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International Journal of Engineering, Science and Technology
Vol. 7, No. 3, 2015, pp. 17-23

INTERNATIONAL JOURNAL OF ENGINEERING, SCIENCE AND TECHNOLOGY

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Model order reduction using eigen algorithm

Jay Singh[1], Kalyan Chatterjee[2], C.B. Vishwakarma[3]

[1&2]Department of Electrical Engineering, Indian School of Mines Dhanbad, INDIA*
[3] Department of Electrical Engineering, Galgotias College of Engineering and Technology Greater Noida, INDIA
[*]Corresponding Author: e-mail: jaysinghism@gmail.comn, Mob. +91-9911306853, 9457457424

Abstract

An order reduction method has been proposed for reducing the order of the large-scale dynamic systems where denominator polynomial determined through Eigen algorithm and numerator polynomial via factor division algorithm. In Eigen algorithm, the most dominant Eigen value of both original and reduced order systems remain same. The proposed mixed method confirm stability of the reduced model if the original system is stable and has been also compared in quality with other existing model order reduction methods.

Keywords: Eigen algorithm; Order reduction; Factor division algorithm; Stability; Transfer function.

DOI: http://dx.doi.org/10.4314/ijest.v7i3.3S

1. Introduction

Any physical system can be transformed into a mathematical model. The mathematical transformation of the system modelling often clues to comprehensive description of a procedure in the form of differential equations of high order which are very difficult to use either for design or analysis. Hence, it is suitable and sometimes needed to find the options of discovery some new similar equation but of lower order type can be considered to reflect the principal characteristics of the original system. Abundant methods are accessible in the literature to reduce the order of dynamic systems in frequency and time domain in Mahmoud *et al.* 1981, Mittal *et al.* 2004, Mukherjee *et al.* 2005, Mukherjee *et al.* 1987 etc. Further, some methods have also been recommended by merging the features of two dissimilar methods (Vishwakarma *et al.* 2008, Dolgin *et al.* 2003, V. Singh *et al.* 2004). Pal *et al.* 1995 proposed pole-clustering method using Inverse distance measure [IDM] criterion and time moment matching. Vishwakarma *et al.* 2011 modified the method of Pal *et al.* 1995 by an iterative method [IM], the complexity with these methods (Pal *et al.* 1995, Vishwakarma *et al.* 2011) is in selecting poles for the required cluster centre. S. Mukherjee *et al.* 1996 has suggested an order reduction method through Eigen spectrum analysis, where system stiffness and pole centroid of the original system and reduced order systems are retained to acquire the reduced order system model. Further, Parmar *et al.* (2007) has proposed a method by combining Eigen spectrum analysis and factor division algorithm to fix the denominator and numerator polynomial respectively. Sometimes difficulty with the developed methods (Mukherjee *et al.* 1996, Parmar *et al.* 2007) is equalization of system stiffness and inclination to turn into non-minimum phase.

Each method has merit and demerit when applied on a certain dynamic system. Major numbers of methods are offered in the literature but no method repetitively gives the best outcomes for all systems. In the proposed work, authors have taken pole directly from the Eigen algorithm while the zeros are determined through factor division algorithm to obtain the reduced order system. Proposed mixed method diminishes the difficulty of non-minimum phase in the reduced models. An order reduction procedure is unassuming and computer leaning.

2. Statement of the problem

Consider the transfer function of original high order system of the order $'n'$ is

$$G(s)=\frac{N(s)}{D(s)}=\frac{a_0+a_1s+a_2s^2+........+a_{n-1}s^{n-1}}{b_0+b_1s+b_2s^2+.......b_ns^n} \tag{1}$$

Where $a_i\,(i=0,1,2...,n-1)$ and $b_i\,(i=0,1,2...,n)$ are identified as known scalar quantities.

Consider the transfer function of the reduced order model of the order 'k' is

$$R_k(s)=\frac{N_k(s)}{D_k(s)}=\frac{c_0+c_1s+c_2s^2+........+c_{k-1}s^{k-1}}{d_0+d_1s+d_2s^2+.......d_ks^k} \tag{2}$$

Where $c_i\,(i=0,1,2...,k-1)$ and $d_i\,(i=0,1,2...,k)$ are identified as unknown scalar quantities.

The prospective of this paper is to develop the reduced order model of form (2) form original high order system (1) such that it retains the important features of the original system.

3. Description of the method

The order reduction procedure has been elaborated through simple steps.

3.1 Determination of Denominators: Reduced order denominator using Eigen algorithm is mentioned in figure 1 with appropriate computer oriented algorithm.

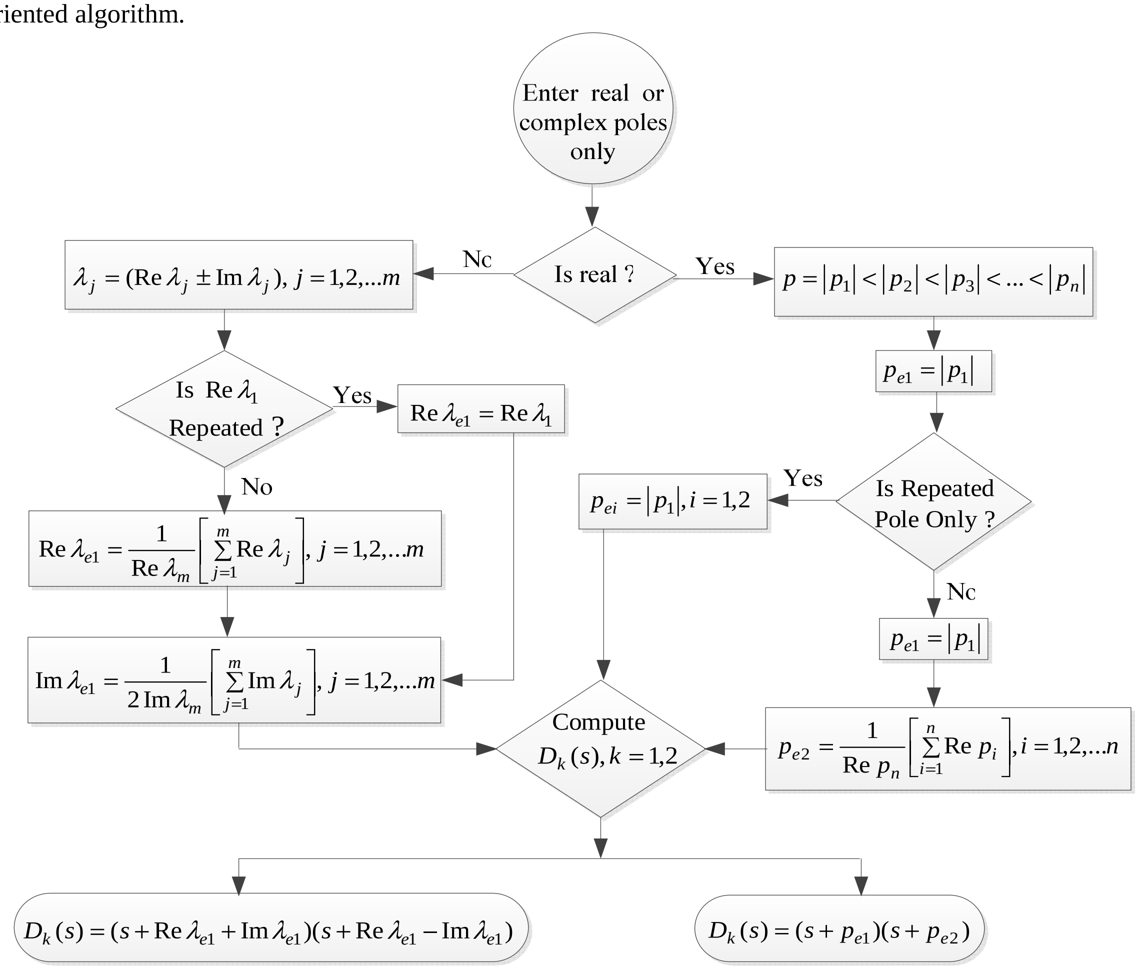

Figure 1. Eigen Algorithm

3.2 Determination of Numerator: Reduced order numerator using factor division algorithm of T.N. Lucas *et al.* 1983 is stated below.

The reduced $k^{th}-$ order system function is considered as

$$R_k(s) = \frac{N_k(s)}{D_k(s)} \tag{3}$$

Where $D_k(s)$ is determined through Eigen algorithm and $N_k(s)$ is designed by matching first $k-$ terms of series expansion about $s=0$ of $G_n(s)$ and $R_k(s)$.

$G_n(s)$ can be considered as

$$G_n(s) = \frac{N_n(s)\times D_k(s)}{D_n(s)\times D_k(s)} = \frac{N_n(s)\times D_k(s)/D_n(s)}{D_k(s)} \tag{4}$$

Therefore, reduced order numerator model $N_k(s)$ of the $R_k(s)$ may be given as

$$\frac{N_n(s)\times D_k(s)}{D_n(s)} = \frac{\sum_{i=0}^{n+k-1} \alpha_i s^i}{\sum_{i=0}^{n} b_i s^i} \text{(about } s=0 \text{, up to } s^{k-1}\text{)} \tag{5}$$

This is can be performed with the help of Routh recurrence formula assumed as follows:

$$\left.\begin{aligned}
c_0 &= \frac{\alpha_0}{b_0}\left\langle\begin{matrix}\alpha_0 & \alpha_1 & \ldots & \alpha_{k-1}\\ b_0 & b_1 & \ldots & b_{k-1}\end{matrix}\right.\\
c_1 &= \frac{\beta_0}{b_0}\left\langle\begin{matrix}\beta_0 & \beta_1 & \ldots & \beta_{k-2}\\ b_0 & b_1 & \ldots & b_{k-2}\end{matrix}\right.\\
c_2 &= \frac{\gamma_0}{b_0}\left\langle\begin{matrix}\gamma_0 & \gamma_1 & \ldots & \gamma_{k-3}\\ b_0 & b_1 & \ldots & b_{k-3}\end{matrix}\right.\\
&\vdots \quad \vdots\\
&\vdots \quad \vdots\\
c_{k-2} &= \frac{u_0}{b_0}\left\langle\begin{matrix}u_0 & u_1\\ b_0 & b_1\end{matrix}\right.\\
c_{k-1} &= \frac{v_0}{b_0}\left\langle\begin{matrix}v_0\\ b_0\end{matrix}\right.
\end{aligned}\right\} \tag{6}$$

Where

$$\left.\begin{aligned}
\beta_i &= \alpha_{i+1} - c_0 b_{i+1} && i = 0,1,2,\ldots k-2\\
\gamma_i &= \beta_{i+1} - c_1 b_{i+1} && i = 0,1,2,\ldots k-3\\
&\vdots\\
&\vdots\\
v_0 &= u_1 - c_{k-2} b_1
\end{aligned}\right\} \tag{7}$$

Therefore, reduced order numerator $N_k(s)$ may be expressed as

$$N_k(s) = c_0 + c_1 s + c_2 s^2 + \ldots + c_{k-1} s^{k-1} \tag{8}$$

4. Numerical Examples

Authors have considered three numerical examples from the literature to make sure the algorithm of the proposed method. All examples are solved in details to find the second order reduced model. Performance error indices (PEE) i.e. integral of square of errors (ISE) as well as integral of absolute error (IAE) have been mentioned in MATLAB environment to show the goodness of proposed method.

$$\text{ISE} = \int_0^\infty \left[g_{ij}(t) - r_{ij}(t)\right]^2 dt; \tag{9}$$

$$\text{IAE} = \int_0^\infty \left|g_{ij}(t) - r_{ij}(t)\right| dt \tag{10}$$

Where, $g_{ij}(t)$ and $r_{ij}(t)$ are the step responses of high order original system and reduced system respectively.

Example 1

Consider a system of sixth order taken from Mahmoud *et al.* 1981.

$$G(s) = \frac{2s^5 + 3s^4 + 16s^3 + 20s^2 + 8s + 1}{2s^6 + 33.6s^5 + 155.9s^4 + 209.5s^3 + 102.4s^2 + 18.3s + 1}$$

Poles of the system are: ($-0.1, -0.2, -0.5, -1, -5 \,\&\, -10$). To find second order system, required poles are calculated using Eigen algorithm as shown in Section 3.1.

$p_{e1} = -0.1$ and $p_{e2} = -1.68$

Therefore, denominator for second order reduced model is written as $D_2(s) = s^2 + 1.78s + 0.168$

Numerator coefficients are obtained using Section 3.2 as follows

$$\alpha_0 = 0.168 \left\langle \begin{matrix} 0.168 & 3.124 \\ 1 & 18.3 \end{matrix} \right.$$

$$\alpha_1 = 0.049 \left\langle \begin{matrix} 0.0496 \\ 1 \end{matrix} \right.$$

So, numerator for second order reduced model is obtained as

$$N_2(s) = 0.0496s + 0.168$$

Finally complete Second order reduced model is obtained as

$$R_2(s) = \frac{N_2(s)}{D_2(s)} = \frac{0.0496s + 0.168}{s^2 + 1.78s + 0.168}$$

The unit step response for the reduced model i.e., $R_2(s)$ and original model $G(s)$ is shown in figure 2. Also, error index ISE, IAE is calculated between transient portions of reduced order model and original high order model as shown in the Table 1.

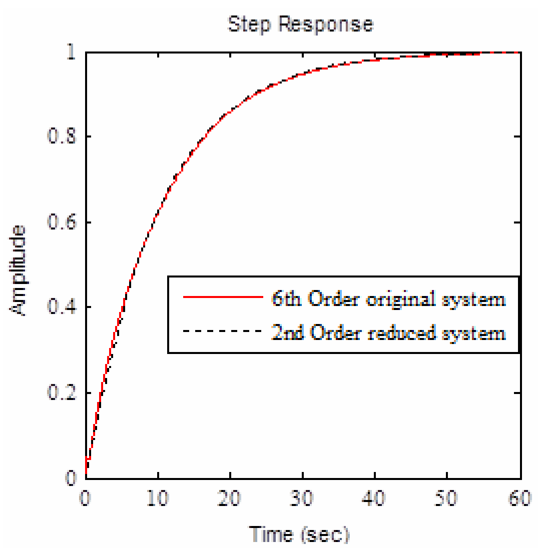

Figure 2. Step response for Example 1

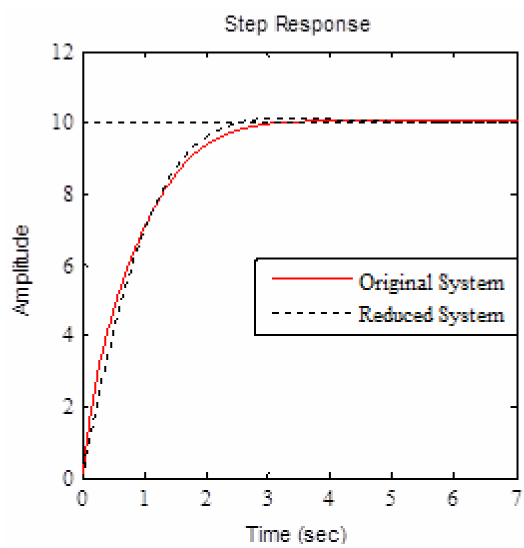

Figure 3. Step response for Example 2

Table I (Comparison of the Proposed Method)

Reduction Methods	Reduced Model	ISE	IAE
Proposed Method	$R_2(s)=\dfrac{0.0496s+0.168}{s^2+1.78s+0.168}$	0.003	0.1396
Vishwakarma *et al.* 2009	$R_2(s)=\dfrac{8s+1}{100.805s^2+16.2254s+1}$	0.843	0.223
Vishwakarma *et al.* 2009	$R_2(s)=\dfrac{100.8048s+1}{100.805s^2+16.2254s+1}$	4.009	22.65

Example 2

Consider a system from Smith *et al.* (1995) of fourth order as mentioned in transfer function form.

$$G(s)=\frac{28s^3+496s^2+1800s+2400}{2s^4+36s^3+204s^2+360s+240}$$

Poles are: $(-1.1967\pm j0.6934)$ and $(-7.8033\pm j1.3576)$

To find second order reduced model, the pole cluster centres are obtained via Eigen algorithm as follows.

$\text{Re}\, p_{e1}=-1.1533$ and $\text{Im}\, p_{e1}=-0.7554$

Using Section 3.1 and Section 3.2, the second order reduced model can be written as

$$R_2(s)=\frac{8.808s+19.0083}{s^2+2.3066s+1.9007}$$

The unit step response is plotted in figure 3 for the reduced model and original model also ISE and IAE is calculated between transient portion of reduced model and original model as shown in the Table II.

Table II (Comparison of the Proposed Method)

Reduction Methods	Reduced Model	ISE	IAE
Proposed Method	$R_2(s)=\dfrac{8.808s+19.0083}{s^2+2.3066s+1.9007}$	0.371	0.9758
Vishwakarma *et al.* 2009	$R_2(s)=\dfrac{1371.048s+2400}{201s^2+317.1498s+240}$	1.763	2.597
Krishnamurthy *et al.* 1978	$R_2(s)=\dfrac{9.046283s+13.043478}{s^2+1.701323s+1.304348}$	1.208	2.265
Prasad *et al.* 2003	$R_2(s)=\dfrac{22.532255s+11.90362}{s^2+3.145997s+1.190362}$	2.743	3.371

Example 3

Consider a system from Shamash *et al.* 1975 of an eight-order described as

$$G(s)=\frac{18s^7+514s^6+5982s^5+36380s.^4+122664s^3+222088s^2+185760s+40320}{s^8+36s^7+546s^6+4536s^5+22449s^4+67284s^3+118124s^2+109584s+40320}$$

The poles are: $(-1,-2,-3,-4,-5,-6,-7,-8)$ To find second order reduced model, the required pole cluster centers are as. $p_{e1}=-1$ and $p_{e2}=-4.5$

Now using Section 3.1 and Section 3.2, the complete reduced order model may be written as.

$$R_2(s)=\frac{14.0097s+4.5}{s^2+5.5s+4.5}$$

The unit step responses are plotted in figure 4 for the reduced model and original model, also error index ISE & IAE have been determined between the transient portion of reduced model and original model as shown in the Table III.

Table III (Comparison of the Proposed Method)

Reduction Methods	Reduced Model	ISE	IAE
Proposed Method	$R_2(s)=\frac{14.0097s+4.5}{s^2+5.5s+4.5}$	0.001	0.128
Parmar *et al.* 2007	$R_2(s)=\frac{24.11429s+8}{s^2+9s+8}$	0.0048	0.3007
Mukherjee *et al.* 2005	$R_2(s)=\frac{11.3909s+4.4357}{s^2+4.2122s+4.4357}$	5.69×10^{-2}	0.4572
Mittal *et al.* 2004	$R_2(s)=\frac{7.0908s+1.9906}{s^2+3s+2}$	2.689×10^{-1}	0.8054
Mukherjee *et al.* 1987	$R_2(s)=\frac{7.0903s+1.9907}{s^2+3s+2}$	2.689×10^{-1}	0.8054

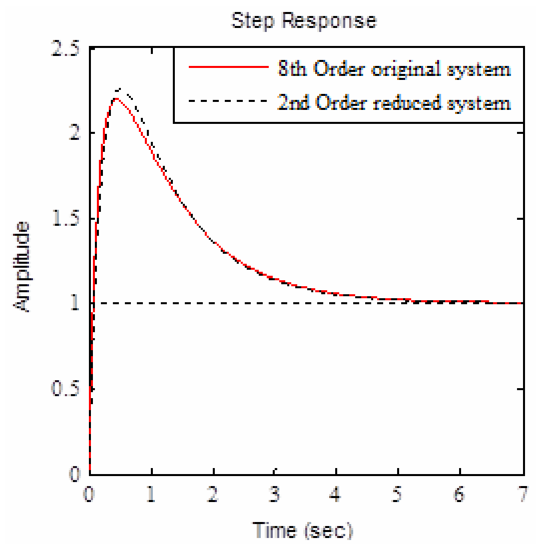

Figure 4. Step response of the reduced model and original model for Example 3

5. Conclusions

The authors have presented a mixed method to reduce the order of original high order system having single input single output (SISO) system. In the proposed method, the reduced order denominator polynomial is determined using Eigen algorithm while the numerator coefficients are determined via factor division algorithm. The method has been deep-rooted on three existing examples taken from the literature. Time response of reduced model and original model are compared graphically and mention in the figure 2, 3 and 4 respectively. From the above comparisons, it is concluded that the proposed method is efficient, simple, and computer oriented also confirmed the stability of the reduced order models, if original high-order model is stable. The proposed method has been compared with some existing order reduction methods using performance error indices, i.e. ISE and IAE.

Acknowledgement

The authors wish to thank the Department of Electrical Engineering, Indian School Mines Dhanbad India for providing the computing facilities during the course of this work.

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Biographical notes

Mr. Jay Singh is a research scholar of Electrical Engineering Department; ISM Dhanbad India. He has completed his M. Tech in Power Electronics Electrical Machines and Drives from Maharshi Dayanand University Rohtak and B.Tech in Electrical Engineering from Madan Mohan Malviya Engineering College Gorakhpur. He is having around Ten years of teaching experience. He has interest in Model Order Reduction, electrical Machines, control systems and Power System etc.

Dr. Kalyan Chatterjee has completed his graduation from Jalipaiguri Government Engineering. College 1997 and Post Graduate from Jadavpur University 1999. He completed his Ph.D from B.I.T,Mesra ,Ranchi India October 2005. Now he is working as Associate Professor in the department of Electrical Engineering, Indian School of Mines, Dhanbad, India. He has about 15 years of teaching and research experience. He has successfully completed one AICTE funded project and one DST sponsored project. He has coordinated one AICTE- ISTE sponsored short term training program on “MATLAB Oriented Electrical System Modelling and Real-time Control”. He has guided two Ph.D students, ten students in Master of Engineering level and presently guiding three Ph.D. students. He trained more than 200 students in MATLAB language through a certificate course. His interests are in Fuzzy logic and ANN applications to power system control and power quality. He has published more than 50 papers in prestigious international journals and conferences.

Dr. Chandra Bhan Vishvakarma has completed his Ph.D on the topic of Model Order Reduction of Linear Dynamic Systems for Control Systems Design from IIT Roorkee India, M.tech in Electrical Engineering with specialization in System Engineering & Operations Research (SEOR) from Department of Electrical Engineering IIT, Roorkee India and B.E. in Electrical Engineering from University of Roorkee, Roorkee (I.I.T Roorkee) . He is having around thirteen years of teaching experience. He has published more than 45 papers in prominent international journals and conferences. He is working as Associate Professor in the department of Electrical Engineering, Galgotias College of Engg & Technology Greater Noida Uttar Pradesh. He has interest in Control Theory, Power Systems Control, Soft Computing and Model Order Reduction.

Received March 2015
Accepted July 2015
Final acceptance in revised form July 2015