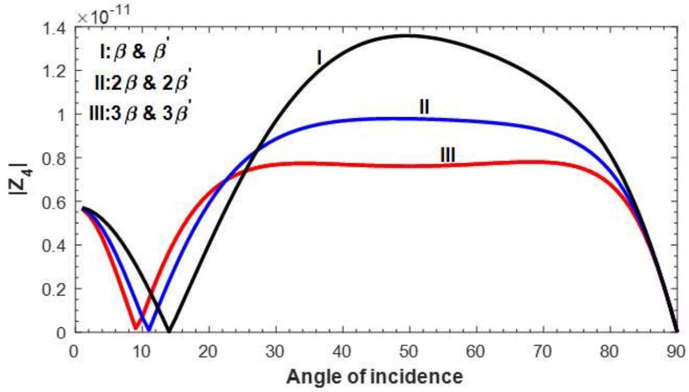

Figure 4: Variation of $|Z_4|$ with angle of incidence for different values of β and β'.

The effect of specific heats on the amplitude ratios $|Z_2|$ and $|Z_4|$ in **Figures 5** and **6** have similar pattern. They have minimum effect of (C_e, C_e') near $\theta_0 = 14°$ and grazing angle of incidence. It is also observed that the values of $|Z_2|$ and $|Z_4|$ decrease with the increase of specific heats. We also noticed very few effects of specific heats on $|Z_1|$ and $|Z_3|$. Thus, the amplitude ratios of the reflected and transmitted shear waves are found to be functions of angle of incidence, elastic and thermal parameters.

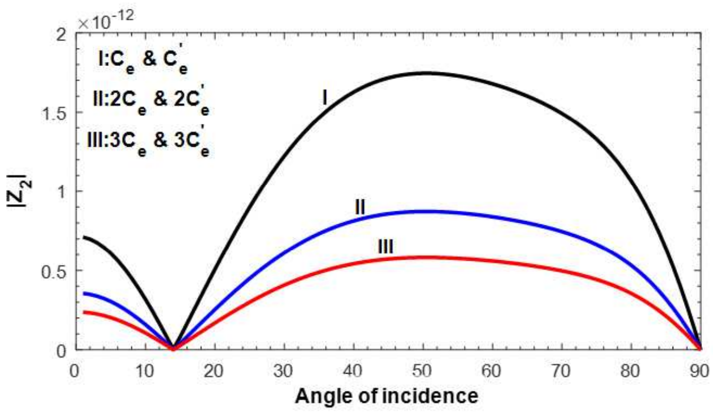

Figure 5: Variation of $|Z_2|$ with angle of incidence for different values of C_e and C_e'.

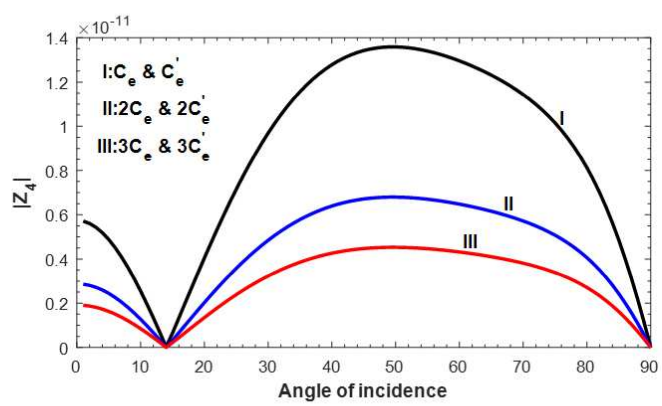

Figure 6: Variation of $|Z_4|$ with angle of incidence for different values of C_e and C_e'.

8. Conclusion

The problem of incident quasi-shear wave at a plane interface between two dissimilar half-spaces of incompressible transversely isotropic thermoelastic materials has been investigated. Due to the incompressibility constraint, we have observed that two coupled quasi-shear waves are propagating in the material. Using the property of continuities of tractions and displacement at the plane interface, the amplitude ratios of the reflected and transmitted shear waves are analytically and numerically obtained to analyze the effect of specific heats and coefficient of linear thermal expansion. We summarize the concluding remarks as

(i) The amplitude ratios of the reflected and transmitted waves are found to be functions of angle of incidence, elastic and thermal parameters of the materials.
(ii) The value of the amplitude ratios $|Z_1|$ and $|Z_3|$ increases and decreases respectively with the increase of θ_0.
(iii) The effect of (β, β') on the amplitude ratios is minimum near the normal and grazing angle of incidence.
(iv) The effects of (C_e, C_e') on $|Z_2|$ and $|Z_4|$ are minimum near $\theta_0 = 14°$ and grazing angle of incidence.
(v) The values of $|Z_2|$ and $|Z_4|$ decrease with the increase of (C_e, C_e').
(vi) The effect of (C_e, C_e') on $|Z_1|$ and $|Z_3|$ is found to be negligible as compared to those on $|Z_2|$ and $|Z_4|$.

The problem of the effect of corrugation on the reflection/refraction of elastic waves between the two dissimilar incompressible transversely isotropic thermoelastic half-spaces can be extended from the present work. One may also work on the surface wave propagation in the incompressible transversely isotropic thermoelastic half-space.

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Biographical notes

T. Lalawmpuia and S.S. Singh are currently in the Department of Mathematics and Computer Science, Mizoram University, Aizawl -796004, Mizoram, India

C. Zorammuana is currently in the Department of Education, Assam University, Silchar - 788 011, Assam, India