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Measuring capability of a binomial process

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Abstract

Many product characteristics are qualitative in nature, e.g. colour, brightness, surface finish etc. The manufacturing process of such products is usually described in terms of fraction nonconforming or conforming which is assumed to follow binomial distribution. Measuring capability of a binomial process implies assessing to what extent the fraction nonconforming or conforming in the continuous stream of lots conform to the specification limits. The C_p or C_{pl} of a binomial process can be estimated using several approaches. However, these approaches generally give widely varying assessment about the capability of a given binomial process. Consequently, a user of the index may inadvertently be led to erroneous decision making based on an inaccurate estimate of the index. In this paper, a procedure is proposed for assessing accuracies of estimates of C_{pu} or C_{pl} obtained by different methods. Subsequently, the best method for evaluating capability of a binomial process is identified based on analysis of multiple case studies, and also the methods giving inaccurate estimates are highlighted.

Keywords: Process capability index, binomial process, fraction nonconforming, nonconforming lot (NL), predicted NL%, prediction error

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1. Introduction

Process capability refers to the ability of a given process to produce outputs according to specified requirements. The basic process capability index is $C_p = (USL - LSL)/6\sigma$, where USL and LSL are the upper and lower specification limits and σ is the population standard deviation. When there is only USL or only LSL for a product characteristic, then the process capability indices are defined as $C_{pu} = (USL - \mu)/3\sigma$ and $C_{pl} = (\mu - LSL)/3\sigma$ respectively, where μ is the population mean. The most widely used other indices are C_{pk} (Kane, 1986), C_{pm} (Hsing and Taguchi, 1985; Chen *et al.*, 2008) and C_{pmk} (Choi and Owen, 1990; Pearn *et al.*, 2005). More detailed information on these indices are available in Kotz and Johnson (1993), English and Taylor (1993), Kotz and Lovelace (1998), Kotz and Johnson (2002), Wu *et al.* (2009), Yum and Kim (2011), Chen *et al.* (2017), Polhemus (2017) and De-Felipe and Benedito (2017). Historically, all these indices are developed for a product characteristic that can be described as a continuous variable and follows normal distribution. The generalization of these indices for continuous non-normal variables are suggested by Clements (1989), Pearn and Kotz (1994), Pearn and Chen (1995), Shore (1998), Chen (2000), Goswami and Dutta (2013), Kovarik and Sarga (2014), Li *et al*. (2015) and Chen *et al*. (2019).

However, in reality, it is observed that many product characteristics are qualitative in nature, e.g. colour, brightness, surface finish etc. The manufacturing process of such products are usually described through a discrete-valued characteristic e.g. fraction defective, fraction nonconforming, fraction conforming etc. The measurement of this kind of characteristic is typically obtained by counting number of defective or nonconforming units (d) within a given number of sample units (n). It is generally assumed that the fraction (proportion) data, $f = d/n$, follows binomial distribution with parameter p (population proportion) and n. It may further be noted that binomial data ideally have one-sided specification limit only. For example, the ideal target value for fraction defective/nonconforming is zero and the value of fraction defective/nonconforming is desired to be less than a specified USL (say,

f_U). Again, the ideal target value for fraction conforming is one and the value of fraction conforming is desired to be more than a specified *LSL* (say, f_L). The main point of concern for a binomial process is the quality of the lots (say, number of items produced in every *T* hours span) that are formed for shipment. Based on the information contained in a sample taken from a lot, it is possible to estimate the expected fraction nonconforming with upper confidence limit or fraction conforming with lower confidence limit in the lot. Accordingly, it can be assessed if fraction nonconforming or fraction conforming in a lot satisfies the specified *USL* or *LSL*. But such analysis, does say nothing about the continuous stream of lots that are produced over long period.

However, the design engineers, process managers, vendors or customers are often interested to know if the process is capable of producing continuous stream of lots having fraction nonconforming or fraction conforming less than f_U or greater than f_L respectively. These queries can be answered satisfactorily by evaluating the appropriate process capability index of the concerned binomial process. When the fraction of interest is smaller-the-better (STB) type, e.g. fraction defective/nonconforming, the capability of the binomial process can be evaluated using C_{pu} index, and when the fraction of interest is larger-the-better (LTB) type, e.g. fraction conforming, the capability of the binomial process can be evaluated using C_{pl} index.

Conventionally, the count data (or fraction data) is assumed to follow a binomial distribution which is often approximated by a standard normal distribution and therefore, process capability index C_{pu} or C_{pl} is computed by using the standard formula for normal data. The normal approximation works well only when the sample size n is large and population proportion (p) is such that both np and $n(1-p)$ are greater than 5. Alternatively, Clement's (1989) percentile based approach (which is developed for continuous non-normal process) may be applied to binomial process (a discrete non-normal process) for obtaining an approximate estimate of C_{pu} or C_{pl}. An estimate of C_{pu} or C_{pl} of a binomial process obtained by percentile based approach will be approximate only because the values of percentile points used in these computations are approximated. Thus, both these approaches can measure C_{pu} or C_{pl} approximately only. Maravelakis (2016) developed a method for measuring process capability of a binomial process. In this method, binomial data are first converted into normally distributed data by using a two-step transformation technique and then, capability of the binomial process is assessed by directly applying the standard formula for C_{pu} or C_{pl} on the transformed data.

Borges and Ho (2001) suggested a new measure of process capability, called *C*-index, which has one-to-one correspondence (mapping) between the proportion of nonconforming items and Z-value of the standard normal distribution. This implies that the process capability will respond to changes in the nonconforming region and not to changes in the distribution of the observed quality characteristic. Thus, the basic process capability index (C_p) for any process can be expressed in terms of *C*-index. In case of unilateral specification, the C_{pu} or C_{pl} of a process can be measured in terms of C_u or C_l respectively.

In the recent past, researchers have proposed some generalized indices for assessment of process capability, which can be used as alternative to C_p. These indices are defined as the ratio of two probabilities instead of ratio of the specification width and actual process width. Thus, these indices can be computed irrespective of distribution of the quality characteristics (normal or non-normal) and data type (continuous or discrete). Yeh and Bhattacharya (1998) proposed C_f index, Perakis and Xekalaki (2002, 2005) presented C_{pc} index and Maiti *et al.* (2010) suggested C_{py} index for assessment of process capability. In case of unilateral specification, the equivalent indices for C_{pu} can be obtained as C_{fu}, C_{pcu} and C_{pyu}, and the equivalent indices for C_{pl} can be obtained as C_{fl}, C_{pcl} and C_{pyl}. Therefore, C_{pu} or C_{pl} index for a binomial process can be measured in terms of these generalized indices.

It has been observed that application of the above discussed methods to a single set of binomial data results in widely varying values for C_{pu} or C_{pl}. This implies that accuracies of the estimates of C_{pu} or C_{pl} obtained by different methods vary widely. No study is reported in literature that attempt to evaluate the accuracies of the estimated $\hat{C}_{pu}$ or $\hat{C}_{pl}$ values that may be obtained by different approaches. Consequently, a user of the index may inadvertently be led to erroneous decision making based on an inaccurate estimate of the index.

In this paper, a procedure is proposed for assessing the accuracies of the estimates of C_{pu} or C_{pl} obtained by different approaches. Then the best method for evaluating capability of a binomial process is identified based on application of the proposed procedure on multiple case study data. The article is organized as follows: Different approaches for computation of C_{pu} or C_{pl} from binomial data are described in Section 2. A procedure for assessing accuracies of estimated $\hat{C}_{pu}$ or C_{pl} values obtained from binomial process using different methods is discussed in Section 3. Analysis of multiple case study data sets and related results are presented in Section 4. Important findings and the issues related to different methods are discussed in Section 5. Section 6 concludes the paper.

2. Different approaches for computation of C_{pu} or C_{pl} from binomial data

Suppose, an item has one or more quality characteristics that are examined by the inspector. If the item does not conform to standard on one or more of these characteristics, it is classified as nonconforming and if the item conforms to standard on all of these characteristics, it is classified as conforming. Let the production process is operating in a stable manner, such that the probability that any unit will be nonconforming (conforming) to specification is p and successive units produced are independent. Suppose a random sample of n units of product is selected from the process. If the random variable D denotes the number of units of product that are nonconforming to the standard, then D has a binomial distribution with parameters n and p, i.e.

$$P\{D=d\} = \binom{n}{d} p^{d}(1-p)^{n-d}\ ;\ d = 0,1,2,\ldots,n \tag{1}$$

Then, the probability distribution of sample fraction nonconforming, $f=d/n$ is also binomial. The cumulative distribution function of f can be obtained by using the binomial distribution as

$$P\{f \le a\} = \mathrm{P}\{d/n \le a\} = \mathrm{P}\{d \le na\} = \sum_{d=0}^{[na]} \binom{n}{d} p^{d}(1-p^{n-d}) \tag{2}$$

where $[na]$ denotes the largest integer less than equal to na. It can be shown that $E(f) = p$ and $E(\sigma_f^2) = p(1-p)/n$ (Montgomery, 2009). Generally, the unknown population proportion (p) is estimated by using the sample fraction of nonconforming items (f).

If the sample fraction (f) is STB type, then it will have only USL (say, $USL = f_1U$), and one will need to estimate C_{pu} as a measure of process capability. On the other hand, if the sample fraction (f) is LTB type, then it will have only LSL (say, $LSL = f_L$) and one will need to estimate C_{pl} as a measure of process capability. For convenience, let us assume that the fraction of interest is the fraction nonconforming which is STB type. Therefore, the task is to estimate the values of C_{pu} from m number of computed sample fractions $f_i = d_i/n_i$ (i = 1,2,3,…,m), where d_i is the observed number of nonconforming items in a sample of size n_i (i = 1,2,3,…,m). Obviously, average sample size is $\bar{n} = \sum_{i=1}^{m} n_i / m$ and average sample fraction nonconforming ($\bar{f}$), given by Equation (3), is the best estimate of the unknown binomial parameter p.

$$\hat{p} = \bar{f} = (\sum_{i=1}^{m} d_i)/(\sum_{i=1}^{m} n_i) \tag{3}$$

It must be noted that $\bar{f}$ is only an overall estimate of the population fraction nonconforming p. The values of sample fraction nonconforming $(f_1 i)(i=1,2,3,\ldots)$ that may be measured from the continuous stream of lots will vary following binomial distribution, and the sample fraction nonconforming (f) in all the lots may not be less than f_U. The expected proportion of nonconforming fractions (PNF_U) or proportion of nonconforming lots (PNL_U) with respect to USL can be estimated as

$$PNF_U = PNL_U = P\{f > f_U\} = 1 - P\{D \le \bar{n} f_U\} = 1 - \sum_{d=0}^{[\bar{n} f_U]} \binom{\bar{n}}{d} \bar{f}^{d}(1-\bar{f})^{\bar{n}-d} \tag{4}$$

where $[\bar{n} f_U]$ denotes the largest integer less than equal to $\bar{n} f_U$. The percentage of nonconforming lots can be obtained by multiplying PNL_U by 100.

If the fraction of interest is LTB type, the parameters $\bar{n}$ and $\bar{f}$ can be estimated from the sample data in the same manner. The expected proportion of nonconforming fractions (PNF_L) or proportion of nonconforming lots (PNL_L) with respect to LSL can be estimated by using Equation (5) and the percentage of nonconforming lots can be obtained by multiplying PNL_L by 100.

$$PNF_L = PNL_L = P\{f > f_L\} = 1 - P\{D \le \bar{n} f_L\} = 1 - \sum_{d=0}^{[\bar{n} f_L]} \binom{\bar{n}}{d} \bar{f}^{d}(1-\bar{f})^{\bar{n}-d} \tag{5}$$

The process capability analysis of a binomial process essentially implies assessing if the process is capable of producing continuous stream of lots having fraction defective/nonconforming less than specified f_U or fraction conforming greater than specified f_L. There are various approaches that can be used for estimating C_{pu} or C_{pl} for binomial process are described in the following sub-sections.

2.1 Normal approximation approach

If sample size n is large and estimate of population parameter $\hat{p}$ (= $\bar{f}$) is such that both $n\bar{f}$ and $n(1-\bar{f})$ are greater than 5, the distribution of f may be approximated by normal distribution with mean $\bar{f}$ and variance $\bar{f}(1-\bar{f})/\bar{n}$ (Montgomery 2009). Thus, the approximate estimates of C_{pu} and C_{pl} can be obtained from the observed sample proportions as follows:

$$\hat{C}_{pu} = (f_U - \overline{f})/(3\text{x}\sqrt{(\overline{f}(1-\overline{f})/\overline{n}}) \quad (6)$$

$$\hat{C}_{pl} = (\overline{f} - f_L)/(3\text{x}\sqrt{(\overline{f}(1-\overline{f})/\overline{n}}) \quad (7)$$

where f_U and f_L are the specified upper and lower limit for STB and LTB type of fraction, respectively.

2.2 Percentile based approach

The percentile based approach (Clements 1989) is developed for estimating C_p, C_{pu} and C_{pl} of a continuous non-normal process. However, the same concept can be used for approximating the estimates of C_{pu} and C_{pl} of a discrete non-normal process. Thus, the approximate estimate of C_{pu} and C_{pl} can be obtained as follows:

$$\hat{C}_{pu} = (\overline{n}\, f_U - M)/(D_{0.99865} - M) \quad (8)$$

$$\hat{C}_{pl} = (\overline{n}\, f_L - M)/(D_{0.00135} - M) \quad (9)$$

Where M is the median (50th percentile point), $D_{0.99865}$ is the 99.865th percentile point and $D_{0.00135}$ is the 0.135th percentile point in the binomial distribution with parameters $\overline{n}$ and $\overline{f}$. It may be noted that M, $D_{0.99865}$ and $D_{0.00135}$ must be integers. So it may not be possible to get exact 50th, 99.865th or 0.135th percentile points.

2.3 Transformation approach

Maravelakis (2016) has proposed a transformation technique by which binomial data, i.e. the sample observations (n_i, d_i)(i=1,2,3,…,m) can be transformed into Q_i (i =2,3,…,m) values. Using the same transformation technique, f_U (or f_L) can also be transformed into Q value. Suppose the transformed f_U (or f_L) is denoted as Q_U (or Q_L). Quesenberry (1991) has shown that if the probability of success p be constant, then Q_i (i =2,3,…,m) are approximately independently and normally distributed. Therefore, C_{pu} or C_{pl} of the original process can be evaluated from the Q_i (i =2,3,…,m) values as follows:

$$\hat{C}_{pu} = (Q_U - \overline{Q})/(3 \text{ x } SD_Q) \quad (10)$$

$$\hat{C}_{pl} = (\overline{Q} - Q_L)/(3 \text{ x } SD_Q) \quad (11)$$

where $\overline{Q}$ and SD_Q are average and standard deviation of the Q_i (i =2,3,…,m) values. Maravelakis (2016) has proposed two different techniques for transformation of sample fractions into Q values for the following two cases.

Case 1: Transformation of sample fractions when P is known

Let probability of success $p = p_0$ (known), and d_i denotes the number of nonconforming items observed in the i^{th} sample of size n_i. At first, the sample observations (n_i,d_i)(i = 1,2,3,…,m) are transformed into cumulative binomial values (u_i) using the binomial cumulative distribution function, as shown in Equation (12). Then, the cumulative binomial values are retransformed into Q_i (i=1,2,3,…,m) values by the inverse of the standard normal distribution, as shown in Equation (13).

$$u_i = F_B(x_i;n_i,p_0) \text{ for } i = 1,2,3,\ldots,m \quad (12)$$

$$Q_i = \phi^{-1}(u_i) \text{ for } i = 1,2,\ldots,m \quad (13)$$

Case 2: Transformation of sample fractions when p is unknown

At first, the sample observations (n_i, d_i)(i=1,2,3,…,m) is transformed into cumulative hypergeometric value (u_i) using the hypergeometric cumulative distribution function as follows:

$$u_i = F_H(d_i;t_i,n_i,N_i);\ i=1,2,3,\ldots,m \quad (14)$$

where, N_i be the sum of all the sample sizes up to sample i, i.e. $N_i = \sum_{j=1}^{i} n_j$ and t_i is the sum of all the nonconforming items in all the samples up to sample i, i.e. $t_i = \sum_{j=1}^{i} d_j$ and m is the number of samples. Then, the cumulative hypergeometric values are retransformed into Q_i (i =2,3,…,m) values by the inverse of the standard normal distribution as shown in Equation (13).

2.4 Mapping based approach

Using Borges and Ho's (2001) mapping based approach, C_{pu} or C_{pl} can be estimated from the observed sample fractions f_i (i=1,2,3,…,m) as follows:

1) Calculate first the values of PNF_U (or PNF_L) for calculation of C_{pu} (or C_{pl}) using Equation (4) (or Equation (5)).

2) Determine the corresponding Z-value of the standard normal distribution that result in probability area equal to PNF_U on the upper tail (or probability area equal to PNF_L on lower tail). Let the Z-value corresponding to PNF_U (or PNF_L) is Z_U (or Z_L). The Z_U (or Z_L) value can be obtained by using inverse cumulative probability of the standard normal distribution function as follows:

$$Z_U = \phi^{-1}(1-PNF_U) \text{ and } Z_L = \phi^{-1}(1-PNF_L)$$

where, $\phi(\bullet)$ denotes the standard normal cumulative distribution function.

3) Then the estimates of C_{pu} or C_{pl} can be obtained as follows:

$$\hat{C}_{pu} = \hat{C}_u = (1/3) \times Z_U \quad (15)$$

$$\hat{C}_{pl} = \hat{C}_l = (1/3) \times Z_L \quad (16)$$

2.5 Process nonconforming based approach

Yeh and Bhattacharya (1998) and Perakis and Xekalaki (2002, 2005) proposed indices C_f and C_{pc} respectively, and these indices measure the process capability by looking directly at the proportion of nonconforming in a process. In case of unilateral specification, the C_f index can be expressed as

$$C_{fu} = \alpha_0^U / \alpha_U \quad (17)$$

$$C_{fl} = \alpha_0^L / \alpha_L \quad (18)$$

where, in case of binomial process, α_0^U and α_0^L are the proportion of nonconforming lots having fractions beyond the specified limits f_U and f_L respectively that the manufacturer can tolerate, and α_U (=PNF_U) and α_L (=PNF_L) are the actual proportion of nonconforming lots having fraction beyond f_U and f_L respectively that can be measured using Equations (4) and (5) respectively. Following the convention for normal distribution, Yeh and Bhattacharya (1998) recommend to consider α_0^U = 0.00135= α_0^L .

For unilateral specification, Perakis and Xekalaki (2002, 2005) proposed C_{pc} index can be expressed as

$$C_{pcu} = (1-p_0^U)/(1-p_U) \quad (19)$$

$$C_{pcl} = (1-p_0^L)/(1-p_L) \quad (20)$$

where, in case of binomial process, p_0^U and p_0^L are the desired proportion of lots having fractions conforming to the specified limits f_U and f_L respectively, and p_U and p_L are actual proportion of lots having sample fractions conforming to f_U and f_L respectively. It may be noted that $1-p_U = \alpha_U$ (=PNF_U) and $1-p_L = \alpha_L$ (=PNF_L), and these can be measured using Equations (4) and (5) respectively. On the other hand, Perakis and Xekalaki (2002, 2005) recommend that 0.9973 is a good choice for the desired proportion of conforming for both sided specifications and thus, a good choice for the desired proportion of conforming for one sided specification is 0.99865. So, $1-p_0^U$ =0.00135=α_0^U and $1-p_0^L$ = 0.00135 = α_0^L . Thus, the indices defined by Yeh and Bhattacharya (1998) and Perakis and Xekalaki (2002, 2005) are essentially the same in case of unilateral specification. Therefore, C_{pu} index of a binomial process can easily be estimated in terms of C_{fu} or C_{pcu} index as $\hat{C}_{pu} = \hat{C}_{fu} = \hat{C}_{pcu}$ and C_{pl} index of a binomial process can easily be estimated in terms of C_{fl} or C_{pcl} index as $\hat{C}_{pl} = \hat{C}_{fl} = \hat{C}_{pcl}$. Here only the C_{pcu} and C_{pcl} indices are considered for further analysis.

2.6 Process yield based approach

Maity *et al.* (2010) proposed C_{py} index as a measure of process capability. For a quality characteristic with both sided specifications, the C_{py} index is defined as follows:

$$C_{py} = \frac{F(U) - F(L)}{1 - \alpha_0^U - \alpha_0^L} \tag{21}$$

where, $F(U)$ and $F(L)$ are cumulative probability distribution function of the quality characteristic at *USL* and *LSL* respectively, and α_0^U and α_0^L are the maximum allowable proportion of nonconforming at upper tail and lower tail of the distribution of the quality characteristic. Here the numerator, $F(U)$ - $F(L)$, gives the measure of the actual process yield (i.e. actual proportion of conforming) and the denominator, $(1-\alpha_0^U - \alpha_0^L)$ gives the measure of the desired process yield (i.e. desired proportion of conforming).

Maiti *et al.* (2010) suggested that in case of unilateral specification, the process target should be taken as μ_e, and the process centre should be located such that $F(\mu_e) = [F(U) + F(L)]/2 = ½ = 0.5$. Therefore, for unilateral specification, C_{py} index can be expressed as

$$C_{pyu} = \frac{F(U) - F(\mu_e)}{1 - \alpha_0^U - F(\mu_e)} = \frac{F(U) - 0.5}{0.5 - \alpha_0^U} \tag{22}$$

$$C_{pyl} = \frac{F(\mu_e) - F(L)}{F(\mu_e) - \alpha_0^L} = \frac{0.5 - F(L)}{0.5 - \alpha_0^L} \tag{23}$$

where the value of α_0^U or α_0^L is conventionally taken as 0.00135, and in case of a binomial process, the cumulative probability $F(U)$ or $F(L)$ can be computed by using Equation (5). Therefore, C_{pu} or C_{pl} index of a binomial process can easily be estimated in terms of C_{pyu} or C_{pyl} and $\hat{C}_{pl} = \hat{C}_{pyl}$.

3. Proposed procedure for assessing accuracies of the estimated $\hat{C}_{pu}$ (or $\hat{C}_{pl}$) values

The most important aspects of process capability indices are that the analysts/users can assess about the products' conformance to the specifications, process centering etc. by examining the values of these indices, and accordingly he/she can take appropriate decision. For example, if a product characteristic *X* of a product follows normal distribution and the estimated process capability index $\hat{C}_p = 1$, it implies that the process is capable to produce 99.730% conforming products with respect to two-sided specifications; if $\hat{C}_p = \hat{C}_{pk}$, it implies that the process must have been centered at the midpoint of the two-sided specifications and thus, production of 99.730% conforming products is ensured. Similarly, $\hat{C}_{pu} = 1$ implies that the process is capable to produce 99.865% conforming products with respect to the *USL* of *X* (say, USL_X). This interpretation is derived from the following relationship:

$$P(X \leq USL_X) = P\left(\frac{X - \hat{\mu}}{\hat{\sigma}} \leq \frac{USL_X - \hat{\mu}}{\hat{\sigma}}\right) = P\left(z \leq 3 \times \frac{USL_X - \hat{\mu}}{3\hat{\sigma}}\right) = P\left(z \leq 3 \times \hat{C}_{pu}\right)$$

$$= \phi\,(3 \text{ x } \hat{C}_{pu})$$

where $\hat{\mu}$ and $\hat{\sigma}$ are the estimates of mean and standard deviation of *X*, respectively. Therefore, percentage of nonconforming products (*NP*%) is predicted as

$$NP\% = 100 \text{ x } P(X \geq USL_X) = 100 \text{ x } [1 - P(X \leq USL_X)] = 100 \text{ x } [1 - \phi\,(3 \text{ x } \hat{C}_{pu})] \tag{24}$$

It may be noted that the concept of prediction of *NP%* is not applicable for a binomial process. This is because a binomial process is described through a discrete-valued characteristic and its measurement is typically obtained by counting number of nonconforming units (d) within a given number of sample units (n). For a binomial process, the main point of concern is the quality of the stream of lots produced, and the basic purpose of process capability analysis for a binomial process is to assess if the process is capable of producing stream of lots having fraction nonconforming less than f_U or fraction conforming greater than f_L. When the fraction of interest is fraction nonconforming, a lot may be called as conforming lot if the fraction nonconforming in the lot is less than f_U, otherwise it may be called as nonconforming lot. Similar to an assessment about the expected *NP%* based on the estimated $\hat{C}_{pu}$ or $\hat{C}_{pl}$ value in case of normal process, the analysts/users of the index should be able to guess about the expected percentage of nonconforming lots (*NL%*) that may be produced in a given binomial process based on the estimated $\hat{C}_{pu}$ or $\hat{C}_{pl}$ value from the concerned binomial process. Otherwise, obtaining the estimate of C_{pu} or C_{pl} from a binomial process would have little or no use.

However to the best of our knowledge no such method is reported in literature which aims to predict expected *NL%* in a binomial process based on the estimated $\hat{C}_{pu}$ or $\hat{C}_{pl}$ value. Equation (24) is also not ideally applicable for prediction of *NL%* based on the estimated $\hat{C}_{pu}$ value from a binomial process because fraction nonconforming (*f*) does not follow normal distribution but follows binomial distribution. But we believed that we can get at least an approximate idea about expected *NL%* in the continuous stream of lots produced in a binomial process by using the same concept for prediction of *NP%* based on the estimated $\hat{C}_{pu}$ value, and so we use Equation (25) for prediction of *NL%* (Approx) based on the estimated $\hat{C}_{pu}$ value. On the other hand, the *True NL%* in the binomial process can be obtained by using Equation (26). Therefore, *Prediction error* (Approx) for an estimated $\hat{C}_{pu}$ value can be obtained by Equation (27). The *Prediction error* (Approx), obtained by Equation (27), may be considered as a metric for comparison of the accuracies of the estimated $\hat{C}_{pu}$ values obtained by different methods.

$$\textit{Predicted NL\%}\ (\text{Approx}) = 100 \times [1 - \phi(3 \times \hat{C}_{pu})] \quad (25)$$

$$\text{True } NL\% = 1 - \sum_{d=0}^{[\bar{n} f_U]} \binom{\bar{n}}{d} \bar{f}^{d} (1-\bar{f})^{\bar{n}-d} \quad (26)$$

$$\textit{Prediction error}\ (\text{Approx}) = |\textit{Expected NL\%}\ (\text{Approx}) - \textit{True NL \%}| \quad (27)$$

Less is the *Prediction error* (Approx) for an estimated $\hat{C}_{pu}$ value, the accuracy of the estimate may be considered more and better is the method of estimation of the C_{pu}. The advantage of the proposed procedure for assessing accuracy of an estimated $\hat{C}_{pu}$ value is that it is simple and the disadvantage is that the procedure is based on an approximate measure of the prediction error.

4. Analysis and related results

Three data sets, published in literature, are analyzed here as three case studies for the purpose of assessing the accuracies of the estimated $\hat{C}_{pu}$ values obtained by different methods. In the three case studies, the sample data are collected from binomial distributions with population proportions about 0.02, 0.06 and 0.10. Sample sizes also varied widely in the three case studies, e.g. 500, 100, 30.

4.1. Case study 1

Hsieh and Tong (2006) carried out process capability analysis of a lead frame manufacturing process. For enhancing yield of the packaging product, the package fabrication department (customer) requires that the number of defective lead frames in their on-line quality control must be less than ten strips per 500 inspection strips, i.e. f_u = 10/500 = 0.02. They carried out the study to assess if the lead frame manufacturing process is capable to satisfy the package fabrication department's requirement. For this purpose, they collected count data on number of defective strips (*d*) per 500 strips for 30 lots produced in 30 days, i.e. n = 500 and m = 30. It is observed that the total number of defective strips $\sum_{i=1}^{m} d_i$ =295. Thus, the average sample fraction ($\bar{f}$) is estimated as

$$\bar{f} = \frac{\sum_{i=1}^{m} d_i}{n \times m} = \frac{295}{500 \times 30} = 0.01967$$

All the sample fractions f_i (i = 1,2,3, …, m) are plotted in a *p*-chart and the chart indicates that the manufacturing process is in control. So this data set is used for estimating C_{pu} using all the six approaches described in Section 2.

Estimation of C_{pu} using normal approximation approach

Since the sample size is large enough (n = 500) and $n\bar{f}$ (= 9.833) is greater than 5, the binomial process data can be approximated by normal distribution with mean $\bar{f}$ = 0.01967 and standard deviation = $\sqrt{(\bar{f}(1-\bar{f})/n}$)= 0.00621. Thus, using Equation (6), the estimate of C_{pu} is obtained as

$$\hat{C}_{pu} = (f_U - \bar{f})/(3\text{x}\sqrt{(\bar{f}(1-\bar{f})/n}) = (0.02-0.01967)/(3 \times 0.00621) = 0.0179$$

Estimation of C_{pu} using percentile based approach

The 50th percentile point (i.e. median M) and the 99.865th percentile point ($D_{0.99865}$) for the fitted binomial distribution with n = 500 and p = 0.01967 are found as 10 and 20 respectively. Thus, using Equation (8), the estimate of C_{pu} is obtained as

$$\hat{C}_{pu} = (nf_U - M)/(D_{0.99865} - M) = (10-10)/(20-10) = 0$$

Estimation of C_{pu} using transformation approach

Here it is assumed that binomial probability p is known to be equal to $\overline{f}$ = 0.01967. Therefore, transformation technique for case 1 described in section 2.3 is applied on the sample observations (500, d_i)(i=1,2,3,…,30) and Q_i(i=1,2,3,…,30) values are obtained. The USL for fraction defective, f_U = 0.02 is also transformed into Q value, which is considered as the equivalent USL for the transformed Q values and it is denoted as Q_U. The Q_U value is obtained as follows:

$$Q_U = \phi^{-1}\{F_B(nf_U]|n,p)\}$$
$$= \phi^{-1}\{F_B([0.02\text{x}500]|n=500,p=0.01967)\} = \phi^{-1}\{F_B([10]|n=500,p=0.01967)\} = \phi^{-1}\{0.604\} = 0.264$$

The $\overline{Q}$ and SD_Q are found to be 0.1841 and 0.75 respectively. Thus, the estimate of C_{pu} is obtained as

$$\hat{C}_{pu} = \frac{Q_U - \overline{Q}}{3 \times SD_Q} = \frac{0.264 - 0.1841}{3 \times 0.75} = 0.0355$$

Estimation of C_{pu} using mapping based approach

Here, n = 500, $\overline{f}$ = 0.01967, f_U = 0.02 and [nf_U] = 10. Therefore, using the procedure described in section 2.4, PNF_U and Z_U are computed first and then, the estimate of C_{pu} is obtained as follows:

$$PNF_U = 1 - \sum_{d=0}^{10} \binom{500}{d} (0.01967)^d (0.98033)^{500-d} = 0.3959$$

$$Z_U = \phi^{-1}(1 - PNF_U) = \phi^{-1}(1 - 0.3959) = \phi^{-1}(0.6041) = 0.264$$

$$\hat{C}_{pu} = \hat{C}_u = (1/3) \text{ x } Z_U = (1/3) \text{ x } 0.264 = 0.088$$

Estimation of C_{pu} using process nonconforming based approach

The value of p_0^U is not specified here and so, as per convention it is taken as 0.99865. This implies that allowable (acceptable) proportion of nonconforming lots is (1− p_0^U) = 0.00135. Here, the sample size n = 500 and the estimate of binomial parameter p is $\overline{f}$ = 0.01967. So, using Equation (4), the actual proportion of nonconforming lots, (1−p_U) = α_U is found to be 0.3959. Therefore, C_{pu} is estimated as

$$\hat{C}_{pu} = \hat{C}_{pcu} = 0.00135/0.3959 = 0.0034$$

Estimation of C_{pu} using process yield based approach

Here, the sample size n = 500 and the estimate of binomial parameter p is $\overline{f}$ = 0.01967, the USL for fraction defective is f_U = 0.02 and allowable proportion of lots having fraction defective more than f_U is 0.00135. Using Equation (5), the cumulative probability for conforming fractions or lots, $F(U)$ is computed as 0.6041. Therefore, C_{pu} is estimated as

$$\hat{C}_{pu} = \hat{C}_{pyu} = (0.6041-0.5)/(0.5-0.00135) = 0.1041/0.49865 = 0.2085$$

It may be noted that estimated $\hat{C}_{pu}$ values obtained by the six approaches varies widely. For the purpose of assessing relative accuracies of these estimates, *Predicted NL%* are obtained separately based on each estimated $\hat{C}_{pu}$ value using Equation (25). On the other hand, the *True NL%* in the current process is computed using Equation (26) and it is found to be 39.59%. Then *Prediction errors* (Approx) are obtained for all the estimated $\hat{C}_{pu}$ values using Equation (27). Table 1 shows the estimated $\hat{C}_{pu}$ values obtained by different methods, *Predicted NL%* (Approx) by these estimates and the *Prediction error* (Approx) for these estimates.

Table 1. $\hat{C}_{pu}$ values obtained by different methods and the *Prediction errors* (Approx) for these estimates

Sl. No.	Approaches for C_{pu} calculation	Estimated $\hat{C}_{pu}$ value	*Predicted NL%* (Approx)	*True NL%*	*Prediction error* (Approx)
1	Normal approximation approach	0.0179	47.86	39.59	8.27
2	Percentile based approach	0.0000	50.00		10.41
3	Transformation approach	0.0355	45.76		6.17
4	Mapping based approach	0.0880	39.59		0
5	Nonconforming based approach	0.0034	49.59		10.00
6	Yield-based approach	0.2085	26.56		13.03

Table 1 shows that the *Prediction error* (Approx) is minimum (zero) for the estimated $\hat{C}_{pu}$ value obtained by Mapping based approach, and maximum (13.03) for the estimated $\hat{C}_{pu}$ value obtained by yield-based approach. This implies that Mapping based approach give the best estimate of C_{pu} and Yield based approach results in the worst estimate of C_{pu}. The *Prediction errors* (Approx) for the estimated $\hat{C}_{pu}$ values obtained by all other approaches are also substantially high, which is indicative that accuracies for these estimates also are quite poor compared to the estimated $\hat{C}_{pu}$ value obtained by Mapping based approach.

4.2 Case study 2

Montgomery (2009) presented in exercise 7.3 (pp. 335), a set of process data on total number of personal computers inspected and total number of nonconforming personal computer observed in each day over last ten consecutive days. Then he wanted to know if the process was in control. The plotted fraction nonconforming control chart exhibited that the process was in control, and therefore, it is decided to use the same data for process capability analysis purpose.

In this data set, sample size (n_i) was variable and the average sample size ($\bar{n}$) is found to 100, and the average fraction nonconforming ($\bar{f}$) is found to be 0.06. Montgomery (2009) did not specify the *USL* for the fraction nonconforming. For the purpose of process capability analysis, here we assume that $\bar{f} + 2 \times \sqrt{\bar{f}(1-\bar{f})/\bar{n}} \approx 0.10$ is the *USL* for the fraction nonconforming, i.e. $f_U = 0.10$.

Now the $\hat{C}_{pu}$ values are computed from the same data set using all the six approaches. The *Predicted NL%* (Approx) are predicted based on these estimated $\hat{C}_{pu}$ values using Equation (25) and the *True NL*% in the current process is computed using Equation (26). The *True NL*% in the current process is found to be 3.76%. Then *Prediction errors* (Approx) for all the estimated $\hat{C}_{pu}$ values are computed using Equation (27). Table 2 shows the estimated $\hat{C}_{pu}$ values obtained by different methods, *Predicted NL*% (Approx) and the *Prediction errors* (Approx) for these estimates.

Table 2. $\hat{C}_{pu}$ values obtained by different methods and the *Prediction errors* (Approx) for these estimates

Sl. No.	Approaches for C_{pu} calculation	Estimated $\hat{C}_{pu}$ value	*Predicted NL%* (Approx)	*True NL%*	*Prediction error* (Approx)
1	Normal approximation approach	0.5614	4.61	3.76	0.85
2	Percentile based approach	0.7143	1.61		2.15
3	Transformation approach	0.6716	2.20		1.56
4	Mapping based approach	0.5931	3.76		0
5	Nonconforming based approach	0.0360	45.71		41.95
6	Yield based approach	0.9273	0.27		3.49

Table 2 shows that the *Prediction error* (Approx) is minimum (zero) for the estimated $\hat{C}_{pu}$ value obtained by Mapping based approach, and maximum (41.95) for the estimated $\hat{C}_{pu}$ value obtained by Nonconforming based approach. This implies that Mapping based approach give the best estimate of C_{pu} and Nonconforming based approach results in the worst estimate of C_{pu}. The *Prediction errors* (Approx) for the estimated $\hat{C}_{pu}$ values obtained by all other approaches are found to be reasonably low and thus may be considered as acceptable.

4.3 Case study 3

Maravelakis (2016) considered a manufacturing process for illustrating his proposed transformation technique for binomial data and subsequent computation of process capability index. Maravelakis (2016) collected a total of m = 100 samples each of size n = 30, and observed the number of nonconforming items (d) in each sample. The fraction nonconforming in these samples are calculated and plotted in p-chart. The plotted p-chart revealed that the process is in control. The *USL* for the fraction nonconforming is f_U = 0.2. The total number of nonconforming items in these samples is found to be $\sum_{i=1}^{m} d_i$ =286 and so, the average fraction nonconforming is computed as $\overline{f}$ = 286/(30 x 100) = 0.09533.

Now the $\hat{C}_{pu}$ values are calculated from the same data set using all the six approaches. The *Predicted NL*% (Approx) are computed based on these calculated $\hat{C}_{pu}$ values using Equation (25) and the *True NL*% in the current process is computed using Equation (26). The *True NL*% in the current process is found to be 2.04%. Then *Prediction errors* (Approx) for estimate of C_{pu} is computed using Equation (27). Table 3 shows the estimated $\hat{C}_{pu}$ values obtained by different methods, *Predicted NL*% (Approx) based on these estimates and the *Prediction errors* (Approx) for these estimates.

Table 3. $\hat{C}_{pu}$ values obtained by different methods and the *Prediction errors* (Approx) for these estimates

Sl. No.	Approaches for C_{pu} calculation	Estimated $\hat{C}_{pu}$ value	*Predicted NL%* (Approx)	*True NL%*	*Prediction error* (Approx)
1	Normal approximation approach	0.651	2.546	2.04	0.51
2	Percentile based approach	0.511	6.250		4.21
3	Mapping based approach	0.682	2.040		0
4	Transformation approach	0.611	3.348		1.13
5	Nonconforming based approach	0.066	42.130		40.09
6	Yield-based approach	0.962	0.195		1.85

Table 3 shows that the *Prediction error* (Approx) is minimum (zero) for the estimated $\hat{C}_{pu}$ value obtained by Mapping based approach, and maximum (40.09) for the estimated $\hat{C}_{pu}$ value obtained by Nonconforming based approach. This implies that Mapping based approach give the best estimate of C_{pu} and Nonconforming based approach results in the worst estimate of C_{pu}. The *Prediction errors* (Approx) for the estimated $\hat{C}_{pu}$ value obtained by Percentile based approach are also noted to be quite high. The *Prediction errors* (Approx) for the estimated $\hat{C}_{pu}$ values obtained by all other approaches are found to be reasonably low and thus may be acceptable.

The *Prediction errors* (Approx) for the estimated $\hat{C}_{pu}$ values obtained by different methods in the three case studies are summarized in Table 4. It can be noted from Table 4 that the *Prediction errors* (Approx) is consistently zero for the $\hat{C}_{pu}$ values obtained by the Mapping based approach. On the other hand, it is found that *Prediction errors* (Approx) is consistently high for the $\hat{C}_{pu}$ values obtained by the Nonconforming based approach. Therefore, it may be concluded that Mapping based approach is the most appropriate one and Nonconforming based approach is the most unsuitable one for assessing capability of a binomial process. For the $\hat{C}_{pu}$ values obtained by the other methods, the *Prediction errors* (Approx) are observed to be inconsistent. It may be noted from Table 4 that the *Prediction errors* (Approx) for the $\hat{C}_{pu}$ values obtained by Normal approximation approach, Percentile based approach, Transformation approach and Yield based approach are quite low in case studies 2 and 3 but substantially high in case study 1. Therefore, it may be concluded that these four approaches are also not reliable for estimation of C_{pu} for a binomial process.

Table 4. $\hat{C}_{pu}$ values obtained by different methods and the *Prediction errors* (Approx) for these estimates

Sl. No.	Approaches for C_{pu} calculation	*Prediction errors* (Approx) for different estimates of C_{pu}		
		Case study 1	Case study 2	Case study 3
1	Normal approximation approach	8.27	0.85	0.51
2	Percentile based approach	10.41	2.15	4.21
3	Transformation approach	6.17	1.56	1.13
4	Mapping based approach	0	0	0
5	Nonconforming based approach	10.00	41.95	40.09
6	Yield-based approach	13.03	3.49	1.85

5. Discussions

The C_{pu} or C_{pl} index for a binomial process can be estimated using several approaches. However, results of analyses of multiple data sets in section 4 show that accuracies of these estimates vary widely. It is observed that the *Predicted NL%* (Approx) based on $\hat{C}_{pu}$ values obtained by Mapping based approach exactly matches with the *True NL%* in the binomial process in all the three case studies, and thus, *Prediction errors* (Approx) are zero for all the three case studies. This happens because of the following facts. In mapping based approach, the C_{pu} value is computed by directly mapping the probability of nonconforming lots to the *Z*-value of standard normal distribution that results in the same probability of nonconforming products and the formula used for prediction of *NL%* (Approx) is also truly applicable for the normally distributed quality characteristic. If a more appropriate formula for prediction of *NL%* in a binomial process can be developed, the *Prediction errors* (Approx) may not be zero. However, there is no doubt that Mapping based approach gives the most accurate estimate of C_{pu} or C_{pl} for a binomial process.

The results in Table 4 reveal that *Prediction errors* (Approx) are always very high for the estimates of C_{pu} obtained by Nonconforming based approach. This implies that *Predicted NL%* based on these estimates differ highly from the *True NL%* in the respective binomial process. The problem with the Nonconforming based approach is that in this approach, C_{pu} (or C_{pl}) is estimated as the ratio of the two very small numbers, where numerator is 0.00135 (acceptable proportion of nonconforming lots) and denominator is actual proportion of nonconforming lots having fraction beyond f_U (or f_L). Thus, the estimate is highly impacted due to a minor deviation in the value of actual proportion from the acceptable proportion. For example, if actual percentage of nonconforming lots is 0.135% then the value of $\hat{C}_{pu}$ is equal to one but if the actual percentage becomes 0.01% then the value of $\hat{C}_{pu}$ would become as high as 13.5, which would give a misleading impression that the process is highly capable although one nonconforming lot per 1000 lots are expected in the process. On the other hand, if the actual percentage becomes 0.5% then the value of $\hat{C}_{pu}$ would become as low as 0.27, which would give again a misleading impression that the process capability is very poor. Due to the same reason the estimated $\hat{C}_{pu}$ values in case studies 2 and 3 give an impression that the capability of those processes are very poor.

The results in Table 4 reveal that *Prediction errors* (Approx) are inconsistent for the estimates of C_{pu} obtained by Normal approximation approach, Percentile based approach, Transformation based approach and Yield based approach. The problem with the Normal approximation approach is that the accuracy of the estimate obtained by this method highly depends on the value of sample size (n) and the average sample fraction ($\overline{f}$) that is used as an estimate of the population proportion. Higher is the sample size (n) and greater is the value of $n\overline{f}$, more accurate will be the estimate of C_{pu} or C_{pl} obtained by Normal approximation approach. The Percentile based approach is a well accepted method for estimation of C_p, C_{pu} or C_{pl} for a process when the quality characteristic can be described as a continuous non-normal variable. But application of this method for obtaining an approximate estimate of C_{pu} or C_{pl} of a binomial process (a discrete non-normal process) does not work well always. This is because the values of percentile points used in the computation are approximated to the nearest integers only and the actual percentile values corresponding to the nearest integers often may differ substantially from the prescribed percentile values. As a result, the estimate of C_{pu} obtained by Percentile based approach sometimes may become quite inaccurate. Among the various approaches considered, the transformation approach is the most complex method for computation of process capability index for a binomial process. However, the results in Table 4 show that accuracy of estimates of C_{pu} obtained by this method are not good always. This is perhaps the technique for transformation of binomial data into normal data does not work well always. In Yield-based approach, the values of the ratios $[F(U)-0.5]/(0.5-\alpha_0^U)$ and $[0.5-F(L)]/(0.5-\alpha_0^L)$ are considered as the estimate of C_{pu} and C_{pl} respectively. Since the values α_0^U and α_0^L are usually taken as 0.00135, the denominator is always equal to 0.49865 in both the ratios. On the other hand, the values of the numerators in both the ratios can be at most 0.5. Therefore, the maximum value of $\hat{C}_{pu}$ or $\hat{C}_{pl}$ in a process can be 0.5/0.49865 = 1.0027. This implies that the estimated $\hat{C}_{pu}$ or $\hat{C}_{pl}$ obtained by Yield-based approach would fail to make distinction among almost capable process, just capable process and highly capable process.

6. Conclusion

Process capability analysis (PCA) is an important analytic tool frequently employed by the design engineers, process managers, vendors as well as customers. The basic purpose of PCA is to assess if a process is capable of meeting the specified requirements. In reality, many product characteristics are qualitative in nature and quality of such products is usually described in terms of fraction nonconforming or fraction conforming in a lot. The fraction nonconforming or fraction conforming is known to follow binomial distribution with parameters *n* (sample size) and *p* (population proportion). Measuring capability of a binomial process implies assessing to what extent the fraction nonconforming (or fraction conforming) in the continuous stream of lots produced comply with *USL* for fraction nonconforming (or *LSL* for fraction conforming). In this paper, a procedure for assessing accuracies of the estimates of C_{pu} or C_{pl} obtained by various methods is discussed. Analysis of multiple case study data reveals that Mapping based approach gives the most accurate estimate of C_{pu} or C_{pl} for a binomial process and Nonconforming based approach gives the most inaccurate estimate of C_{pu} or C_{pl} obtained by other methods like Normal approximation approach, Percentile based approach, Transformation based approach and Yield based approach are inconsistent and therefore, these estimates are unreliable. Thus, only the Mapping based approach is appropriate for estimating C_{pu} or C_{pl} for a binomial process.

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