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International Journal of Engineering, Science and Technology
Vol. 6, No. 1, 2014, pp. 58-70

INTERNATIONAL JOURNAL OF ENGINEERING, SCIENCE AND TECHNOLOGY

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Effect of radiation and porosity parameter on hydromagnetic flow due to exponentially stretching sheet in a porous media

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Abstract

The aim of this paper is to study the momentum and the heat transfer characteristics in incompressible electrically conducting boundary layer flow over an exponentially stretching sheet under the effect of magnetic field with thermal radiation through porous medium. The governing boundary layer equations are converted into self-similar nonlinear ordinary differential equations, using similarity transformations in exponential form and then solved numerically using shooting method. The velocity profile, skin friction-co-efficient and rate of heat transfer are computed numerically and then graphically studied with respect to similarity variable (η) for different cases of velocity ratio parameter (β)

Keywords: Boundary layer, unsteady flow, exponentially stretching surface, porous medium, thermal radiation.

DOI: http://dx.doi.org/10.4314/ijest.v6i1.7

1. Introduction

Flow of incompressible viscous fluid and heat transfer phenomena over stretching sheets have received great attention due to the abundance of practical application in the chemical and manufacturing process like Aerodynamics, Extrusion of plastic sheets, Continuous Casting of metals, glass fibers and paper production. The study of Magneto hydrodynamics flow and Heat transfer of an electrically conducting fluid has attracted the interest of many researchers due to extensive industrial application. The study of hydrodynamic flow of an electrically conducting fluid caused by the deformation of the wall of a vessel containing a fluid is of considerable interest in a modern metallurgical and metal-working process.

Crane (1970) was the first to consider the boundary layer flow caused by a stretching sheet which moves with a velocity varying linearly with the distance from a fixed point. The heat transfer aspect of this problem was investigated by Carragher and Crane (1982), under the conditions when the temperature difference between the surface and the ambient fluid is proportional to a power of the distance from a fixed point. Numerous studies such as Pavlov (1977), Gupta et al. (1977), Chen et al. (1988), Pop et al.(1996), Vajrvelu (2001), Cortell (2007), Zhang et al.(2011) and Bhatacharyya (2011) have been conducted later to extend the pioneering work of Crane (1970). Last few decades in almost all investigations on the flow over a stretching sheet, the flow occurs because of linear variation of stretching velocity of the flat sheet. So the boundary layer flow induced by an exponentially stretching sheet is not studied much through it is very important and realistic flow frequently appeared in many engineering process. Magyari and Keller (1999) first considered the boundary layer flow due to an exponential stretching sheet and they also studied the heat transfer in the flow taking exponentially varied wall temperature. Elbashbeshy (2001) numerically examined the flow and heat transfer over an exponentially stretching surface considering wall mass suction. Khan and Sanjayanand (2005) studied the flow of viscoelastic fluid sheet with viscous dissipation effect. Partha et al. (2005) obtained a similarity solution for mixed convection flow past an exponentially stretching surface by taking into account the influence of viscous dissipation on the convective transport. Sanjayanand and Khan (2006) discussed the effects of heat and mass transfer on the boundary layer flow of viscoelastic fluid. Al-Odat et al. (2006) explained the effect magnetic field on thermal boundary layer on an exponentially

stretching continuous surface with an exponential temperature distribution. Recently, Sajid and Hayat (2008) showed the influence of thermal radiation on the boundary layer flow past an exponentially stretching sheet and they reported a series solution for velocity and temperature using HAM.

Hiemenz (1911) first studied the steady flow in the neighbourhood of a stagnation-point. Chiam. (1994)considered a problem which is a combination of the works of Hiemenz (1911) and Crane (1970), i.e. the stagnation-point flow towards a stretching sheet taking identical stretching rate of the sheet and strain rate of the stagnation-point flow and he found no boundary layer structure near the sheet. Mahapatra and Gupta(2001) reinvestigated stagnation-point flow towards a stretching surface. They reported in their research work that a boundary layer is formed when stretching velocity is less than the free stream velocity. As the stretching velocity exceeds the free stream velocity than an inverted boundary layer is formed. Mahapatra *et al.* (2002) studied heat transfer in stagnation-point flow on stretching sheet with viscous dissipation effect. Viscous incompressible fluid striking on a permeable stretching surface with heat generation or absorption has been studied by Attia (2000).the same stagnation-point flow towards a stretching sheet with different stretching and straining rates and found two kinds of boundary layer near the sheet depending on the ratio of the stretching and straining rates.

Many researchers have analyzed magnetic field parameter on two-dimensional stagnation point flow of a viscous incompressible stagnation point flow of a viscous incompressible fluid. Attia (2007) investigated effect of increasing magnetic field on velocity boundary. Effects of Ohmic heating and viscous dissipation on steady flow with variable free stream has been investigated by Singh *et al*.(2009). Many other phenomenon for effects of heat transfer flows have been discussed in detail by Bejan *et al*. (2006), and White (2006). Free convection heat transfer with radiation effect near the isothermal stretching sheet and over a flat sheet near the stagnation point have been investigated, respectively, by Ghaly *et al*. (2002) and Pop *et al.* (2004). They found that a boundary layer thickness increases with radiation. The radiative effect on the heat transfer from an arbitrarily stretching surface with non-uniform surface temperature in a porous medium has been studied by Rashad(2007) .

Motivated by the above investigations and possible applications, it is of interest in the present work to study the problem of the effect of thermal radiation and heat transfer over an unsteady two dimensional viscous, incompressible flow of an electrically conducting fluid over an exponentially stretching surface embedded in a porous medium . The approach uses boundary layer theory along with the similarity procedure to find and compare feasible solution. The motion of the fluid is generated due to exponentially stretching of the sheet with the application of two equal & opposite forces which are applied along the x-axis so that the wall is stretched keeping the origin fixed. Fluid is considered in the influence of transverse magnetic field and thermal radiation. The linear stretching of the sheet is assumed because of its simplicity in the modeling of the flow and heat transfer over stretching surface which also permits the similarity solution and are useful in understanding the interaction of flow field with temperature field.

2. Formulation of the Problem

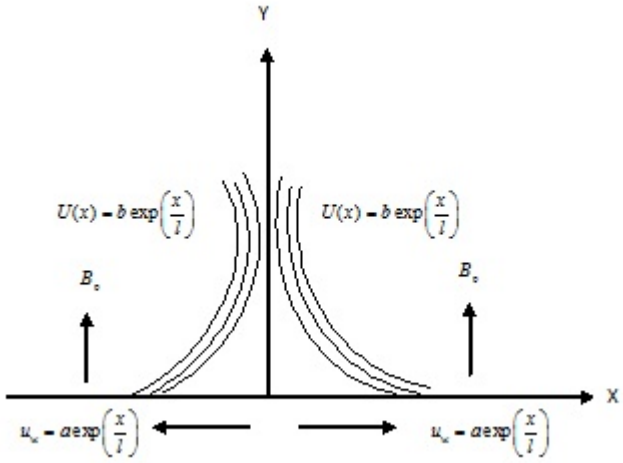


Fig. 1 Geometry of the problem

A steady two-dimensional flow of an incompressible viscous electrically conducting fluid over a continuous exponentially stretching sheet with velocity $U_w(x)$ in an exponential free stream with velocity $U(x)$, assuming that the plate has a surface temperature T_0 and is placed in a fluid of uniform ambient temperature T_∞ , has been considered as shown in Figure 1 .The sheet coincides with the plane $y = 0$ and the flow confined to $y > 0$. The motion of the fluid is generated due to exponentially stretching of the sheet with the application of two equal & opposite forces which are applied along the x-axis so that the wall is stretched

keeping the origin fixed. Here, in the present analysis there is no lateral velocity or pressure gradient far from the stretching surface. we have taken x-axis along the wall in the direction of motion of the flow, the y-axis being normal to it and *u* and *v* are tangential and normal velocity components respectively. A uniform magnetic field of strength B_0 is applied normally to the stretching surface which produces magnetic effect. It is also assumed that the fluid is weakly electrically conducting so that the induced magnetic field is negligible, which is justified for MHD flow at small magnetic Reynolds number. Under the usual boundary layer approximations, the flow and heat transfer with the radiation effects (Bansal, 1977; Schlichting *et al.*,1999.) are governed by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u-\frac{\upsilon}{\lambda}u \tag{2}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y}=0 \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{4}$$

Here, eq. (3) implies that pressure is independent of y, means pressure can be taken from the potential of flow i.e. in the free stream because pressure is independent of thickness of the boundary layer i.e. pressure is same at any point of the line perpendicular to the stretching sheet inside the boundary layer.

The stretching velocity $U_w(x)$ and free stream velocity $U(x)$ are respectively given by (Bhattacharyya and Moukhopadhyay. 2011)

$$u=U_w=a\exp\left(\frac{x}{l}\right),$$

$$u=U(x)=b\exp\left(\frac{x}{l}\right),$$

Here, in the free stream $u=U(x)=b\exp\left(\frac{x}{l}\right)$, eq. (2) reduces to.

$$U\frac{\partial U}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{\sigma B_0^2}{\rho}U-\frac{\upsilon}{\lambda}U \tag{5}$$

Eliminating $-\frac{1}{\rho}\frac{\partial p}{\partial x}$ from eq. (2) and (5), we get,

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=U\frac{\partial U}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}(u-U)-\frac{\upsilon}{\lambda}(u-U) \tag{6}$$

For the present problem, where the stretching of the boundary surface is assumed to be such that the flow directional velocity is of exponential order of the flow directional, we employ the following boundary conditions

$$u=U_w=a\exp\left(\frac{x}{l}\right),\quad v=0,\quad T=T_\infty+T_0\exp\left(\frac{x}{2l}\right)\qquad \text{at } y=0 \tag{7}$$

$$u=U(x)=b\exp\left(\frac{x}{l}\right),\qquad T=T_\infty\qquad \text{at } y=\infty \tag{8}$$

Here, eq. (7) implies that stretching sheet velocity and temperature varies exponentially with a distance from a fixed point with no slip condition and eq. (8) implies that outside of the boundary layer the stretching sheet velocity attained the full stream velocity U and temperature attained the uniform ambient temperature T_∞ .

where ρ is the fluid density, υ is the kinematic viscosity, μ is the dynamic viscosity, T is the temperature, k is the thermal conductivity, c_p is the specific heat and q_r is the radiative heat flux. T_0 and T_∞ are respectively the temperatures at and far away from the plate and l is a constant.

Employing the Rosseland approximation for radiation [Chen. et al (2008)] expressed as,

$$q_r = -\frac{4\sigma}{3\alpha}\frac{\partial T^4}{\partial y} \tag{9}$$

where a is mean absorption co-efficient and σ is the Stefen Boltzman constant. It is assumed that the temperature difference within the flow sufficiently small such that T^4 can be expressed as a linear function of temperature which after expanding using Taylor's series about T_∞ and neglecting the higher order term, reduces to

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \tag{10}$$

Hence the change in radiative flux with respect to y has been obtained as,

$$\frac{\partial q_r}{\partial y} = -\frac{16}{3\alpha}\frac{T_\infty^3\sigma}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{11}$$

and equation (3) reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{16}{3\alpha}\frac{T_\infty^3}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{12}$$

The equation of continuity is satisfied if a stream function $\psi(x, y)$ is chosen such that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}$$

By introducing the following dimensionless co-ordinate variable (Khan and Sanjayanand. 2005) viz;

$$\eta = y\sqrt{\frac{a}{2\upsilon l}}\exp\left(\frac{x}{2l}\right) \tag{13}$$

$$\psi(x, y) = \sqrt{2\upsilon l}\exp\left(\frac{x}{2l}\right) f(\eta) \tag{14}$$

$$T = T_\infty + T_0\exp\left(\frac{x}{2l}\right)\theta(\eta) \tag{15}$$

where η is the similarity variable, ψ is the stream function, θ is the dimensionless temperature and f is the dimensionless stream function. The above momentum and energy equations by using equations (13) to (15) are made free from dimension & reduces to

$$f''' + ff'' - 2f'^2 - 2(M + N)(f - \beta) + 2\beta^2 = 0 \tag{16}$$

$$\left(1 + \frac{4}{3R}\right)\theta'' + P_r\left[f\theta' - f'\theta + Ef'^2\right] = 0 \tag{17}$$

with the boundary condition;

$$f = 0,\ f' = 1,\ \theta = 1 \qquad \text{at} \quad \eta = 0 \tag{18}$$

$$f' = \beta,\ \theta = 0 \qquad \text{at} \quad \eta = \infty \tag{19}$$

where, dash represent differentiation with respect to η and $\beta = \frac{b}{a}$ is the velocity parameter, $M = \frac{\sigma B_0^2 l}{\rho u_w}$ is magnetic parameter, $N = \frac{\upsilon l}{u_w \lambda}$ is the porosity parameter, $R = \frac{k\alpha}{4\alpha T_\infty^3}$ is the radiation parameter $P_r = \frac{\mu c_p}{k}$ is the Prandtl number, $E = \frac{\alpha^2}{T_0 c_\rho}$ is the Eckert number.

The skin friction co-efficient c_f and the Nusselt number Nu_x are two important physical quantity and are defined as

$$c_f = \frac{2\tau_w}{(u_w)^2}, Nu_x = \frac{xq_w}{T_w - T_\infty} \tag{20}$$

where the skin friction τ_w and heat transfer from the sheet q_w are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial x}\right)_{y=0}, q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{21}$$

By using equation (21) and (20), the skin friction co-efficient c_f and the Nusselt number N_{u_x} reduces to

$$c_f \sqrt{R_{e_x}} = f''(0) \tag{22}$$

$$Nu_x \frac{N}{\sqrt{R_{e_x}}} = -\theta'(0) \tag{23}$$

To assess the accuracy of the present method, comparison with previously reported data available in the literature has been made. It is clear from Table 1 that the numerical values of $-\theta'(0)$ in the present paper for $\Pr = 1$ when $M = 0, N = 0, R = 0$ and $E = 0$ are in good agreement with results obtained by Mahapatra *et al*. (2002). From Table 1 it is clear that the scheme used in this paper is stable and accurate.

3. Numerical Simulation

In order to solve the set of non linear differential equations (16) and (17) subjected to the boundary conditions (18) and (19), the most efficient numerical shooting technique with Runge Kutta scheme is been used where it is most important to choose the appropriate finite values of $\eta \to \infty$ .The solution process is repeated with another large value of η_∞ until two successive values of $f''(0)$ and $\theta'(0)$ up to the desired significant value. The last value of η_∞ is chosen as appropriate value of the limit $\eta \to \infty$ for that particular set of parameters. The differential equations (16) and (17) were first converted into a set of five first-order simultaneous equations. To solve this system five initial conditions are required but there are only three initial conditions, $f(0)$ and $f'(0)$ on $f'(\eta)$ and $\theta(0)$ on $\theta(\eta)$. Still there are two initial conditions $f''(0)$ and $\theta'(0)$ are required, which are not prescribed. However the values $f''(\eta)$ and $\theta'(\eta)$ are known at $\eta \to \infty$. Shooting technique has been employed to find the two unknown initial values utilizing these two ending boundary conditions. After finding the required boundary conditions, the problem has been solved numerically using Runge-Kutta scheme.

4. Results and Discussion

To analyse the results numerical computations have been carried out by using the method described in the previous section for various value of velocity parameter β , magnetic parameter M , porosity parameter N , Prandtl number P_r , Radiation parameter R and for Eckert number E .

It is evident from Fig 2 (b), that when $\beta > 1$the flow has a boundary layer structure and the thickness of the boundary layer decreases with the increase in β . Moreover the straining motion near the stagnation motion for $\beta > 1$, increase so the acceleration of the external stream increases which causes to decrease the thickness of the boundary layer with increase in β and as a result the horizontal velocity increases with the increase of β .

On the other hand, when $\beta < 1$ the flow has an inverted boundary layer as shown in Fig 2(a). These results agree with those reported by Mahapatra and Gupta (2002). In this case, the stretching velocity of the sheet exceeds the velocity of the external stream. It is also observed that no boundary layer is formed when $\beta = 1$.

The magnetic parameter M represent the importance of magnetic field on the flow. The presence of transverse magnetic field sets in Lorentz force, which results in retarding force on the velocity field and there for as M increases, so does the retarding force and hence velocity decreases as shown in Fig.3 (a), when $\beta < 1$.When $\beta > 1$which just opposite of $\beta < 1$, expected velocity increases with increase of magnetic parameter as shown in Fig.3 (b). This explain the results presented in Table.4 is that the shear stress at the sheet decreases due to increase in magnetic parameter when $\beta < 1$, while it increases with the increase in magnetic parameter when $\beta > 1$.

Fig. 4(a) and Fig. 4(b) represent the influence of the porous medium on horizontal velocity. In these figure the graphical representation of horizontal velocity for different value of the porosity parameter for two set of value $\beta < 1$and $\beta > 1$ is obtained. It is found that for $\beta < 1$, the horizontal velocity $f'(\eta)$ decreases with the increase of N i.e. porosity parameter resistance to the flow since it restricts the motion of the fluid along the surface, with increasing the value of N, the thickness of the velocity boundary layer increase, so the velocity decreases with the increases of the value of N.This is due to the fact that the effect of porous medium which opposes the flow also increases and leads to enhance deceleration of the flow, but horizontal velocity increases with the increase of N when $\beta > 1$.

Fig. 5(a) and Fig. 5(b) represent the effect of Prandtl number P_r on temperature profile for both cases $\beta < 1$and $\beta > 1$respectively. Prandtl number defines the ratio of momentum diffusivity to the thermal diffusivity. It is noticed that as P_r increases, the temperature decreases for both cases. This is because, physically, if P_r increases, the thermal diffusivity decreases and these phenomena lead to the decreasing of energy ability that reduced the thermal boundary layer thickness.

From Fig. 6(a) and Fig. 6(b) it is very clear that with increasing value of the radiation parameter R, the thermal boundary layer thickness decreases and the heat flux of the surface increases since the temperature profiles become steeper. The effect of radiation parameter for both cases $\beta < 1$and $\beta > 1$, at a particular point is to reduce the temperature significantly in the flow region. The increase in radiation parameter is equivalent to the release of heat energy from the flow region and so the fluid temperature decreases as the thermal boundary layer thickness become thinner.

From Fig.7 (a) and Fig.7 (b) we observe that with increasing value of Eckert number E temperature increases for both cases $\beta < 1$and $\beta > 1$ respectively. Eckert number, physically, is a measure of frictional heat in the system. Hence the thermal regime with larger E values is subjected to rather more frictional heating causing a source of rise in temperature.

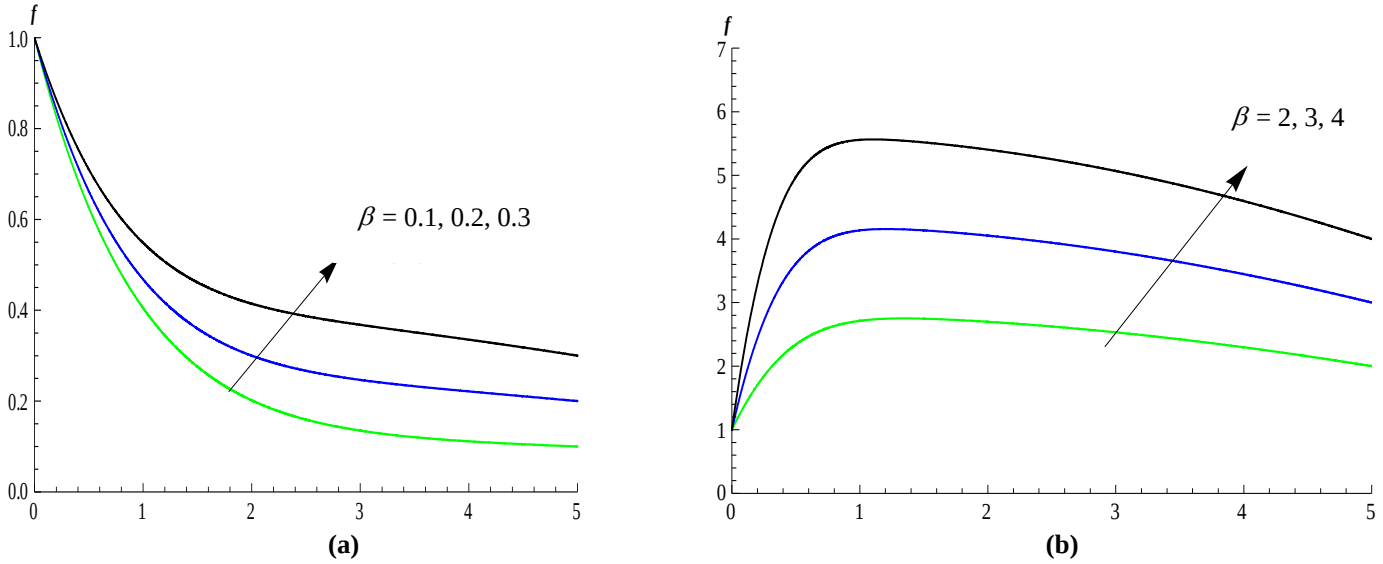


Fig. 2. Velocity profile $f'(\eta)$ for different values of (a) $\beta < 1$and (b) $\beta > 1$ for fixed value of $N = 0, M = 0, P_r = 0, R = 0$ and $E = 0$.

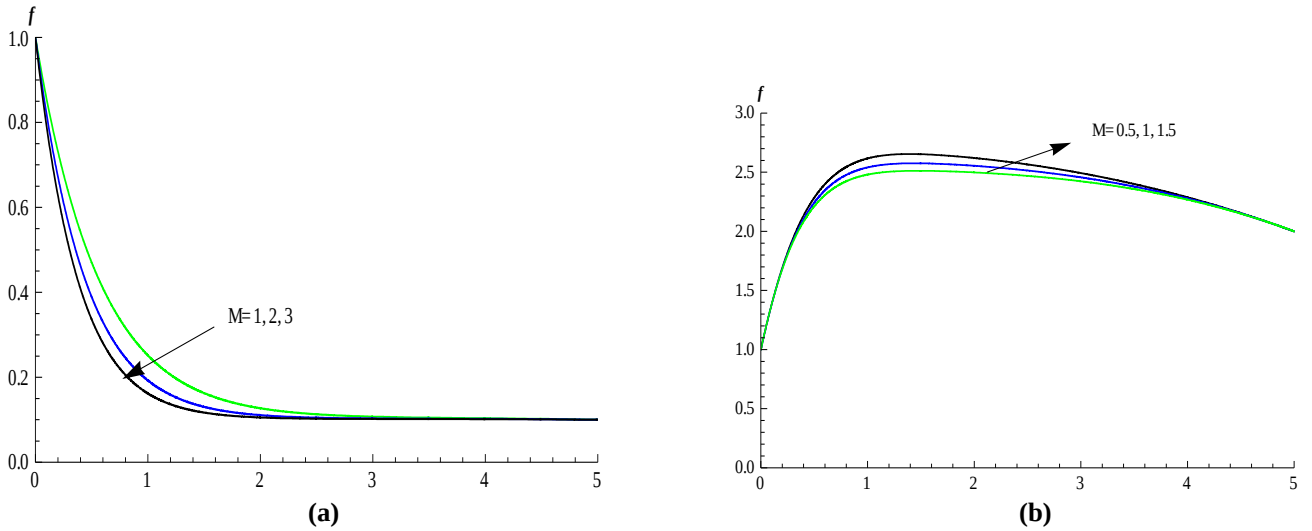


Fig. 3. Velocity profile $f'(\eta)$ for different values of M by taking (a) $\beta = 0.1$ and (b) $\beta = 2$ for fixed value of $N = 1, P_r = 1, R = 1$ and $E = 0.1$.

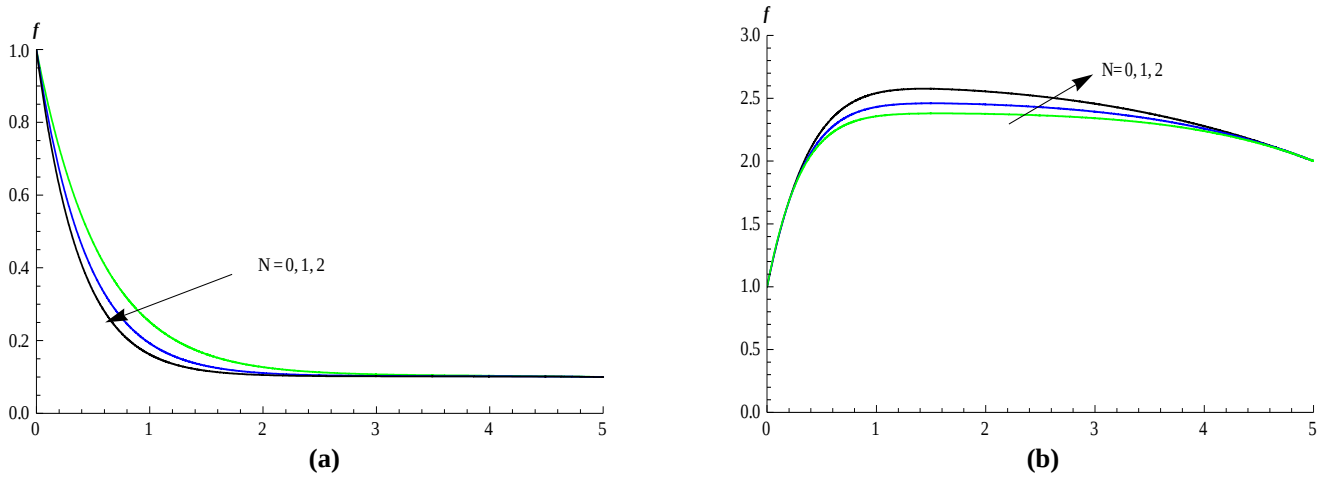


Fig.4. Velocity profile $f'(\eta)$ for different values of N by taking (a) $\beta = 0.1$ and (b) $\beta = 2$ for fixed value of $M = 1, P_r = 1, R = 1$ and $E = 0.1$

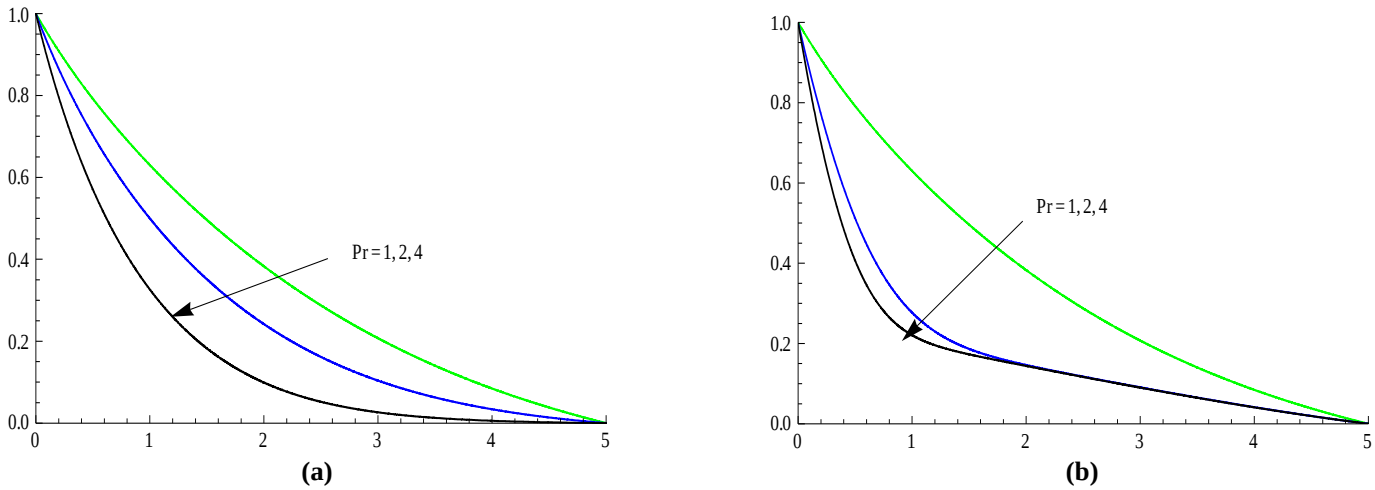
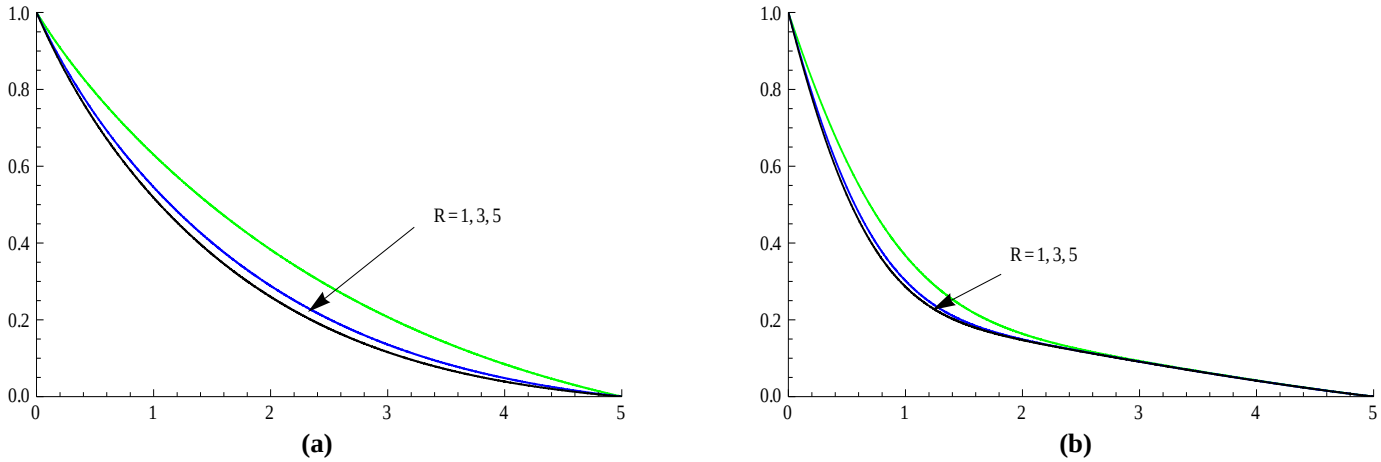
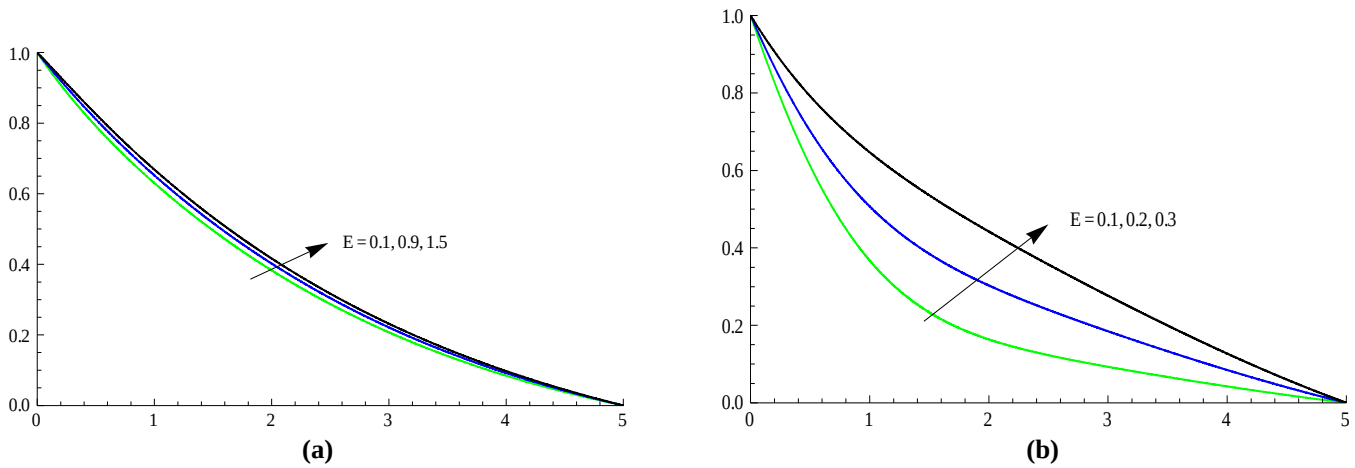


Fig.5. Temperature profile $\theta(\eta)$ for different values of P_r by taking (a) $\beta = 0.1$ and (b) $\beta = 2$ for fixed value of $M = 1, N = 1, R = 1$ and $E = 0.1$



(a) **(b)**

Fig.6. Temperature profile $\theta(\eta)$ for different values of R by taking (a) $\beta = 0.1$ and (b) $\beta = 2$ for fixed value of $M = 1, N = 1, P_r = 1$ and $E = 0.1$



(a) **(b)**

Fig.7. Temperature profile $\theta(\eta)$ for different values of E by taking (a) $\beta = 0.1$ and (b) $\beta = 2$ for fixed value of $M = 1, N = 1, R = 1$ and $P_r = 1$

The numerical value of skin friction co-efficient and Nusselt number are computed using the Mathematica software and are displayed in the tabular form in Tables 2 to 9 for various value of β, N, M, P_r, R and E. Table 2. shows that with increase of β, numerical values of wall shear stress decreases provided that $\beta < 1$, but increases provided that $\beta > 1$ respectively. This is because of the nature of the value of $f''(0)$ which is positive when $\beta > 1$ and negative when $\beta < 1$.

Table 3 and Table 4 give the computed value of the dimensionless shear stress for various value of M for the cases $\beta < 1$ and $\beta > 1$ respectively. It may observe that for a fixed value of β the wall shear stress decreases with increase in the value of M for the case $\beta < 1$, but increases for the case $\beta > 1$.

Table 5 gives the computed value of the dimensionless shear stress for various value of N for the cases $\beta < 1$ and $\beta > 1$ respectively. It may observe that for a fixed value of β the wall shear stress decreases with increase in the value of N provided that $\beta < 1$, but increases provided that $\beta > 1$.

Table 6 display a decrease of thermal boundary layer thickness with increasing value of Prandtl number for the cases $\beta < 1$ and $\beta > 1$ respectively. In heat transfer problems, the Prandtl number P_r controls the relative thickening of momentum and the thermal boundary layers. Fluid with lower Prandtl number wills posses higher thermal conductivities so that heat can diffuse from the sheet faster than for higher P_r fluids. Hence Prandtl number can be used to increase the rate of cooling.

Table 7 displays a decrease of Nusselt number with increasing value of Radiation parameter for the cases $\beta < 1$ and $\beta > 1$ respectively because Radiation/absorption has tendency to reduce rate of heat transfer. Therefore, Nusselt number (rate of

heat transfer) decreases with increase in radiation parameter. Table 8 and Table 9 display an increase in Nusselt number with increasing value of Eckert number because Heat transfer increases with increase in viscous dissipation which is measured by Eckert number. So, the Nusselt number increases with increase in Eckert number for the cases $\beta < 1$ and $\beta > 1$ respectively

Table. 1. Comparison of $\theta'(0)$ for $\Pr = 1$ when $M = 0, N = 0, R = 0$ and $E = 0$

β	$\theta'(0)$	
	Mahapatrta *et al* (2002)	Present paper
0.1	- 0.603	- 0.603142
0.2	- 0.625	- 0.625147
2	- 0.974	- 0.973912
3	- 1.124	-1.134276

Table. 2. Value of $-f''(0)$ for various value of $\beta, M, N, \Pr, R,$ and E.

Parameter (fixed value)	Parameter (Different Value)		$-f''(0)$
$M = 0, N = 0, \Pr = 0, R = 0, E = 0$	β	0.1	1.25366
		0.2	1.19512
		0.3	0.11145
		2	5.29462
		3	9.87654
		4	14.41795

Table.3. Value of $-f''(0)$ for various value of M for fixed $\beta = 0.1, N = 1, \Pr = 1, R = 1, E = 0.1$ when $\beta > 1$.

Parameter (fixed value)	Parameter (Different Value)		$-f''(0)$
$N = 1, P_r = 1, R = 1, E = 0.1$	M	1	1.79345
		2	3.34267
		3	4.20286

Table.4. Value of $-f''(0)$ for various value of M for fixed value of $\beta = 2, N = 1, \Pr = 1, R = 1, E = 0.1$ when $\beta < 1$.

Parameter (fixed value)	Parameter (Different Value)		$-f''(0)$
$N = 1, P_r = 1, R = 1, E = 0.1$	M	0.5	4.39678
		1.0	4.43828
		1.5	4.69732

Table. 5. Value of $-f''(0)$ for various value of N for fixed value of $M = 1, \Pr = 1, R = 1, E = 0.1$ for both cases.

Parameter (fixed value)	Parameter (Different Value)		$-f''(0)$	
			$\beta < 1$	$\beta > 1$
$N = 1, P_r = 1, R = 1, E = 0.1$	N	0	1.76718	4.53688
		1	2.54609	4.65972
		2	4.58755	4.86378

Table. 6. Value of $-\theta'(0)$ for various value of $\Pr$ for fixed value of $M=1, N=1, R=1, E=0.1$ for both cases.

Parameter (fixed value)	Parameter (Different Value)		$-\theta'(0)$	
			$\beta<1$	$\beta>1$
$N=1, P_r=1, R=1, E=0.1$	Pr	1	0.75903	0.97818
		2	0.84548	1.37410
		3	1.71293	1.703891

Table. 7. Value of $-\theta'(0)$ for various value of R for fixed value of $M=1, N=1, \Pr=1, E=0.1$ for both cases.

Parameter (fixed value)	Parameter (Different Value)		$-\theta'(0)$	
			$\beta<1$	$\beta>1$
$N=1, P_r=1, R=1, E=0.1$	R	1	0.75903	0.97876
		2	0.84361	1.24063
		3	0.98323	1.34015

Table. 8. Value of $-\theta'(0)$ for various value of E for fixed value of $M=2, N=1, \Pr=1, R=1$ for the case $\beta<1$.

Parameter (fixed value)	Parameter (Different Value)		$-\theta'(0)$
$N=1, P_r=1, R=1, E=0.1$	E	0.1	0.75903
		0.9	0.47281
		0.15	0.39754

Table. 9. Value of $-\theta'(0)$ for various value of E for fixed value of $M=2, N=1, \Pr=1, R=1$ for the case $\beta>1$.

Parameter (fixed value)	Parameter (Different Value)		$-\theta'(0)$
$N=1, P_r=1, R=1, E=0.1$	E	1	0.98651
		2	0.83803
		3	0.67685

4. Conclusions

The effect of thermal radiation and heat transfer over an unsteady two dimensional viscous, incompressible fluid flow of an electrically conducting fluid on exponentially stretching surface embedded in a porous medium has been investigated. Fluid is considered in the influence of transverse magnetic field and thermal radiation. Numerical solution for governing equation has been obtained. Which allow the computation of the flow and heat transfer characteristic for various values of the magnetic parameter, permeability parameter, radiation parameter, Eckert number and Prandtl number.

The main result of the paper can be summarized as follows:

- The boundary layer thickness decreases as β increases. The increase in the value of β implies that free stream velocity increases in comparison to stretching velocity, which results in the increase in pressure and straining motion near stagnation point.
- When Hartman number increases the velocity profiles decreases when $\beta < 1$. In case when $\beta > 1$, which is just opposite to $\beta < 1$, the velocity profiles increase with the increase in the Hartmann number. The shear stress at the sheet decreases

due to increase in the Hartmann number when $\beta < 1$, while it increases with the increase in the Hartmann number when $\beta > 1$which explains the results presented in tab. 4.

- The effect of porosity parameter N depends on β. When $\beta < 1$, velocity profile decreases as porosity parameter increases. The velocity profiles increase with the increase in the porosity parameter when $\beta > 1$.
- The temperature profile decreases with an increase in the Prandtl number. This is in agreement with the physical fact that at higher Prandtl number, fluid has a thinner thermal boundary layer .
- With increase in radiation parameter the temperature profiles decrease but with increase in Eckert number the temperature profiles increases.

Nomenclature

β	Velocity ratio parameter
ρ	Fluid density
υ	Kinematic viscosity
μ	Dynamic viscosity
T	Temperature
k	Thermal conductivity
c_p	Specific heat and
q_r	Radiative heat flux.
T_0 and T_∞	Temperatures at and far away from the plate
l	Constant.
a	Mean absorption co-efficient
σ	Stefen Boltzman constant
a	Stretching sheet parameter
b	free stream velocity parameter
M	Magnetic parameter.
N	Permeability parameter
R	Radiation parameter
P_r	Prandtl number
E	Eckert number.
ψ	Stream function
η	Similarity variable

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Received February 2013
Accepted November 2013
Final acceptance in revised form November 2013