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Hall effect on MHD flow of visco-elastic micro-polar fluid layer heated from below saturating a porous medium

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Abstract

The problem of visco-elastic micro-polar fluid layer heated from blow in the presence of uniform vertical magnetic field with Hall current in porous medium is discussed here and obtained a dispersion relation using normal mode analysis. From this dispersion relation, the medium permeability K_1 has destabilizing effect, the coupling parameter K has stabilizing effect. the micro-polar coefficient A has stabilizing effect under certain conditions, the magnetic field has stabilizing effect when $\bar{\delta} < \frac{\in}{A}$ and $\in > \frac{1}{2}$, the micro-polar heat conduction parameter has stabilizing effect when $\in > \frac{1}{2}$ and Hall parameter has destabilizing effect under $\bar{\delta} < \frac{\in}{A}$ and $\in > \frac{1}{2}$ and a necessary condition for oscillatory mode is obtained with $\bar{\delta} < \frac{\in}{A}$ and $Q < \frac{P_r}{\pi^2}$ and $F > \frac{Q\pi^2 K_1}{\in P_r \bar{j}}$

Keywords: Visco-elastic, Micro-polar fluid, Magnetic field, Hall current, Porous Medium.

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1. Introduction

B' Walters (1964) and Beard and Walters (1964) investigated the behavior of visco-elastic prototype fluid and Sen (1978) studied the behavior of visco-elastic fluid over an infinite porous plate with constant suction. Singh and Singh (1983) have studied the magneto-hydrodynamic flow of visco-elastic fluid past an accelerated plate. The flow of visco-elastic and electrically conducting fluid past an infinite plate has been studied by Sherief and Ezaat (1994). Gupta (1967) studied the stability of a small amplitude falling flim of visco-elastic. Shaqfeh *et al.* (1989) had shown that the visco-elastic property has destabilizing effect for small Reynolds number. However, the viscoelstic property possesses a primarily stabilizing effect on the film flow for moderate Reynolds numbers. Omokhuale *et al.* (2012) studied the effects of concentration and Hall current on unsteady flow of a visco-elastic fluid in a fixed plate. Chaudhary and Das (2013) studied viscoelastic unsteady MHD flow between two horizontal parallel plates with Hall current.. In view of the fact that the study of visco-elastic fluid in a porous medium may find applications in geophysics and chemical technology. However, in this paper, an attempt has been made to examine the effect of Hall current on MHD flow of visco-elastic (Rivlin-Ericksen type) micro-polar fluid layer heated from below on porous medium and the nature of the components like medium permeability, heat conduction, visco-elasticity, Hall current and Magnetic field are analyzed and to the best of my knowledge this problem is uninvestigated so far.

2. Mathematical Formulation

Consider an infinite, horizontal, incompressible electrically non-conducting visco-elastic micro-polar fluid layer of thickness d. A cartesian coordinate system (x, y, z) is chosen such that origin is at the lower boundary and the z-axis is vertically upward. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity $\in$ and medium permeability κ. The lower boundary at z=0 and the upper boundary at $z = d$ are maintained at constant but different temperature T_0 and T_1 such that a steady adverse temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ is maintained. The whole system is acted upon by a gravity field $\vec{\mathbf{g}} = (0, 0, -g)$ and a strong uniform magnetic field $\vec{\mathbf{H}} = (0, 0, H_0)$ is applied along z-axis.

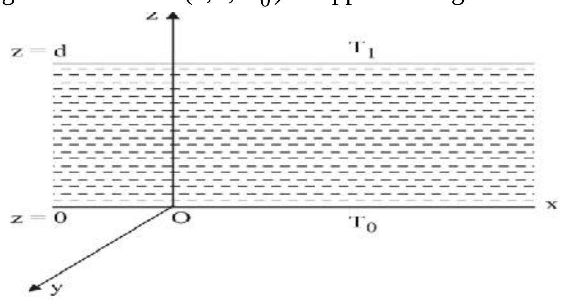

Fig. 1: Geometry of the problem

Here, we have taken Rivlin-Ericksen visco-elastic fluid in which when the fluid permeates a porous medium, the gross effect is represented by Darcy's law and the usual viscous term in the momentum equation is replaced by the resistance term $\left[-\frac{1}{\kappa}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\vec{\mathbf{q}}\right]$. Also both boundaries are considered to be free and perfect conductor of heat. For an isotropic medium the surface porosity is $\in$ so that $1- \in$ is the fraction that is occupied by solid.

Within Boussinesq approximation, the equations governing the motion of a micro-polar fluid saturating porous medium following (Lebon, 1981).(Lukaszewicz 1999, Kirti *et al.* 1999) for above model are as follows :

The equation of continuity for an incompressible fluid is

$$\nabla.\vec{\mathbf{q}} = 0 \tag{1}$$

The equation of momentum is

$$\frac{\rho_0}{\in}\left[\frac{\partial \vec{\mathbf{q}}}{\partial t} + \frac{1}{\in}(\vec{\mathbf{q}}.\nabla)\vec{\mathbf{q}}\right] = -\nabla p + (\mu + \zeta)\nabla^2\vec{\mathbf{q}} - \frac{1}{\kappa}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\vec{\mathbf{q}} + \zeta\nabla\times\vec{\mathbf{N}} - \rho g\hat{\mathbf{e}}_z + \frac{\mu_e}{4\pi}\left(\nabla\times\vec{\mathbf{H}}\right)\times\vec{\mathbf{H}} \tag{2}$$

The equation of internal angular momentum is

$$\rho_0 j\left[\frac{\partial \vec{\mathbf{N}}}{\partial t} + \frac{1}{\in}(\vec{\mathbf{q}}.\nabla)\vec{\mathbf{N}}\right] = (\alpha' + \beta' + \gamma')\nabla\left(\nabla.\vec{\mathbf{N}}\right) - \gamma'\nabla\times\nabla\times\vec{\mathbf{N}} + \zeta\left(\frac{1}{\in}\nabla\times\vec{\mathbf{q}} - 2\vec{\mathbf{N}}\right) \tag{3}$$

Where p, ρ, ρ_0, $\vec{\mathbf{q}}$, $\vec{\mathbf{N}}$, μ, ζ, μ', κ , α', β', γ', j, μ_e and $\hat{\mathbf{e}}_z$ denote respectively, pressure, fluid density, reference density, filter velocity, micro-rotation, viscosity, dynamic micro-rotation viscosity, visco-elasticity, medium permeability, micro-polar viscosity coefficients, micro-inertia constant, magnetic permeability and unit vector in z-direction.

The equation of energy is

$$\left[\rho_0 C_v \in + \rho_s C_s (1-\in)\right]\frac{\partial T}{\partial t} + \rho_0 C_v (\vec{\mathbf{q}}.\nabla)T = \chi_T \nabla^2 T + \delta\left(\nabla\times\vec{\mathbf{N}}\right).\nabla T \tag{4}$$

and the equation of state of the problem is

$$\rho = \rho_0\left[1 - \alpha(T - T_0)\right] \tag{5}$$

Where $C_v, C_s, \chi_T, \rho_s, \delta, \alpha, T,$ and T_0 denote respectively specific heat at constant volume, heat capacity of solid (porous material matrix), thermal conductivity, density of solid matrix, coefficient giving account of coupling between the spin flux and heat flux, coefficient of thermal expansion, temperature and reference temperature.

The Maxwell's equation in the presence of Hall current yield

$$\in\frac{\partial \vec{\mathbf{H}}}{\partial t} = \nabla\times\left(\vec{\mathbf{q}}\times\vec{\mathbf{H}}\right) + \in\gamma_m\nabla^2\vec{\mathbf{H}} - \frac{\in}{4\pi e n_e}\nabla\times\left[\left(\nabla\times\vec{\mathbf{H}}\right)\times\vec{\mathbf{H}}\right] \tag{6}$$

and $\qquad \nabla.\vec{\mathbf{H}} = 0 \tag{7}$

Where $\vec{\mathbf{H}} = (0, 0, H_0)$, H_0 is a constant, n_e = electron density and e = charge on electron and γ_m is the magnetic viscosity.

3. Basic State of the Problem

The basic state is defined by these equations $\vec{\mathbf{q}} = \vec{\mathbf{q}}_b(0,0,0), \vec{\mathbf{N}} = \vec{\mathbf{N}}_b(0,0,0), \rho = \rho_b(z)$, and $p = p_b(z)$
Under this basic state equations (1) to (7) become

$$\frac{dp_b}{dz} + \rho_b g = 0 \tag{8}$$

$$T = -\beta z + T_0 \tag{9}$$

and

$$\rho_b = \rho_0(1 + \alpha\beta z) \tag{10}$$

4. Perturbation Equations

Using (8),(9),(10) and the following perturbed variables are

$$\vec{\mathbf{q}} = \vec{\mathbf{q}}_b + \vec{\mathbf{q}}' = \vec{\mathbf{q}}', \vec{\mathbf{N}} = \vec{\mathbf{N}}_b + \vec{\mathbf{N}}' = \vec{\mathbf{N}}', p = p_b + p', \rho = \rho_b + \rho', \vec{\mathbf{H}} = \vec{\mathbf{H}}_b + \vec{\mathbf{h}}, T = T_b + \theta$$

where $\vec{\mathbf{q}}', \vec{\mathbf{N}}, p', \rho', \vec{\mathbf{h}}$ and θ are the perturbations in $\vec{\mathbf{q}}, \vec{\mathbf{N}}, P, \rho, \vec{\mathbf{H}}$ and T respectively and after that using the following non-dimensional transformations

$$x = dx^*, y = dy^*, z = dz^*, \vec{\mathbf{q}}' = \frac{K_T}{d}\vec{\mathbf{q}}^*, \vec{\mathbf{N}} = \frac{K_T}{d^2}\vec{\mathbf{N}}^*, t = \frac{\rho_0 d^2}{\mu}t^*, \theta = \beta d\theta^*, p' = \frac{\mu K_T}{d^2}p^*, \quad \vec{\mathbf{h}} = H_0\vec{\mathbf{h}}^*,$$

where $K_T = \dfrac{\chi_T}{\rho_0 C_v}$ is the thermal diffusivity, the equations from (1) to (7) in linear form after dropping stars become

$$\nabla.\vec{\mathbf{q}} = 0 \tag{11}$$

$$\frac{1}{\in}\frac{\partial \vec{\mathbf{q}}}{\partial t} = -\nabla p + (1+K)\nabla^2\vec{\mathbf{q}} - \frac{1}{K_1}\left(1 + F\frac{\partial}{\partial t}\right)\vec{\mathbf{q}} + R\theta\hat{e}_z + K\nabla\times\vec{\mathbf{N}} + Q(\nabla\times\vec{\mathbf{h}})\times\hat{e}_z \tag{12}$$

$$\bar{j}\frac{\partial \vec{\mathbf{N}}}{\partial t} = C_0\nabla(\nabla.\vec{\mathbf{N}}) - C\nabla\times(\nabla\times\vec{\mathbf{N}}) + K\left(\frac{1}{\in}\nabla\times\vec{\mathbf{q}} - 2\vec{\mathbf{N}}\right) \tag{13}$$

$$EP_r\frac{\partial\theta}{\partial t} = \nabla^2\theta + w - \bar{\boldsymbol{\delta}}\xi \tag{14}$$

$$\in P_r\frac{\partial\vec{\mathbf{h}}}{\partial t} = \frac{\partial\vec{\mathbf{q}}}{\partial z} + \frac{\in P_r}{P_m}\nabla^2\vec{\mathbf{h}} - \in\beta_e^{1/2}\frac{\partial}{\partial z}\left(\nabla\times\vec{\mathbf{h}}\right) \tag{15}$$

Where $R = \dfrac{\rho_0 g\alpha\beta d^4}{\mu K_T}$ is the thermal Rayleigh number, $Q = \dfrac{\mu_e H_0^2 d^2}{4\pi\mu K_T}$ is the Chandrasekhar number,

$C_0 = \dfrac{\alpha' + \beta' + \gamma'}{\mu d^2}, C = \dfrac{\gamma'}{\mu d^2}, K = \dfrac{\zeta}{\mu}, K_1 = \dfrac{\kappa}{d^2}, E = \in + \dfrac{\rho_s C_s(1-\in)}{\rho_0 C_v}, P_r = \dfrac{\mu}{\rho_0 K_T}$ is the Prandtl number, $P_m = \dfrac{\mu}{\rho_0\gamma_m}$ is the magnetic prandtl number, $F = \dfrac{\mu'}{\rho_0 d^2}$ is the visco-elastic parameter, $\beta_e = \left(\dfrac{H_0}{4\pi K_T e n_e}\right)^2$ is the Hall parameter, $\bar{\boldsymbol{\delta}} = \dfrac{\delta}{\rho_0 C_v d^2}$ is the coupling parameter and $w = \vec{\mathbf{q}}.\hat{e}_z, \xi = \left(\nabla\times\vec{\mathbf{N}}\right).\hat{e}_z$

5. Boundary Condition

Both boundary are taken to be free and perfectly heat conducting, then we have

$$w = \frac{d^2 w}{dz^2} = 0, \vec{\mathbf{N}} = 0, \theta = 0, \xi = 0 \text{ at z = 0 and z = 1} \tag{16}$$

6. Dispersion Relations

Taking curl twice on both sides (12), and using (17) and (19) and taking z- component, we have

$$\left[\frac{1}{\in}\frac{\partial}{\partial t}+\frac{1}{K_1}\left(1+F\frac{\partial}{\partial t}\right)-(1+K)\nabla^2\right]\nabla^2 w=R\nabla_1^2\theta+K\nabla^2\xi+QD\left(\nabla^2 h_z\right) \quad (17)$$

Taking curl on both sides (12), and taking z-component, we have

$$\left[\frac{1}{\in}\frac{\partial}{\partial t}+\frac{1}{K_1}\left(1+F\frac{\partial}{\partial t}\right)-(1+K)\nabla^2\right]\zeta_z = K\left(\nabla\times\nabla\times\vec{\mathbf{N}}\right).\hat{e}_z+QDm_z \quad (18)$$

Taking curl on both sides of equation (13) and then z-component, we have

$$\left[\bar{\mathbf{j}}\frac{\partial}{\partial t}+2K-C\nabla^2\right]\xi=-\frac{K}{\in}\nabla^2 w \quad (19)$$

Taking curl twice on both sides of equation (13) and then z-component, we have

$$\left[\bar{\mathbf{j}}\frac{\partial}{\partial t}+2K-C\nabla^2\right]\left(\nabla\times\nabla\times\vec{\mathbf{N}}\right).\hat{e}_z=-\frac{K}{\in}\nabla^2\zeta_z \quad (20)$$

Eliminating $(\nabla\times\nabla\times\vec{\mathbf{N}}).\hat{e}_z$ between (18) and (20), we have

$$\left[\frac{1}{\in}\frac{\partial}{\partial t}+\frac{1}{K_1}\left(1+F\frac{\partial}{\partial t}\right)-(1+K)\nabla^2\right]\left[\bar{\mathbf{j}}\frac{\partial}{\partial t}+2K-C\nabla^2\right]\zeta_z=-\frac{K^2}{\in}\nabla^2\zeta_z+Q\left[\bar{\mathbf{j}}\frac{\partial}{\partial t}+2K-C\nabla^2\right]Dm_z \quad (21)$$

From (14), we have

$$\left[EP_r\frac{\partial}{\partial t}-\nabla^2\right]\theta=w-\bar{\boldsymbol{\delta}}\xi \quad (22)$$

Taking curl on both sides of equation (15), and taking z-component, we have

$$\in P_r\frac{\partial m_z}{\partial t}=D\zeta_z+\frac{\in P_r}{P_m}\nabla^2 m_z+\in\beta_e^{1/2}D\left(\nabla^2 h_z\right) \quad (23)$$

Taking z-component on both sides of equation (15), we have

$$\in P_r\frac{\partial h_z}{\partial t}=Dw+\frac{\in P_r}{P_m}\nabla^2 h_z-\in\beta_e^{1/2}Dm_z \quad (24)$$

where $\nabla_1^2\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}, h_z=\vec{\mathbf{h}}.\hat{e}_z, D=\frac{\partial}{\partial z},\ \zeta_z=\left(\nabla\times\vec{\mathbf{q}}\right).\hat{e}_z$ and $m_z=\left(\nabla\times\vec{\mathbf{h}}\right).\hat{e}_z$

Boundary condition (16) now become

$$w=D^2w=0=\xi=\zeta_z=D\zeta_z=h_z=Dm_z=m_z=\theta \text{ at } z=0 \text{ and } z=1 \quad (25)$$

7. Normal Mode Analysis

Consider $[w,\zeta_z,\xi,\theta,h_z,m_z]=\left[W(z),X(z),G(z),\Theta(z),B(z),M(z)\right]\exp.\left[ik_x x+ik_y y+\sigma t\right]$

Applying above normal mode analysis to the equations (17) to (19) and (21) to (24), we have

$$\left[\frac{\sigma}{\in}\frac{\partial}{\partial t}+\frac{1}{K_1}(1+F\sigma)-(1+K)(D^2-a^2)\right](D^2-a^2)W=-Ra^2\Theta+K(D^2-a^2)G+QD(D^2-a^2)B \quad (26)$$

$$\left[\bar{\mathbf{j}}\sigma+2K-C(D^2-a^2)\right]G=-\frac{K}{\in}(D^2-a^2)W \quad (27)$$

$$\left[\frac{\sigma}{\in}\frac{\partial}{\partial t}+\frac{1}{K_1}(1+F\sigma)-(1+K)(D^2-a^2)\right]\left[\bar{\mathbf{j}}\sigma+2K-C(D^2-a^2)\right]X=-\frac{K^2}{\in}(D^2-a^2)X+Q\left[\bar{\mathbf{j}}\sigma+2K-C(D^2-a^2)\right]DM \quad (28)$$

$$\left[EP_r\sigma-(D^2-a^2)\right]\Theta=W-\bar{\boldsymbol{\delta}}G \quad (29)$$

$$\left[EP_r\sigma-\frac{\in P_r}{P_m}(D^2-a^2)\right]\mathrm{M}=DX+\in\beta_e^{1/2}(D^2-a^2)B \quad (30)$$

and

$$\left[EP_r\sigma-\frac{\in P_r}{P_m}(D^2-a^2)\right]\mathrm{B}=DW-\in\beta_e^{1/2}DM \quad (31)$$

Where $a^2 = \kappa_x^2 + \kappa_y^2$ is the wave number and $\sigma = \sigma_r + i\sigma_i$ is the stability parameter. Now the boundary conditions become

$$W = D^2W = 0 = G = X = DX = B = M = DM,\ \Theta=0 \text{ at } z=0 \text{ and } z=1 \tag{32}$$

Thus, the proper solution satisfying (32) can be taken as

$$W = W_0 \sin \pi z, \text{ Where } W_0 \text{ is a constant} \tag{33}$$

Eliminating Θ, G, B, X and M from (26) to (31), and substituting the value of W from (33) and using $b=\pi^2 + a^2$, we have

$$\Bigg[\Bigg\{\left[\frac{\sigma}{\in} + \frac{1}{K_1}(1+F\sigma) + b(1+K)\right]\left[\bar{\mathbf{j}}\sigma + 2K + Cb\right] + \frac{K^2 b}{\in}\Bigg\}.\Bigg\{\left[\in P_r\sigma + \frac{\in P_r b}{P_m}\right]^2 + \in^2 \pi^2 b\beta_e\Bigg\}$$

$$+Q\pi^2\left(\bar{\mathbf{j}}\sigma + 2K + Cb\right)\left(\in P_r\sigma + \frac{\in P_r b}{P_m}\right)\Bigg] \times \Bigg[\left\{\frac{\sigma}{\in} + \frac{1}{K_1}(1+F\sigma) - b(1+K)\right\}\left\{EP_r\sigma + b\right\}\left\{\bar{\mathbf{j}}\sigma + 2K + Cb\right\}(-b)$$

$$+Ra^2\left\{\bar{\mathbf{j}}\sigma + 2K + Cb - \frac{\bar{\boldsymbol{\delta}}Kb}{\in}\right\} + \frac{K^2b^2}{\in}\left(EP_r\sigma + b\right)\Bigg]$$

$$= \Bigg[\Bigg\{\left[\frac{\sigma}{\in} + \frac{1}{K_1}(1+F\sigma) + b(1+K)\right]\left[\bar{\mathbf{j}}\sigma + 2K + Cb\right] - \frac{K^2 b}{\in}\Bigg\}.\left\{\in P_r\sigma + \frac{\in P_r b}{P_m}\right\} + Q\pi^2\left(\bar{\mathbf{j}}\sigma + 2K + Cb\right)\Bigg]\ Q\pi^2 b\left(EP_r\sigma + b\right)\ \left(\bar{\mathbf{j}}\sigma + 2K + Cb\right) \tag{34}$$

8. Stationary Convection

For stationary convection we take σ = 0 in (34), we get

$$R = \frac{b^2\left[\frac{\in P_r b}{P_m}\left\{\left(\frac{1}{K_1} + b + bK\right)(2K+Cb) - \frac{K^2b}{\in}\right\} + Q\pi^2(2K+Cb)\right]^2 + \in^2 \pi^2 b^3\beta_e\left[\left(\frac{1}{K_1} + b + bK\right)(2K+Cb) - \frac{K^2b}{\in}\right]^2}{a^2\left(2K + Cb - \frac{\bar{\boldsymbol{\delta}}Kb}{\in}\right)\left[\left\{\left(\frac{1}{K_1} + b + bK\right)(2K+Cb) - \frac{K^2b}{\in}\right\} \times \left\{\left(\frac{\in P_r b}{P_m}\right)^2 + \in^2 \pi^2 b\beta_e\right\} + Q\pi^2 \frac{\in P_r b}{P_m}(2K+Cb)\right]} \tag{35}$$

When ($H_0 = 0$) i.e. Q = 0 and $\beta_e = 0$, equation (35) becomes

$$R = \frac{b^2\left[\left(\frac{1}{K_1} + b + bK\right)(2K+Cb) - \frac{K^2b}{\in}\right]}{a^2\left(2K + Cb - \frac{\bar{\boldsymbol{\delta}}Kb}{\in}\right)} \tag{36}$$

In absence of porous medium [$K_1 \to \infty$ and $\in=1$] equation (36) reduces to

$$R = \frac{b^3\left[(1+K)(2K+Cb) - K^2\right]}{a^2\left(2K + Cb - \bar{\boldsymbol{\delta}}Kb\right)} = \frac{b^3\left[Cb(1+K) + 2K + K^2\right]}{a^2\left(2K + Cb - \bar{\boldsymbol{\delta}}Kb\right)} \tag{37}$$

Which is the same as discussed by Payne and Straughan (1989) in their paper.
Also if $\bar{\delta} = 0$, then (37) reduces to

$$R = \frac{b^3\left[Cb(1+K) + 2K + K^2\right]}{a^2(2K+Cb)} = \frac{b^3}{a^2}\left[1 + K\left(\frac{Cb+K}{Cb+2K}\right)\right] \tag{38}$$

Which is similar equation as given by Data and Sastry (1976) . Also if K = 0, then (38) reduces to

$$R = \frac{b^3}{a^2}$$

Which is the classical value of R for Newtonian fluid obtained by Lebon and Perez-Garcia (1981).

Equation (35) can be rewritten as

$$R = \frac{b^2\left[\frac{\in P_r b}{P_m}\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}+Q\pi^2(2A+b)\right]^2 + \in^2\pi^2 b^3\beta_e\left[\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right]^2}{a^2\left(2A+b-\frac{\bar{\boldsymbol{\delta}}Ab}{\in}\right)\left[\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}.\times\left\{\left(\frac{\in P_r b}{P_m}\right)^2+\in^2\pi^2 b\beta_e\right\}+\frac{Q\pi^2 \in P_r b}{P_m}(2A+b)\right]} \tag{39}$$

Where $A=\frac{K}{C}$ denotes the micropolar coefficient.

For the behavior of K_1 (Medium permeability), K (coupling parameter), A (Micropolar coefficient), Q (Magnetic field), β_e (Hall Current) and $\bar{\boldsymbol{\delta}}$ (microplar heat conduction parameter), we examine the nature of $\frac{dR}{dK_1},\frac{dR}{dK},\frac{dR}{dA},\frac{dR}{dQ},\frac{dR}{d\beta_e}$ and $\frac{dR}{d\bar{\boldsymbol{\delta}}}$ respectively.

From (39), we have

$$\frac{dR}{dK_1}=\left(\frac{b^2}{a^2}\right)\frac{\begin{array}{c}\left[\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}\left\{\frac{\in^2 P_r^2 b^2}{P_m^2}+\in^2\pi^2 b\beta_e\right\}+\frac{Q\pi^2 \in P_r b}{P_m}(2A+b)\right] \\ \times\left[2\left\{\frac{-\in P_r b}{P_m K_1^2}(2A+b)\right\}\left[\frac{\in P_r b}{P_m}\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}+Q\pi^2(2A+b)\right]\right. \\ \left. -\frac{2\in^2\pi^2 b\beta_e(2A+b)}{K_1^2}\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}\right] \\ +\frac{(2A+b)}{K_1^2}\left\{\frac{\in^2\pi^2 b^2}{P_m^2}+\in^2\pi^2 b\beta_e\right\}\left\{\left[\frac{\in P_r b}{P_m}\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}\right.\right. \\ \left.\left.+Q\pi^2(2A+b)\right]^2+\in^2\pi^2 b\beta_e\left[\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right]^2\right\}\end{array}}{\begin{array}{c}\left(2A+b-\frac{\bar{\boldsymbol{\delta}}Ab}{\in}\right)\left[\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}\right. \\ \left.\times\left\{\frac{\in^2 P_r^2 b^2}{P_m^2}+\in^2\pi^2 b\beta_e\right\}+Q\pi^2(2A+b)\right]^2\end{array}}$$

Clearly, $\frac{dR}{dK_1}<0$ when $\bar{\boldsymbol{\delta}}<\frac{\in}{A}$, $\beta_e<\frac{P_r^2 b}{\pi^2 P_m^2}$ and $K_1<\frac{\in}{KA}\left(\text{or} \in>\frac{1}{2}\right)$

Thus, the medium permeability K_1 has destabilizing effect because R decreases as K_1 increases when

$$\bar{\boldsymbol{\delta}}<\frac{\in}{A},\ \beta_e<\frac{P_r^2 b}{\pi^2 P_m^2} \text{ and } \left[K_1<\frac{\in}{KA}\left(\text{or } \in>\frac{1}{2}\right)\right]$$

In particular, if there is no heat conduction $\bar{\boldsymbol{\delta}}=0$ and $\beta_e=0$ $[i.e.\ H_0=0]$, then

$$\frac{dR}{dK_1}=\frac{-b^2}{a^2K_1^2}$$

Which is always negative, thus the medium permeability has destabilizing effect without any condition.
From (39), we get

$$\frac{dR}{dK}=\frac{b^2\left(2Ab+b^2-\frac{Ab}{\in}\right)}{a^2\left(2A+b-\frac{\bar{\boldsymbol{\delta}}Ab}{\in}\right)}\;\frac{\left(\frac{\in^2 P_r^2 b^2}{P_m^2}-\in^2\pi^2 b\beta_e\right)\left[\frac{\in P_r b}{P_m}\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}+Q\pi^2(2A+b)\right]^2+\in^2\pi^2 b\beta_e\left(\in^2\pi^2 b\beta_e+\frac{3\in^2 P_r^2 b^2}{P_m^2}\right)\left[\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right]^2+\frac{4Q^2\pi^4\in^3 P_r b^2\beta_e(2A+b)}{P_m}\left[\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right]}{\left[\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}.\left\{\frac{\in^2 P_r^2 b^2}{P_m^2}+\in^2\pi^2 b\beta_e\right\}+\frac{Q\pi^2\in P_r b}{P_m}(2A+b)\right]^2}$$

Clearly, $\frac{dR}{dK}>0$ when $\bar{\boldsymbol{\delta}}<\frac{\in}{A},\ \frac{1}{2}<\in<1$ and $\beta_e<\frac{P_r^2 b}{\pi^2 P_m^2}$ .Thus, R increases as K increases therefore, the coupling parameter *K* has stabilizing effect when

$\bar{\boldsymbol{\delta}}<\frac{\in}{A},\frac{1}{2}<\in<1$ and $\beta_e<\frac{P_r^2 b}{\pi^2 P_m^2}$ Also $\frac{dR}{dK}<0$ when, $\bar{\boldsymbol{\delta}}<\frac{\in}{A},0<\in<\frac{1}{2}$ and $\frac{\beta_e\pi^2 P_m^2}{P_r^2}<b<A\left|2-\frac{1}{\in}\right|$

Thus, the coupling parameter *K* has destabilizing effect because R decreases as K increases when $\bar{\boldsymbol{\delta}}<\frac{\in}{A},\ 0<\in<\frac{1}{2}$ and $\frac{\beta_e\pi^2 P_m^2}{P_r^2}<b<A\left|2-\frac{1}{\in}\right|$

In particular, when $\bar{\boldsymbol{\delta}}=0$ and $\beta_e=0$, we have

$$\frac{dR}{dK}=\frac{b^3\left(2A+b-\frac{A}{\in}\right)}{a^2(2A+b)}$$

Clearly, $\frac{dR}{dK}>0$ when $\frac{1}{2}<\in<1$ and $\frac{dR}{dK}<0$ when $0<\in<\frac{1}{2}$ and $b<A\left|2-\frac{1}{\in}\right|$

If there is no heat conduction and Hall current, R increases as K increases when $\frac{1}{2}<\in<1$ thus, the coupling parameter K has stabilizing effect and R decreases as K increases when $0<\in<\frac{1}{2}$ and $b<A\left|2-\frac{1}{\in}\right|$ thus, K has destabilizing effect.

From (39), we get

$$\frac{dR}{dA}=\frac{b^2}{a^2}\;\frac{\begin{gathered}\left(2A+b-\frac{\bar{\delta}Ab}{\in}\right)\left[\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}.\left\{\frac{\in^2 P_r^2b^2}{P_m^2}+\in^2\pi^2 b\beta_e\right\}+\frac{Q\pi^2\in P_r b(2A+b)}{P_m}\right]\\ \times\left\{2\left[\frac{\in P_r b}{P_m}\left(\frac{2}{K_1}+2b+2bK-\frac{Kb}{\in}\right)+2Q\pi^2\right]\left[\frac{\in P_r b}{P_m}\left\{\left(\frac{1}{K_1}+b+bK\right)\times(2A+b)-\frac{KAb}{\in}\right\}+Q\pi^2(2A+b)\right]\right.\\ \left.+2\in^2\pi^2 b\beta_e\left(\frac{2}{K_1}+2b+2bK-\frac{Kb}{\in}\right)\left[\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right]\right\}\\ -\left\{\left[\frac{\in P_r b}{P_m}\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}+Q\pi^2(2A+b)\right]^2+\in^2\pi^2 b\beta_e\left[\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right]^2\right\}\\ \times\left\{\left(2-\frac{\bar{\delta}b}{\in}\right)\left[\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}\left\{\frac{\in^2 P_r^2b^2}{P_m^2}+\in^2\pi^2 b\beta_e\right\}+\frac{Q\pi^2\in P_r b}{P_m}(2A+b)\right]\right.\\ \left.+\left(2A+b-\frac{\bar{\delta}Ab}{\in}\right)\left[\left(\frac{2}{K_1}+2b+2bK-\frac{Kb}{\in}\right)\left\{\frac{\in^2 P_r^2b^2}{P_m^2}+\in^2\pi^2 b\beta_e\right\}+\frac{2Q\pi^2\in P_r b}{P_m}\right]\right\}\end{gathered}}{\left(2A+b-\frac{\bar{\delta}Ab}{\in}\right)^2\left[\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}.\left\{\frac{\in^2 P_r^2b^2}{P_m^2}+\in^2\pi^2 b\beta_e\right\}+\frac{Q\pi^2\in P_r b}{P_m}(2A+b)\right]^2}$$

Now $\frac{dR}{dA}>0$, when $\in>\frac{1}{2},\ K>\frac{5}{4},\ \bar{\delta}>K_1,\ \max.\left\{\left(\frac{36\in}{K^2K_1\bar{\delta}}\right)^{1/3},\frac{6}{\bar{\delta}}\left(\frac{\in}{K}\right)^{1/2}\right\}<b<\frac{1}{\in^2\pi^2\beta_e}$,

$\beta_e<\min\left\{\frac{4P_r^2}{9\pi^2K_1P_m^2},\ \frac{QP_r\bar{\delta}}{4\in^2P_m}\right\}$, $\max.\left\{\frac{9}{2K\bar{\delta}},\frac{\in}{KK_1},\frac{2}{K_1(4K-5)},\frac{9K_1}{4\beta_e\in\pi^2\bar{\delta}},\frac{2\in^2P_r^2}{\pi^2KK_1\bar{\delta}\beta_eP_m^2}\right\}<A<\min.\left\{K,\frac{\pi^2\beta_eP_m^2}{2P_r^2},\frac{2\in^2P_r^2}{\bar{\delta}KP_m^2}\right\}$,

Thus, R increases as A increases when above conditions hold, therefore, the micro-polar coefficient has stabilizing effect.

In Particular, when $\bar{\delta}=0,\beta_e=0\,[i.e.\,Q=0]$, we have

$$\frac{dR}{dA}=\left(\frac{b^2}{a^2}\right)\left[\frac{-(2A+b)\frac{Kb}{\in}+\frac{2KAb}{\in}}{(2A+b)^2}\right]=\frac{b^2}{a^2}\left[\frac{-\frac{Kb^2}{\in}}{(2A+b)^2}\right]=\frac{-Kb^4}{a^2\in(2A+b)^2}$$

$\Rightarrow\frac{dR}{dA}<0$, thus, if there is no heat conduction and hall current, the Micropolar coefficient has destabilizing effect.

Again, if $\beta_e=0$, then

$$\frac{dR}{dA}=\frac{\in P_rb^3}{a^2P_m}\left[\frac{\frac{\bar{\delta}b^3}{\in}+\frac{\bar{\delta}Kb^3}{\in}+\frac{b^2}{\in}\left(\frac{\bar{\delta}}{K_1}-K\right)}{\left(2A+b-\frac{\bar{\delta}Ab}{\in}\right)^2}\right]$$

$\Rightarrow\frac{dR}{dA}>0$ when $\frac{\bar{\delta}}{K_1}>K$

Thus, if there is no Hall current, R increases as A increases when $\bar{\delta}>KK_1$. Therefore, the micropolar coefficient has stabilizing effect.

From (39), we get

$$\frac{dR}{dQ}=\left(\frac{b^2}{a^2}\right)\left[\frac{\begin{array}{c}\frac{\pi^2 \in P_r b}{P_m}(2A+b)\left[\frac{\in P_r b}{P_m}\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}+Q\pi^2(2A+b)\right]^2\\ +\frac{\pi^2 \in P_r b}{P_m}(2A+b)\in^2 \pi^2 b\beta_e\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}^2\\ +2Q\pi^4(2A+b)^2 \in^2 \pi^2 b\beta_e\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}\end{array}}{\left(2A+b-\frac{\bar{\delta}Ab}{\in}\right)\left[\begin{array}{c}\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}\\ \times\left\{\frac{\in^2 P_r^2 b^2}{P_m^2}+\in^2 \pi^2 b\beta_e\right\}+\frac{Q\pi^2 \in P_r b}{P_m}(2A+b)\end{array}\right]^2}\right]$$

Clearly $\frac{dR}{dQ}>0$ when $\bar{\delta}<\frac{\in}{A}$ and $\in>\frac{1}{2}$ Thus, the Rayleigh number R increases as magnetic parameter Q increases when $\bar{\delta}<\frac{\in}{A}$ and $\in>\frac{1}{2}$, therefore, the magnetic field has stabilizing effect.

From (39), we get

$$\frac{dR}{d\beta_e}=\frac{-b^2 Q\pi^2(2A+b)}{a^2\left(2A+b-\frac{\bar{\delta}Ab}{\in}\right)}\left[\frac{\begin{array}{c}\in^2 \pi^2 b\left[\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right]\\ \times\left[\frac{\in P_r b}{P_m}\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}+Q\pi^2(2A+b)\right]\end{array}}{\left[\begin{array}{c}\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}.\\ \times\left\{\frac{\in^2 P_r^2 b^2}{P_m^2}+\in^2 \pi^2 b\beta_e\right\}+\frac{Q\pi^2 \in P_r b}{P_m}(2A+b)\end{array}\right]^2}\right]$$

Clearly $\frac{dR}{d\beta_e}<0$ when $\bar{\delta}<\frac{\in}{A}$ and $\in>\frac{1}{2}$ Thus, the Rayleigh number R decreases as ${}_e$ increases when $\bar{\delta}<\frac{\in}{A}$ *and* $\in>\frac{1}{2}$ Therefore, the Hall current parameter has destabilizing effect.

From (39), we get

$$\frac{dR}{d\bar{\delta}}=\frac{Ab^3}{\in a^2\left(2A+b-\frac{\bar{\delta}Ab}{\in}\right)^2}\left[\frac{\begin{array}{c}\left[\frac{\in P_r b}{P_m}\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}+Q\pi^2(2A+b)\right]^2\\ +\in^2 \pi^2 b\beta_e\left[\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right]^2\end{array}}{\left[\begin{array}{c}\left\{\left(\frac{1}{K_1}+b+bK\right)(2A+b)-\frac{KAb}{\in}\right\}.\\ \times\left\{\frac{\in^2 P_r^2 b^2}{P_m^2}+\in^2 \pi^2 b\beta_e\right\}+\frac{Q\pi^2 \in P_r b}{P_m}(2A+b)\end{array}\right]}\right]$$

Clearly, $\frac{dR}{d\bar{\delta}} > 0$ when $\in > \frac{1}{2}$, thus, the Rayleigh number R increases as $\bar{\mathbb{C}}$ increases when $\frac{1}{2} < \in < 1$ therefore, the micro-polar heat conduction parameter shows stabilizing effect.

9. Oscillatory Convection

For the oscillatory convection we put $\sigma = i\sigma_i$ in (34) and separating real and imaginary parts and then eliminating R between real and imaginary parts we have

$$f_0 s^4 + f_1 s^3 + f_2 s^2 + f_3 s + f_4 = 0 \qquad \text{Where } s = \sigma_i^2 \tag{40}$$

$$f_0 = a^2\left(2K + cb - \frac{\hat{\delta}Kb}{\in}\right)\left[\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\left(b + bK + \frac{1}{K_1}\right)\left(-\frac{2\in^4 EP_r^5 \bar{j}^3 b^2}{P_m}\right)\right.$$
$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^3\left(-\frac{4\in^4 EP_r^5 \bar{j}^3 b^3}{P_m^2} - \in^4 EP_r^5 b\,\bar{j}^3\right) + \left(\frac{1}{\in}+\frac{F}{K_1}\right)\left(b + bK + \frac{1}{K_1}\right)^2\left(-\in^4 EP_r^5 b\,\bar{j}^3\right)$$
$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\left(b + bK + \frac{1}{K_1}\right)(2K + cb)\left(-\in^4 EP_r^5 b\,\bar{j}^2\right) + \left(\frac{1}{\in}+\frac{F}{K_1}\right)^3 (2K + cb)^2\left(-3\in^4 EP_r^5 b\,\bar{j}\right)$$
$$\left.+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\left(Q\pi^2 b\in^3 EP_r^4 \bar{j}^2 - K^2\in^2 EP_r^4 b^2 \bar{j}^2 + Q\pi^2\in^3 EP_r^4 \bar{j}^3 b\right)\right]$$

Thus $f_0 < 0$ when $\bar{\delta} < \frac{\in C}{K}\left(\text{or } \bar{\delta} < \frac{\in}{A}\right)$, $Q < \frac{P_r}{\pi^2}$ and $F > \frac{Q\pi^2 K_1}{\in P_r \bar{j}}$.

$$f_1 = a^2\left(2K + Cb - \frac{\bar{\delta}Kb}{\in}\right)\left[b^5\left\{\left(\frac{1}{\in}+\frac{F}{K_1}\right)(1+K)^2 C\left(\frac{4\in^2 EP_r^5 \bar{j}}{P_m}(2\in-1) + 4P_r^4 \bar{j}^2 \in^2\right)\right.\right.$$
$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)(1+K)^2\left(\frac{4\in^2 EP_r^5 \bar{j}^3}{P_m}\left(\in - \frac{1}{P_m}\right) + \in^4 P_r^4 \bar{j}^3\left(2 - \frac{\in EP_r}{P_m^2}\right)\right)$$
$$+(1+K)^2\in^4 P_r^4 \bar{j}^2\left((1+K)\bar{j} - C\left(\frac{1}{\in}+\frac{F}{K_1}\right)\right) + \left(\frac{1}{\in}+\frac{F}{K_1}\right)^2 C(1+K)\left(\frac{2\in^3 P_r^2 \bar{j}^2}{P_m}(3\in P_r^2 - 1)\right.$$
$$\left.+\frac{2\in^3 EP_r^5 \bar{j}^2}{P_m}(5\in - 4)\right) + \left(\frac{1}{\in}+\frac{F}{K_1}\right)^2 (1+K)C^2\left(\frac{4\in^4 EP_r^5 \bar{j}}{P_m} + 2\in^3 P_r^4 \bar{j}\left(\in - \frac{2EP_r}{P_m}\right)\right)$$
$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^3 C^2\left(\frac{2\in^4 EP_r^5 \bar{j}}{P_m^2} + \frac{2\in^3 P_r^4 \bar{j}}{P_m}\left(\in - \frac{2EP_r}{P_m}\right)\right) + \left(\frac{1}{\in}+\frac{F}{K_1}\right)^2 (1+K)\left(\frac{4\in^4 P_r^4 \bar{j}^3}{P_m^2} + \frac{2\in^4 EP_r^5 \bar{j}^3}{P_m^3}\right)$$
$$\left.+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^3 C\left(\frac{4\in^4 P_r^4 \bar{j}^2}{P_m^2} + \frac{2\in^4 EP_r^5 \bar{j}^2}{P_m^3}\right) + \left(\frac{1}{\in}+\frac{F}{K_1}\right)^3 C^3\in^4 P_r^4 + (1+K)^3 C\in^4 EP_r^5 \bar{j}^2\right\}$$
$$+b^4\left\{\left(\frac{1}{\in}+\frac{F}{K_1}\right)2(1+K)\left(K^2 + K + \frac{C}{K_1}\right)\left(\frac{4\in^2 EP_r^5 \bar{j}^2}{P_m}(2\in - 1) + 4P_r^4\in^2 \bar{j}^2\right)\right.$$
$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)\frac{2(1+K)}{K_1}\left(\frac{4\in^3 EP_r^5 \bar{j}^3}{P_m}\left(\in - \frac{1}{P_m}\right) + \in^4 P_r^4 \bar{j}^3\left(2 - \frac{\in EP_r}{P_m^2}\right)\right)$$
$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)\left(2K + 2K^2 + \frac{C}{K_1}\right)\left(\frac{2\in^3 P_r^2 \bar{j}^2}{P_m}(3\in P_r^2 - 1) + \frac{2\in^3 EP_r^5 \bar{j}^2}{P_m}(5\in - 4)\right)$$
$$+\in^4 P_r^4 \bar{j}^2\frac{(1+K)}{K_1}\left[(1+K)\bar{j} - 2C\left(\frac{1}{\in}+\frac{F}{K_1}\right)\right] + \in^4 P_r^4 \bar{j}^2 (1+K)^2\left[\frac{C}{K_1}EP_r - 2K\left(\frac{1}{\in}+\frac{F}{K_1}\right)\right]$$
$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)(1+K)^2\in^4 EP_r^3 \bar{j}^3\left(4\pi^2\beta_e - \frac{P_r^2}{P_m^2}\right) + \left(\frac{1}{\in}+\frac{F}{K_1}\right)(1+K)\in^4\pi^2 EP_r^3 \bar{j}^2\beta_e\left[2C - \in\bar{j}(1+K)\right]$$
$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)C(1+K)\left[\in^3 EP_r^5 K^2 \bar{j} + 2\in^3 EP_r^4 K^2 \bar{j}\right] + \left(\frac{1}{\in}+\frac{F}{K_1}\right)(1+K)\left[\frac{2\in^2 EP_r^4 K^2 \bar{j}^2}{P_m} + \in^3 P_r^3 K^2 \bar{j}^2\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\left(\frac{C^2}{K_1}+4KC(1+K)\right)\left[\frac{4\in^4 EP_r^5\,\bar{j}}{P_m}+2\in^3 P_r^4\,\bar{j}\left(\in-\frac{2EP_r}{P_m}\right)\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^3 C^2\left(2\in^4\pi^2 EP_r^3\,\bar{j}\beta_e\right)+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^3 4KC\left[\frac{2\in^4 EP_r^5\,\bar{j}}{P_m^2}+\frac{2\in^3 P_r^4\,\bar{j}}{P_m}\left(\in-\frac{2EP_r}{P_m}\right)\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2 C\left(\frac{2\in^2 EP_r^4K^2\,\bar{j}}{P_m}+\in^3 P_r^3K^2\,\bar{j}\right)+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\left(\frac{\in^3 EP_r^6K^2\,\bar{j}^2}{P_m^2}+\frac{\in^3 EP_r^4K^2\bar{j}^2}{P_m^2}\right)$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2(1+K)\frac{2\in^4\pi^2 EP_r^3\,\bar{j}^3\beta_e}{P_m}+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\frac{4\in^4 P_r^4\,\bar{j}^3}{K_1P_m^2}+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\frac{2\in^3 EP_r^5\,\bar{j}^2}{P_m^2}\left[-K^2+\frac{\in\bar{J}}{K_1P_m}\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^3 C\left(\frac{2\in^4\pi^2 EP_r^3\,\bar{j}^2\beta_e}{P_m}\right)+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^3 2K\left[\frac{4\in^4 P_r^4\,\bar{j}^2}{P_m^2}+\frac{2\in^4 EP_r^5\,\bar{j}^2}{P_m^3}\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^3 2KC^2(\in^4 P_r^4)+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\left(\frac{C^3}{K_1}+C^2(2K+2K^2)\right)(\in^4 EP_r^5)+2K(1+K)^3\in^4 EP_r^5\,\bar{j}^2\quad\Big\}$$

$$+b^3\Bigg\{\left(\frac{1}{\in}+\frac{F}{K_1}\right)\left(\frac{4K(1+K)}{K_1}+\frac{C}{K_1^2}\right)\left[\frac{4\in^3 EP_r^5\,\bar{j}^2}{P_m}(2\in-1)+4\in^2 P_r^4\,\bar{j}^2\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)\frac{1}{K_1^2}\left[\frac{4\in^3 EP_r^5\,\bar{j}^3}{P_m}\left(\in-\frac{1}{P_m}\right)+\in^4 P_r^4\,\bar{j}^3\left(2-\frac{\in EP_r}{P_m^2}\right)\right]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)\frac{2(1+K)}{K_1}\left[\in^4 EP_r^3\,\bar{j}^3\left(4\pi^2\beta_e-\frac{P_r^2}{P_m^2}\right)\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\frac{2K}{K_1}\left[\frac{2\in^3 P_r^2\,\bar{j}^2}{P_m}(3\in P_r^2-1)+\frac{2\in^3 EP_r^5\,\bar{j}^2}{P_m}(5\in-4)\right]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)\in^4\pi^2 EP_r^3\,\bar{j}^2\beta_e$$

$$\times\left[\left(\frac{1}{\in}+\frac{F}{K_1}\right)(2K+2K^2)-\frac{2(1+K)}{K_1}\bar{j}\right]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\frac{C}{K_1}(2\in^4\pi^2 EP_r^3\,\bar{j}^2\beta_e)$$

$$+\in^4 P_r^4\,\bar{j}^2\left[EP_r(1+K)\left(\frac{2K(1+K)}{K_1}+\frac{C}{K_1^2}\right)-\left(\frac{1}{\in}+\frac{F}{K_1}\right)\left(\frac{4K(1+K)}{K_1}+\frac{C}{K_1^2}\right)\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)C(1+K)Q\pi^2\in^3 EP_r^4\,\bar{j}\,(\bar{j}-1)+\left(\frac{1}{\in}+\frac{F}{K_1}\right)(1+K)\frac{Q\pi^2\in^3 EP_r^4\,\bar{j}^2}{P_m}(6-5\bar{j})$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\left(4K^2+4K^3+\frac{4KC}{K_1}\right)\left[\frac{4\in^4 EP_r^5\,\bar{j}}{P_m}+2\in^3 P_r^4\,\bar{j}\left(\in-\frac{2EP_r}{P_m}\right)\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^3 4K^2\left[\frac{2\in^4 EP_r^5\,\bar{j}}{P_m^2}+\frac{2\in^3 P_r^4\,\bar{j}}{P_m}\left(\in-\frac{2EP_r}{P_m}\right)\right]+(1+K)Q\pi^2\in^3 EP_r^4\,\bar{j}^3\left[(1+K)-\left(\frac{1}{\in}+\frac{F}{K_1}\right)\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2 CQ\pi^2\in^3 EP_r^4\,\bar{j}\,[3C-\bar{j}]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\left[\frac{Q\pi^2\in^3 EP_r^4\,\bar{j}^2}{P_m^2}(5\bar{j}-1)+\frac{Q\pi^2\in^3 P_r^3\,\bar{j}^2}{P_m}(\bar{j}-1)\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2 C\left[\frac{Q\pi^2\in^2 EP_r^4\,\bar{j}^2}{P_m}(5\in-2)+\frac{Q\pi^2\in^2 EP_r^4\,\bar{j}}{P_m}(2\in-1)\right]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\in^3 P_r^3\,\bar{j}\,[2K^3-2Q\pi^2C\,\bar{j}]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)\in^2 P_r^3K^2\,\bar{j}\left[\frac{\in\bar{j}}{K_1}-EP_rK^2\right]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\left[\in^3\pi^2 P_r^3\,\bar{j}^2(EP_rK^2\beta_e-Q\,\bar{j})+\in^3\pi^2 EP_r^2K^2\,\bar{j}^2\beta_e\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)(1+K)\in^3 P_r^3\,\bar{j}^3\left[\in P_r\left(\frac{1}{\in}+\frac{F}{K_1}\right)-Q\pi^2\right]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)(1+K)\frac{2\in^2 EP_r^4\,\bar{j}^2}{P_m}\left[\in P_r\,\bar{j}\left(\frac{1}{\in}+\frac{F}{K_1}\right)-Q\pi^2\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)\left(2K+2K^2+\frac{C}{K_1}\right)\left[\in^3 EP_r^5K^2\,\bar{j}+2\in^3 EP_r^4K^2\,\bar{j}\right]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)\frac{1}{K_1}\left(\frac{2\in^2 EP_r^4K^2\,\bar{j}^2}{P_m}\right)$$

$$+8\in^4\pi^2 KCEP_r^3\,\bar{j}\beta_e\left(\frac{1}{\in}+\frac{F}{K_1}\right)^3+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\frac{4\in^2 EP_r^4K^3\,\bar{j}}{P_m}+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\frac{2\in^4\pi^2 EP_r^3\,\bar{j}^3\beta_e}{K_1P_m}$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^3\frac{4\in^4\pi^2 KEP_r^3\,\bar{j}^2\beta_e}{P_m}+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^3 C\left[\in^4 P_r^4\,\bar{j}^2+\frac{2\in^3 EP_r^5\,\bar{j}^2}{P_m}\right]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^3\left(\frac{2\in^4 EP_r^5\,\bar{j}^3}{P_m^2}\right)$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^3 4K^2C\in^4 P_r^4+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\left(\frac{2KC^2}{K_1}+4K^2C+4K^3C\right)(\in^4 EP_r^5)+\frac{(1+K)}{K_1^2}\in^4 P_r^4\,\bar{j}^3$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2(1+K)C\left(2\in^4 EP_r^5\,\bar{j}^2\right)\Bigg\}$$

$$+b^2\Bigg\{\left(\frac{1}{\in}+\frac{F}{K_1}\right)\frac{2K}{K_1^2}\left[\frac{4\in^3 EP_r^5\,\bar{j}^2}{P_m}(2\in-1)+4\in^2 P_r^4\,\bar{j}^2\right]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)\frac{1}{K_1^2}\left[\in^4 EP_r^3\,\bar{j}^3\left(4\pi^2\beta_e-\frac{P_r^2}{P_m^2}\right)\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)\frac{1}{K_1}\in^4\pi^2 EP_r^3\,\bar{j}^2\beta_e\left[4K\left(\frac{1}{\in}+\frac{F}{K_1}\right)-\frac{1}{K_1}\in\bar{j}\right]+\frac{\in^4 P_r^4\,\bar{j}^2}{K_1^2}\left[\frac{\bar{j}}{K_1}-2K\left(\frac{1}{\in}+\frac{F}{K_1}\right)\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)\left(2K+2K^2+\frac{C}{K_1}\right)Q\pi^2\in^2 EP_r^4\,\bar{j}\left(\bar{j}-1\right)+\left(\frac{1}{\in}+\frac{F}{K_1}\right)\frac{1}{K_1}\left[\frac{Q\pi^2\in^3 EP_r^4\,\bar{j}^2}{P_m}(6-5\bar{j})\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\frac{4K^2}{K_1}\left[\frac{4\in^4 EP_r^5\,\bar{j}}{P_m}+2\in^3 P_r^4\,\bar{j}\left(\in-\frac{2EP_r}{P_m}\right)\right]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2 2K\Bigg[\frac{Q\pi^2\in^2 EP_r^4\,\bar{j}^2}{P_m}(5\in-2)$$

$$+\frac{Q\pi^2\in^2 EP_r^4\,\bar{j}}{P_m}(2\in-1)\Bigg]\Bigg]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2 2K.Q\pi^2\in^3 EP_r^4\,\bar{j}[6C-\bar{j}]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)\frac{1}{K_1}\frac{2\in^2 EP_r^4\,\bar{j}^2}{P_m}\begin{bmatrix}\left(\frac{1}{\in}+\frac{F}{K_1}\right)\in P_r\,\bar{j}\\ -Q\pi^2\end{bmatrix}$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)Q\pi^2\in^2 P_r^3\,\bar{j}^2\left[K^2E-\left(\frac{1}{\in}+\frac{F}{K_1}\right)2K\in\right]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)Q\pi^2\in^2 EP_r^4\,\bar{j}^2\left[1-\left(\frac{1}{\in}+\frac{F}{K_1}\right)\in(1+K)\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)Q\pi^2\in^2 EP_r^4\,\bar{j}^2\left[K^2-\frac{1}{K_1}\in\bar{j}\right]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)Q\pi^2\in^2 P_r^3\,\bar{j}\left[EP_r\,K^2-\frac{1}{K_1}\in\bar{j}^2\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\in^3 P_r^4\,\bar{j}^2\left[\frac{1}{K_1}\in\bar{j}-EP_r\,K^2\right]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\in^3\pi^2 EP_r^2\,\bar{j}^3\beta_e\left[\left(\frac{1}{\in}+\frac{F}{K_1}\right)2\in P_r-Q\pi^2\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\in^3\pi^2 EP_r^2\,\bar{j}\beta_e\left[16K^3\in P_r\left(\frac{1}{\in}+\frac{F}{K_1}\right)-Q\pi^2\,\bar{j}\right]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\left(2K+2K^2+\frac{C}{K_1}\right)\left(2\in^4 P_r^5\,\bar{j}^2\right)$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)\frac{2K}{K_1}\left(\in^3 EP_r^5 K^2\,\bar{j}+2\in^3 EP_r^4 K^2\,\bar{j}\right)+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^3 2K\left(\in^4 P_r^4\,\bar{j}^2+\frac{2\in^3 EP_r^5\,\bar{j}^2}{P_m}\right)$$

$$+8K^3\in^4 P_r^4\left(\frac{1}{\in}+\frac{F}{K_1}\right)^3+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\in^4 EP_r^5\left[8K^3+8K^4+\frac{4K^2C}{K_1}\right]+\frac{2(1+K)}{K_1}Q\pi^2\in^3 EP_r^4\,\bar{j}^3$$

$$+\in^4 EP_r^5\,\bar{j}^2\left[\frac{2K+2K^2}{K_1}+\frac{C}{K_1^2}\right]\Bigg]\Bigg\}+b\Bigg\{\left(\frac{1}{\in}+\frac{F}{K_1}\right)\frac{2K}{K_1}Q\pi^2\in^3 EP_r^4\,\bar{j}\,(\bar{j}-1)$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)Q\pi^2\in^3 EP_r^4\,\bar{j}^2\left[\left(\frac{1}{\in}+\frac{F}{K_1}\right)\bar{j}-\frac{1}{K_1}\right]+Q\pi^2\in^2 EP_r^3\,\bar{j}^3\left[\frac{\in P_r}{K_1^2}-2Q\pi^2\left(\frac{1}{\in}+\frac{F}{K_1}\right)\right]$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)Q\pi^2\in^2 EP_r^3\,\bar{j}\left[\left(\frac{1}{\in}+\frac{F}{K_1}\right)4K^2.3\in P_r-Q\pi^2\bar{j}\right]+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\frac{2K}{K_1}\in^4 EP_r^5\,\bar{j}^2$$

$$+\left(\frac{1}{\in}+\frac{F}{K_1}\right)^2\frac{8K^3}{K_1}(\in^4 EP_r^5)+\frac{2K}{K_1^3}(\in^4 EP_r^5\,\bar{j}^2)\Bigg\}$$

and f_2, f_3, f_4 have usual expressions.

From (40), we notice that $s=\sigma_i^2$ which is always positive, therefore the sum of roots of (40) must be positive but it is $\left(-\frac{f_1}{f_0}\right)$.

Now $f_0<0$ when $\bar{\delta}<\frac{\in C}{K}\left(\text{or}\,\bar{\delta}<\frac{\in}{A}\right)$ and $Q<\frac{P_r}{\pi^2}$ and $F>\frac{Q\pi^2 K_1}{\in P_r\,\bar{j}}$

$$f_1>0 \text{ when } \frac{4}{5}<\in<1,\ 1<\bar{j}<\frac{6}{5},\ \sqrt{\frac{1}{3\in}}<P_r<\frac{P_m}{2E},\ P_m>\max.\left\{1,\sqrt{\frac{2EP_r}{5}}\right\},$$

$$C > \max.\left\{\frac{1}{3}, \frac{2(1+K)}{5}\right\}, \beta_e > \max.\left\{\frac{1}{12\pi^2 P_m^2}, \frac{5QK_1}{3P_m}\right\},$$

$$\max.\left\{\left(\frac{Q\pi^2}{3}\right)^{1/3}, \left(\frac{4}{5EP_r K_1}\right)^{1/2}\right\} < K < \min.\left\{\left(\frac{6}{5K_1}\right)^{1/2}, \left(\frac{6}{5EP_r K_1}\right)^{1/2}\right\}$$

$$\max.\left\{\frac{1}{2KK_1}, \frac{Q\pi^2}{P_r}, \frac{5Q\pi^2 K_1}{36KP_r}, \frac{5}{6K_1}, \frac{5Q\pi^2 K_1}{72P_r}\right\} < \left(\frac{1}{\in} + \frac{F}{K_1}\right) < \psi$$

where $$\psi = \min.\left\{\frac{EP_r}{2KK_1}, \frac{9(1+K)}{5}, \frac{E\delta P_r(1+K)}{2}, (1+K), \frac{3}{5KK_1}, \frac{5EK}{8}, \frac{5}{4(1+K)}, \frac{P_r}{2Q\pi^2 K_1^2}\right\}$$

This gives the necessary condition if the oscillatory modes exist.

10. Result and Discussions

1. For Stationary Convection:

(i) As medium permeability K_1 increases, Rayleigh number R decreases with the increment in wave number a=1 to 2.5 when A=0.5, $\in$=0.6, P_r=2, P_m=4, $\bar{\delta}$=0.05, β_e=0.2, K=1, Q=10, therefore, medium permeability K_1 has destabilizing effect (see fig.2). If there is no heat conduction and Hall current, as medium permeability increases, Rayleigh number decreases but at wave number *a=2* and *a=2.5* the change in Rayleigh number is approximately the same thus the medium permeability has destabilizing effect without any condition (see fig.3)

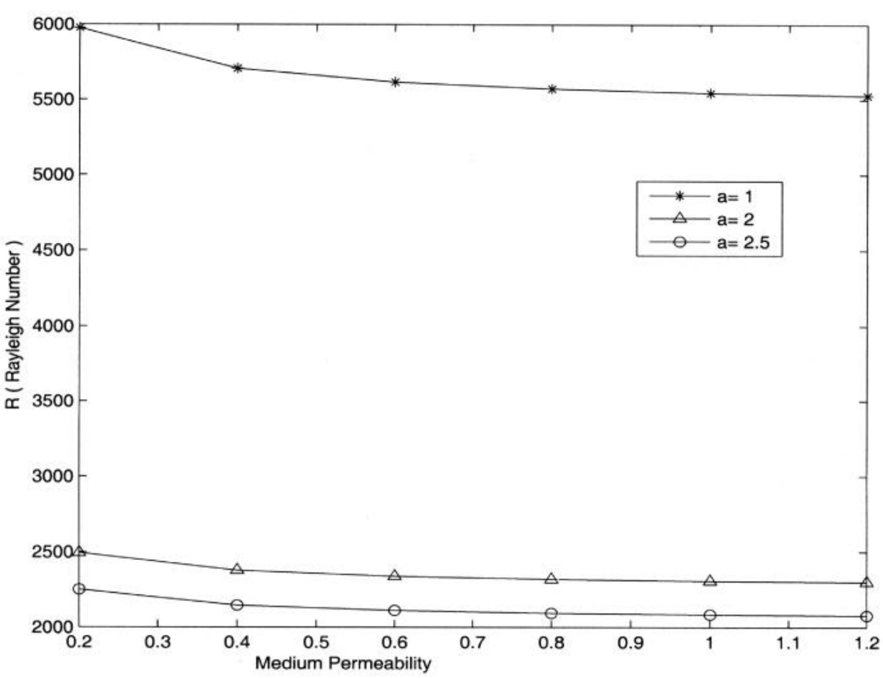

Fig.2: Stability curves for the variation of R v/s K_1 (Medium permeability) for A=0.5, Č=0.6, P_r=2, P_m=4, $\bar{\delta}$=0.05, β_e=0.2, K=1, Q=10.

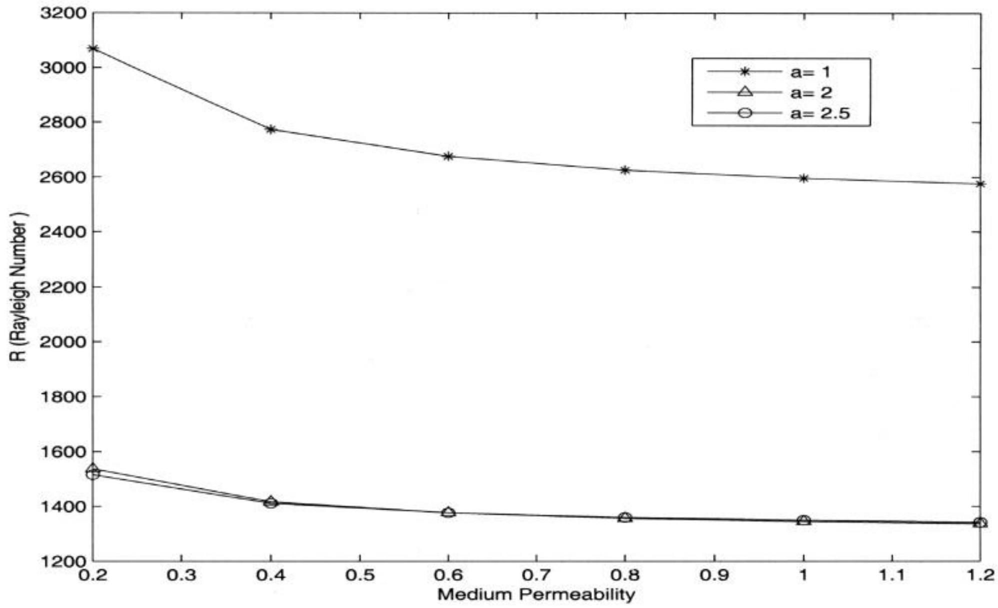

Fig.3: Stability curves for the variation of R v/s K_1 (Medium permeability) for A=0.5, Č=0.6, P_r=2, P_m=4, K=1, $\bar{\delta}$ =0, β_e =0, Q=0.

(ii) As the coupling parameter *K* increases, Rayleigh number R increases with the wave number from *a*=1 to 2.5 when A=0.5, $\in$=0.6, $\bar{\delta}$ =0.05, β_e =0.2, P_r=2, P_m=4, K_1=0.03, Q=10. therefore, the coupling parameter *K* has stabilizing effect and at *a=2* and *a*=2.5 the change in Rayleigh number is approximately the same (see fig. 4). As the coupling parameter *K* increases, Rayleigh number R decreases with the increment in magnetic parameter Q, therefore, coupling parameter *K* has destabilizing effect when A=0.5, $\in$=0.026, a=1, P_r=2, P_m=4, β_e =0.3, K_1=0.03, $\bar{\delta}$ =0.05 (see fig. 5).

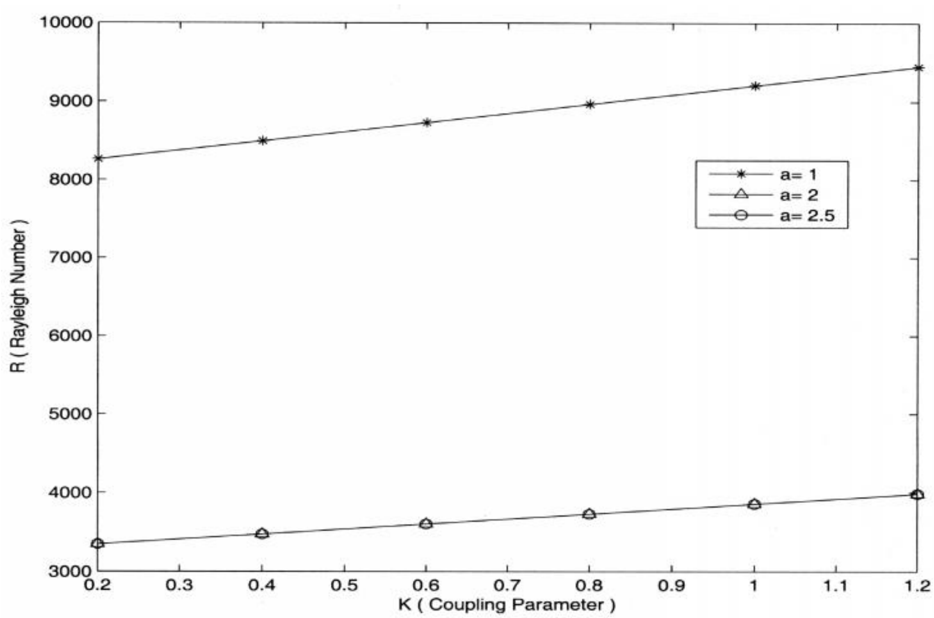

Fig. 4: Stability curves for the variation of R v/s K under stationary convection for A=0.5, Č=0.6, $\bar{\delta}$ =0.05, β_e =0.2, P_r=2, P_m=4, K_1=0.03, Q=10.

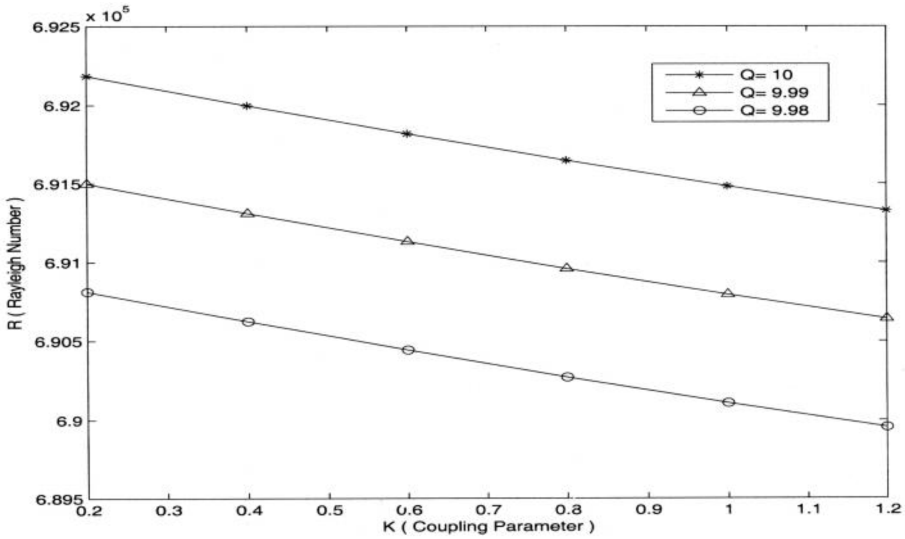

Fig. 5: Stability curves for the variation of R v/s K under stationary convection for A=0.5, Č=0.026, a=1, P_r=2, P_m=4, β_e =0.3, K_1=0.03, $\bar{\delta}$ =0.05.

If there is no heat conduction and Hall current, as the coupling parameter *K* increases, Rayleigh number R decreases when A=0.5, $\in$=0.026, P_r=2, P_m=4, β_e =0, $\bar{\delta}$ =0, Q=0, K_1=0.03, therefore, the coupling parameter *K* has destabilizing effect(see fig. 6) and as the coupling parameter *K* increases, Rayleigh number R increases with the increment in wave number *a*=1 to 1.11 Thus, the coupling parameter *K* has stabilizing effect when A=0.5, $\in$=0.6, P_r=2, P_m=4, β_e =0, $\bar{\delta}$ =0, Q=0, K_1=0.03 (see fig. 7)

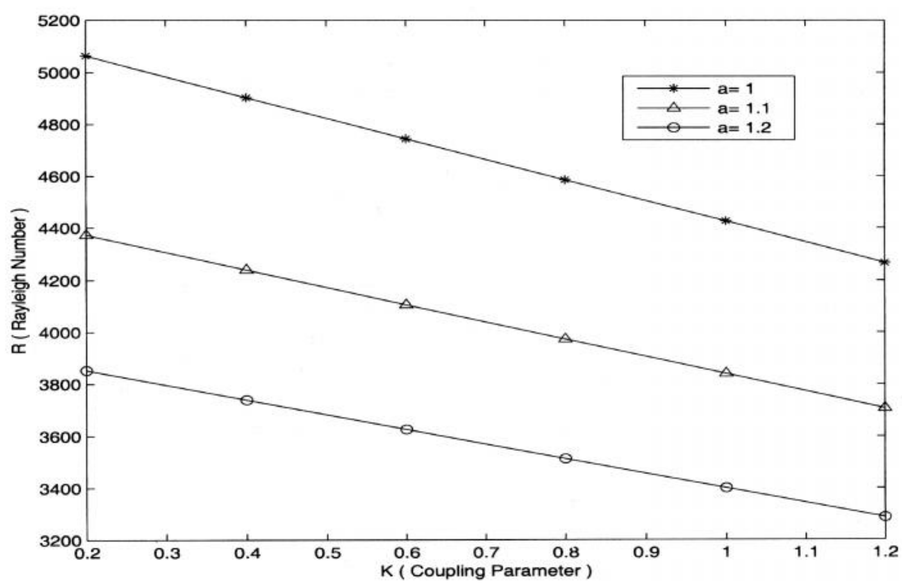

Fig. 6: Stability curves for the variation of R v/s under stationary convection K for A=0.5, Č=0.026, P_r=2, P_m=4, β_e =0, $\bar{\delta}$ =0, Q=0, K_1=0.03.

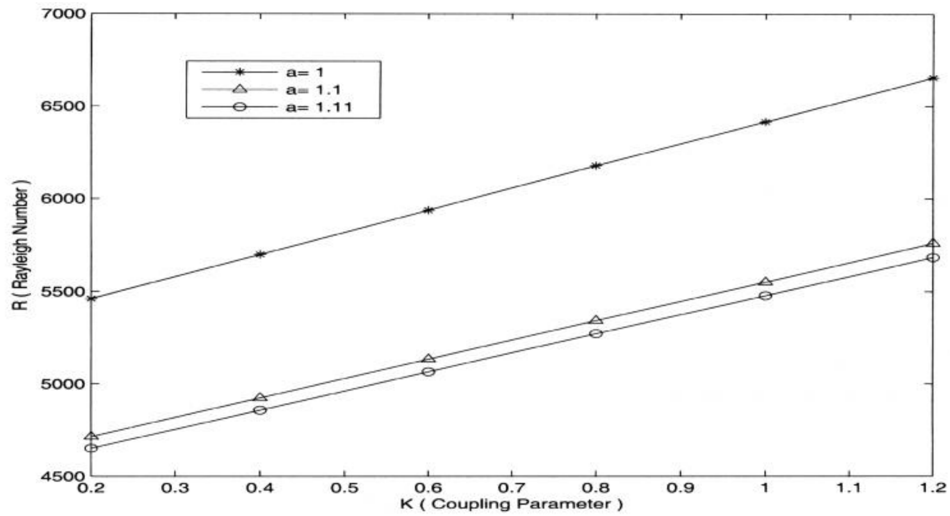

Fig. 7: Stability curves for the variation of R v/s K under stationary convection for A=0.5, Č=0.6, P_r=2, P_m=4, β_e =0, $\bar{\delta}$ =0, Q=0, K_1=0.03.

(iii) When $\in > \frac{1}{2},\ K > \frac{5}{4},\ \bar{\delta} > K_1,\quad \max.\left\{\left(\frac{36\in}{K^2K_1\bar{\delta}}\right)^{1/3},\ \frac{b}{\bar{\bar{\delta}}}\left(\frac{\in}{K}\right)^{1/2}\right\} < b < \frac{1}{\in^2\pi^2\beta_e},$

$$\beta_e < \min.\left\{\frac{4P_r^2}{9\pi^2K_1P_m^2},\ \frac{QP_r\bar{\delta}}{4\in^2 P_m}\right\},\ \text{and}$$

$$\max.\left\{\frac{9}{2K\bar{\delta}},\ \frac{\in}{KK_1},\ \frac{2}{K_1(4K-5)},\ \frac{9K_1}{4\beta_e\in\pi^2\bar{\delta}},\ \frac{2\in^2P_1^2}{\pi^2KK_1\bar{\delta}\beta_eP_m^2}\right\} < A < \min.\left\{K,\ \frac{\pi^2\beta_eP_m^2}{2P_r^2},\ \frac{2\in^4P_r^2}{\bar{\delta}KP_m^2}\right\}$$

Thus, the micro-polar coefficient *A* has stabilizing effect under above conditions.
If there is no heat conduction and Hall current, the micro-polar coefficient has destabilizing effect and when there is no Hall current, the micro-polar coefficient has stabilizing effect when $\bar{\delta} > KK_1$.

(iv) As the magnetic parameter Q increases, Rayleigh number R increases with the increment in wave number a=1 to 1.2 when A=0.5, $\in$=0.6, K=1, K_1= 0.03, $\bar{\delta}$ =0.05, P_r=2, P_m=4, β_e = 0.3, thus the magnetic field has stabilizing effect. (see fig. 8).

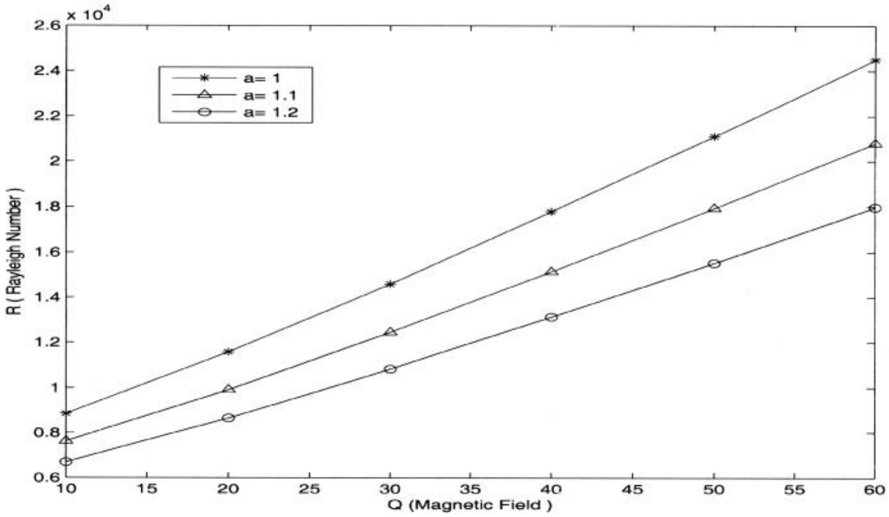

Fig. 8: Stability curves for the variation of R v/s Q under stationary convection for A=0.5, Č=0.6, K=1, K_1= 0.03, $\bar{\delta}$ =0.05, P_r=2, P_m=4, β_e = 0.3.

(v) As the Hall parameter increases, Rayleigh number R decreases with the increment in a=1 to 1.2 when A=0.5, K=1, K_1=0.03, $\bar{\delta}$ =0.05, $\in$=0.6, P_r= 2, P_m=4, Q=10, thus the Hall parameter has destabilizing effect . (see fig. 9).

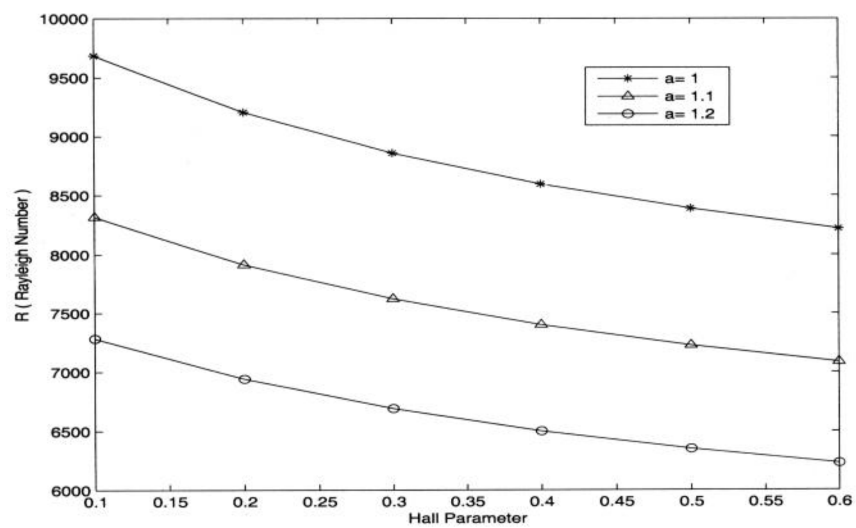

Fig. 9: Stability curves for the variation of R v/s β_e under stationary convection for A=0.5, K=1, K_1=0.03, $\bar{\delta}$ =0.05, Č=0.6, P_r= 2, P_m=4, Q=10.

(vi) As the micro-polar heat conduction parameter increases, Rayleigh number R increases with the increment of wave number a=1 to 1.11 when A=0.5, K=1, K_1=0.03, P_r=2, P_m=4, $\in$=0.6, Q=10, β_e =0.2, therefore, the micro-polar heat conduction has stabilizing effect (see fig. 10).

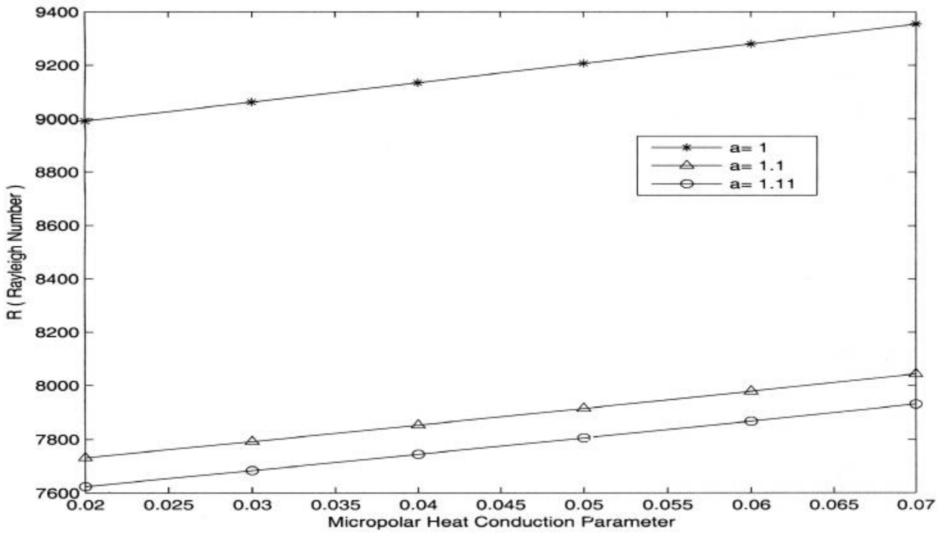

Fig. 10:Stationary convection stability curves for the variation of R v/s $\bar{\delta}$ for A=0.5, K=1, K_1=0.03, P_r=2, P_m=4, Č=0.6, Q=10, β_e =0.2.

(vii) In case of stationary convection visco-elasticity does not matter.

(2) For Oscillatory Convection: The necessary condition for the oscillatory modes is given by

$$\bar{\delta} < \frac{\in}{A},\ Q < \frac{P_r}{\pi^2},\ F > \frac{5Q\pi^2 K_1}{6P_r} \text{ and } \frac{4}{5} < \in < 1,\ 1 < \bar{j} < \frac{6}{5},\ \sqrt{\frac{1}{3\in}} < P_r < \frac{P_m}{2E},\ P_m > \max.\left\{1, \sqrt{\frac{2EP_r}{5}}\right\}$$

$$C > \max.\left\{\frac{1}{3}, \frac{2(1+K)}{5}\right\},\ \beta_e > \max.\left\{\frac{1}{12\pi^2 P_m^2}, \frac{5QK_1}{3P_m}\right\}$$

$$\max.\left\{\left(\frac{Q\pi^2}{3}\right)^{1/3}, \left(\frac{4}{5EP_r K_1}\right)^{1/2}\right\} < K < \min.\left\{\left(\frac{6}{5K_1}\right)^{1/2}, \left(\frac{6}{5EP_r K_1}\right)^{1/2}\right\}$$

and
$$\max.\left\{\frac{1}{2KK_1}, \frac{Q\pi^2}{P_r}, \frac{5Q\pi^2 K_1}{36KP_r}, \frac{5}{6K_1}, \frac{5Q\pi^2 K_1}{72P_r}\right\} < \left(\frac{1}{\in} + \frac{F}{K_1}\right) < \psi$$

where
$$\psi = \min.\left\{\frac{EP_r}{2KK_1}, \frac{9(1+K)}{5}, \frac{EP_r(1+K)}{2}, (1+K), \frac{3}{5KK_1}, \frac{5EK}{8}, \frac{5}{4(1+K)}, \frac{P_r}{2Q\pi^2 K_1^2}\right\}$$

11. Conclusion

Medium permeability shows destabilizing effect whether Hall current is present or not. For fixed magnetic field coupling parameter shows stabilizing effect whereas as magnetic field increases coupling parameter turns to destabilizing effect, and if there is no heat conduction and Hall current, coupling parameter has destabilizing effect when porosity is less than 0.5 and it will have stabilizing effect when porosity is greater than 0.5. Magnetic field has stabilizing effect whereas Hall current has destabilizing effect when porosity is greater than 0.5, and micropolar heat conduction has stabilizing effect.

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