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Double-diffusive convection of compressible rotating Walters' (B′) fluid with Hall currents saturating a porous medium

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Abstract

Keeping in view the conflicting tendencies of rotation and Hall currents (magnetic field) while acting together; combined effects of Hall currents and rotation are considered on the hydromagnetic stability of a compressible Walters' (Model B′) elastico-viscous fluid heated and soluted from below saturating a porous medium. Boussinesq approximation is used to simplify the complex hydromagnetic equations and the perturbation equations are analyzed in terms of normal modes. A dispersion relation governing the effects of visco-elasticity, salinity gradient, rotation, Hall currents and medium permeability is derived. It has been found that for stationary convection, Walters' (Model B′) fluid behaves like an ordinary Newtonian fluid due to the vanishing of the visco-elastic parameter. Compressibility, solute gradient, rotation and magnetic field postpone the onset of instability as such their effect is to stabilize the system. Hall currents and medium permeability are found to hasten the onset of instability for permissible range of values of various parameters. The dispersion relation is analyzed numerically and the effects of various parameters for permissible range of values are depicted graphically. The visco-elasticity, solute gradient and Hall currents (hence magnetic field) introduce oscillatory modes in the system which were non-existent in their absence. Also the case of overstability is discussed and sufficient conditions for the non-existence of overstability are derived.

Keywords: Walters' (Model B′) fluid, rotation, Hall currents, thermosolutal instability, compressibility, porous medium.

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1. Introduction

Chandrasekhar (1981) in his celebrated monograph considered a detailed account of the theoretical and experimental results for the onset of thermal instability (Bénard convection) for Newtonian viscous/inviscid fluids under varying assumptions of hydrodynamics and hydromagnetics. In the standard Bénard problem instability is driven by density difference caused by a temperature difference between upper and the lower planes bounding the fluid. If the fluid additionally has salt dissolved in it then there are potentially two destabilizing sources for the density difference, the temperature field and the salt field. The heat and solute being two diffusing components, double-diffusive convection/thermosolutal convection is a general term dealing with such phenomenon. This double-diffusive phenomenon has been extensively studied recently due to its direct relevance in the field of chemical engineering, astrophysics and oceanography.

The investigation of flow of fluids through porous medium has become an important topic due to the recovery of crude oil from the pores of reservoir rocks. A great number of applications of such a flow in geophysics are found in a book by Philips (1991). The effect of the earth's magnetic field on the stability of such a flow is of interest in geophysics particularly in the study of earth's core where the earth's mantle, which consists of conducting fluids, behaves like a porous medium. When fluid flow is considered in a porous medium, some additional complexities arise which are principally due to the interactions between the fluid and the porous material. We will consider those fluids for which Darcy's law is applicable, which states that the gross effect, as the fluid slowly percolates through the pores of rock, is that usual viscous term in the equation of elastico-viscous fluid motion will be replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\boldsymbol{q}\right]$, where μ and μ' are the coefficients of viscosity and viscoelasticity , k_1 is the medium permeability and $\boldsymbol{q}$ is the Darcian (filter) velocity of the fluid. The stability of flow of a single component fluid through porous medium taking into account the Darcy's resistance has been studied by Lapwood (1948) and Wooding (1960). When the fluids are compressible, the equations governing the system become quite complicated. Spiegel and Veronis (1960) simplified the set of equations governing the flow of compressible fluids assuming that the depth of the fluid layer is much smaller than the scale height, as defined by the author's, and the motions of infinitesimal amplitude are considered. Sharma and Gupta (1993) investigated the effect of porosity on the thermal instability of compressible fluid with Hall currents and suspended particles. Thermal instability of compressible, finite Larmor radius Hall plasma has been studied by Sharma and Sunil (1996) in a porous medium. The Hall current is important in flows of laboratory plasmas as well as in many geophysical and astrophysical situations. Sherman and Sutton (1962) have considered the effect of Hall currents on the efficiency of a magneto-hydro dynamic (MHD) generator. Numerous problems on effect of Hall currents under varying assumptions of hydrodynamics and hydromagnetics for Newtonian fluids have been attempted by many researchers in the past, e.g. Gupta (1967), Sharma and Gupta (1990), Sharma and Sunil (1995), Chauhan and Agrawal (2011), Guchhait *et al.*(2011) and Prasad and Kumar (2012) to name a few among several others. In all the above mentioned studies, fluids have been considered to be Newtonian.

In the last two decades with the advancement of studies for polymeric solutions and other viscoelastic fluids, many scientists and researchers focused their attention to study non-Newtonian fluid flow problems. The pioneering and fundamental works are of Bhatia and Steiner (1972), Oldroyd's (1958), Rivlin and Ericksen (1955) and Walters (1960) on various viscoelastic fluids. Walters (1962) reported that the mixture of polymethyl methacrylate and pyridine at 25^0C containing 30.5 g of polymer per liter behaves very nearly as the Walters' (Model B′) fluid. In the last decade, quite a number of authors investigated fluid flow problems on these viscoelastic fluids. Some of these are Sunil *et al.* (2000a), Sharma *et al.* (2006), Gupta and Sharma (2007, 2008), Kumar *et al.* (2004), Gupta and Kumar (2010), Gupta and Aggarwal (2011). But none of the authors have studied the combined effect of rotation and Hall currents on thermosolutal instability problem for Walters' (Model B′) fluid. This combined effect in the study of Walters' (Model B′) fluid is very important and interesting due to the interacting and conflicting effects of rotation and Hall currents when applied together. It is worth while to mention here that magnetic field has stabilizing effect where as Hall currents and permeability have destabilizing effects in the absence of rotation. But in the presence of rotation the effects of magnetic field, Hall currents and permeability are stabilizing / destabilizing depending upon the conditions as derived later in the results and discussion section. Therefore keeping in view the conflicting tendencies of Hall currents, and rotation applied together and various applications of viscoelastic fluids in chemical technology and paper industry, the present problem of double-diffusive convection for compressible Walters' fluid has been investigated. Some earlier known results have been recovered from the present formulation.

2. Mathematical Formulation of the Problem and Perturbation Equations

We have considered an infinite, horizontal, compressible electrically conducting Walters' (Model B′) fluid layer of thickness d in a homogenous medium of porosity ε and medium permeability k_1 which is heated and soluted from below ($z=0$) so that temperature and concentration at bottom is T_0 and C_0 and at the upper layer ($z=d$) is T_d and C_d respectively, as shown in Figure 1. A uniform temperature gradient $\beta(=\left|dT/dz\right|)$ and concentration gradient $\beta'(=\left|dC/dz\right|)$ are maintained. The fluid is acted upon by the gravity force $\mathbf{g}=(0,0,-g)$, uniform vertical magnetic field $\mathbf{H}=(0,0,H)$ and uniform vertical rotation $\mathbf{\Omega}=(0,0,\Omega)$.

Let $T, p, \rho, C, \alpha, \alpha', g, \eta, \mu_e, N, e, \nu, \nu', \kappa, \kappa'$ and $\mathbf{q}=(u,v,w)$ denote, respectively, temperature, pressure, density, concentration, thermal coefficient of expansion, solvent coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability, electron number density, charge on an electron, kinematic viscosity, kinematic viscoelasticity, thermal diffusivity, solute diffusivity and fluid velocity.

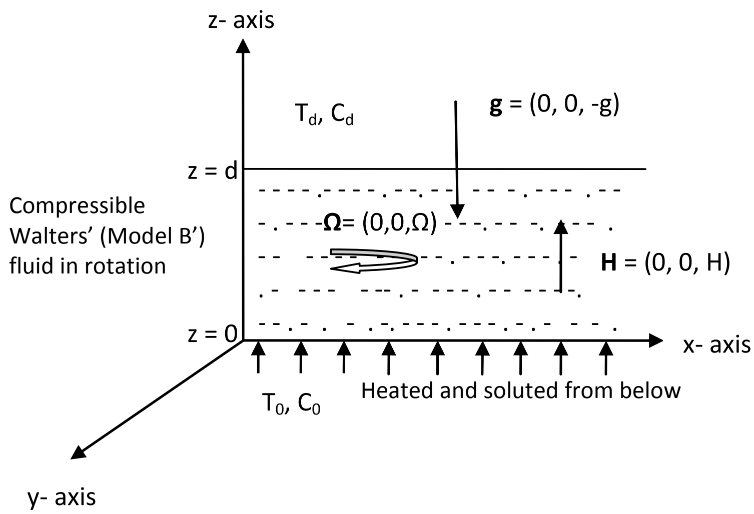

Figure 1. Geometrical Configuration

The equations expressing conservation of momentum, mass, temperature, solute concentration and equation of state after using Boussinesq approximation (see Chandrasekhar, 1981; Walters, 1960 and Joseph, 1976) are

$$\frac{1}{\varepsilon}\left\{\frac{\partial \boldsymbol{q}}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{q}.\nabla)\boldsymbol{q}\right\}=-\left(\frac{1}{\rho_m}\right)\nabla p-\frac{1}{k_1}\left(\nu-\nu'\frac{\partial}{\partial t}\right)\boldsymbol{q}+\boldsymbol{g}\left(1+\frac{\delta\rho}{\rho_m}\right)+\frac{\mu_e}{4\pi\rho_m}\left(\nabla\times\boldsymbol{H}\right)\times\boldsymbol{H}+\frac{2}{\varepsilon}(\boldsymbol{q}\times\boldsymbol{\Omega}), \tag{1}$$

$$\nabla.\boldsymbol{q} = 0, \tag{2}$$

$$E\frac{\partial T}{\partial t}+\left(\boldsymbol{q}.\nabla\right)T=\kappa\nabla^2 T, \tag{3}$$

$$E'\frac{\partial C}{\partial t}+\left(\boldsymbol{q}.\nabla\right)C=\kappa'\left(\nabla^2 C\right), \tag{4}$$

$$\rho=\rho_m\left[1-\alpha(T-T_0)+\alpha'(C-C_0)\right], \tag{5}$$

where $E=\varepsilon+(1-\varepsilon)\left(\rho_s c_s/\rho_0 c_f\right)$ is a constant and E' is a constant analogous to E but corresponding to solute rather than heat and ρ_s, c_s, ρ_0 and c_f denote the density and heat capacity of solid (porous) matrix and fluid matrix, respectively.

In the present model, we have ignored the non-Newtonian effects of second-order fluids on heat transportation in comparison to other terms in heat equation and assume that viscoelastic effects influence the heat transport only through velocity. From Maxwell's equations for a porous medium, we have

$$\varepsilon\frac{d\boldsymbol{H}}{dt}=(\boldsymbol{H}.\nabla)\boldsymbol{q}+\varepsilon\eta\nabla^2\boldsymbol{H}-\frac{\varepsilon}{4\pi Ne}\nabla\times[(\nabla\times\boldsymbol{H})\times\boldsymbol{H}], \tag{6}$$

$$\nabla.\boldsymbol{H}=0, \tag{7}$$

where $\frac{d}{dt}=\frac{\partial}{\partial t}+q.\nabla$ stands for convective derivative. The state variables pressure, density, and temperature are expressed in the form [Speigal and Veronis, (1960)]

$$f(x,y,z,t) = f_m + f_0(z) + f'(x,y,z,t), \tag{8}$$

f_m stands for constant space distribution of f, f_0 is the variation in the absence of motion and $f'(x,y,z,t)$ is the fluctuation resulting from motion. For initial state, we have

$p = p(z), \rho = \rho(z), T = T(z), C = C(z), \mathbf{q} = (0,0,0)$ and $\mathbf{H} = (0,0,H)$, where

$$p(z) = p_m - g\int_0^z (\rho_m + \rho_0)dz,$$

$$\rho(z) = \rho_m\left[1 - \alpha_m(T - T_0) + \alpha'_m\left(C - C_0\right) + K_m(p - p_m)\right],$$

$$T(z) = -\beta z + T_0, C(z) = -\beta' z + C_0,$$

$$\alpha_m = -\left(\frac{1}{\rho}\frac{\partial \rho}{\partial T}\right)_m (= \alpha, say), \quad \alpha'_m = -\left(\frac{1}{\rho}\frac{\partial \rho}{\partial C}\right)_m (= \alpha', say), K_m = \left(\frac{1}{\rho}\frac{\partial \rho}{\partial p}\right)_m . \tag{9}$$

Here p_m and ρ_m stand for a constant space distribution of p and ρ. Linearized stability theory and normal mode analysis method is used to study infinitesimal perturbation and depth of fluid layer is assumed to be much less than the scale height as defined by Spiegal and Veronis (1960). Using these assumptions and results for compressible fluids, the flow equations are found to be the same as those for incompressible fluids except that the static temperature gradient β is replaced by its excess over the adiabatic $\left(\beta - g/C_p\right)$. In our analysis we have considered a small perturbation on steady state solution and let $\delta p, \delta\rho, \theta, \gamma$, $\mathbf{h} = (h_x, h_y, h_z)$ and $\mathbf{q} = (u, v, w)$ denote the perturbations in pressure, density, temperature, solute concentration, magnetic field and velocity respectively. The change in density $\delta\rho$ is given by

$$\delta\rho = -\rho_m(\alpha\theta - \alpha'\gamma). \tag{10}$$

Then the linearized hydromagnetic perturbation equations are

$$\frac{1}{\varepsilon}\left\{\frac{\partial \boldsymbol{q}}{\partial t}\right\} = -\frac{1}{\rho_m}(\nabla\delta p) - \frac{1}{k_1}\left(\nu - \nu'\frac{\partial}{\partial t}\right)\boldsymbol{q} - \boldsymbol{g}(\alpha\theta - \alpha'\gamma) + \frac{\mu_e}{4\pi\rho_m}(\nabla\times\boldsymbol{h})\times\boldsymbol{H} + \frac{2}{\varepsilon}(\boldsymbol{q}\times\boldsymbol{\Omega}), \tag{11}$$

$$\nabla . \boldsymbol{q} = 0, \tag{12}$$

$$E\frac{\partial\theta}{\partial t} = \left(\beta - \frac{g}{C_p}\right)w + \kappa\nabla^2\theta, \tag{13}$$

$$E'\frac{\partial\gamma}{\partial t} = \beta' w + \kappa'\left(\nabla^2\gamma\right), \tag{14}$$

$$\nabla . \boldsymbol{h} = 0, \tag{15}$$

$$\varepsilon\frac{\partial\boldsymbol{h}}{\partial t} = (\boldsymbol{H}.\nabla)\boldsymbol{q} + \varepsilon\eta\nabla^2\boldsymbol{h} - \frac{\varepsilon}{4\pi Ne}\nabla\times[(\nabla\times\boldsymbol{h})\times\boldsymbol{H}]. \tag{16}$$

3. Normal Mode Analysis Method and Dispersion Relation

In the present study, we have used normal mode analysis method and assumed that perturbation quantities are of the form

$$[w, h_z, \theta, \gamma, \zeta, \xi] = [W(z), K(z), \Theta(z), \Gamma(z), Z(z), X(z)] \times \exp(ik_x x + ik_y y + nt), \tag{17}$$

where k_x and k_y are the wave numbers along x and y directions and resultant wave number is given by $k = \left(k_x^2 + k_y^2\right)^{1/2}$ and n is the growth rate. Also, $\zeta = \partial v/\partial x - \partial u/\partial y$ is the z-component of vorticity and $\xi = \partial h_y/\partial x - \partial h_x/\partial y$ is the z-component of current density.

Using expression (17), equations (11) – (16) can be rewritten as

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_1}(1-\sigma F)\right](D^2 - a^2)W + \frac{ga^2d^2}{\nu}(\alpha\Theta - \alpha'\Gamma) - \frac{\mu_e H d}{4\pi\rho_m \nu}(D^2 - a^2)DK + \frac{2\Omega d^3}{\varepsilon \nu} DZ = 0, \tag{18}$$

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_1}(1-\sigma F)\right]Z - \frac{\mu_e H d}{4\pi\rho_m \nu} DX - \frac{2\Omega d}{\varepsilon \nu} DW = 0, \tag{19}$$

$$[D^2 - a^2 - Ep_1\sigma]\Theta + \frac{\beta d^2}{\kappa}\left(\frac{G-1}{G}\right)W = 0, \tag{20}$$

$$[D^2 - a^2 - E'q\sigma]\Gamma + \left(\frac{\beta' d^2}{\kappa'}\right)W = 0, \tag{21}$$

$$[D^2 - a^2 - p_2\sigma]K + \left(\frac{Hd}{\varepsilon\eta}\right)DW - \frac{Hd}{4\pi N e \eta} DX = 0, \tag{22}$$

$$[D^2 - a^2 - p_2\sigma]X + \left(\frac{Hd}{\varepsilon\eta}\right)DZ + \frac{H}{4\pi N e \eta}(D^2 - a^2)K = 0, \tag{23}$$

where various non-dimensional parameters used are as follows

$$a = kd,\quad \sigma = \frac{nd^2}{\nu},\quad p_1 = \frac{\nu}{\kappa},\quad p_2 = \frac{\nu}{\eta},\quad q = \frac{\nu}{\kappa'},\quad F = \frac{\nu'}{d^2},\quad P_1 = \frac{k_1}{d^2}, G = \left(\frac{C_p}{g}\right)\beta, x^* = \frac{x}{d}, y^* = \frac{y}{d}, z^* = \frac{z}{d}, D = \frac{d}{dz^*}.$$

Consider the case of two free boundaries which are perfect conductors of both heat and solute concentration. For the case of free boundaries the boundary conditions are (see Chandrasekhar, 1981)

$W = D^2W = 0,\ DZ = 0,\quad \Theta = 0,\quad \Gamma = 0$ at $z = 0$ and 1, $K = 0$, on perfectly conducting boundaries (24)

and h_x, h_y, h_z are continuous. Since the components of magnetic field are continuous and the tangential components are zero outside the fluid, we have

$DK = 0$, on the boundaries. (25)

Using the boundary conditions (24) and (25), it can be shown that all the even order derivatives of W must vanish for $z = 0$ and 1. Therefore the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z, \tag{26}$$

where W_0 is a constant. After eliminating θ, X, Z, Γ and K between equations (18) - (23), we obtain

$$R_1 = \left(\frac{G}{G-1}\right)\left\langle\left(\frac{1+x}{x}\right)\left[\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}(1-i\sigma_1\pi^2F)\right](1+x+i\sigma_1Ep_1)+S_1\frac{(1+x+iE\sigma_1p_1)}{(1+x+iE'\sigma_1q)}+\left(\frac{(1+x+i\sigma_1p_1)}{x}\right)\times\right.$$
$$\times\left[Q_1(1+x)\left\{\left[\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}(1-i\sigma_1\pi^2F)\right]\left(1+x+i\sigma_1p_2\right)+Q_1+2\sqrt{T_1M}\right\}+T_1\{(1+x+i\sigma_1p_2)^2+M(1+x)\}\right]\times$$
$$\left.\times\left\{(1+x+i\sigma_1p_2)^2\left[\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}(1-i\sigma_1\pi^2F)\right]+Q_1(1+x+i\sigma_1p_2)+M(1+x)\left[\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}(1-i\sigma_1\pi^2F)\right]\right\}^{-1}\right\rangle, \tag{27}$$

where

$$R_1=\frac{g\alpha\beta d^4}{\nu\kappa\pi^4},\ S_1=\frac{g\alpha'\beta'd^4}{\nu\kappa'\pi^4},\ Q_1=\frac{\mu_eH^2d^2}{4\pi^3\rho_m\nu\eta\varepsilon},\ M=\left(\frac{H}{4\pi Ne\eta}\right)^2,\ T_1=\frac{4\Omega^2d^4}{\nu^2\pi^4\varepsilon^2},\ P=\pi^2P_1,\ x=\frac{a^2}{\pi^2}\ \text{and}\ \ i\sigma_1=\frac{\sigma}{\pi^2}.$$

Equation (27) is the dispersion relation including the effects of rotation, Hall currents, compressibility and solute gradient on the thermosolutal instability of Walters' (Model B′) fluid in porous medium.

4. Results and Discussion

(i) Case of Stationary Convection

Consider the case when instability sets in the form of stationary convection. For stationary convection, $\sigma_1 = 0$ and the dispersion relation (27) reduces to

$$R_1=\left(\frac{G}{G-1}\right)\left[\frac{1}{P}\frac{\left(1+x\right)^2}{x}+S_1+\left(\frac{1+x}{x}\right)\times\left\{\frac{Q_1\left[\left(1+x\right)+PQ_1+2P\sqrt{T_1M}\right]+PT_1\left[1+x+M\right]}{[\left(1+x\right)+PQ_1+M]}\right\}\right]. \tag{28}$$

The above equation expresses the modified Rayleigh number R_1 as a function of dimensionless wave number, x, and the parameters S_1, Q_1, G, M and T_1. For stationary convection, the viscoelastic parameter F vanishes with σ_1 and the Walters' (Model B′) fluid behaves like an ordinary Newtonian fluid. Keeping the non-dimensional number G (accounting for compressibility) as fixed, we get

$$\overline{R_c}=\left(\frac{G}{G-1}\right)R_c, \tag{29}$$

where R_c and $\overline{R_c}$ denote, respectively , the critical Rayleigh numbers in the absence and presence of compressibility. Thus, the effect of compressibility is to postpone the onset of thermosolutal instability. The cases $G < 1$ and $G = 1$ correspond to negative and infinite values of Rayleigh numbers due to compressibility which are not relevant in the present study. To investigate the effect of combined presence of Hall currents and rotation for thermosolutal convection in porous medium, we examine the natures of dR_1/dS_1, dR_1/dQ_1, dR_1/dM, dR_1/dT_1 and dR_1/dP analytically and numerically. Equation (28) yields

$$\frac{dR_1}{dS_1}=\left(\frac{G}{G-1}\right), \tag{30}$$

which shows that solute gradient has a stabilizing effect on thermosolutal convection. Numerically, R_1 as given in equation (28), is plotted against x for $G=10, Q_1=100, M=10, T_1=10^3$ and for different values of S_1 = 100, 200, 300, 400, 500 in Figure 2. Here we would like to mention that the values of R_1 are calculated in MS-Excel for different values of other parameters involved. It is clear from the figure that the Rayleigh number increases with an increase in S_1 and establishes the stabilizing effect of solute gradient. To analyze the effect of magnetic field, expression (28) yields

$$\frac{dR_1}{dQ_1}=\left(\frac{G}{G-1}\right)\left(\frac{1+x}{x}\right)\left\{\left[1+x+M\right]\left[\left(1+x\right)+P(2Q_1+2\sqrt{T_1M}-PT_1)\right]+P^2Q_1^2\right\}\left\{[(1+x)+PQ_1+M)]^2\right\}^{-1}, \tag{31}$$

which implies that magnetic field has stabilizing/destabilizing effect depending upon whether

$$\left[\left(1+x\right)+P(2Q_1+2\sqrt{T_1M})\right]>P^2T_1 \quad \text{or} \quad \left[\left(1+x\right)+P(2Q_1+2\sqrt{T_1M})\right]<P^2T_1.$$

This destabilizing effect of magnetic field occurs in the presence of rotation. For $T_1 = 0$,

$$\frac{dR_1}{dQ_1}=\left(\frac{G}{G-1}\right)\left(\frac{1+x}{x}\right)\frac{\left\{\left[1+x+M\right]\left[\left(1+x\right)+2PQ_1\right]+P^2Q_1^2\right\}}{\left[(1+x)+PQ_1+M)\right]^2}, \tag{32}$$

which shows the stabilizing effect of magnetic field. We have plotted R_1 against the scaled wavenumber, x, for $G = 10$, $P = 0.001$, $S_1 = 100$, $M = 10$, $T_1=10^3$ and for various values of $Q_1 = 100$, 150, 200, 250 and 300 in Figure 3. It is clear from the figure that R_1 increases with an increase in Q_1, confirming the stabilizing effect of magnetic field. Here, it is worthwhile to mention that the former condition mentioned above, i.e.,

$$\left[\left(1+x\right)+P(2Q_1+2\sqrt{T_1M})\right]>P^2T_1$$

holds true for the values of various parameters under consideration. The result is in agreement with that of Gupta and Sharma (2008) for Rivlin-Ericksen fluids. In Figure 4, R_1 is plotted against the scaled wavenumber, x, for $T_1 = 0$. The figure clearly exhibits the stabilizing effect of magnetic field. The earlier work of Sunil *et al*. (2004) is a particular case of the present work in the absence of rotation for a non-porous medium.

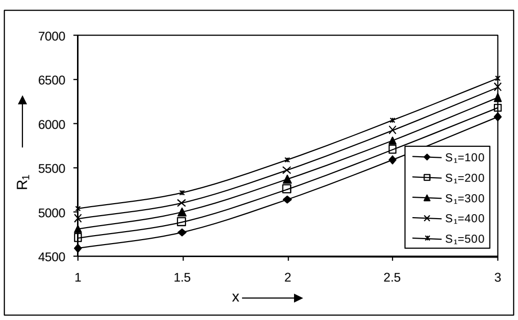

Figure 2. Variation of Rayleigh number R_1 with wavenumber x for fixed $G=10$, $P = 0.001$, $Q_1 = 100$, $M=10$, $T_1 = 1000$ and for various values of $S_1 = 100, 200, 300, 400$ and 500.

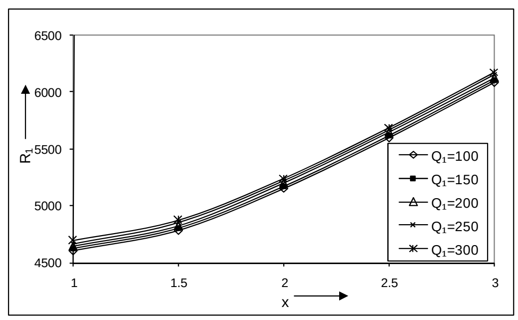

Figure 3. Variation of Rayleigh number R_1 with wavenumber x for fixed $G=10$, $P = 0.001$, $S_1 = 100$, $M=10$, $T_1 = 1000$ and for various values of $Q_1 = 100, 150, 200, 250$ and 300.

In the absence of Hall currents, the above expression for the derivative reduces to

$$\frac{dR_1}{dQ_1}=\left(\frac{G}{G-1}\right)\left(\frac{1+x}{x}\right)\frac{\left\{\left[(1+x)+PQ_1\right]^2-P^2T_1(1+x)\right\}}{\left[(1+x)+PQ_1)\right]^2}, \tag{33}$$

which is in agreement with the counterpart presented by Sharma and Bhardwaj (1993), reflecting the stabilizing/destabilizing effect of magnetic field in the presence of rotation. Thus, in the absence of Hall currents and presence of rotation, magnetic field has stabilizing or destabilizing effect depending on whether

$$\left[(1+x)+PQ_1)\right]^2>P^2T_1(1+x) \quad \text{or} \quad \left[(1+x)+PQ_1)\right]^2<P^2T_1(1+x).$$

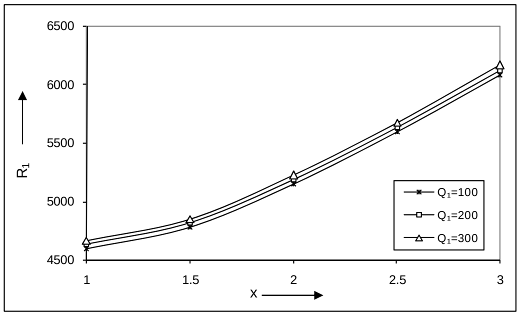

Figure 4. Variation of Rayleigh number R_1 with wavenumber x for fixed $G=10$, $P=0.001$, $S_1=100$, $M=10$, $T_1=0$ and for various values of $Q_1=100, 200$ and 300.

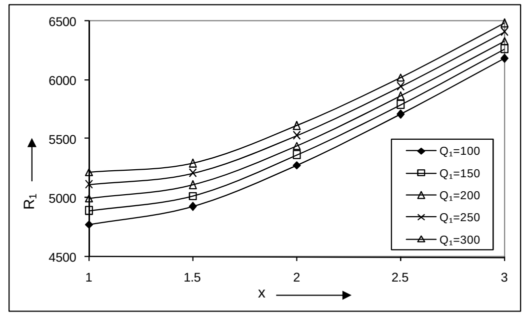

Figure 5. Variation of Rayleigh number R_1 with wavenumber x for fixed $G=10$, $P=0.001$, $S_1=100$, $M=0$, $T_1=1000$ and for various values of $Q_1=100, 150, 200, 250$ and 300.

But for permissible values of various parameters involved, the already mentioned effect is stabilizing as

$$\left[(1+x)+PQ_1)\right]^2 > P^2 T_1 (1+x),$$

is the only condition which is satisfied. This can also be seen graphically as shown in Figure 5, where R_1 is plotted against x for different values of Q_1 in the absence of Hall currents and presence of rotation.

For analyzing the effect of Hall currents, we obtain the expression

$$\frac{dR_1}{dM} = -\left(\frac{G}{G-1}\right)\left(\frac{1+x}{x}\right)Q_1\left\{\left[\left(1+x\right)+PQ_1+P\sqrt{T_1 M}\right]\left[1-P\sqrt{T_1/M}\right]\right\}\left\{[(1+x)+PQ_1+M)]^2\right\}^{-1}, \tag{34}$$

which states that Hall currents have stabilizing/destabilizing effect depending on whether $P\sqrt{T_1/M} > 1$ or $P\sqrt{T_1/M} < 1$. But for the permissible range of values of various parameters under consideration, this effect is destabilizing since $P\sqrt{T_1/M} < 1$ is the only condition which is satisfied. In the absence of rotation, the above expression reduces to

$$\frac{dR_1}{dM} = -\left(\frac{G}{G-1}\right)\left(\frac{1+x}{x}\right)Q_1\left\{\frac{\left[(1+x)+PQ_1\right]}{\left[(1+x)+PQ_1+M)\right]^2}\right\}, \tag{35}$$

establishing the usual destabilizing effect of Hall currents. These results are in agreement with the numerical/graphical results of Figures 6 and 7 where R_1 is plotted against the scaled wavenumber, x, for $G=10$, $Q_1=100$, $S_1=100$ and for $T_1=10^3$ and $T_1=0$, respectively, for various values of $M\left(=10, 30, 50\right)$. Expression for observing effect of rotation is obtained as

$$\frac{dR_1}{dT_1} = \left(\frac{G}{G-1}\right)\left(\frac{1+x}{x}\right)P\left\{\frac{\left[1+x+M+Q_1\sqrt{M/T_1}\right]}{\left[(1+x)+PQ_1+M)\right]}\right\}, \tag{36}$$

which reflects the stabilizing influence of rotation. This is in agreement with the corresponding result for Rivlin-Erickson fluids as derived by Gupta and Sharma (2007). In Figure 8, R_1 increases with the increase in T_1 which confirms the aforementioned result.

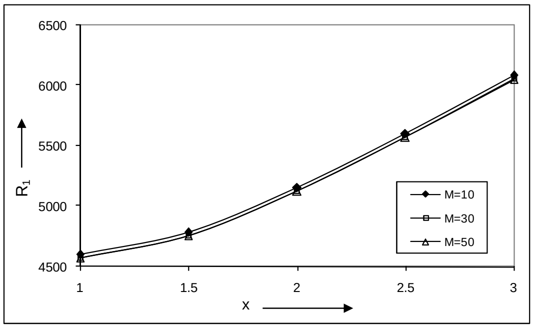

Figure 6. Variation of Rayleigh number R_1 with wavenumber x for fixed G =10, P = 0.001, Q_1 = 100, S_1 = 100, T_1 =1000 and for various values of M = 10, 30 and 50.

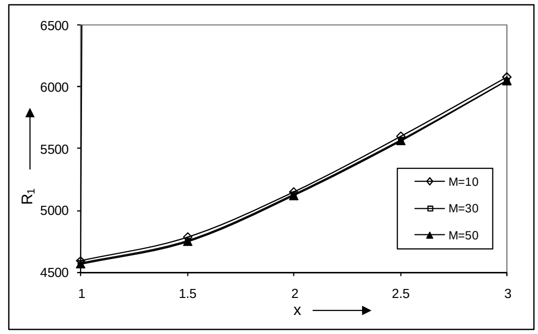

Figure 7. Variation of Rayleigh number R_1 with wavenumber x for fixed G =10, P = 0.001, Q_1 = 100, S_1 = 100, T_1 = 0 and for various values of M = 10, 30 and 50.

For analyzing the effect of medium permeability, we obtain

$$\frac{dR_1}{dP} = \left(\frac{G}{G-1}\right)\left(\frac{1+x}{x}\right)\frac{1}{P^2}\Big\{P^2\left[Q_1^{\,2}M + 2Q_1\sqrt{T_1M}\left(1+x+M\right) + T_1M^2 + 2T_1M(1+x)\right] + $$
$$+(1+x)\left[P^2T_1(1+x) - \left\{(1+x)+PQ_1+M\right\}^2\right]\Big\}\left\{(1+x)+PQ_1+M\right\}^{-2}. \tag{37}$$

Pondering this equation meticulously, it is seen that permeability has a stabilizing/destabilizing effect depending upon whether $P^2T_1(1+x) > \left\{(1+x)+PQ_1+M\right\}^2$ or $P^2T_1(1+x) < \left\{(1+x)+PQ_1+M\right\}^2$.

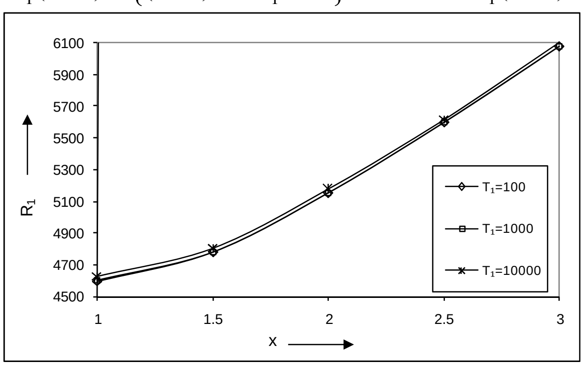

Figure 8. Variation of Rayleigh number R_1 with wavenumber x for fixed G = 10, P = 0.001, Q_1 = 100, S_1 = 100, M = 10 and for fixed values of T_1 = 100, 1000 and 10000.

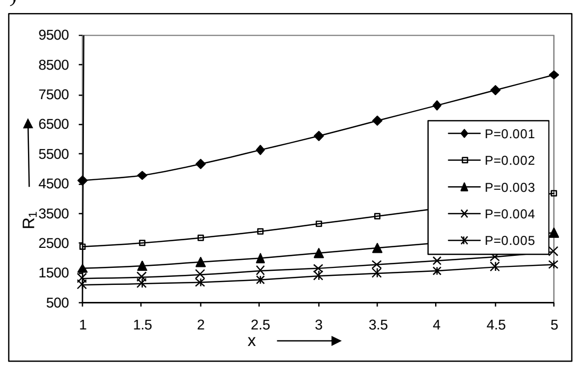

Figure 9. Variation of Rayleigh number R_1 with wavenumber x for fixed G =10, Q_1 = 100, S_1=100, M =10, T_1 =1000 and for various values of P = 0.001, 0.002, 0.003, 0.004, 0.005.

In Figure 9, R_1 is plotted against the scaled wavenumber x for values of P = (0.001, 0.002, 0.003, 0.004, 0.005). One may confirm that R_1 decreases with the increase in P. This shows the destabilizing influence of medium permeability for the values of parameters under consideration (satisfying the latter condition) i.e.

$$P^2T_1(1+x) \quad < \quad \left\{(1+x)+PQ_1+M\right\}^2.$$

In the absence of rotation, the expression reduces to

$$\frac{dR_1}{dP} = \left(\frac{G}{G-1}\right)\left(\frac{1+x}{x}\right)\frac{1}{P^2}\,\frac{\left\{P^2MQ_1^2 - (1+x)\left[(1+x)+PQ_1+M\right]^2\right\}}{\left\{(1+x)+PQ_1+M\right\}^2}, \tag{38}$$

which shows that the permeability has destabilizing/stabilizing effect depending on whether

$$P^2MQ_1^2 < (1+x)\left[(1+x)+PQ_1+M\right]^2 \text{ or } P^2MQ_1^2 > (1+x)\left[(1+x)+PQ_1+M\right]^2.$$

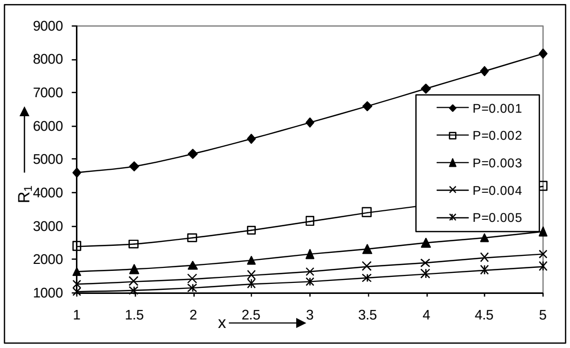

Figure 10. Variation of Rayleigh number R_1 with wavenumber x for fixed G =10, Q_1 = 100, S_1=100, M =10, T_1 = 0 and for various values of P = 0.001, 0.002, 0.003, 0.004, 0.005.

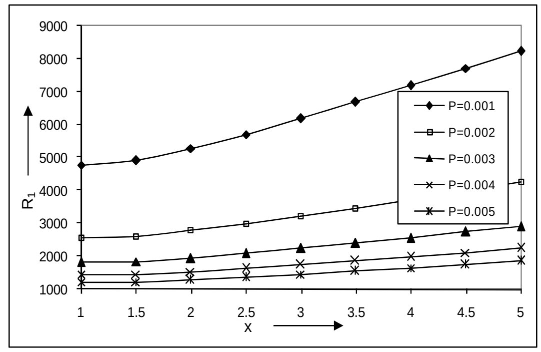

Figure 11. Variation of Rayleigh number R_1 with wavenumber x for fixed G =10, Q_1 = 100, S_1=100, M = 0, T_1 =1000 and for various values of P = 0.001, 0.002, 0.003, 0.004, 0.005.

Figure 10 confirms the destabilizing influence of permeability in the absence of rotation as the later of the two inequalities does not hold true for the permissible values of various parameters. This result is identical to that of Gupta and Sharma (2008). Further, in the absence of Hall currents, the expression reduces to

$$\frac{dR_1}{dP}=\left(\frac{G}{G-1}\right)\left(\frac{(1+x)^2}{x}\right)\frac{1}{P^2}\frac{\left\{P^2T_1(1+x)-\left[(1+x)+PQ_1\right]^2\right\}}{\left\{(1+x)+PQ_1\right\}^2}, \tag{39}$$

which shows that the permeability has destabilizing/stabilizing effect depending on whether

$$P^2T_1(1+x)<\left[(1+x)+PQ_1\right]^2 \quad \text{or} \quad P^2T_1(1+x)>\left[(1+x)+PQ_1\right]^2 .$$

Again Figure 11 confirms the destabilizing influence of medium permeability in the absence of Hall currents as the later of the two inequalities does not hold true for the considered values of various parameters. In the absence of both rotation and Hall currents, the expression becomes

$$\frac{dR_1}{dP}=-\left(\frac{G}{G-1}\right)\left(\frac{(1+x)^2}{x}\right)\frac{1}{P^2}, \tag{40}$$

which accounts for the usual destabilizing influence of medium permeability in the absence of rotation and Hall currents. This is in agreement with the result of Figure 12, where R_1 is plotted against x for various values of P = 0.001, 0.002, 0.003, 0.004, 0.005.

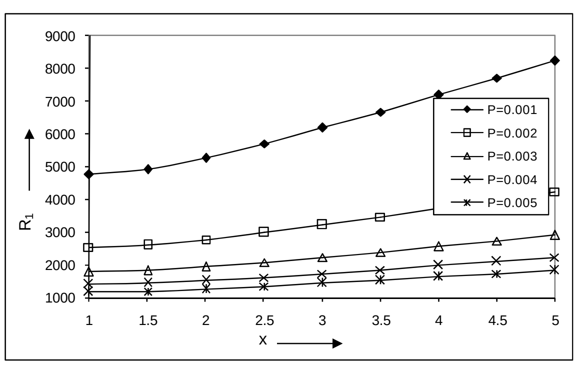

Figure 12. Variation of Rayleigh number R_1 with wavenumber x for fixed G = 10 , Q_1 = 100, S_1=100, M = 0, T_1 =0 and for various values of P = 0.001, 0.002, 0.003, 0.004, 0.005.

(ii) Stability of the System and Oscillatory Modes

To determine the possibility of oscillatory modes we multiply equation (18) by W^*, the complex conjugate of W and using equations (19) - (23) together with the boundary conditions (24) and (25), we obtain

$$\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_1}(1-\sigma F)\right]I_1+\left(\frac{G}{G-1}\right)\frac{g\alpha\kappa a^2}{\nu\beta}\left[I_2+Ep_1\sigma^* I_3\right]+\frac{g\alpha'\kappa' a^2}{\nu\beta'}\left[I_4+Eq'\sigma^* I_5\right]+\frac{\mu_e\eta\varepsilon}{4\pi\rho_m\nu}\left[I_6+p_2\sigma^* I_7\right]+$$
$$+d^2\left[\frac{\sigma^*}{\varepsilon}+\frac{1}{P_1}(1-\sigma^* F)\right]I_8+\frac{\mu_e d^2\eta\varepsilon}{4\pi\rho_m\nu}\left[I_9+p_2\sigma I_{10}\right]=0, \tag{41}$$

where

$$I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz,\qquad I_2=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,$$

$$I_3=\int_0^1\left(|\Theta|^2\right)dz,\qquad I_4=\int_0^1\left(|D\Gamma|^2+a^2|\Gamma|^2\right)dz,\qquad I_5=\int_0^1\left(|\Gamma|^2\right)dz,$$

$$I_6=\int_0^1\left(\left|D^2K\right|^2+2a^2|DK|^2+a^4|K|^2\right)dz,$$

$$I_7=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz,\qquad I_8=\int_0^1\left(|Z|^2\right)dz,$$

$$I_9=\int_0^1\left(|DX|^2+a^2|X|^2\right)dz,\qquad I_{10}=\int_0^1\left(|X|^2\right)dz,$$

where integrals $I_1, I_2 \ldots\ldots\ldots\ldots I_{10}$ are all positive and definite. Putting $\sigma=\sigma_r+i\sigma_i$ and equating real and imaginary parts of equation (41), we get

$$\left\{\left[\frac{1}{\varepsilon}-\frac{F}{P_1}\right]I_1-\left(\frac{G}{G-1}\right)\frac{g\alpha\kappa a^2}{\nu\beta}Ep_1I_3+\frac{g\alpha'\kappa' a^2}{\nu\beta'}E'qI_5+\frac{\mu_e\eta\varepsilon}{4\pi\rho_m\nu}p_2I_7+d^2\left[\frac{1}{\varepsilon}-\frac{F}{P_1}\right]I_8+\frac{\mu_e\eta d^2\varepsilon}{4\pi\rho_m\nu}p_2I_{10}\right\}\sigma_r=$$
$$-\left\{\frac{I_1}{P_1}-\left(\frac{G}{G-1}\right)\frac{g\alpha\kappa a^2}{\nu\beta}I_2+\frac{g\alpha'\kappa' a^2}{\nu\beta'}I_4+\frac{\mu_e\eta\varepsilon}{4\pi\rho_m\nu}I_6+\frac{d^2}{P_1}I_8+\frac{\mu_e\eta d^2\varepsilon}{4\pi\rho_m\nu}I_9\right\}, \tag{42}$$

$$\left\{\left[\frac{1}{\varepsilon}-\frac{F}{P_1}\right]I_1+\left(\frac{G}{G-1}\right)\frac{g\alpha\kappa a^2}{\nu\beta}Ep_1I_3-\frac{g\alpha'\kappa' a^2}{\nu\beta'}E'qI_5-\frac{\mu_e\eta\varepsilon}{4\pi\rho_m\nu}p_2I_7-d^2\left[\frac{1}{\varepsilon}-\frac{F}{P_1}\right]I_8+\frac{\mu_e\eta d^2\varepsilon}{4\pi\rho_m\nu}p_2I_{10}\right\}\sigma_i=0. \tag{43}$$

It is inferred from equation (42) that σ_r may be positive or negative, which means that the system may be stable or unstable. Also, from equation (43), σ_i may be zero or non-zero, signifying that the modes may be non-oscillatory or oscillatory. The oscillatory modes appear due to the presence of viscoelasticity, solute gradient and magnetic field (hence Hall currents) which, would not exist in the absence of such effects. This result is in agreement with the result from the study of Sunil *et al.* (2000b) where the effect of Hall currents has been investigated on thermal instability of Walters' (Model B′) fluid.

(iii) Case of Overstability

Let us now discuss the possibility whether instability may occur as an overstability. Since for overstability, we wish to find Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which equation (27) will admit solutions with real values of σ_1. Equating real and imaginary parts of equation (27) and eliminating R_1 between them, we obtain

$$A_4 c_1^4 + A_3 c_1^3 + A_2 c_1^2 + A_1 c_1 + A_0 = 0, \tag{44}$$

where we have set $c_1 = \sigma_1^2$, $b = 1 + x$, and

$$A_4 = p_2^4 E'^2 q^2 b \left[\frac{Ep_1}{P} + b\left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right) \right] \left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right)^2, \tag{45}$$

$$\begin{aligned} A_3 = {} & E'^2 q^2 p_2^2 b^2 \left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right) \left[Q_1 \left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right)(Ep_1 - p_2) + \frac{p_2^2}{P^2} + 2b\left(b - M\right)\left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right)^2 \right] + p_2^4 b \left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right)^2 \times \\ & \times \left[b^2 \frac{Ep_1}{P} + \frac{p_2}{P} + \left(2 + b^3\right)\left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right) \right] + \frac{p_2^5}{P^2} E' q \left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right) + 2 p_2^2 b^2 E'^2 q^2 \frac{Ep_1}{P} \left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right) \times \\ & \times \left\{ 2b \left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right) - (b + M) \right\} + p_2^3 b \left[S_1 (b-1) p_2 (Ep_1 - E'q) \left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right)^2 + p_2 E'^2 q^2 \left\{ \frac{Ep_1}{P^3} - T_1 \left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right) \right\} \right] + \\ & + \left[4E' q p_2^3 b \left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right)^2 \left\{ \frac{1}{P} + \frac{1}{2} Q_1 E p_1 b + b \left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right) \right\} \right] + T_1 E p_1 E'^2 q^2 \frac{p_2^4}{P}. \end{aligned} \tag{46}$$

Since σ_1 is real for overstability, the four values of $c_1 (= \sigma_1^2)$ should be positive. The sum of roots of equation (44) is $-A_3 / A_4$ and this should be positive if each of the roots is positive. Now, it is clear from expression (45) and (46) that A_3 and A_4 are always positive if

$$\frac{\pi^2 F}{P} < \frac{1}{\varepsilon}, \quad Ep_1 > p_2, \quad Ep_1 > E'q, \quad b > M, \quad b > 1, \quad 2b\left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right) > \left(b + M\right) \text{ and } \quad \frac{Ep_1}{P^3} > T_1 \left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right),$$

i.e. if

$$\kappa < \min \left\{ E\eta, \frac{E}{E'} \kappa', E \frac{\nu^3 \varepsilon^3 d^2}{4\Omega^2 k_1^2 \pi^2 (k_1 - \varepsilon \nu')} \right\} \tag{47}$$

and

$$k > \max \left\{ \frac{H}{4Ne\eta d} \left(\frac{\varepsilon k_1}{2(k_1 - \varepsilon \nu') - \varepsilon k_1} \right)^{1/2}, \left(\frac{H}{4Ne\eta d} \right) \right\}. \tag{48}$$

Thus, for conditions (47) and (48), overstability cannot occur and the principle of exchange of stabilities is valid. Hence, these are the sufficient conditions for the non-existence of over stability, the violation of which does not necessarily imply the occurrence of overstability. The analogous conditions $\kappa < \min \left\{ E\eta, \frac{E}{E'} \kappa', E \frac{\nu^3 \varepsilon^3 d^2}{4\Omega^2 k_1^2 \pi^2 (k_1 + \varepsilon \nu')} \right\}$ and $k > \max \left\{ \left(\frac{\mu_e H^2 E'^2 \pi}{2\rho_m \kappa'^2 d^2} \right)^{1/4}, \left(\frac{cH}{4Ne\eta d} \right) \right\}$ are derived by Gupta and Sharma (2008) for the case of Rivlin-Erickson elastico-viscous fluid. Further, in the absence of rotation, magnetic field (and hence Hall currents) and viscoelasticity, the above conditions, as expected, reduce to $\kappa < \min \left\{ \eta, \kappa' \right\}$ (see Chandrasekhar, 1981 and Veronis, 1965).

5. Concluding Remarks

In the present paper, the combined effect of Hall currents and rotation on the stability of a compressible Walters' (Model B′) elastico-viscous fluid heated and soluted from below in porous medium is considered. The effects of various parameters such as magnetic field, compressibility, Hall currents, rotation and medium permeability have been investigated analytically as well as numerically. The main results from this paper are as follows:

1. The expressions dR_1/dS_1, dR_1/dQ_1, dR_1/dM, dR_1/dT_1 and dR_1/dP are examined analytically and it has been found that the solute gradient (S_1) and rotation (T_1) have stabilizing effect. Very interestingly, magnetic field which has stabilizing effect for $T_1 = 0$ has stabilizing/destabilizing influence for $T_1 \neq 0$, while Hall currents and permeability have destabilizing effect for $T_1 = 0$ and also have stabilizing/destabilizing influence for $T_1 \neq 0$. Various conditions for stabilizing/destabilizing influence of magnetic field, Hall currents and permeability are derived.
2. The effects of the above mentioned parameters are also studied numerically for permissible range of values of various parameters through Figures (2) - (12). It is found that magnetic field postpones the onset of instability, while Hall currents and permeability hasten the same for the considered allowed range. The reason for stabilizing effects of magnetic field and rotation are accounted for by Chandrasekhar (1981) and for solute gradient by Veronis (1965). These are found to be valid for second-order fluids as well.
3. The effect of compressibility is to postpone the onset of instability as is clear from equation (29).
4. The oscillatory modes appear due to the presence of viscoelasticity, solute gradient and Hall currents. In the absence of these effects, the principle of exchange of stabilities is found to hold well.
5. The conditions $\kappa < \min\left\{E\eta, \frac{E}{E'}\kappa', E\frac{\nu^3\varepsilon^3 d^2}{4\Omega^2 k_1{}^2\pi^2(k_1-\varepsilon\nu')}\right\}$ and $k > \max\left\{\frac{H}{4N e\eta d}\left(\frac{\varepsilon k_1}{2(k_1-\varepsilon\nu')-\varepsilon k_1}\right)^{1/2}, \left(\frac{H}{4Ne\eta d}\right)\right\}$ are sufficient for the non-existence of overstability. In the absence of viscoelasticity, rotation, magnetic field (hence Hall currents) and permeability, the above conditions, as expected, reduce to $\kappa < \min\{\eta, \kappa'\}$.

Nomenclature

C_p	specific heat of the fluid at constant pressure,
c_s	heat capacity of solid matrix,
c_f	heat capacity of the fluid,
C	concentration,
C_0	concentration at bottom layer,
C_d	concentration at upper layer,
d	depth of fluid layer,
D	derivative with respect to z = (d/dz),
E	constant due to porous medium for heat,
E'	constant due to porous medium for solute,
e	charge of an electron,
F	factor due to kinematic viscoelasticity,
f	an arbitrary function of x, *y, z, t.*
f_m	constant space distribution of f,
f_0	variation of f in absence of motion,
$G = \left(\frac{C_p}{g}\right)\beta$	factor due to compressibility,
$\boldsymbol{g} = (0,0,-g)$	acceleration due to gravity,
$\boldsymbol{h} = (h_x, h_y, h_z)$	perturbation in magnetic field $\boldsymbol{H} = (0,0,H)$,
$\boldsymbol{H} = (0,0,H)$	magnetic field vector having components $(0,0,H)$,
$k = \left(k_x^2 + k_y^2\right)^{1/2}$	wave number of the disturbance,
k_x, k_y	wavenumbers in x and y directions respectively,

Greek Letters

α	thermal coefficient of expansion,
α'	solvent coefficient of expansion,
$\beta\left(=\lvert dT/dz\rvert\right)$	temperature gradient,
$\beta'\left(=\lvert dC/dz\rvert\right)$	concentration gradient,
∂	curly operator,
∇	del operator,
δ	perturbation in the respective physical quantity,
η	resistivity,
ε	porosity,
θ	perturbation in temperature,
γ	perturbation in solute concentration,
κ	thermal diffusivity,
κ'	solute diffusivity,
ξ	z-component of current density,
ζ	z-component of vorticity,
μ	viscosity of the fluid,
μ'	viscoelasticity of the fluid,
μ_e	magnetic permeability,
ν	kinematic viscosity,
ν'	kinematic visco-elasticity,
ρ	density of the fluid,

k_1	medium permeability,
M	Hall current parameter,
n	growth rate of the disturbance,
N	electron number density,
P	factor due to permeability,
p	fluid pressure,
p_m	constant space distribution of p,
p_1	thermal Prandtl number,
p_2	magnetic Prandtl number,
q	effective thermal conductivity of the pure fluid,
Q_1	Chandrasekhar number,
R_1	Rayleigh number,
S_1	solute Rayleigh number,
T_1	Taylor number,
T	temperature,
T_0	temperature at bottom layer,
T_d	temperature at upper layer,
$\boldsymbol{q} = (u, v, w)$	fluid velocity vector having components (u, v, w),
W_0	constant,
(x, y, z)	x, y, z directions,
x	wavenumber.
ρ_0	density of fluid matrix,
ρ_s	density of porous matrix,
ρ_m	constant space distribution of ρ,
$\boldsymbol{\Omega} = (0, 0, \Omega)$	rotation vector having components $(0, 0, \Omega)$.

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