

The impact of awareness programs by media on the spreading and control of non-communicable diseases

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Abstract

In this paper a non-linear mathematical model for the effects of awareness programs on the burden of non-communicable diseases is analyzed. Even as infections and nutritional deficiencies are receding as leading contributors to death and disability, cardiovascular diseases (CVDs), cancers, diabetes, neuropsychiatric ailments, and other chronic diseases are becoming major contributors to the burden of disease. Complications of diseases constitute a burden for the individuals and the whole society. But the diseases can be avoided or delayed in many cases if awareness and sensitization are raised to inform people in the susceptible stage. In the modeling process the growth rate of awareness programs impacting the population is assumed to be proportional to the numbers of diseased with and without complications. The model is analyzed by using stability theory of differential equations. Our analysis shows that diseased or diseased with complication can be prevented, controlled or at least delayed by introducing effective awareness programmes. The numerical simulation analysis of the model confirms the analytical results.

Keywords: Epidemic model, awareness programs, aware susceptible class, local stability, global stability..

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1. Introduction

Over the past year the pandemic of non-communicable diseases (NCDs) has become a key focus of global political agenda. At the United Nations' high-level meeting on the prevention and control of NCDs in September 2011, a general consensus has been reached that NCDs were already the leading causes of death in all world regions and that their burden is increasing rapidly (WHO, 2011). The rate of this increase is particularly striking in low- and middle-income countries (LMICs), where life expectancy is increasing as a result of improved socio-economic conditions (Alwan *et al.*, 2010). It is expected that by the year 2030, NCDs could become responsible for 52 million deaths. In LMICs, health systems will face considerable challenge in adjusting to the rapidly growing demand for services, and this could in turn become an additional significant barrier to achieving the Millennium Development Goals (Alwan *et al.*, 2010). As a result, many parallel advocacy efforts for tackling NCDs are taking place, with a particular focus on heart disease, cancer, respiratory diseases, diabetes and stroke (Beaglehole *et al.*, 2011). A number of interventions have been outlined that could have immediate preventive effect and slow down the pandemic, such as tobacco control, improved diet, exercise and decreased alcohol intake (Beaglehole *et al.*, 2011).

In the coming decades, however, the noticeable impact will be on adults of working age in low- and middle-income countries. The program stresses that 3.2 million deaths can be attributed to complications associated with non-communicable diseases each year making the burden of premature death similar to that of HIV/AIDS. Despite these stark facts, the problem seems to be largely unrecognized. The growing trend of non-communicable diseases indicates the need, especially in developing countries, to adopt urgent measures and efficient strategies for the prevention and management of diseases and its complications (Boutayeb *et al.*, 2005). Statistical analysis on AIDS awareness programs shows that public awareness can play an appreciable role in preventing the AIDS epidemic (Cui *et al.*, 2008). Numerical simulations and mathematical models are useful tools for the elaboration of

health strategies. They offer interesting possibilities for building and testing assumptions, estimating and controlling parameters, understanding the population’s dynamics and proposing pragmatic actions based on different scenarios. Misra et al. (2011) investigated the effects of awareness programs on the spread of infectious diseases and concluded that the spread of an infectious disease can be controlled by using awareness programs but the disease remains endemic due to immigration. Keeping this research in mind, we formulate and analyze a nonlinear mathematical model for non-communicable diseases to study the impact of awareness programs conducted by a media campaign on susceptible class, diseased class with and without complication in the population. The main purpose is to attract health policy makers and public attention to the need to place more emphasis in health care on the prevention of illness and the promotion of good health, instead of focusing predominantly on treating ill-health.

The rest of this paper is organized as follows: In section 2 gives the formulation of the model. In Section 3, we discuss the boundedness of the solution. In Sections, 4 our model is analyzed with regard to equilibrium point and its local and global stability. In section 5, we present numerical simulation to illustrate the applicability of results obtained. We conclude with a short discussion in section 6.

2. Mathematical Model

Let S, D, C, S_m and M represent the numbers of susceptible class, diseased class without and with complications, aware susceptible class in a population and the cumulative density of awareness programs driven by the media in that region at time t respectively. $A, \lambda_1, d, \lambda, \lambda_2, \gamma, \nu, \delta, \lambda_0, \mu$ and μ_0 are positive parameters. A denote the incidence of susceptible class, λ_1 the probability of developing disease, d the natural mortality rate, λ_2 the probability of a diseased class developing a complication, γ the rate at which complications are cured, ν the rate at which diseased patients with complications become severely disabled, δ the mortality rate due to complications, λ the dissemination rate of awareness among susceptible class in a population due to which they form a different class, μ the rate with which awareness programs are being implemented, μ_0 the depletion rate of these programs due to ineffectiveness, social problems in the population, λ_0 the rate of transfer of aware class to susceptible class, respectively.

Keeping above consideration and assumption in mind the dynamics of model is governed by the following system of nonlinear ordinary differential equations:

$$\begin{aligned}
 \frac{dS}{dt} &= A - (\lambda_1 + d)S - \lambda SM + \lambda_0 S_m, \\
 \frac{dD}{dt} &= \lambda_1 S - (\lambda_2 + d)D + \gamma C, \\
 \frac{dC}{dt} &= \lambda_2 D - (\gamma + d + \nu + \delta) C, \\
 \frac{dS_m}{dt} &= \lambda SM - dS_m - \lambda_0 S_m, \\
 \frac{dM}{dt} &= \mu(D + C) - \mu_0 M.
 \end{aligned}
 \tag{2.1}$$

where, $S(0) > 0, D(0) \geq 0, C(0) \geq 0, S_m(0) \geq 0, M(0) \geq 0$.

The system shows that in a time interval of length t , A cases enter the stage of susceptible class. In that same interval, the number of susceptible class S , is seen to decrease by the amount dS (natural mortality), $\lambda_1 S$ (patients who develop disease without complications). During this time interval, the number of diseased class without complications is increased by the aforementioned amount $\lambda_1 S$ and by the amount γC (patients whose complications are cured), and to decrease by dD and $\lambda_2 D$. The number of diseased class with complications C increases by $\lambda_2 D$ and decreases by $\gamma C, dC, \nu C$ and δC . The number of aware class in a population increased by the amount λSM and decrease by the amount dS_m and $\lambda_0 S_m$. Similarly, the growth rate of awareness program increased by $\mu D, \mu C$ and decrease by the amount $\mu_0 M$.

3. Boundedness of the System

In analogy to the population dynamics, it is very important to observe the consequences that restrict the growth of the population. In this sense, study of boundedness of the solution of system around different steady states is very much needed. For this, we find boundedness of the system in the following lemma:

Lemma 3.1: The set

$$\Omega = \left\{ (S, D, C, S_m, M) : 0 \leq S + D + C + S_m \leq \frac{A}{d}, 0 \leq M \leq \frac{2\mu A}{d\mu_0} \right\},$$

is the region of attraction for all solutions initiating in the interior of the positive octant.

Proof: Let (S, D, C, S_m, M) be any solutions with positive initial conditions $(S_0, D_0, C_0, S_{m0}, M_0)$.

Define a function

$$N = S + D + C + S_m.$$

Computing the time derivative of N along solutions of system (2.1), we get

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dD}{dt} + \frac{dC}{dt} + \frac{dS_m}{dt},$$

$$\frac{dN}{dt} \leq A - dN.$$

Applying a Lemma on differential inequalities (Birkhoff and Rota, 1982), we obtain

$$0 \leq N(S, D, C, S_m) \leq \frac{A}{d} + \frac{N(S_0, D_0, C_0, S_{m0})}{\exp(dt)},$$

and for $t \rightarrow \infty$, $0 \leq N \leq \frac{A}{d}$.

From the last equation of the system (2.1) and using the upper bound of the population N , we get

$$\frac{dM}{dt} \leq \mu \left(\frac{A}{d} + \frac{A}{d} \right) - \mu_0 M.$$

According to comparison principle, it follows that

$$M_{\max} = \frac{2\mu A}{d\mu_0}.$$

Therefore all solutions of system (2.1) enter into the region

$$\Omega = \left\{ (S, D, C, S_m, M) : 0 \leq S + D + C + S_m \leq \frac{A}{d}, 0 \leq M \leq \frac{2\mu A}{d\mu_0} \right\}.$$

This completes the proof of lemma.

4. Equilibrium and Stability Analysis

4.1. Existence of Equilibrium Point

Theorem 4.1 There exists only one equilibrium point of the system (2.1) as $E_0(S^*, D^*, C^*, S_m^*, M^*)$ if only if conditions $\lambda_0 > \lambda(d + \lambda_0)$ and $\gamma\lambda_2 > (\lambda_2 + d)(\gamma + d + \nu + \delta)$ hold for the system (2.1).

For proof of this theorem see Appendix A.

4.2 Stability Analysis

4.2.1 Local Stability

Theorem 4.2.1 The equilibrium point E_0 is locally asymptotically stable if following conditions are satisfied:

1. P_1, P_2, P_3, P_4 and P_5 are positive,
2. $P_1 P_2 - P_3 > 0$,

3. $(p_1 p_2 - p_3) p_3 - p_1^2 p_4 > 0,$
4. $(p_1 p_2 - p_3)(p_3 p_4 - p_2 p_5) - (p_1 p_4 - p_5)^2 > 0.$

For proof of this theorem see Appendix B.

4.2.2 Global Stability

Theorem 4.2.2 The equilibrium point E_0 is non-linearly stable if the following inequalities are satisfied:

$$(b_{12})^2 < \frac{4}{9} b_{11} b_{22}, (b_{15})^2 < \frac{1}{3} b_{11} b_{55}, (b_{14})^2 < \frac{2}{3} b_{11} b_{44}, (b_{23})^2 < \frac{2}{3} b_{22} b_{33}, (b_{25})^2 < \frac{1}{3} b_{22} b_{55}, (b_{35})^2 < \frac{1}{2} b_{33} b_{55},$$

$$(b_{45})^2 < \frac{1}{2} b_{44} b_{55},$$

where

$$b_{11} = \lambda_1 + d + \lambda M, b_{12} = \lambda_1, b_{22} = \lambda_2 + d, b_{15} = \lambda E^*, b_{55} = \mu_0, b_{33} = \gamma + d + \nu + \delta,$$

$$b_{14} = \lambda M^*, b_{44} = d + \lambda_0, b_{23} = \gamma + \lambda_2, b_{25} = \mu, b_{35} = \mu, b_{45} = \lambda E^*.$$

For proof of this theorem see Appendix C.

5. Numerical simulation

We now integrate system (2.1) to substantiate the above analytical finding. The model is studied numerically using fourth order Runge-Kutta method under the following set of compatible parameters with the help of MATLAB software package.

First, we consider following set of parameter values of study the system (2.1) numerically:

$$A = 2000, \lambda_1 = 0.6, d = 0.6, \lambda = 0.0002, \lambda_2 = 1.5, \gamma = 0.005, \nu = 1.9, \delta = 1.9, \lambda_0 = 0.2, \mu = 0.005,$$

$$\mu_0 = 0.8. \tag{5.1}$$

The interior equilibrium point E_0 for above set of parameters is as follows:

$$S^* = 1665.84, D^* = 476.339, C^* = 162.204, S_m^* = 1.66204, M^* = 3.9909.$$

The characteristic polynomial and eigenvalues corresponding to equilibrium point E_0 are obtained as:

$$\psi^5 + 9.3058 \psi^4 + 30.0233 \psi^3 + 43.3172 \psi^2 + 28.6772 \psi + 7.10571 = 0.$$

$$\psi_1 = -4.40819, \psi_2 = -2.09838, \psi_3 = -1.19501, \psi_4 = -0.825687, \psi_5 = -0.778532..$$

Since all the eigenvalues corresponding to E_0 be negative, therefore E_0 is locally asymptotically stable. Further, to illustrate the global stability of the equilibrium point graphically, numerical simulation is performed for different initial starts and the result is displayed in Figure 4. It is found that all inequalities are satisfied for above parameter values. In Figure1 susceptible class, diseased class, diseased class with complication, aware class and awareness program in a population are plotted against time, from this figure it is noted that for a given initial values, all the population tend to their corresponding value of equilibrium point E_0 and hence coexist in the form of stable steady state, assuring the local stability of E_0 . This shows that the system (2.1) is locally and globally stable. Figures have been plotted between dependent variables and time for different parameter values to show changes occurring in population with time under different conditions. The variation of susceptible class, diseased class, diseased class with complication, aware susceptible class in a population and awareness programs with respect to time t for different values of rate of dissemination ' λ ' is shown in Figure 2. From this figure, it can be noted that as the rate of dissemination ' λ ' increases susceptible class, diseased class, diseased class with complication and awareness programs decrease. This means diseased and diseased with complication classes decreases with an increase in the value of the dissemination rate of awareness. At the same time, aware susceptible class in a population increases with rate of dissemination ' λ ' increases.

Further, the variation of susceptible class, diseased class, diseased class with complication, aware susceptible class in a population and awareness programs with respect to time t for different values of rate of implementation of awareness programs ' μ ' is shown in Figure.3. From this figure, it can be noted that as the rate of implementation of awareness programs ' μ ' increases susceptible class, diseased class, diseased class with complication decrease. At the same time, aware class in a population and awareness programs both increase with rate of implementation of awareness programs ' μ ' increases. These figures show that due to awareness programs diseases and diseases with complication can be controlled.

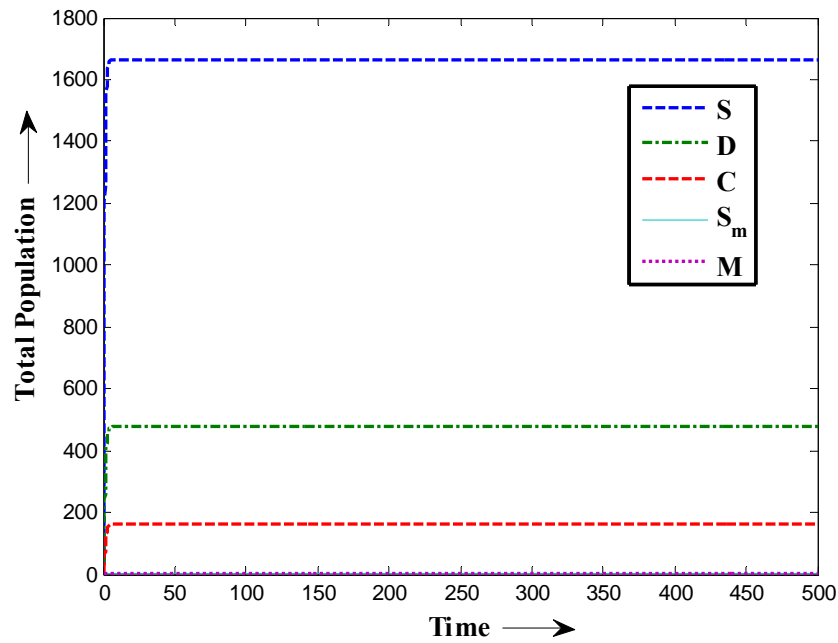
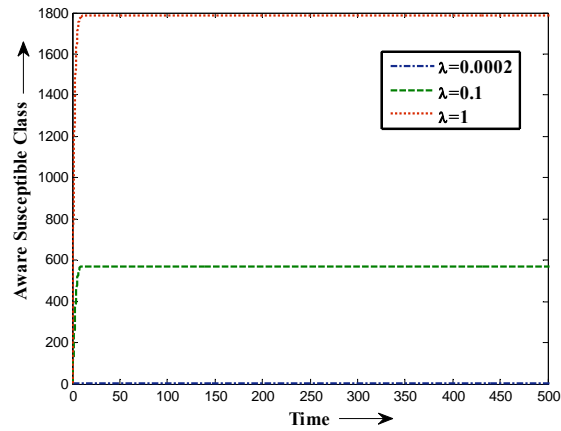
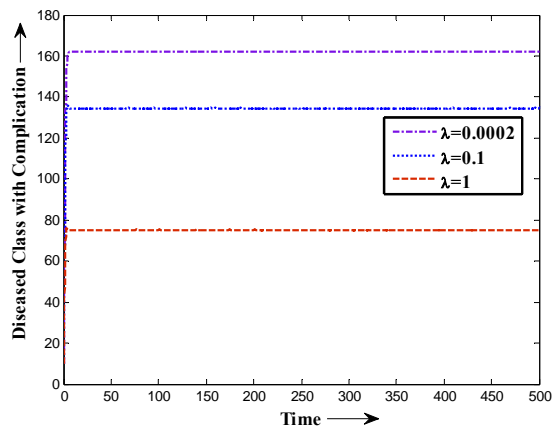
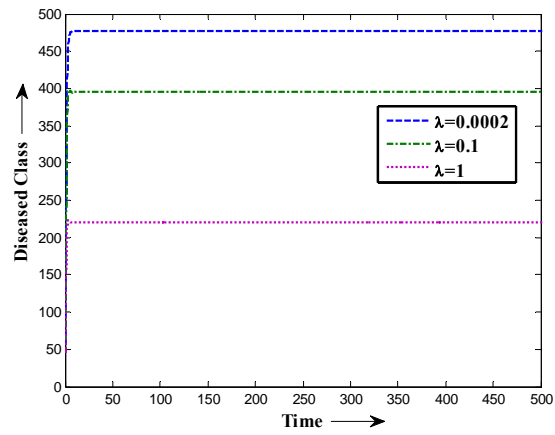
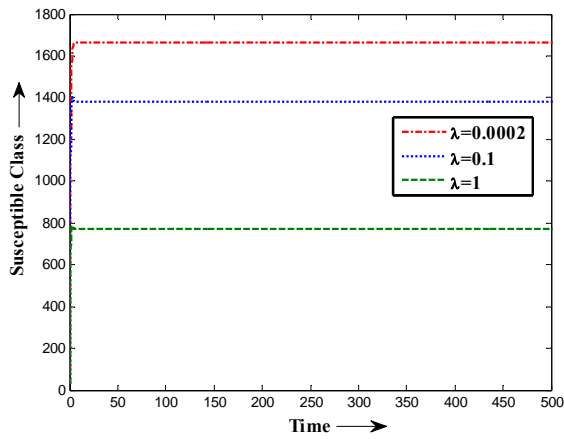


Figure.1. Stable behavior of S, D, C, S_m and M with time and other parameter values are same as (5.1).



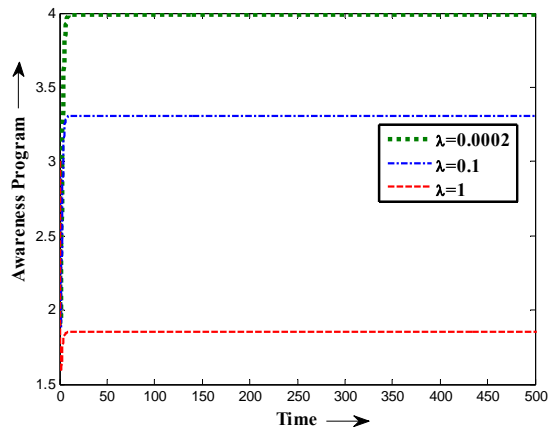
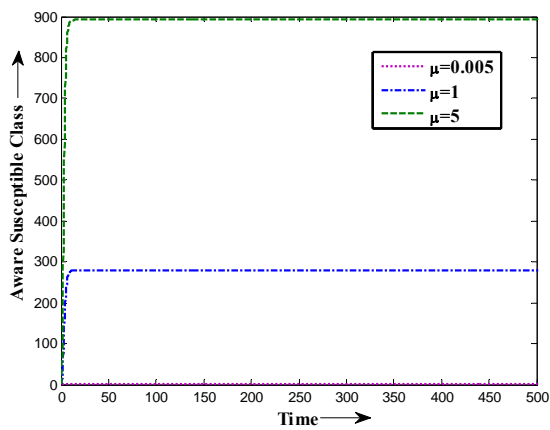
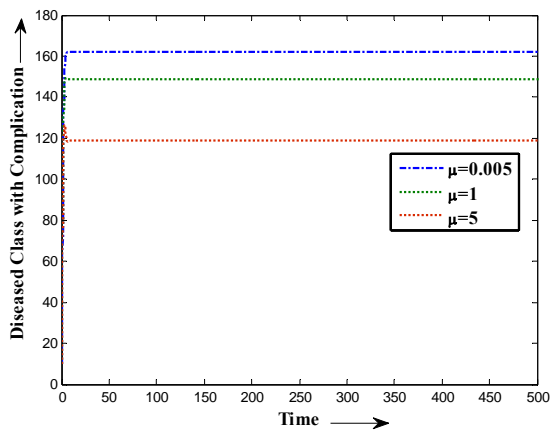
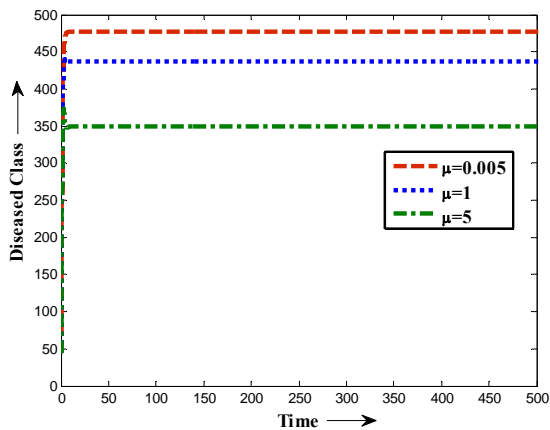
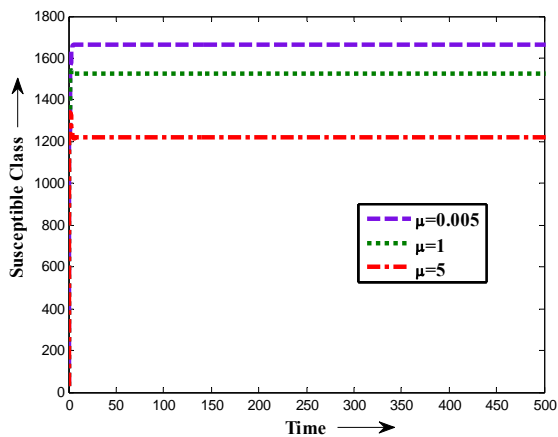


Figure.2. Graph of S versus t , D versus t , C versus t , S_m versus t and M versus t for different λ and other values of parameters are same as (5.1). These figures show that equilibrium number of diseased and diseased with complication classes decreases with an increase in the value of the dissemination rate of awareness.



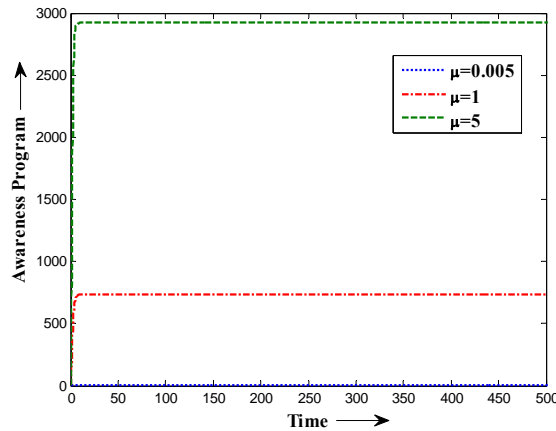


Figure.3. Graph of S versus t , D versus t , C versus t , S_m versus t and M versus t for different μ and other values of parameters are same as (5.1). These figures show that equilibrium number of diseased and diseased with complication classes decreases with an increase in the value of the implementation rate of awareness programs.

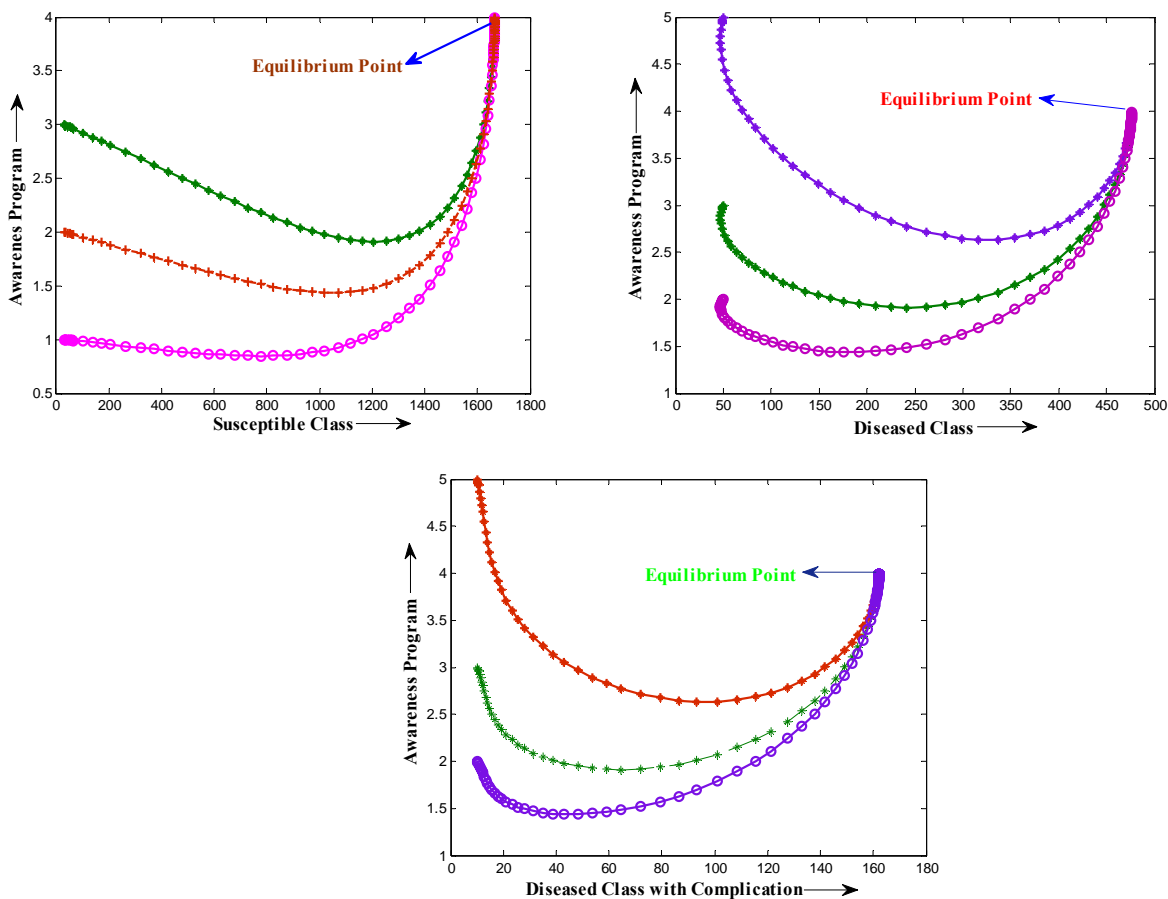


Figure.4. Graph of S versus M , D versus M and C versus M and other parameters are same as (5.1).

In Fig.4, we may see that all the trajectories initiating inside the region of attraction approach towards the equilibrium value $(S^*, D^*, C^*, S_m^*, M^*)$ for different initial starts, indicating the global stability of $(S^*, D^*, C^*, S_m^*, M^*)$.

6. Conclusion

In this paper, a nonlinear mathematical model is proposed and analyzed to see the effects of awareness programs driven by the media on susceptible class, diseased class, and diseased class with complication in a variable population with immigration. It has been considered that the growth rate of awareness programs impacting the population is assumed to be proportional to the numbers of diseased class with and without complications. It has been further assumed that due to the effect of media, susceptible class individuals form a separate class of aware susceptible class individuals. The analysis demonstrates that an endemic equilibrium is locally as well as globally stable under certain conditions. The analysis showed that awareness programs through the media campaigning are helpful in decreasing the diseased class in a population. By awareness programs, at least we could have prevented diseases whenever possible and, where not possible, to minimize complications and maximize quality of life.

Appendix A

There exists only one equilibrium point of the system (2.1) as $E_0(S^*, D^*, C^*, S_m^*, M^*)$

where

$$S^* = \frac{\{(\lambda_2 + d)(\gamma + d + \nu + \delta) - \gamma \lambda_2\}C^*}{\lambda_1 \lambda_2}, D^* = \frac{(\gamma + d + \nu + \delta)C^*}{\lambda_2},$$

$$S_m^* = \frac{C^{*2} \mu \lambda \{(\lambda_2 + d)(\gamma + d + \nu + \delta) - \gamma \lambda_2\}(\gamma + d + \nu + \delta + \lambda_2)}{\mu_0 \lambda_1 \lambda_2^2 (d + \lambda_0)}, M^* = \frac{\mu(\gamma + d + \nu + \delta + \lambda_2)C^*}{\mu_0 \lambda_2},$$

and

$$\lambda_0 > \lambda(d + \lambda_0) \text{ and } \gamma \lambda_2 > (\lambda_2 + d)(\gamma + d + \nu + \delta). \tag{A.1}$$

$$A_1 C^{*2} + A_2 C^* + A_3 = 0, \tag{A.2}$$

where

$$A_1 = \mu \lambda \{ \gamma \lambda_2 - (\lambda_2 + d)(\gamma + d + \nu + \delta) \} (\gamma + d + \nu + \delta + \lambda_2) \{ \lambda_0 - \lambda(d + \lambda_0) \},$$

$$A_2 = -(\lambda_1 + d)(d + \lambda_0) \mu_0 \lambda_2 \{ \gamma \lambda_2 - (\lambda_2 + d)(\gamma + d + \nu + \delta) \},$$

$$A_3 = -(d + \lambda_0) \mu_0 \lambda_1 \lambda_2^2 A.$$

In this way C^* is the unique positive root of the equation (A.2) with conditions (A.1).

Appendix B

To discuss the local stability of the system (2.1), we compute the variational matrix of the system (2.1). The entries of the general variational matrix are given by differentiating the right side of the system (2.1) with respect to S, D, C, S_m and M , i.e.

$$V(E_0) = \begin{bmatrix} -(\lambda_1 + d) - \lambda M^* & 0 & 0 & \lambda_0 & \lambda S^* \\ \lambda_1 & -(\lambda_2 + d) & \gamma & 0 & 0 \\ 0 & \lambda_2 & -(\gamma + d + \nu + \delta) & 0 & 0 \\ \lambda M^* & 0 & 0 & -(d + \lambda_0) & \lambda S^* \\ 0 & \mu & \mu & 0 & -\mu_0 \end{bmatrix}.$$

The eigenvalues of the matrix $V(E_0)$ are the roots of the characteristic equation given below

$$\psi^5 + p_1 \psi^4 + p_2 \psi^3 + p_3 \psi^2 + p_4 \psi + p_5 = 0,$$

where

$$p_1 = 4d + \gamma + \delta + M^* \lambda + \lambda_0 + \lambda_1 + \lambda_2 + \mu_0 + \nu,$$

$$p_2 = 3d(2d + \gamma + \delta + M^* \lambda + \lambda_0 + \lambda_1 + \lambda_2 + \nu) + M^* \lambda(\gamma + \delta + \lambda_2 + \nu + \mu_0) + \gamma(\lambda_0 + \lambda_1 + \mu_0) + \delta(\lambda_0 + \lambda_1 + \lambda_2 + \mu_0) + \mu_0(\lambda_0 + \lambda_1 + \lambda_2 + 4d + \nu) + \nu(\lambda_0 + \lambda_1 + \lambda_2) + \lambda_0(\lambda_1 + \lambda_2) + \lambda_1 \lambda_2,$$

$$p_3 = 3d^2(\gamma + \delta + M^* \lambda + \lambda_0 + \lambda_1 + \lambda_2 + 2\mu_0 + \nu) + 3d\mu_0(\gamma + \delta + M^* \lambda + \lambda_0 + \lambda + \lambda_2 + \nu) + M^* \lambda(2d\gamma + 2d\delta + 2d\lambda_2 + \delta\lambda_2 + \gamma\mu_0 + \delta\mu_0 + \lambda_2\mu_0 + 2d\nu + \lambda_2\nu + \mu_0\nu) + 2d(\gamma\lambda_0 + \delta\lambda_0 + \gamma\lambda_1 + \delta\lambda_1 + \lambda_0\lambda_1 + \delta\lambda_2 + \lambda_0\lambda_2 + \lambda_1\lambda_2 + \lambda_0\nu + \lambda_1\nu + \lambda_2\nu) + \lambda_0(\gamma\lambda_1 + \delta\lambda_1 + \delta\lambda_2 + \lambda_1\lambda_2 + \gamma\mu_0 + \delta\mu_0 + \mu_0\lambda_1 + \mu_0\lambda_2 + \nu\lambda_1 + \nu\lambda_2 + \mu_0\nu)$$

$$4d^3 + \lambda_1(\delta\lambda_2 + S^* \lambda\mu + \gamma\mu_0 + \delta\mu_0 + \lambda_2\mu_0 + \lambda_2\nu + \mu_0\nu) + \lambda_2\mu_0(\delta + \nu),$$

$$\begin{aligned}
 p_4 = & d^4 + d^3(\gamma + \delta + M * \lambda + \lambda_0 + \lambda_1 + \lambda_2 + 4\mu_0 + \nu) + d^2(M * \gamma\lambda + M * \delta\lambda + \gamma\lambda_0 + \delta\lambda_0 + \gamma\lambda_1 + \delta\lambda_1 \\
 & + \lambda_0\lambda_1 + \delta\lambda_2 + M * \lambda\lambda_2 + \lambda_0\lambda_2 + \lambda_1\lambda_2 + 3\gamma\mu_0 + 3\delta\mu_0 + 3M * \lambda\mu_0 + 3\lambda_0\mu_0 + 3\lambda_1\mu_0 + \lambda_2\mu_0 + \\
 & M * \lambda\nu + \lambda_0\nu + \lambda_1\nu + \lambda_2\nu + 3\mu_0\nu) + d(\gamma\lambda_0\lambda_1 + \delta\lambda_0\lambda_1 + M * \delta\lambda\lambda_2 + \delta\lambda_0\lambda_2 + \delta\lambda_1\lambda_2 + \lambda_0\lambda_1\lambda_2 + \\
 & 2S * \lambda\lambda_1\mu + 2M * \lambda\gamma\mu_0 + 2M * \delta\lambda\mu_0 + 2\gamma\lambda_0\mu_0 + 2\delta\lambda_0\mu_0 + 2\gamma\lambda_1\mu_0 + 2\delta\lambda_1\mu_0 + 2\lambda_0\lambda_1\mu_0 + \\
 & 2\delta\lambda_2\mu_0 + 2M * \lambda\lambda_2\mu_0 + 2\lambda_0\lambda_2\mu_0 + 2\lambda_1\lambda_2\mu_0 + \lambda_0\lambda_1\nu + M * \lambda\lambda_2\nu + \lambda_0\lambda_2\nu + \lambda_1\lambda_2\nu + 2M * \lambda\mu_0\nu \\
 & + 2\lambda_0\mu_0\nu + 2\lambda_1\mu_0\nu + 2\lambda_2\mu_0\nu) + \delta\lambda_0\lambda_1\lambda_2 + S * \lambda\lambda_1\mu(\gamma + \delta + \lambda_2 + \nu) + \gamma\lambda_0\lambda_1\mu_0 + \delta\lambda_0\lambda_1\mu_0 + \\
 & M * \delta\lambda\lambda_2\mu_0 + \delta\lambda_0\lambda_2\mu_0 + \delta\lambda_1\lambda_2\mu_0 + \lambda_0\lambda_1\lambda_2\mu_0 + \lambda_0\lambda_1\lambda_2\nu + \lambda_0\lambda_1\mu_0\nu + M * \lambda\lambda_2\mu_0\nu + \\
 & \lambda_0\lambda_2\mu_0\nu + \lambda_1\lambda_2\mu_0\nu,
 \end{aligned}$$

$$\begin{aligned}
 p_5 = & d^4\mu_0 + d^3(\gamma\mu_0 + \delta\mu_0 + M * \lambda\mu_0 + \lambda_0\mu_0 + \lambda_1\mu_0 + \lambda_2\mu_0 + \mu_0\nu) + d^2(S * \lambda\lambda_1\mu + M * \gamma\lambda\mu_0 + \gamma\lambda_0\mu_0 \\
 & + \delta\lambda_0\mu_0 + \gamma\lambda_1\mu_0 + \delta\lambda_1\mu_0 + \lambda_0\lambda_1\mu_0 + \delta\mu_0\lambda_2 + M * \lambda\lambda_2\mu_0 + \lambda_0\lambda_2\mu_0 + \lambda_1\lambda_2\mu_0 + M * \lambda\mu_0\nu + \lambda_0\mu_0\nu \\
 & + \lambda_1\mu_0\nu + \lambda_2\mu_0\nu) + d\{S * \lambda\lambda_1\mu(\gamma + \delta + \lambda_2 + \nu) + \gamma\lambda_0\lambda_1\mu_0 + \delta\lambda_0\lambda_1\mu_0 + M * \delta\lambda\lambda_2\mu_0 + \delta\lambda_0\lambda_2\mu_0 \\
 & + \delta\lambda_1\lambda_2\mu_0 + \lambda_0\lambda_1\lambda_2\mu_0 + \lambda_0\lambda_1\mu_0\nu + M * \lambda\lambda_2\mu_0\nu + \lambda_0\lambda_2\mu_0\nu + \lambda_1\lambda_2\mu_0\nu\} + \lambda_0\lambda_1\lambda_2\mu_0(\delta + \nu).
 \end{aligned}$$

By Routh-Hurwitz criteria, this equilibrium point is locally asymptotically stable if following conditions are satisfied:

1. p_1, p_2, p_3, p_4 and p_5 are positive,
2. $p_1p_2 - p_3 > 0$,
3. $(p_1p_2 - p_3)p_3 - p_1^2p_4 > 0$,
4. $(p_1p_2 - p_3)(p_3p_4 - p_2p_5) - (p_1p_4 - p_5)^2 > 0$.

Appendix C

Consider the following positive definite function about equilibrium point E_0 ,

$$G = \frac{1}{2}(S - \hat{S})^2 + \frac{1}{2}(D - \hat{D})^2 + \frac{1}{2}(C - \hat{C})^2 + \frac{1}{2}(S_m - \hat{S}_m)^2 + \frac{1}{2}(M - \hat{M})^2.$$

Differentiating G with respect to time t along the solutions of model (2.1), we get

$$\dot{G} = (S - \hat{S}) \frac{dS}{dt} + (D - \hat{D}) \frac{dD}{dt} + (C - \hat{C}) \frac{dC}{dt} + (S_m - \hat{S}_m) \frac{dS_m}{dt} + (M - \hat{M}) \frac{dM}{dt}.$$

Using system of equation (2.1), we get after some algebraic manipulations as

$$\begin{aligned}
 \dot{G} = & -(\lambda_1 + d + \lambda M)(S - S^*)^2 - \lambda S^*(S - S^*)(M - M^*) + (\lambda M - \lambda_0)(S - S^*)(S_m - S_m^*) - \\
 & (\lambda_2 + d)(D - D^*)^2 + \lambda_1(S - S^*)(D - D^*) + (\gamma + \lambda_2)(C - C^*)(D - D^*) - (\gamma + d + \nu + \delta)(C - C^*)^2 \\
 & + \lambda S^*(S_m - S_m^*)(M - M^*) - (d + \lambda_0)(S_m - S_m^*)^2 + \mu(M - M^*)(C - C^*) \\
 & + \mu(D - D^*)(M - M^*) - \mu_0(M - M^*)^2.
 \end{aligned}$$

The above equation can further be written as sum of the quadratics

$$\begin{aligned}
 \dot{G} = & -\frac{1}{3}b_{11}(S - S^*)^2 + b_{12}(S - S^*)(D - D^*) - \frac{1}{3}b_{22}(D - D^*)^2 - \frac{1}{3}b_{11}(S - S^*)^2 + \\
 & b_{14}(S - S^*)(S_m - S_m^*) - \frac{1}{2}b_{44}(S_m - S_m^*)^2 - \frac{1}{3}b_{11}(S - S^*)^2 + b_{15}(S - S^*)(M - M^*) \\
 & - \frac{1}{4}b_{55}(M - M^*)^2 - \frac{1}{3}b_{22}(D - D^*)^2 + b_{23}(D - D^*)(C - C^*) - \frac{1}{2}b_{33}(C - C^*)^2 - \frac{1}{3}b_{22}(D - D^*)^2 \\
 & + b_{25}(D - D^*)(M - M^*) - \frac{1}{4}b_{55}(M - M^*)^2 - \frac{1}{2}b_{33}(C - C^*)^2 + b_{35}(C - C^*)(M - M^*) \\
 & - \frac{1}{4}b_{55}(M - M^*)^2 - \frac{1}{2}b_{44}(S_m - S_m^*)^2 + b_{45}(S_m - S_m^*)(M - M^*) - \frac{1}{4}b_{55}(M - M^*)^2.
 \end{aligned}$$

where

$$b_{11} = \lambda_1 + d + \lambda M, \quad b_{12} = \lambda_1, \quad b_{22} = \lambda_2 + d, \quad b_{15} = \lambda S^*, \quad b_{55} = \mu_0, \quad b_{33} = \gamma + d + \nu + \delta,$$

$$b_{14} = \lambda M^*, \quad b_{44} = d + \lambda_0, \quad b_{23} = \gamma + \lambda_2, \quad b_{25} = \mu, \quad b_{35} = \mu, \quad b_{45} = \lambda S^*$$

Sufficient condition for \dot{G} to be negative definite are that the following inequalities hold,

$$(b_{12})^2 < \frac{4}{9} b_{11} b_{22}, \quad (b_{15})^2 < \frac{1}{3} b_{11} b_{55}, \quad (b_{14})^2 < \frac{2}{3} b_{11} b_{44}, \quad (b_{23})^2 < \frac{2}{3} b_{22} b_{33}, \quad (b_{25})^2 < \frac{1}{3} b_{22} b_{55}, \quad (b_{35})^2 < \frac{1}{2} b_{33} b_{55},$$

$$(b_{45})^2 < \frac{1}{2} b_{44} b_{55}.$$

Hence G is a Liapunov function with respect to equilibrium point E_0 , whose domain contains the region of attraction Ω .

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