Power system observability with minimum phasor measurement units placement

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Abstract

This paper presents optimal phasor measurement units (PMUs) placement algorithms for power system observability. The optimal placement problem (OPP) is formulated such that minimizing the number of PMU installations for full network observability. Three approaches, in this paper, are introduced aiming at reducing the computational burden in Optimal Placement problems. Depth First Search, Simulated Annealing and Minimum Spanning Tree as well as their differences and relations are discussed in details. The OPP methodologies applied include the system observablility during normal operating conditions, as well as during single branch forced outages. In order to improve the speed of convergence, an initial PMU placement is provided by graph-theoretic procedure. The IEEE 14-bus, 118-bus standard test power systems and New England 39-bus test systems are used for simulation purposes.

Keywords: Phasor Measurement Unit (PMU); Observability; Depth First Search; Simulated Annealing Method; Minimum Spanning Tree; Optimal PMU Placement (OPP).

1. Introduction

Phasor measurement units (PMUs) are considered as a promising tool for future monitoring, protection and control of power systems. The PMU is a power system device capable of measuring voltage and current phasor in a power system. Synchronism among phasor measurements is achieved by same-time sampling of voltage and current waveforms using a common synchronizing signal from the global positioning satellite (GPS) (Meliopoulos et al.,2006; Phadke et al.,2006; Phadke et al.,1986). Since the voltage and current phasors are measured, the state estimation equations become linear, and it is easier to find the solution than the nonlinear system state estimation (Gamm et al., 2008). The PMUs can enhance many present applications such as measurement loss and branch outage (Rakpenthai et al., 2007), bad data detection (Jian Chen and Ali Abur, 2006), stability studies (Berkestedt, 2007) and fault location studies (Kai-Ping Lien et al., 2006). The way power systems are controlled is ought to be revolutionized by such a new technology. However, the overall cost of the metering system will limit the number and locations of PMUs. In addition to the cost factor, different criteria are suggested for the proper allocation of PMUs in a given system. Network observability, state estimation accuracy and robustness present samples of such criteria.

Network observability determines whether a state estimator will be able to determine a unique state solution for a given set of measurements, their location, and a specified network topology. The problem is independent of the measurement errors (or even the measurement values), branch parameters as well as the operating state of the system. There are three basic approaches to conduct network observability analysis; namely numerical, topological, and hybrid approaches (Shahraeini et al., 2011; Saini et al., 2012). The numerical observability approach is based on the fact that a unique solution for the state vector can be estimated if the gain matrix is non singular or equivalently if the measurement Jacobian matrix has a full column rank and well conditioned. The topological observability approach is based on the fact that a network is fully observable if the set of measurements can form at least one measurement spanning tree of full rank (Ivatloo, 2009; Baldwin et al., 1993). It is neither economical nor necessary to

install a PMU at each bus of wide area power network. As a result, the problem of optimal PMU placement (OPP) concerns with where and how many PMUs should be implemented on a power system to achieve full observability at a minimum number of PMUs.

The placement of a minimal set of phasor measurement units to make the system measurement model observable is the main objective of this paper. Due to the high cost of having a PMU at each node, some of the studies performed in the mid 1980s focused on PMU placement and pseudo measurements for complete or partial observability of the system. One of these studies suggested a placement of PMU by Simulated Annealing technique (Kerdchuen et al., 2007). This study used topologically observable concept to check the observvbility. An optimal PMU placement by Tabu Search algorithm for power system observability was developed in Jiangnan Peng et al. (2006). Ahmadi et al. (2011) introduced a binary particle swarm optimization (BPSO) based methodology for the optimal placement of phasor measurement units (PMUs) when using a mixed measurement set. In addition, the optimum number of PMUs placement, which makes the power system observable has been reached through a genetic-based algorithm (sevic et al., 2003). A spanning tree search of multiple solutions for the minimal PMU positioning issue was introduced in Denegri et al. (2002). An algorithm based on integer programming was successfully implemented to find the optimum number and the locations of the installed PMUs of the power system (Venugopal et al., 2010; Bei Gou, 2008). In Najafabadi and Alouani (2011), an algebraic method is developed to solve OPP problem to minimize the number of PMUs considering a limitation on PMUs channels. In Nuqui and Phadke (2005), the search tree and SA techniques were used for the OPP problem.

The objective of the present work is to find the minimum number of PMUs to make the system observable, as well as the optimal locations of these PMUs. The OPP methodologies applied include the system observablility during normal operating conditions, as well as during single branch forced outages. The schemes are discussed and compared to obtain OPP solutions with less calculation effort and less time- consuming. The methods are applied to IEEE 14-bus test system, New England (NE) 39- bus system and IEEE118-bus test system.

2. Phasor Measurement Unit (PMU) Technology

A phasor measurement unit is a device that uses state-of-the-art digital signal processors that can measure 50/60Hz AC waveforms (voltages and currents) typically at a rate of 48 samples per cycle. A phase-locked oscillator along with a Global Positioning System (GPS) reference source provides the needed high speed synchronized sampling with 1 microsecond accuracy. Line frequencies are also calculated by the PMU at each site. This method of phasor measurement yields a high degree of resolution and accuracy. The resultant time tagged phasors can be transmitted to a local or remote receiver at rates up to 50/60 samples per second. PMUs come in different sizes. Some of the larger ones can measure up to 10 phasors plus frequency while others only measure from one to three phasors plus frequency. Figure 1 shows the PMU hardware block diagram for the previous illustrated procedures.
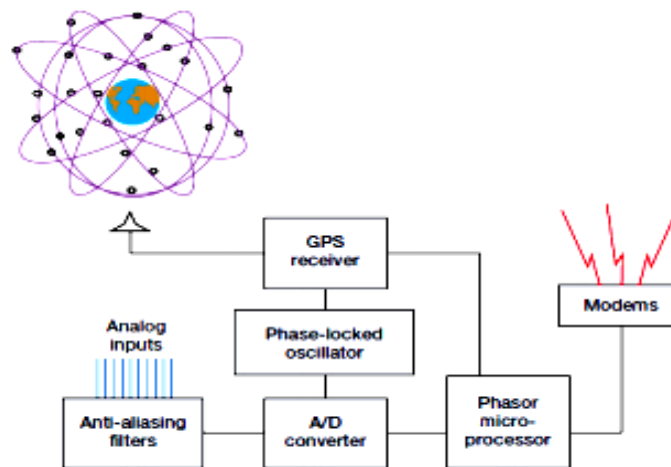

Figure 1. Phasor Measurement Unit (PMU) hardware block diagram (Singh et al., 2011)

2.1 Wide-Area Measurement Systems (WAMS) and Phasor Data Concentrators (PDC)

Wide area measurement systems (WAMS) focus on collecting the synchronized system measurements in real-time and distribute them further to applications that make use of the data. A basic structure of a WAMS is illustrated in Figure 2. The figure shows a WAMS consisting of PMUs, communication links, and data concentrators which are needed to fully exploit the benefits of synchronized phasors measurements. The PMU measurements are transmitted immediately to a receiving unit, usually a phasor

data concentrator (PDC). The data are then sent without delay to real-time applications and to data storage for off-line use. The WAMS provides the possibility of serving all measurement applications through different choice of data rates. (Jóhannsson, 2010)
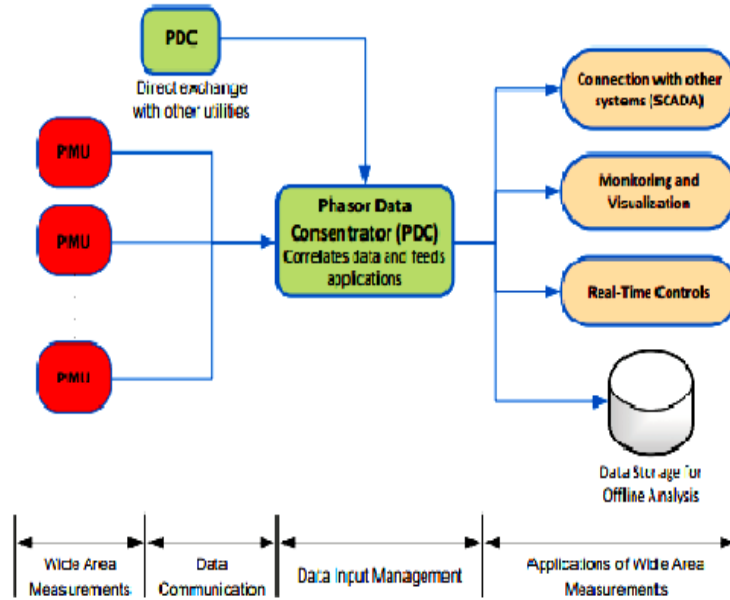

Figure 2. General Overview of Wide Area Measurement Systems.

Figure 3 shows PMUs geographically dispersed to form a wide area monitoring system (WAMS) in which the PMUs deliver GPS time-tagged measurements to a Phasor Data Concentrator (PDC). The PDC sorts the incoming phasor measurements before signal processing converts PMU data into actionable information that can be presented to an operator in the form of a Human Machine Interface (HMI).This HMI provides an operator with critical information about the state of power system (SAINI et al., 2012).
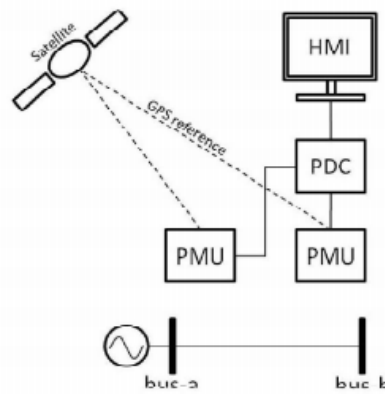

Figure 3. PMU Layout with GPS time stamped Signal

2.2Main Strategy of PMU Placement Based on Power Systems Intrinsic Characteristics

For the purposes of real-time dynamic performance monitoring the power system operating conditions, WAMS should have following monitoring functions as in Singh et al. (2011):

- Key lines and links reflect the main system characteristics
- Substations on key system interties and major load areas
- Key system generating plants
- Key system substations
- Special protection systems and remedial action schemes.

Overall measurement facilities must support:
- Real-time observation of system performance

• Recording and analysis of system disturbances.

3. Power System Obsevability Analysis and PMU Placement Rules

3.1 Observability Analysis

The estimate principle of PMU optimal placement in a power system mostly is power system observability. After placing a new PMU, whatever method is used, the observability of the power system must be checked. If the system is observable, the placement stops, else the placement must be continued.

As in Ivatloo et al. (2009), there are two major algorithms for power network observability analysis, topology based algorithms and numerical methods.

(1) *Topology Observability:* They use the decoupled measurement model and graph theory. In these methods, decision is based on logical operations. Thus, they require only information about network connectivity, measurement types and their locations, if a full rank spanning tree can be constructed with the current measurement set; the system will be observable.

(2) *Numerical Observability:* They use either fully coupled or decoupled measurement models. These methods are based on numerical factorization of the measurement Jacobean or measurement information gain matrix. If any of these matrices is full rank, the system will be observable.

$$Z = Hx + V \tag{1}$$

where,

Z: is the metrical vector of m dimensions.
H: is the Jacobian matrix of m × (2N – 1) dimensions.
X: is the voltage vector of (2N – 1) dimension.
V: is the metrical noise vector of m dimensions.

3.2. PMU Placement Rules

The objective of the OPP problem is the strategic choice of the minimum number of PMUs (*nPMU* min) and the optimal allocation *AL (nPMU* min *)* of the total number of PMUs (*nPMU*) in order to ensure complete observability and satisfy a preset redundancy criterion. The OPP problem can be formulated as in Manousakis et al. (2011).

$$\min \{\max R(nPMU, AL (nPMU))\} \tag{2}$$
$$\text{Subject to } Obs (nPMU, AL (nPMU)) = 1 \tag{3}$$

where R (*nPMU* , *AL (nPMU)*) is the redundancy measurement index and *Obs* is the observability evaluation logical function. The optimal solution (*nPMU* min) and *AL (nPMU* min *)* is difficult to be obtained directly, due to the large-scale nature of the OPP combinatorial optimization problem and the dependence of system observability on the number of PMUs and the placement set and also there is no means to obtain the minimum number of PMUs *nPMU* min directly up to the present, it depends on iterative algorithm to plough around this kind of problem. Computationally, the OPP problem is highly nonlinear, discontinuous and multimodal, having a non convex, and non smooth objective function so it is impossible to apply the routine of optimal methods to find the complete optimal solution for it always has a great number of local extreme. It must use modern optimal approaches especially DFS and SA methods which have the capability of overall optimization and independent on the information such as grads to solve this kind of complex problem. The unobservable buses are made observable by solving the OPP considering the following rules

(1) For buses with PMUs, voltage and current phasors for all incident branches are known. These are called direct measurements.

(2) If voltage and current phasors at one end of a branch are known, then voltage phasor at the other end of the branch can be obtained. These are called pseudo measurements.

(3) If voltage phasors of both ends of a branch are known, then the current phasor of this branch can be obtained directly. These measurements are also called pseudo measurement.

(4) For a zero-injection bus i in an *N*-bus system we have:

$$\sum_{j=1}^{N} Y_{ij} V_j \tag{4}$$

Where, Y_{ij} is the *ij*-th element of the admittance matrix of the system, and V_j is the voltage phasor of *j*-th bus.

Therefore, if there is zero injection bus without PMU whose incident branches current phasors are all known but one, then the current phasor of the unknown one could be obtainable by KCL equations.

(5)If a zero-injection bus with unknown voltage phasor and voltage phasors of its adjacent buses are all known, then the voltage phasor of the zero-injection bus can be found using the nodal equations.

(6)If a group of adjacent zero-injection buses exists, whose voltage phasors are unknown but the voltage phasors of all adjacent buses to the group are known, then the voltage phasors of zero-injection buses can be obtained using the nodal equations (Grigoras et al., 2009).

Finally Figure 4 shows the flowchart of observability analysis based on aforementioned rules.
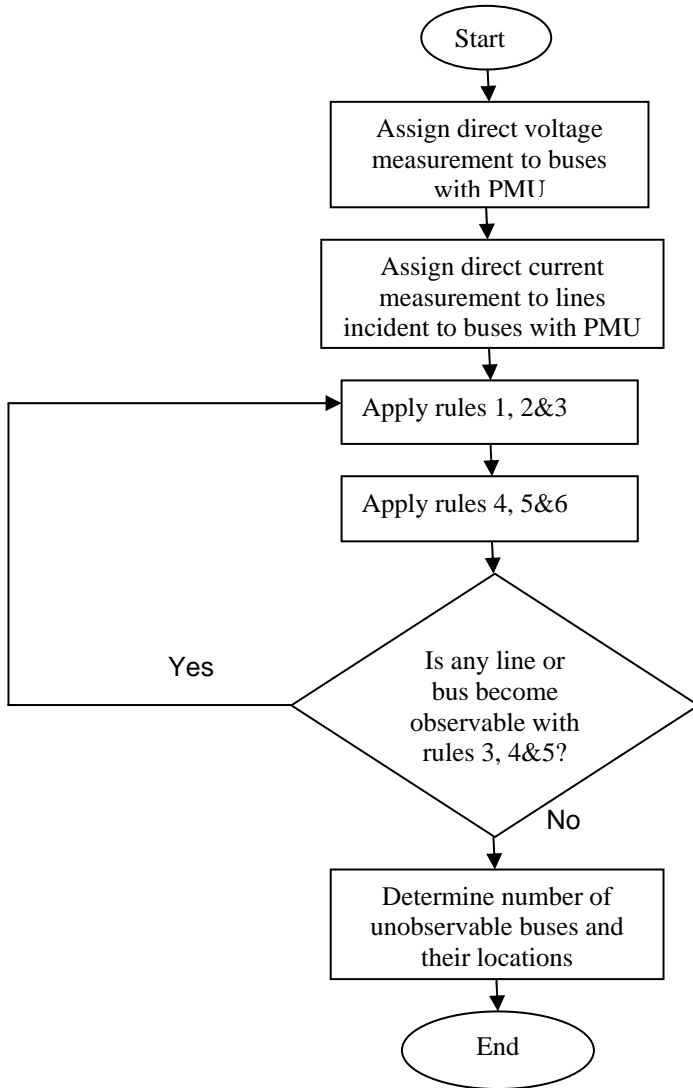

Figure 4. Flow chart of topological observability analysis

4. Optimal PMU placement considering normal and contingency conditions

The presented approaches for the optimal PMU placement are implemented in two sequences. At first, the optimal placement is carried out with the goal of minimizing the total number of required PMUs for complete system observability during normal conditions, i.e. no PMU failure or line outage. Then, the single branch outage is discussed by two approaches.

4.1 Optimal PMU placement in normal conditions
Before beginning the search, an initial PMU placement which makes system observable, is provided by graph theoretic search procedure (Baldwin et al., 1993). It should be noted that a reasonable starting point can greatly accelerate the speed of convergence in complex optimization problems. The initial placement is built based on the following steps.

Step 1: Place a PMU at the bus located in the unobservable region which has the maximum number of incident lines.
Step 2: Implement topological observability flowchart, depicted in fig. 3, determine numbers and locations of unobserved buses (if exist).
Step 3: If there is an unobservable region yet, go to step 1, otherwise stop.

For any system, this placement scheme leads to a PMU configuration which satisfies complete system observability. However, as shown in simulation results, this scheme is not the optimal one and an optimization tool is required for deriving the minimum number of required PMUs.

For any system, this placement scheme leads to a PMU configuration which satisfies complete system observability. However, as shown in simulation results, this scheme is not the optimal one and an optimization tool is required for deriving the minimum number of required PMUs.

Then we have to minimize the total number of PMUs required for complete system observability assuming normal conditions. The optimization problem can be described mathematically as:

$$\text{Minimize} \qquad \sum_{i=1}^{N} PMU_i \tag{5}$$

$$\text{subject to} \quad N_{nobs} = 0 \tag{6}$$

Where PMU_i indicates the installation of PMU at bus i, it is 1, if a PMU is installed on bus i and 0 vice versa. N_{nobs} represents the number of unobserved buses in the network.

In an actual power system, there may be certain buses that are strategically important, such as a bus connected to a heavily loaded or economically important area, or a bus anticipated to be a future expansion point. In such a case, PMUs may be placed at the preferred buses. The rest of the unobservable phase angles are made observable by placing a minimum number of additional PMUs. The radial buses are excluded from the list of potential locations for placing a PMU because of two reasons: 1) A PMU placed at a radial bus can measure the phasors at that bus and only one additional bus that is connected to it and 2) a PMU placed at the bus connected to the radial bus can measure the phase angle of the radial bus by using the measurement of the current through the radial line. When zero-injection measurements are considered, zero-injection buses having only two incident lines are also excluded from the list since their voltage phasors can be determined by using KCL. A PMU is preassigned to each bus connected to a radial bus, provided it is not a zero-injection bus, since in that case it may be possible to determine the voltage phasor of the corresponding radial bus by using KCL. Preassigning PMUs to certain buses in this manner reduces the total number of possible combinations of PMU locations, thereby reducing the computational burden. Three methods are used to solve the problem.

4.1.1 Depth First Search Method

Depth First Search method (DFS) is applied extensively in earlier time, which is one of the tree search methods of PMU placement. It marks all vertices in a directed graph in the order they are discovered and finished, while partitioning the graph into a forest, This method uses only rules from 1 to 3 (it does not consider pure transit nodes). The first PMU is placed at the bus with the largest number of connected branches if there is more than one bus with this characteristic, one is randomly chosen. The following PMUs are placed with the same criterion, until the complete network visibility is obtained. DFS merely considered the "depth" through the process of expanding, which makes the observational topologies, increases the unwanted redundancy (Dongjie et al.,2004).

4.1.2 Simulated Annealing Method

Simulated annealing method is put forward by Metropolis in 1953. Simulated Annealing (SA) is a technique that finds a good solution to an optimization problem, by trying random variations of the current solution. A worse variation is accepted as the new solution with a probability that decreases as the computation proceeds. The slower the cooling schedule, or rate of decrease, the more likely the algorithm is to find an optimal or near-optimal solution (Manousakis et al., 2011). It solves the problems of combination optimization by simulating the physical anneal process of the solid matter (such as metal). By analogy with this physical process, each step of the SA algorithm replaces the current solution by a random" nearby" solution, chosen with a probability that depends both on the difference between the corresponding function values and also on a global parameter T (called the temperature), that is gradually decreased during the process. The dependency is such that the current solution changes almost randomly when T is large, but increasingly "downhill" as T goes to zero.

The procedures can be subdivided into several main steps, as follows:

(1) Select an initial condition X_0 from the approve solution space randomly, calculate its target function value $f(X_0)$, and select the initial control temperature $T0$ and the length of Markov Chain;
(2) Engender a random disturbance in the approve solution space, then gain a new state X_1, calculate its target function value $f(X_1)$;
(3) Judge, whether it satisfies $f(X_1) < f(X_0)$, if yes, then accept the new state X_1 as the current state; if not, then judge whether it satisfy X_1 according to Metropolis rule, if yes, then accept the new state X_1 as the current state, else accept the state X_0 as the current state;

(4) Judge, whether the sample process ends according to some convergence rule, if yes, then turn to (5); if not, then turns to (2);
(5) lower the control temperature T according to some annealing project;
(6) Judge, whether the annealing process ends according to some convergence rule, if yes, then turn to (7); if not, then turns to (2);
(7) Output the current solution as the best solution.
The code of the application of simulated annealing method for PMU optimal placement in a power system is in
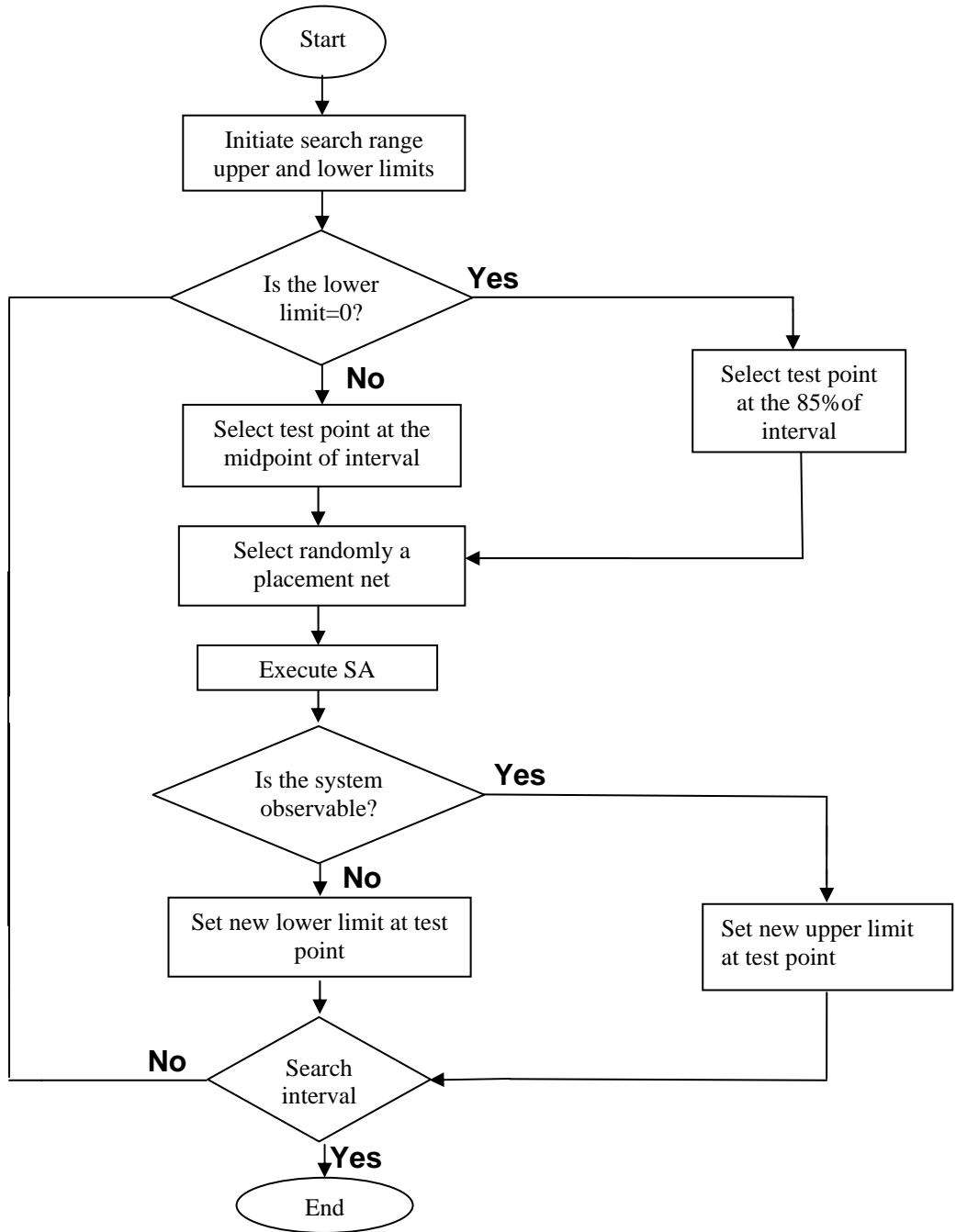Nuqui (2001).

Figure 5. Flow chart of SA method

4.1.3 Minimum Spanning Tree Method

   Minimum Spanning Tree method (MST) is a modified depth first approach. The procedure can be subdivided into three main steps (Farsadi et al., 2009):

(1) Generation of N minimum spanning trees:
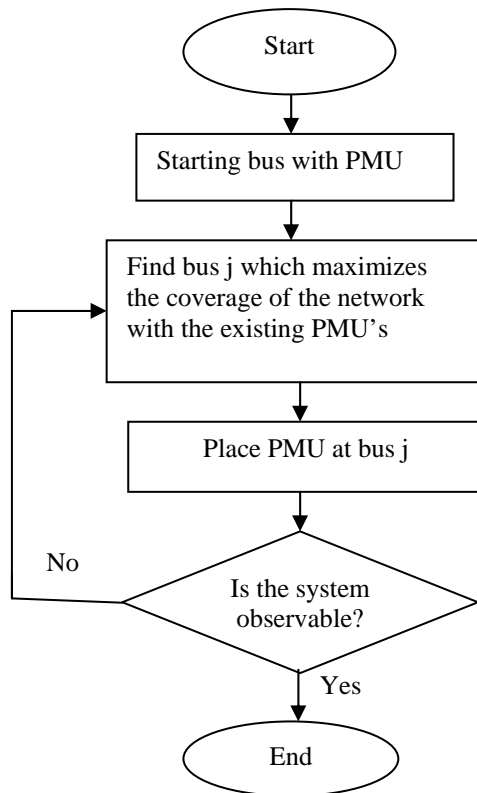Figure 6 depicts the flowchart of the minimum spanning tree generation algorithm.

Figure 6. Flow chart of MST method

The algorithm is performed N times (N being the number of buses), thus using all the nodes as starting bus. PMU's location ends when the entire network is observable, and thus a minimum spanning tree is built.
(2) Search of alternative patterns:
The PMU sets obtained with the step (1) are reprocessed as follows: one at a time, each PMU of each set is replaced at the buses connected with the node where a PMU was originally set, as depicted in Figure 7. PMU placements which lead to a complete observability are retained.
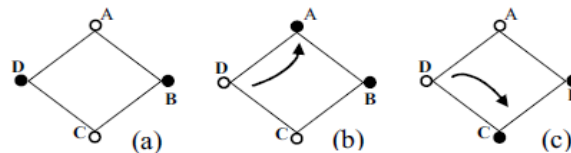

Figure 7. Search of alternative placement sets

(3) Reducing PMU number in case of pure transit nodes:
In this step, it is verified if the network remains observable taking out one PMU at a time from each set, as depicted in Figure 8. If no pure transient nodes are present in the network, the procedure ends at step 2.
This method finds the bus which maximizes the coverage of the network with the existing PMU's, then sets a PMU at the bus. It can reach the complete convergence, and also make the optimization best.
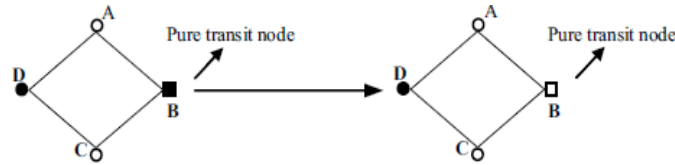
Figure 8. Pure transit node filtering.

4.2 Optimal PMU placement in contingency conditions

In this stage, two algorithms are introduced to maintain network observability during a single PMU loss or line outage. The objective function of this part can be expressed mathematically as follows:

$$\text{Minimize} \quad \sum_{i=1}^{N} PMU_i \tag{7}$$

$$\text{Subject to} \quad N_{nobs} = 1 \quad \text{single PMU loss or branch outage} \tag{8}$$

The details of PMU placement in contingency conditions are described in the following sections.

4.2.1 Recursive and Single-Shot Security N-1 Algorithms

The rules for minimal PMU placement assume a fixed network topology and a complete reliability of measurement devices. Simple criteria which yield a complete visibility in case of line outages (N-1 security) are proposed in Denegri et al. (2002) and are based on the following definition:
A bus is said to be observable if at least one of the two following conditions applies:
Rule 1: a PMU is placed at the node;
Rule 2: the node is connected at least to two nodes equipped with a PMU.
Rule 2 is ignored if the bus is connected to single-end line.
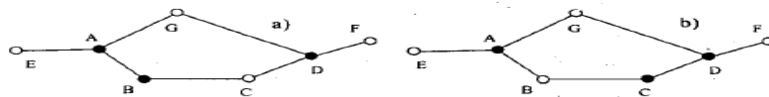

Figure 9. PMU placement with an N-1 criterion.

Figure 9 shows a graphical representation of an N-1 security placement. For a minimal placement, positioning a PMU at nodes A and B would be enough for obtaining a complete spanning tree of the network. But, if a line A-B   (or C-D) is lost, the voltage of B (or C) cannot be estimated. An additional PMU should be added either at B or C, to ensure N-1 observability. Note that if the line A-E or D-F are lost, buses E or F are non longer observable. However the remaining system is still fully observable.
Figure 10 and Figure 11 depict two possible algorithms for obtaining the N-1 security placement, by means of a modified depth first search and a single shot method respectively. In the first method, the procedure starts from one bus and builds the spanning tree assigning a PMU at the closest bus connected to the buses already observed. The procedure is then repeated starting from each bus of the network and finally selecting the minimal PMU placement sets.The second method is based only on topological rules and provides a spanning tree by means of a "single shot" technique. Each bus is associated with an observability index w which is set to zero at the beginning of the procedure. Observability indexes are used as flags for possible measurement and/or pseudo-measurement of node voltage magnitudes and phase angles, according to the PMUs monitoring properties. First, the buses interconnected to only one other bus (interconnection index h=1) are searched and PMU is set on the latter buses and continue as shown in Figure 11   (Denegri et al., 2002).
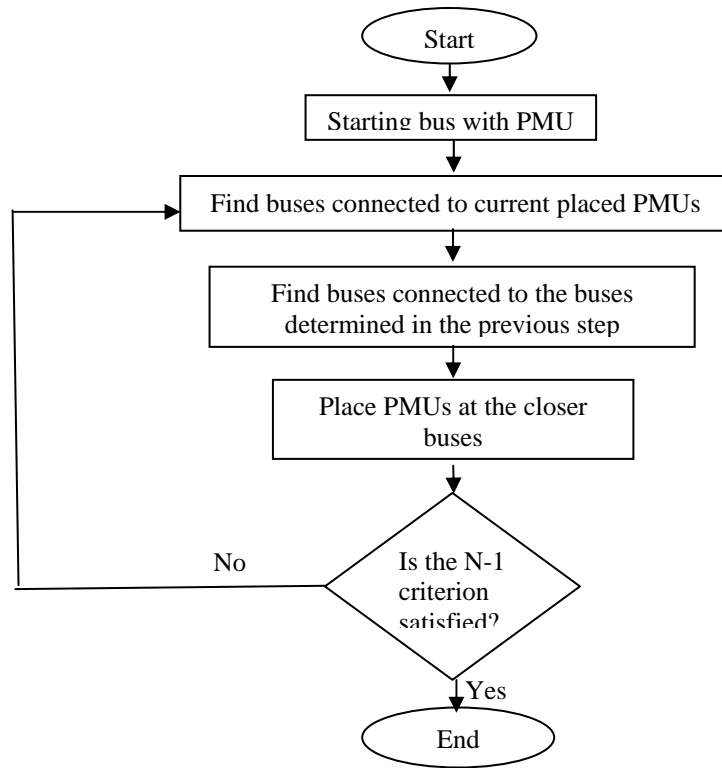
Figure 10. Recursive N-1 Security Method

Since this method doesn't ensure necessarily that each bus is observed at least by two PMUs, a final fulfillment of the resulting spanning tree is needed. While being very fast, this method generally leads to slightly higher PMU numbers than the first method does. For this method, it's not performed a search for alternative patterns, thus, the single shot method will provide only one solution in this case.

5. Simulation Results and Analysis

5.1. Simulation Example 1

The first system under study is the IEEE 14-bus system. It has five synchronous machines, three of which are synchronous condensers used for reactive power support and twenty branches. Single line diagram of this test system is shown in Figure 12. Node 7 is called a pure transit node; we also call it no load node. There are no loads to consume the power, no generators to inject the power either. The power injected by node 8 and node 4 transmits to node 9 completely that is why it is named pure transit node. According to PMU placement rule (4), for node7, as long as two current branches are known; the left current branch can be calculated by pseudo measurement. The initial placement results in installing 5 PMUs and the minimum output of the introduced algorithms is only 3 PMUs.
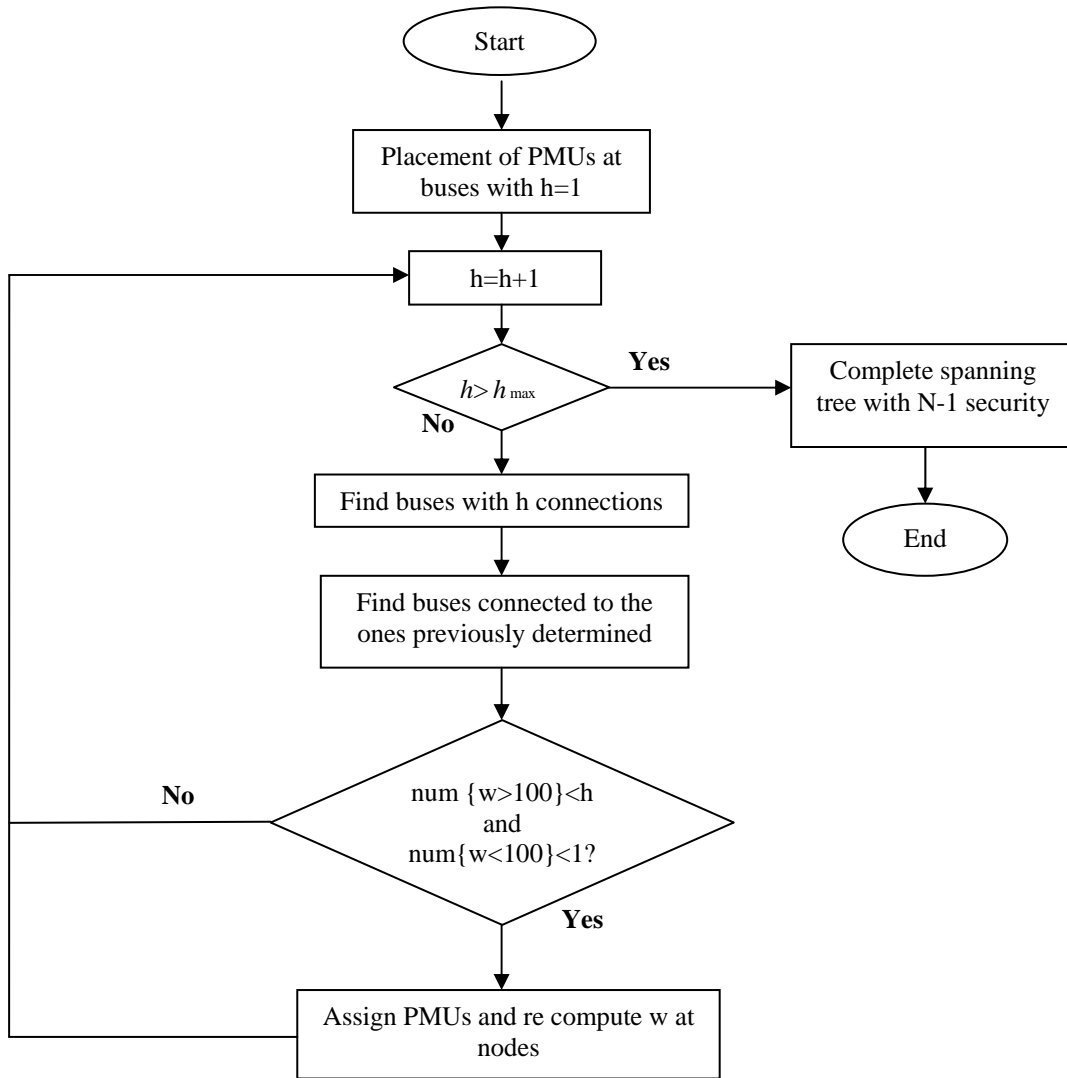
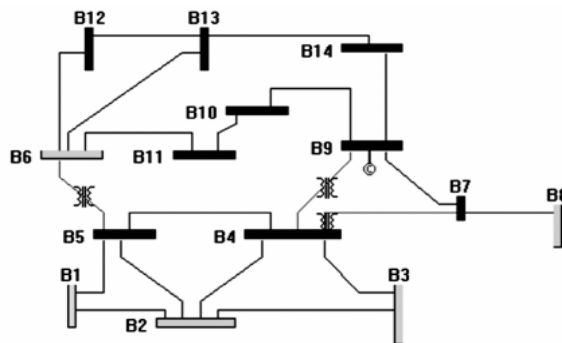**Figure 11.** Single Shot N-1 Security Method.

Figure 12. IEEE 14-Bus System.

5.2. Simulation Example 2

The second system under study is the IEEE 39- bus system, which is shown in Figure 13. This system is well known as 10-machine New-England Power System, having 10 generators, 19 loads, 36 transmission lines, and 46 branches   represents a reduced model of the NE power system. The initial placement results in installing 11 PMUs and the minimum output of the introduced algorithms is only 9 PMUs.

Figure 13. IEEE 39- New England System

5.3. Simulation Example 3

The third system under study is the IEEE 118- bus system which is shown in Figure 14. The system consists of 41 synchronous generators and 27 synchronous compensators with 186 branches. The initial placement results in installing 39 PMUs and the minimum output of the introduced algorithms is only 29 PMUs.
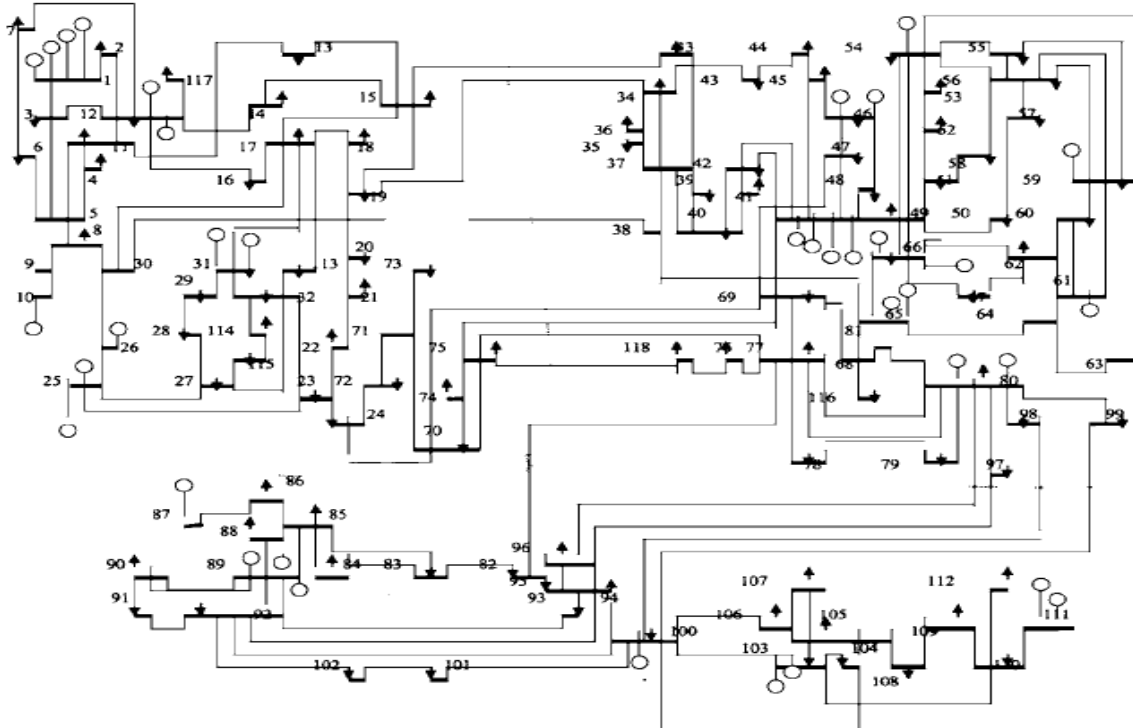
Figure 14. 118 bus test system

Table 1 shows the number of radial buses and the number of zero-injection buses, for the test systems.

Table 1. Radial and Zero-Injection Buses for the Test Systems

Test system	No. of radial buses	No. of Zero injection buses
IEEE 14-bus	1	1
New England 39-bus	9	12
IEEE 118-bus	7	10

Table 2 shows the buses where PMUs must be placed in order to make all the radial buses observable.

Table 2. Preassigned PMU Locations

Test system	Buses where PMU must be placed considering zero injection buses
IEEE 14-Bus	None
New England 39-bus	20,23,25,29

In the case of large power systems, the search method is computationally intensive. However, due to the high capital cost of PMUs, the normal practice is to either install a limited number of PMUs or install them in an incremental fashion and combine them with existing conventional measurements. For this reason, the introduced PMU placement methods can be used in practical scenarios. DFS, SA and MST methods are used separately to complete the simulation. The results are shown in Table 3,4 and 5.

Table 3. Results of IEEE 14-Bus System

Method	Elapsed Time	Number of PMUs	Placement
DFM	0.094 s	6	1, 4, 6, 8, 10, 14
MST	0.452 s	3	2, 6, 9
SA	0.359 s	4	1, 4, 11, 13

Table 4. Results of IEEE 39-Bus System

Method	Elapsed time	Number of PMU's	Placement
DFM	0.125 s	16	2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 23, 26, 33, 35, 37, 38, 39
MST	3.853 s	9	1, 3, 8, 10, 16, 20, 23, 25, 29 (2 sets) 2, 3, 8, 10, 16, 20, 23, 25, 29
SA	28 m 26.083 s	9	2, 3, 8, 12, 16, 20, 23, 25, 29

Table 5. Results of IEEE 118-Bus System

Method	Elapsed time	Number of PMU's	Placement
DFM	0.11 S	41	1, 5, 9, 12, 13, 17, 19, 22, 24, 25, 29, 32, 36, 37, 41, 44, 46, 49, 53, 57, 58, 59, 62, 64, 68, 71, 75, 76, 78, 80, 83, 86, 89, 91, 93, 95, 100, 102, 105, 110, 115
MST	1min 27.111 s	31	2, 5, 9, 12, 13, 17, 21, 27, 29, 32, 34, 37, 40, 45, 49, 53, 56, 59, 66, 68, 71, 77, 80, 85, 86, 90, 94, 101, 105, 110, 118 (10 sets)
SA	3h 6min 39.653 s	29	1, 8, 11, 12, 19, 22, 27, 31, 32, 34, 37, 40, 45, 49, 52, 56, 62, 65, 72, 75, 77, 80, 85, 86, 91, 94, 101, 105 ,110

The simulation results are obtained using a microcomputer with 2.4 G Hz processor, 4 GB memories. The elapsed time is different for the different computers, but proportion of time is the same. The speed of DFS method is the fastest, but it needs the most PMUs. The results of the first three methods in section 3 can be summarized as follow in Figure 15.
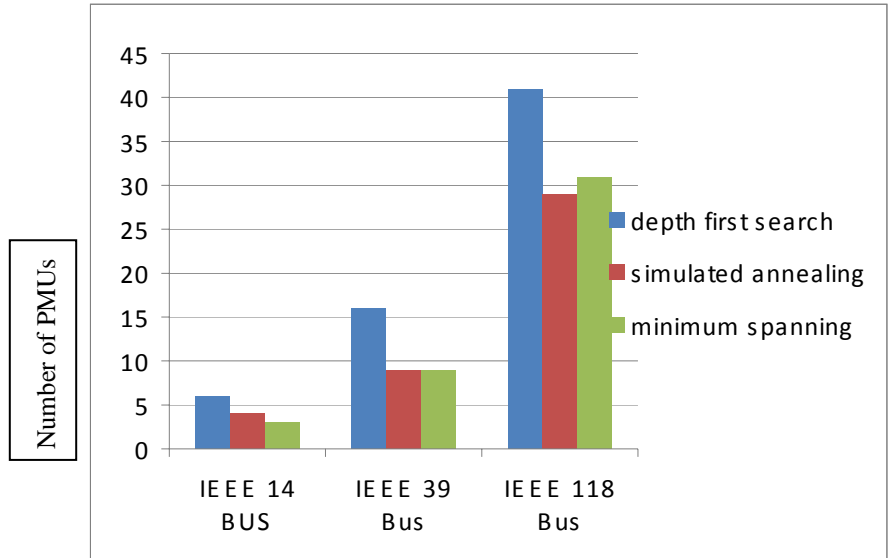
Figure 15. Summary of results of three algorithms

   The convergence of SA method has the close relation with the initial value. It must calculate the whole nodes after setting a PMU because it may engender a new disturbance during the calculation process which makes the parameter matrix huge, thus the speed of placement is restricted. Furthermore, the diversity of the solution is very poor in DFS method and SA method; they only have one setting project. MST method can overcome these shortcomings and hold their excellencies at the same time. In the process of optimization, MST method assimilates the thought of DFS, but differing from this method; it finds the bus which maximizes the coverage of the network with the existing PMU's not the bus with the largest number of connected branches. When the system reaches a complete observability, it can reduce PMU number in case of pure transit nodes, which can reduce the redundancy and increase economic efficiency. At the same time, MST method saves each bus, which has the same coverage of the network during the process of optimization, then find the best setting project separately, which leads to the multiformity of the result. Also Table 6 shows the result of our simulation on the test systems, where N is the total number of buses, n_{min} is the minimum number of PMUs required for complete system observability. n_{min} / N is the ratio of the number of the required PMUs to the number of buses. It can be observed that the required number of PMUs for complete system observability is ranged from 21 to 25 percent. However it should be noted that the number of required PMUs for complete system observability is strictly dependent on network topology and the number of zero-injection buses.

Table 6. Minimum Number oF PMUs for Complete System Obsrevability

Test system	N	n_{min}	n_{min} / N
IEEE 14-Bus	14	3	0.2143
IEEE 39-Bus	39	9	0.2308
IEEE 118-Bus	118	29	0.2458

5.4 Results of Recursive N-1 Algorithm for Different Test Systems
Results of the N-1 methods for the examples explained before are shown in Tables 7 and 8.

Table 7. Results of Recursive N-1 Algorithm

Test system	Elapsed time	Number of PMU's		Placement
IEEE 14-Bus	0.09 s	8	[+0]	1,2,4,6,7,10,12,14 (13 sets)
IEEE 39-Bus	0.637 s	18	[+4]	2,4,6,8,10,12,14,16,18,19,20,22,23,25,26,27,29,39 (4 sets)
IEEE 118-Bus	9.098 s	63	[+10]	1,4,5,7,8,9,12,13,14,17,19,21,23,26,27,29,32,33,34,35, 37, 38,40,44,46,49,52,53,55,56,57,58,59,60,62,64,66,68,70,71, 72,75,76,77,79,80,83,85,86,88,89,91,92,93,94,96,100,102,1 05,107,109,110,115 (6 sets)

Table 8. Results of Single Shot N-1 Security Algorithm

Test system	Elapsed time	Number of PMU's		Placement
IEEE 14-Bus	0.037	8	[2]	2,3,5,6,7,10,13,14.
IEEE 39-Bus	0.054	18	[4]	2,4,6,8,10,12,15,16,18,19,20,22,23, 25, 27,28,29,39.
IEEE 118-Bus	0.738	72	[10]	1,4,6,8,9,12,13,14,17,19,21,22,23, 24,25,27,29,30,32,33,34,35,38,39, 41,42,44,45,46,47,48,50,51,53,54, 56,57,59,61,62,64,66,68,69,71,74, 75,77,79,80,82,84,86,88,89,90,92, 94,96,98,99,101,102,103,104,106, 107,108,110,115,118.

Also the results of N-1 methods can be summarized as in Figure 16
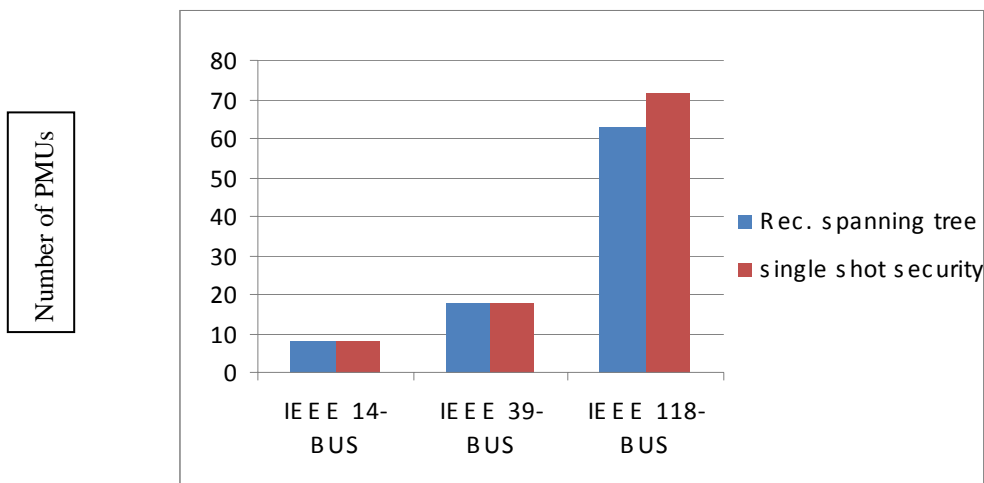

Figure16. Summary of results of N-1 algorithms

   The single shot method seems to provide generally slightly higher number of PMUs (about 10%).
For the N-1 methods, the quantities in square brackets indicate the additional PMUs required for accomplishing a complete observability also in case of measurement device outages. The N-1 security criteria lead to PMU sets about the 50%, which may increase up to 75% when considering also PMU outages.

6. Comparison between Algorithms

   The optimization problem of PMU placement means the minimum number of PMUs (Cai and Ai, 2005). It is impossible to apply the routine of optimal methods to find the complete optimal solution for it always has a great number of local extreme. It must use modern optimal approaches, especially DFS and SA methods, which have the capability of overall optimization and independent on the information such as grads to solve this kind of complex problem.
   DFS method finds the first bus with the largest number of connected branches, and then continues to search in the solution place from the initial solution with the same criterion, until the complete network observability is obtained.
The steps are as follows:
(1) Select the bus with the highest dimension to place PMU first;
(2) Search in the same way as step (1) and find the measurements, pseudo- measurements and extend measurements, then estimate the degree of system observability;
(3) If the complete network observability is not obtained, do the steps (1) and (2) again to the unobservable area, until the complete observability is obtained. DFS method only considers the criterion of 'depth' in the process, and doesn't do the repetitive operation, so its operand is minimal, which leads to the highest speed compared with the other two methods. But its result is not the optimum because its optimization criterion is unitary; furthermore, it doesn't consider that many measurements of the network system can be obtained by pseudo- measurements. Also it doesn't match the results given in Table 2
   SA approach has the complementary effect to DFS method. This algorithm starts from a given initial solution, produces an aggregation of candidate solution. It accepts the whole candidate solutions, which are better than current solution in the process of optimization, and accepts inferior solution in a certain probability. This characteristic has the key effect on jumping out of local

optimization in the search process, but it makes its operand and solution space increase leading to the low speed. From the simulation results, we can see that the elapsed time of SA approach is the longest compared with DFS especially in the instance of that the number of system' buses is large, and the connection of network is complex. Anneal strategy that can get the ideal structure finally has the same important influence on the optimization, which also has the important effect on PMU optimal placement. SA algorithm can find the project, which can reach the system observability and need the least PMUs at strategic locations. MST method is a modified DFS approach; it holds the excellence of high-speed of DFS, improves the shortcomings of the poor systematization and complex arithmetic at the same time. MST method improves the optimization rule using pseudo-measurement, which can find manifold placement strategy in the context of ensuring the solution space. The simulation results have demonstrated that MST method is excelling the other two methods in entirety and multiformity of the results. Comparison of the convergence speed, the whole convergence and the multiformity of the results of the three methods (normal operation) are given in Table 9.

Table 9. Comparison between the Three Algorithms

Algorithms	Convergent speed	Whole convergence	Multiformity
DFS	excellent	poor	Poor
SA	poor	excellent	Poor
MST	excellent	excellent	Excellent

Table 10 compares the results of the given algorithms with that of TS, Graph theoretic, Iterated local search, Binary Particle Swarm (BPSO), Modified Invasive Weed Optimization (MIWO), Artificial bee colony algorithm and integer programming (IP). The algorithms are implemented and tested for IEEE 14, 118, and New England 39 bus systems. The results are given and compared to other methods. The first method is very fast in providing a solution as compared to other search methods, but not the optimal solution as the other algorithms. Simulation results for different network show the effectiveness of the methods in finding the minimum optimal number of PMU bus locations for full observability assessment of Power Systems.

Table 10. Comparison of Total Number of PMUS Obtained by Several Methods

Test System	14-Bus	39-Bus	118-Bus
Depth first search	6	16	41
Simulated Annealing	4	9	29
Minimum Spanning Tree	3	9	31
Recursive N-1 Algorithm	8	18	63
Single shot N-1 security	8	18	72
Tabu search (Jiangnan Peng et al.,2006)	3	10	N/A
Integer linear programming (Cai and Ai, 2005)	3	N/A	29
Graph theoretic (Farsadi et al.,2009)	5	N/A	N/A
Iterated local search (Dua et al.,2008)	4	N/A	32
BPSO (Ahmadi et al.,2011)	3	N/A	29
MIWO (Sudha et al.,2012)	4	N/A	N/A
Artificial bee colony algorithm (Tankasala et al.,2012)	4	N/A	32

N/A: not available

Compared to the algorithms in Tankasala et al., (2012) and Dua et al.,(2008) Minimum spanning tree can guarantee the minimum number of PMUs, and results in relatively smaller measurement. Compared to the algorithm in Jiangnan Peng et al., (2006) the algorithms are simpler and easier to be implemented.

The results of Table 10 indicate that:
1. No specified technique could be considered as absolute optimal technique for all cases;
2. The Depth First Search gives the highest PMUs number because it does not consider the zero injections and further its solution is not optimal;
3. In case of not considering the zero-injections, Simulated Annealing Method (SA) is giving the minimum numbers of PMUs, but it suffers from inconvenience regarding complete observability in large systems;
4. Less number of PMUs will be required if the zero-injections buses are considered;
5. In case of any single branch forced outage, higher number of the PMUs is required and hence the cost will be increased.
6. In case of studying single branch outage, the Rec. (N-1) Spanning Tree is better than the single shot security method.

7. Conclusions

In this paper, the problem of determining the minimum number of the PMUs and their placement to ensure that a power system is a topologically observable has been presented. An observability assessment based on topological analysis was introduced. Single branch outages as well as normal operating condition have been efficiently studied. The algorithms has been applied to test different algorithms to solve the OPP for IEEE standard 14-bus, 39-bus and 118-bus test systems. The OPP results have been discussed and compared to demonstrate the effectiveness of the different applied methods. Several factors such as the number and location of installed PMUs, branch outage and the system topology have been accomplished the OPP. The obtained information for PMUs can be used as a new way provided for the security analysis and stability control in power system operation. The optimal PMU placement decreases the number of PMUs which redounds cost declining, and obtains better power network operation and monitoring.

Nomenclature

PMU	Phasor Measurement Unit	GPS	Global Positioning System
OPP	Optimal PMU Placement	WAMS	Wide Area Measurement System
PDC	Phasor Data Concentrtor		

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