# Transformation Parameters Derivation between Global, National and Mine Grid Reference Systems*

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## Abstract

In mining operations, coordinate transformation plays a key role in transforming coordinates acquired in the Global Navigation Satellite System (GNSS) into the national and local mine grid systems. It has often been known that most mining sites have transformation parameters determined using only a few common points or the minimum co-located points. However, these determined parameters only fit within a limited extent of the mine concession. Hence, allowing for extrapolation and incorrect transformation results when the existing transformation parameters are utilised beyond the existing co-located points. As the mine expands beyond it operationalised zones, there is the need to redefine a new set of transformation parameters that are devoid of extrapolation and apply to a wider coverage of the mine concession. This study applied, evaluated, and compared the Two-dimensional (2D) conformal similarity model and 2D affine model to facilitate the transformation of the Local Mine Grid (LMG) coordinates to the Ghana National Projected Grid (GNG), Universal Transverse Mercator (UTM), and vice versa. To guarantee the consistency of the transformation results between the models tested on all the grid systems, similar transformation performance was revealed. In transforming between GNG and LMG, the 2D conformal results vary from the 2D Affine by 0.0079 m, -0.0128 m, 0.0079 m and 0.0261 m in $RMSE_{HPE}$, $SD_{HPE}$, $Max_{HPE}$, and $Min_{HPE}$. Similar observation was made for transforming between UTM and LMG, and UTM and GNG, respectively. Based on the results obtained it can be stated that the two models are applicable in connecting mine grid system into a national grid system (non-geocentric), and UTM.

**Keywords:** 2D conformal model, 2D affine model, transformation parameters, coordinate transformation, UTM zonal parameters

## 1 Introduction

The use of non-geocentric datum for geodetic survey and mapping activities dominates the mining industry in Ghana. This is because most existing mine sites use an established Local Mine Grid (LMG) system. The LMG is established based on a particular reference point located in a stable area on the mine and assigned an arbitrary set of coordinates (Walker and Awange, 2020). A second reference point is then chosen to orient the grid coordinate system in a particular direction. This orientation is usually achieved by considering the strike of the orebody, true north, magnetic north, or any other direction that is feasible at the time (Walker and Awange, 2020). Therefore, for the purposes of land, engineering, and cadastral surveying, it is important to ensure that the LMG is connected to the accepted national mapping system of a country and the Universal Transverse Mercator (UTM) which is a globally applicable grid system (Younis, 2019). This connection between the different grid systems can be achieved through coordinate transformation.

Coordinate transformation plays a crucial role in mining operations where the Global Navigation Satellite System (GNSS) such as the Global Positioning System (GPS) is used for precise positioning. Thus, the ubiquitous nature of the GPS has led to its successful adoption and usage in the mining industry for various geodetic survey works. However, localising the GPS acquired UTM data which is based on a geocentric datum into the LMG, and a non-geocentric classical geodetic datum requires the estimation of transformation parameters between the systems to bring harmony and homogeneity. It is because of this that most mining sites have coordinate transformation parameters determined (Walker and Awange, 2020).

However, there are couple of challenges encountered when implementing the existing transformation parameters of a mine. For instance, the mine considered in this study had the existing transformation parameters determined using only a few co-located points skewed at a particular location on the mine concession. The reason is that the mine has expanded its operations and thus the initial common points utilised do not cover the greater extent of the mine concession. Moreso, only the minimum co-located points were used to determine those existing transformation parameters. By the foregoing challenges, a general understanding that can be inferred is as follows:

i. The existing transformation parameters are restricted in their use to only areas covered by the respective co-located points utilised in their determination. Hence, the parameters are not applicable across the entire mine concession. Thus, subjecting the existing transformation parameters to extrapolate when applied beyond the co-located points utilised to determine them.
ii. The transformation results (transformed coordinates) produced when such existing

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transformation parameters are utilised may have distortions and data incompatibility issues due to the extrapolation scenario usually encountered.

Therefore, there is a need for a redefinition of the transformation parameters between the LMG, classical geodetic network, and GNSS global grid systems. Although several related 2D coordinate transformation works have been reported in literature (Gargula and Gawronek, 2023; Hong, 2021; Qin *et al*., 2020; Alcaras *et al*. 2020; Lu *et al*., 2019; Rofatto *et al*., 2019; Eteje *et al*., 2019; Bremner and Santos, 2019; Lehmann and Lösler, 2018; Öcalan, 2018; Goudarzi and Landry, 2017; Ampatzidis and Melachroinos, 2017; Ampatzidis and Demirtzoglou, 2017; Ansari *et al*. 2017; Soycan *et al*., 2017), none of the existing studies have applied and compared the suitability of 2D conformal and 2D affine models to unify a mine grid, national mapping grid and UTM. It was noticed that most of the studies focused on cadastral coordinate transformation, direct projection of geocentric system to local topocentric coordinates and geological map transformation. The present study therefore bridged the literature gap by applying, evaluating, and comparing the 2D conformal and 2D affine models to ascertain their suitability for transforming between an LMG, national grid, UTM and vice versa. In this study, the national grid considered is the Ghana National Grid (GNG) coordinate based on the War Office 1926 ellipsoid which is non-geocentric.

To assess the performance of the transformation models, the calculated Horizontal Positional Error (HPE), the Standard Deviation HPE ($SD_{HPE}$), Root Mean Square Error HPE ($RMSE_{HPE}$), and maximum and minimum HPE were employed. These evaluation metrics were selected because they conform to scholarly practice in coordinate transformation (Ziggah *et al*., 2019). To this end, the investigation performed in this study demonstrates that the 2D conformal and 2D affine models are applicable in connecting mine grid system into a national grid system (non-geocentric), and UTM. This study will therefore create the opportunity for mine surveyors, exploration geologists, and mine geologists to perform coordinate transformation on site and to know the most adequate transformation model to be used for transforming ore blocks from one grid system to the other.

## 2 Resources and Methods Used

### 2.1 Resources

The LMG coordinate system applied in this study is for a Mine (hereafter Mine X) located in the Western Region of Ghana. It was established based on the orebody strike with arbitrary coordinates assigned to two reference points. The LMG can be described as an assumed system that works only within the confines of Mine X. Mine X has a total concession size of 83924.89986 acres. It references the Transverse Mercator with False Northing and False Easting values of 13514.012 m and 6205.196 m, respectively. The latitude of origin is $5^0$ 21’ 27.42354″ N and the central meridian is $2^0$ 01’ 26.81616″ W. The LMG has a scale factor of 1.0. The origin of the Easting and Northing is 5976.486 m and 9515.363 m, respectively.

The GNG coordinate system is defined by the War Office 1926 ellipsoid. This grid system uses the Transverse Mercator projection based on the Accra 1929 datum and is the official grid system used for surveying and mapping works in Ghana (Kumi-Boateng and Ziggah, 2020; Kotzev, 2013). The following are the defined ellipsoidal and projection parameters. The *a*, *b* and *f* are the semi-major axis, semi-minor axis, and flattening, respectively.

Projection: Transverse Mercator
Ellipsoid: War Office 1926

Ellipsoid properties: $a$ = 20,926,201 ft
$b$ = 20,855,505 ft
$f$ = 1/296
Unit of measurement: Foot
Meridian of origin: 01°00' West of Greenwich
Latitude of origin: 04° 40' North
Scale factor at central meridian: 0.99975
False coordinates at origin: 900,000 ft. Easting
Nil Northing
Maximum angular distance of a point from central meridian: 2° 23'

The UTM, which is an extension of the Transverse Mercator was established to define 2D horizontal positions on the Earth surface (Baqir and Loay, 2020; Lu *et al*., 2014). In 1975, Ghana adopted the UTM system placed in UTM zones 30 N and 31 N. The following are the defined zonal parameters for Ghana (Kotzev, 2013).

**Zone 30N Parameters**

Scale factor at the central meridian: 0.99960
Grid origin at latitude: 0°00´
Grid origin at longitude: 3°00´W
False easting = 500000.00 m
False northing = 0.00 m
Ellipsoid: World Geodetic System 1984 (WGS84)
Ellipsoid properties: a = 6378137 m
f = 1/298.257223563
The maximum angular distance of a point from the central meridian: 3°00´.

**Zone 31N Parameters**
Scale factor at the central meridian: 0.99960
Grid origin at latitude: 0°00´
Grid origin at longitude: 3°00´ E
False easting = 500000.00 m.
False northing = 0.00 m.
Ellipsoid: World Geodetic System 1984 (WGS84)
Ellipsoid properties: a = 6378137 m
f = 1/298.257223563
The maximum angular distance of a point from the central meridian: 3°00´.

To evolve a set of datum transformation parameters that has a wider coverage of unifying the LMG, GNG and UTM grid systems, a set of 29 control points were established across the entire operational zones of the mine. This exercise was important because of the limited number of existing common points and non-availability of some of the ground controls. Therefore, primary data of grid coordinates in LMG, GNG and UTM systems were observed.

Data acquisition for the controls was done independently with the use of dual frequency Stonex S900A GPS/GNSS static receivers, antennas, antenna cables, and communication accessories. The Differential Global Positioning System (DGPS) technique was employed with reference to beacons established in the GNG coordinate system and the UTM WGS 1984, Zone 30N.

With respect to the observations in the LMG system, the two established Continuously Operating Reference Stations (CORS) about 70 km apart based on the Network Real Time Kinematic (NRTK) was employed to provide positioning information. It must be indicated that there is stable internet connectivity with no interference from other sources of electromagnetic waves, etc.

Here, the CORS which have been configured in the LMG system by Mine X served as the reference base stations. The LMG configuration of the CORS has been done using all the reported information in Section 2.1 including a semi-major axis and flattening values of 6378137.00 m and 1/298.2572229329. Interestingly, the semi-major axis is equivalent to those defined on the WGS84. The LMG grid coordinates observations were supported by the survey team of Mine X. Fig. 1 shows the spatial distribution of the co-located points used to determine the various transformation parameters between the grid systems (LMG, GNG and UTM). Table 1 presents a sample of the coordinates in the LMG, GNG and UTM systems

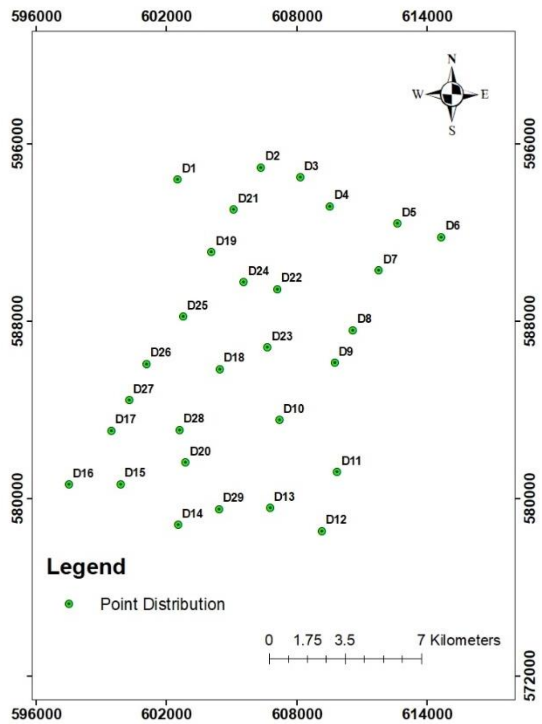

**Fig. 1 Location of Co-located Points across Mine X Concession**

**Table 1 Sample Co-located Points (Unit: metres)**

| PT ID | LMG East | LMG North | GNG East | GNG North | UTM East | UTM North |
|---|---|---|---|---|---|---|
| P1 | 3056.827 | 14172.557 | 159041.444 | 78860.470 | 606356.631 | 594965.884 |
| P2 | 4654.103 | 15118.788 | 160840.183 | 78400.064 | 608156.205 | 594511.435 |
| P3 | 6549.509 | 15155.486 | 162206.111 | 77085.688 | 609526.381 | 593201.718 |

## 2.2 Methods

### 2.2.1 2D Conformal Transformation Model

The 2D conformal transformation model was applied to determine the various transformation parameters suitable to transform coordinates between the different grid systems (LMG, GNG and UTM). Walker and Awange (2020) emphasised that the model has the characteristic of retaining the true shape and angles after transformation and hence is a useful tool for mapping activities.

The 2D conformal model is composed of two translations of the coordinate origin, one scale factor and one rotation parameter. In Walker and Awange

(2020), the observation equation for the 2D conformal transformation model can be expressed as

$$E = (S\ \cos\theta)e - (S\ \sin\theta)n + D_E$$
$$N = (S\ \sin\theta)e + (S\ \cos\theta)n + D_N \quad (1)$$

where (E, N) are the target grid coordinates, (e, n) are the source grid coordinates, *S* is the scale factor, $\theta$ is the rotational angle, and $(D_E, D_N)$ are the shifts in Easting and Northing (translation parameters).

Simplifying Equation (1) by putting $S\cos\theta = a$, $S\sin\theta = b$, $D_E = c$, and $D_N = d$ gives Equation (2).

$$E = ae - bn + c$$
$$N = be + an + d \quad (2)$$

where *a*, *b*, *c*, and *d* are the unknown transformation parameters to be determined.

The $\theta$ and *S* in Equation (1) is computed using Equations (3) and (4).

$$\theta = \arctan\left(\frac{b}{a}\right) \quad (3)$$

$$S = \left(\frac{a}{\cos\theta}\right) \quad (4)$$

2.2.2 2D Affine Transformation Model

The 2D affine transformation model was utilised to transform coordinates between LMG, GNG and UTM after determining their respective transformation parameters. This method is widely known in photogrammetry for interior orientation to counter irregular film shrinkage, or in nonconformal maps. The affine model consists of two translation parameters, two scale factors, one skew angle and one rotation. Therefore, the 2D affine transformation model (Walker and Awange, 2020) can be expressed as in Equation (5).

$$E = (S_X \cos\theta)X + (S_Y \cos\theta\sin\beta + S_Y \sin\theta\cos\beta)Y + T_X$$
$$N = (-S_X \sin\theta)X + (-S_Y \sin\theta\sin\beta + S_Y \cos\theta\cos\beta)Y + T_Y \quad (5)$$

where (E, N) are the target grid coordinates, (X, Y) are the source grid coordinates, $S_X$ and $S_Y$ are the scale factors in *x* and *y* direction, $\theta$ is the angle of rotation, $\beta$ is the skew angle, *Tx* and *Ty* are the shift parameters.

For simplicity, Equation (5) can be expressed into Equation (6) by letting

$a = S_X \cos\theta$,

$b = S_Y(\cos\theta\sin\beta + \sin\theta\cos\beta)$,

$d = -S_X \sin\theta$,

$e = S_Y(\cos\theta\sin\beta - \sin\theta\cos\beta)$.

$$E = ax + by + T_x$$
$$N = dx + ey + T_y \quad (6)$$

where the *a*, *b*, *d*, *e*, $T_x$ and $T_y$ are the unknown parameters to be determined. Due to the orthogonality of the reference axes, the skew angle $\beta$ becomes zero (0). To calculate for $S_X$, $S_Y$, and $\theta$, Equations (7), (8) and (9) are used.

$$S_X = \sqrt{a^2 + d^2} \quad (7)$$

$$S_Y = \sqrt{b^2 + e^2} \quad (8)$$

$$\theta = \arctan\left(\frac{b}{a}\right) \quad (9)$$

2.2.3 Ordinary Least Squares Solution to the Coordinate Transformation Models Applied

The 2D conformal transformation model requires a minimum of two co-located grid coordinates to uniquely determine the four unknown transformation parameters. Similarly, the 2D affine transformation model needs a minimum of three co-located grid coordinates to compute the unique six transformation parameters. Practically, there are often more observations than the minimum co-located points required. This creates an overdetermined system of linear equations emanating from a set of redundant observables. This leads to the ordinary least squares solution to determine the unknown transformation parameters. The choice of the ordinary least squares is due to its simplicity of application and achievable results. Moreover, methods like total least squares and generalised least squares when assessed previously by other researchers in Ghana showed similar results (Ziggah *et al*., 2016; Laari *et al*., 2016). Hence, this study relied on the ordinary least squares to determine the transformation parameters.

Applying the least squares method, the 2D conformal model (Equation (2)) could be represented in matrix form as

$$BX + V = L \quad (10)$$

where $V$ is the residual, $B$ is the designed matrix, $L$ is the observation vector matrix and $X$ is the vector of the unknown transformation parameters to be determined.

Hence, expressing Equation (2) in the form of Equation (10) gives Equation (11).

$$\begin{bmatrix} e & -n & 1 & 0 \\ n & e & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} + \begin{bmatrix} V_e \\ V_n \end{bmatrix} = \begin{bmatrix} E \\ N \end{bmatrix} \quad (11)$$

where

$$B = \begin{bmatrix} e & -n & 1 & 0 \\ n & e & 0 & 1 \end{bmatrix},\ X = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix},$$

$$V = \begin{bmatrix} V_e \\ V_n \end{bmatrix} \text{ and } L = \begin{bmatrix} E \\ N \end{bmatrix}.$$

To determine the unknown transformation parameters $X$, Equation (12) (Ghilani, 2010) was used.

$$X = (B^T B)^{-1} \times (B^T L) \quad (12)$$

In the implementation of the 2D affine model, Equation (6) was expressed into the form of Equation (10) yielding Equation (13).

$$\begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} + \begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} E \\ N \end{bmatrix} \quad (13)$$

where

$$B = \begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix},\ X = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix},$$

$$V = \begin{bmatrix} V_x \\ V_y \end{bmatrix} \text{ and } L = \begin{bmatrix} E \\ N \end{bmatrix}.$$

To determine the six unknown transformation parameters, $X$ for the 2D affine model, Equation (12) was used. After determining values for the transformation parameters, any points in the source grid system can be transformed into the target system and vice versa.

2.2.4 Coordinate Transformation Model Assessment Criteria

The accuracy and precision of the applied coordinate transformation models were assessed using statistical indices. This was done by quantifying the horizontal positional residuals obtained when the transformation models' transformed coordinates were subtracted from the existing grid coordinates. The statistical indices used were the Horizontal Positional Error (HPE), Root Mean Square HPE ($RMSE_{HPE}$), and Standard Deviation of HPE ($SD_{HPE}$). They are defined by Equations (14) to (16) (Ziggah *et al*., 2019) respectively. The maximum and minimum HPE were also considered.

$$HPE = \sqrt{(E_{O_i} - E_{T_i})^2 + (N_{O_i} - N_{T_i})^2} \quad (14)$$

$$RMSE_{HPE} = \sqrt{\frac{\sum_{i=1}^{N} (HPE_i)^2}{N}} \quad (15)$$

$$SD_{HPE} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} \left( HPE - \overline{HPE} \right)^2} \quad (16)$$

where ($E_O$, $N_O$) are the observed grid coordinates, and ($E_T$, $N_T$) are the transformed grid coordinates given by the 2D conformal and 2D affine models. The $RMSE_{EASTING}$ and $RMSE_{NORTHING}$ are the Root Mean Square Error (RMSE) in Easting and Northing coordinates. The $\overline{HPE}$ is the average of the HPE.

## 3 Results and Discussion

### 3.1 New Transformation Parameters Determined

The application of the 2D conformal and 2D affine transformation models requires the determination of four and six unknown transformation parameters. This study deduced these parameters from a sequence of co-located points in LMG, GNG and UTM grid systems. A total of 29 co-located points were utilised for the coordinate transformation work. To apply the two transformation models, 20 evenly distributed co-located points that have a wider coverage of the mine concession served as the reference points for the parameter determination. The remaining 9 co-located points were used as check points to evaluate the performance of the transformation models. Fig. 2 shows the spatial distribution of the reference and check points utilised. To determine the transformation parameters, the least squares approach as described in Section 2.2.3 was employed. The reason was that the 20 co-located points generated more equations than the unknown transformation parameters to be determined which resulted into an over-determined system. The determined parameters and their associated standard deviations for the 2D conformal and 2D affine models that unified the LMG, GNG and UTM systems for Mine X are presented in Tables 2, 3, 4, 5, 6 and 7. The standard deviation values indicate the precision of the transformation and how well the transformed coordinates agree with the existing coordinates. Consequently, the relatively smaller standard deviation values indicate better precision.

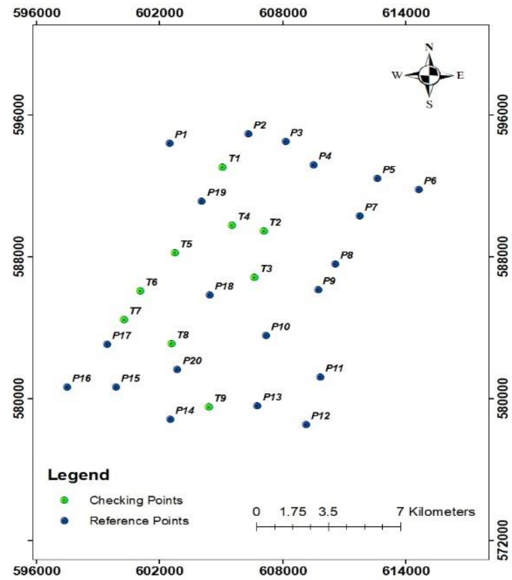


**Fig. 2 Spatial Distribution of Reference and Check Points**

**Table 2 2D Conformal Model Derived Transformation Parameters between LMG and GNG**

| Parameter | GNG to LMG | LMG to GNG | Unit |
|---|---|---|---|
| a | 0.7071563457±6.78817×10$^{-06}$ | 0.707095017±6.79×10$^{-06}$ | m |
| b | 0.7071185427±6.78817×10$^{-06}$ | -0.707057217±6.79×10$^{-06}$ | m |
| c | -53646.7050466247±1.1824 | 146859.17823008±0.0972 | m |
| d | -154055.2268466890±1.1824 | 71000.39343835±0.0972 | m |

**Table 3 2D Affine Model Derived Transformation Parameters between LMG and GNG**

| Parameter | GNG to LMG | LMG to GNG | Unit |
|---|---|---|---|
| a | 0.7071273586±1.11×10$^{-05}$ | 0.707171154±8.92×10$^{-06}$ | m |
| b | -0.7071052027±8.92×10$^{-06}$ | 0.707116894±1.11×10$^{-05}$ | m |
| c | -53643.0309541927±1.6171 | -154056.00700845±1.6171 | m |
| d | 0.7071168938±1.11×10$^{-05}$ | -0.70710520±8.92×10$^{-06}$ | m |
| e | 0.7071711545±8.92×10$^{-06}$ | 0.707127359±1.11×10$^{-05}$ | m |
| f | -154056.0070084580±1.6171 | -53643.03095±1.6171 | m |

**Table 4 2D Conformal Model Derived Transformation Parameters between LMG and UTM**

| Parameter | UTM to LMG | LMG to UTM | Unit |
|---|---|---|---|
| a | 0.7095801290±2.03×10$^{-06}$ | 0.709200461±2.03×10$^{-06}$ | m |
| b | 0.7050045300±2.03×10$^{-06}$ | -0.70462731±2.03×10$^{-06}$ | m |
| c | -7748.3225962614±1.7144 | 594202.51032279±0.0291 | m |
| d | -835487.6230172810±1.7144 | 587068.52750188±0.0291 | m |

**Table 5 2D Affine Model Derived Transformation Parameters between LMG and UTM**

| Parameter | UTM to TGM | TGM to UTM | Unit |
|---|---|---|---|
| a | $0.7095745356 \pm 3.42 \times 10^{-06}$ | $0.709203629 \pm 3.67 \times 10^{-06}$ | m |
| b | $-0.7049995682 \pm 2.74 \times 10^{-06}$ | $0.704625074 \pm 2.40 \times 10^{-06}$ | m |
| c | -7747.8404434132±2.0404 | 594202.49993717±0.0450 | m |
| d | $0.7050092062 \pm 3.42 \times 10^{-06}$ | $-0.70463471 \pm 3.67 \times 10^{-06}$ | m |
| e | $0.7095805572 \pm 2.74 \times 10^{-06}$ | $0.709197611 \pm 2.40 \times 10^{-06}$ | m |
| f | -835490.71022443±2.0404 | 587068.6185±0.0450 | m |

**Table 6 2D Affine Model Derived Transformation Parameters between UTM and GNG**

| Parameter | GNG to UTM | UTM to GNG | Unit |
|---|---|---|---|
| a | $0.9997706426 \pm 2.1158 \times 10^{-06}$ | $1.000219114 \pm 7.24493 \times 10^{-06}$ | m |
| b | $0.0032071225 \pm 2.1158 \times 10^{-06}$ | $-0.003208561 \pm 7.24493 \times 10^{-06}$ | m |
| c | 447604.7223292120±0.36854 | -449357.17575046±6.1128 | m |
| d | 515613.4231521060±0.36854 | -514290.23419260±6.1128 | m |

**Table 7 2D Conformal Model Derived Transformation Parameters between GNG and UTM**

| Parameter | GNG to UTM | UTM to GNG | Unit |
|---|---|---|---|
| a | $0.9997495820 \pm 1.23 \times 10^{-05}$ | $1.000240194 \pm 1.23 \times 10^{-05}$ | m |
| b | $-0.0031910490 \pm 9.85 \times 10^{-06}$ | $0.003192536 \pm 9.84 \times 10^{-06}$ | m |
| c | 447606.9424757220±1.7866 | -449360.56115218±7.3320 | m |
| d | $0.0032191327 \pm 1.23 \times 10^{-05}$ | $-0.00322063 \pm 1.23 \times 10^{-05}$ | m |
| e | $0.9997749604 \pm 9.85 \times 10^{-06}$ | $1.000214811 \pm 9.84 \times 10^{-06}$ | m |
| f | 515611.2077531450±1.7866 | -514280.3894±7.3320 | m |

## 3.2 Statistical Evaluation of the New Transformation Parameters

Mathematically, the confidence in any prediction model is ascertained by how close the model's outputs agree with the observed or actual targets. Hence, this study applied a two-way analytical step to assess the performance of the transformation models implemented. First, the initial transformation results from the 20 reference points that were used to determine the parameters were analysed. This was closely followed by the 9 check points which were used to independently verify the efficiency and accuracy of the derived transformation parameters. To achieve the two-way analysis, Equations (14) to (16) were applied to quantify the estimated residuals generated between the observed and transformed grid coordinates.

Tables 8, 9 and 10 present the summary statistical results when the derived transformation parameters were applied to the 20 reference points to transform coordinates between LMG and GNG, LMG and UTM and GNG and UTM. It is noteworthy that the reverse transformation (GNG and LMG, UTM and LMG, and UTM and GNG) produced identical results. It was only the arithmetic sign of the error differences between the existing and transformed coordinates that was different. Moreover, because the HPE squared the errors, the transformation results were identical. Hence, the reverse transformation results were not reported.

**Table 8 Reference Points Summary HPE Statistic Results for Transforming between GNG and LMG**

| Statistical Indicator | 2D Conformal | 2D Affine | Unit |
|---|---|---|---|
| $RMSE_{HPE}$ | 0.3721 | 0.3415 | m |
| $SD_{HPE}$ | 0.2488 | 0.1986 | m |
| $Max_{HPE}$ | 1.1022 | 0.8958 | m |
| $Min_{HPE}$ | 0.0614 | 0.1278 | m |

**Table 9 Reference Points Summary HPE Statistic Results for Transforming between UTM and LMG**

| Statistical Indicator | 2D Conformal | 2D Affine | Unit |
|---|---|---|---|
| $RMSE_{HPE}$ | 0.1114 | 0.1101 | m |
| $SD_{HPE}$ | 0.0435 | 0.0506 | m |
| $Max_{HPE}$ | 0.1971 | 0.2419 | m |
| $Min_{HPE}$ | 0.0447 | 0.0267 | m |

**Table 10 Reference Points Summary HPE Statistic Results for Transforming between UTM and GNG**

| Statistical Indicator | 2D Conformal | 2D Affine | Unit |
|---|---|---|---|
| $RMSE_{HPE}$ | 0.3969 | 0.3773 | m |
| $SD_{HPE}$ | 0.2635 | 0.2242 | m |
| $Max_{HPE}$ | 1.2224 | 1.0424 | m |
| $Min_{HPE}$ | 0.0767 | 0.0693 | m |

In Tables 8, 9 and 10, the $RMSE_{HPE}$ values is the transformation accuracy indicator which provides the level of uncertainty associated with the transformation results. In Tables 8, 9 and 10, it was established that to transform coordinates from GNG to LMG, UTM to LMG, GNG to UTM and vice versa, the 2D conformal model could produce approximately 0.3721 m, 0.1114 m, 0.3969 m. The 2D affine could achieve 0.3415, 0.1101, and 0.3773 respectively. Based on these outcomes, it can be established that both the 2D conformal and 2D affine models can transform coordinates between the grid systems in Mine X with satisfactory accuracy.

The practicality of the determined parameters was carried out using the $SD_{HPE}$ results (Tables 8, 9 and 10). The computed $SD_{HPE}$ values provide the degree of precision of the transformed coordinates when compared with the observed. A precision of 0.2488 m, 0.0435 m, 0.2635 m were achieved by the 2D conformal model while 0.1986 m, 0.0506 m and 0.2242 m were achieved by the 2D affine. These obtained SD HE values are in consonance with transforming from GNG to LMG, UTM to LMG, GNG to UTM and vice versa. Based on the $SD_{HPE}$ values obtained, it can be established that the individual transformed coordinates vary slightly from the most probable value. Hence, the 2D conformal and 2D affine models produced transformed coordinates that are precise.

Considering the results in Tables 8, 9 and 10, the $Max_{HPE}$ and $Min_{HPE}$ signify the largest and lowest dispersion in horizontal distance between observed and transformed coordinates. Given the $Max_{HPE}$ and $Min_{HPE}$ values produced it can be stated that the 2D conformal model when applied to transform between GNG and LMG, UTM and LMG, and UTM and GNG could produce horizontal dispersion in metre interval of [0.0614, 1.1022], [0.0447, 0.1971] and [0.0767, 1.224]. Similarly, 2D affine model could achieve [0.1278, 0.8958], [0.0267, 0.2419], and [0.0693, 1.0424].

The authenticity of the reference points derived transformation parameters was verified using the 9 validation points (check points). Using the check points enable one to assess the transformation strength of the models employed. The reason is that the check points were not used in determining the transformation parameters. Tables 11, 12, and 13 show the difference in positional shifts when the 2D conformal and affine models were applied to the check points. These positional shifts can additionally be viewed in Figs. 3, 4 and 5. Comparatively, it was noticed that both the 2D conformal and affine models produced no significant difference in their transformation results. The summary statistic results (Tables 14, 15 and 16) of the horizontal residuals confirmed that assertion. Thus, based on the estimated variations between the 2D conformal and affine models' performance indicators ($RMSE_{HPE}$, $SD_{HPE}$, $Max_{HPE}$ and $Min_{HPE}$), it was obvious that both methods could produce compatible results. Examining Tables 14, 15 and 16 results indicate no superiority between the 2D conformal and 2D affine model. This means that similar transformation accuracy and precision could be achieved within Mine X if any of the models is used to perform coordinate transformation between the LMG, GNG and UTM.

**Table 11 Positional Shifts for Transforming between GNG and LMG (unit: metres)**

| | 2D Conformal | | | 2D Affine | | |
|---|---|---|---|---|---|---|
| Check Point | ΔE | ΔN | HPE | ΔE | ΔN | HPE |
| T1 | 0.0203 | 0.1513 | 0.1526 | -0.1068 | 0.0519 | 0.1188 |
| T2 | -0.0740 | -0.0165 | 0.0758 | -0.0953 | -0.0593 | 0.1123 |
| T3 | -0.0067 | 0.0332 | 0.0339 | -0.0061 | 0.0287 | 0.0293 |
| T4 | -0.1303 | 0.0507 | 0.1399 | -0.2005 | 0.0007 | 0.2005 |
| T5 | 0.0470 | 0.1834 | 0.1893 | -0.0840 | 0.1517 | 0.1734 |
| T6 | 0.1570 | 0.0085 | 0.1573 | 0.0056 | 0.0054 | 0.0078 |
| T7 | 0.1565 | 0.0550 | 0.1659 | 0.0036 | 0.0748 | 0.0749 |
| T8 | 0.1780 | 0.0691 | 0.1910 | 0.1101 | 0.1127 | 0.1576 |
| T9 | 0.3164 | -0.1850 | 0.3666 | 0.3484 | -0.0855 | 0.3587 |

**Table 12 Positional Shifts for Transforming between UTM and LMG (unit: metres)**

| Check Point | 2D Conformal | | | 2D Affine | | |
|---|---|---|---|---|---|---|
| | ΔE | ΔN | HPE | ΔE | ΔN | HPE |
| T1 | -0.0554 | 0.1146 | 0.1273 | -0.0957 | 0.1183 | 0.1522 |
| T2 | -0.0774 | 0.0514 | 0.0929 | -0.0887 | 0.0473 | 0.1005 |
| T3 | -0.0617 | -0.0004 | 0.0617 | -0.0624 | -0.0013 | 0.0624 |
| T4 | -0.0494 | 0.0170 | 0.0523 | -0.0708 | 0.0199 | 0.0736 |
| T5 | -0.1486 | -0.0086 | 0.1488 | -0.1780 | 0.0081 | 0.1781 |
| T6 | -0.1326 | -0.0708 | 0.1503 | -0.1608 | -0.0453 | 0.1671 |
| T7 | -0.0523 | -0.0034 | 0.0524 | -0.0768 | 0.0265 | 0.0813 |
| T8 | -0.0517 | -0.0254 | 0.0576 | -0.0566 | -0.0058 | 0.0569 |
| T9 | 0.0294 | 0.0988 | 0.1031 | 0.0523 | 0.1115 | 0.1232 |

**Table 13 Positional Shifts for Transforming between UTM and GNG (unit: metres)**

| Check Point | 2D Conformal | | | 2D Affine | | |
|---|---|---|---|---|---|---|
| | ΔE | ΔN | HPE | ΔE | ΔN | HPE |
| T1 | -0.0794 | 0.0276 | 0.0841 | 0.0548 | 0.0391 | 0.0673 |
| T2 | 0.0456 | 0.0505 | 0.0680 | 0.0801 | 0.0706 | 0.1068 |
| T3 | -0.0626 | 0.0152 | 0.0645 | -0.0609 | 0.0186 | 0.0637 |
| T4 | 0.0334 | -0.0810 | 0.0876 | 0.1053 | -0.0781 | 0.1311 |
| T5 | -0.2741 | 0.0025 | 0.2741 | -0.1680 | -0.0351 | 0.1716 |
| T6 | -0.2609 | 0.1488 | 0.3003 | -0.1536 | 0.0817 | 0.1740 |
| T7 | -0.1889 | 0.1064 | 0.2168 | -0.0910 | 0.0227 | 0.0938 |
| T8 | -0.2293 | 0.0955 | 0.2484 | -0.2017 | 0.0340 | 0.2045 |
| T9 | -0.0023 | 0.4036 | 0.4036 | -0.0701 | 0.3486 | 0.3556 |

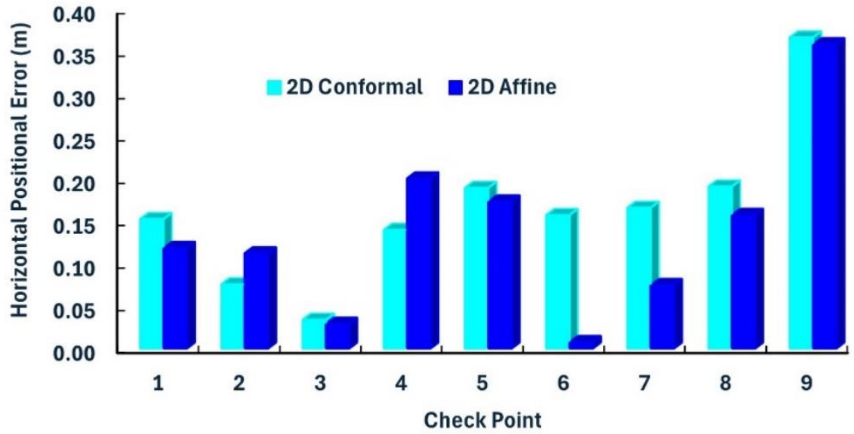


**Fig. 3 HPE for Transforming between GNG and LMG (unit: metres)**

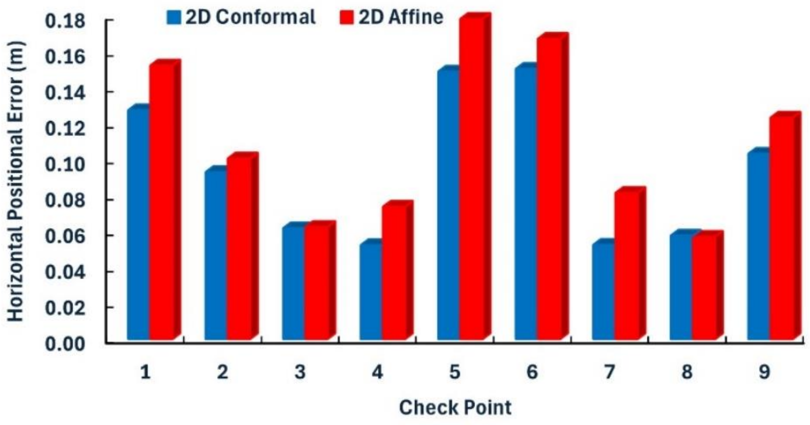


**Fig. 4 12 HPE for Transforming between UTM and LMG (unit: metres)**

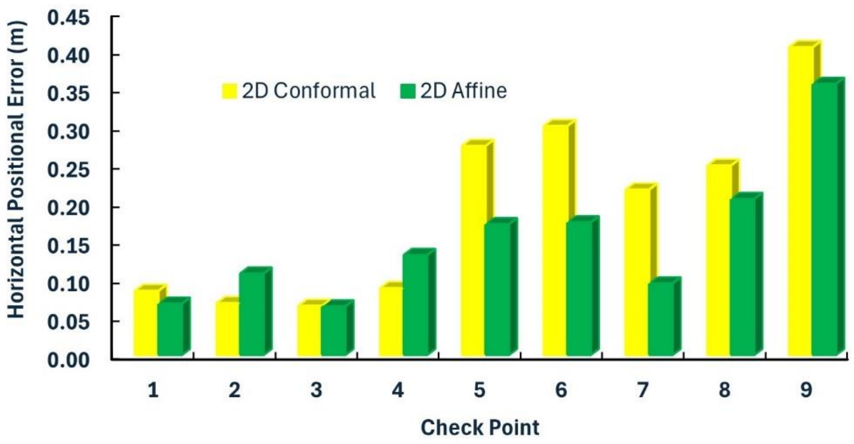


**Fig. 5 HPE for Transforming between UTM and GNG (unit: metres)**

**Table 14 Check Points Summary HPE Statistic Results for Transforming between GNG and LMG (unit: metres)**

| Statistical Indicator | 2D Conformal | 2D Affine | Variation |
|---|---|---|---|
| $RMSE_{HPE}$ | 0.3666 | 0.3587 | 0.0079 |
| $SD_{HPE}$ | 0.0921 | 0.1049 | -0.0128 |
| $Max_{HPE}$ | 0.3666 | 0.3587 | 0.0079 |
| $Min_{HPE}$ | 0.0339 | 0.0078 | 0.0261 |

**Table 15 Check Points Summary HPE Statistic Results for Transforming between UTM and LMG (unit: metres)**

| Statistical Indicator | 2D Conformal | 2D Affine | Variation |
|---|---|---|---|
| $RMSE_{HPE}$ | 0.1031 | 0.1232 | -0.0201 |
| $SD_{HPE}$ | 0.0406 | 0.0463 | -0.0057 |
| $Max_{HPE}$ | 0.1503 | 0.1781 | -0.0278 |
| $Min_{HPE}$ | 0.0523 | 0.0569 | -0.0046 |

**Table 16 Reference Points Summary HPE Statistic Results for Transforming between UTM and GNG (unit: metres)**

| Statistical Indicator | 2D Conformal | 2D Affine | Variation |
|---|---|---|---|
| $RMSE_{HPE}$ | 0.4036 | 0.3555 | 0.0481 |
| $SD_{HPE}$ | 0.1231 | 0.0907 | 0.0323 |
| $Max_{HPE}$ | 0.4036 | 0.3555 | 0.0481 |
| $Min_{HPE}$ | 0.0644 | 0.0637 | 0.0007 |

### 3.3 Comparison between Newly Derived and Existing Transformation Parameters

This section provides a comparative assessment between the newly derived transformation parameters and Mine X existing parameters. The essence of this analysis was to assess the effectiveness of the existing parameters across the entire Mine X operational size of 83924.89986 acres. Furthermore, the analysis will justify the need to redefine the transformation parameters for Mine X since the Mine is now operating beyond its initial operationalised area. It is noteworthy that Mine X existing transformation parameters are based on the 2D conformal model. Hence, to carry out the comparison, the 2D conformal model was used and the existing parameters were applied to the 9 check points used for validation in this study.

Figs. 6, 7 and 8 present the horizontal residuals achieved for each of the check points when the new and existing parameters were tested.

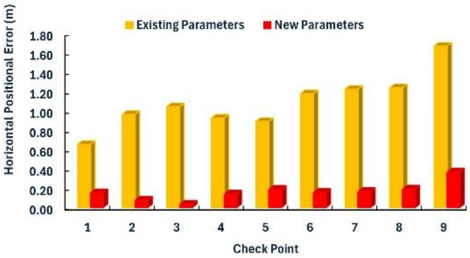


**Fig. 6 HPE between Existing and New Parameters for Transforming between GNG and LMG**

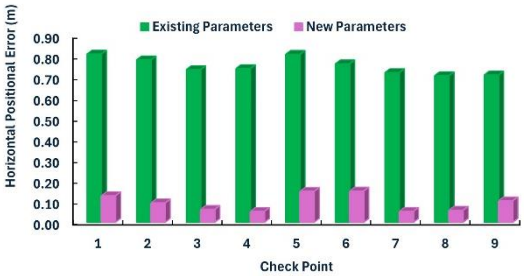


**Fig. 7 HPE between Existing and New Parameters for Transforming between UTM and LMG**

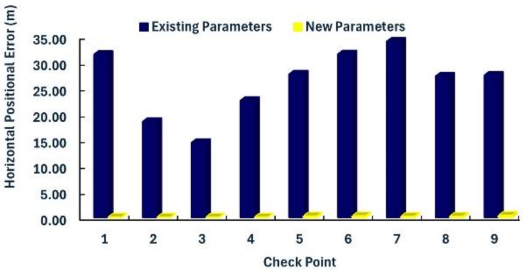


**Fig. 8 HPE between Existing and New Parameters for Transforming between UTM and GNG**

A closer examination of the figures indicates that the positional shifts produced by the existing transformation parameters were high as compared to the new parameters. Hence, it can be inferred that the existing parameters achieved limited transformation accuracy and could provide the best fit transformation results for Mine X. To arrive at the reasons for such limitations, the following questions must be answered.

i. How many co-located points were used at the time of determining the existing parameters?
ii. How were the co-located points spatially distributed? Was it an even distribution across the Mine or skewed at a particular location on the Mine?
iii. What were the survey techniques utilised for the observation and the accuracy of the controls?

The above questions are critical because the strength of any determined transformation parameters is based on the actual field surveyed data. In this case, any errors within the reference systems will be evident in the calculated transformation parameters. A careful review of the historical information at Mine X showed that the existing parameters were determined using a limited number of co-located points (Table 17). It was evident that for LMG and GNG, the co-located points covered only the limited space the Mine was operating at the time which do not meet the current operationalised areas of Mine X. Between the LMG, GNG and UTM only two-co-located points were utilised.

Given the two co-located points, it defeats the argument made by scholars that for adequate transformation parameters, the common points must be evenly distributed and be a network of interconnected points scattered across the area of interest. Joining these two co-located points can only create a line and the transformation results are only applicable within the scope of coverage of the points. This was shown in the summary statistical results of the horizontal errors for both the new and existing parameters (Tables 18, 19 and 20). This means that the existing transformation parameters are geodetically handicapped in terms of accuracy. Hence, deficient transformation accuracy is achievable should the existing parameters be continually used for transformation in Mine X. Ultimately, the results provided by the existing transformation parameters confirmed the assertion made in Walker and Awange (2020) that it is practically good to determine parameters based on co-located points outside the area of interest. Thus, it is important to work from without to within.

**Table 17 Number of Co-located Points used to Determine Mine X Existing Transformation Parameters**

| Datum Transformation | Number of Co-located Points |
|---|---|
| LMG and GNG | 5 |
| LMG and UTM | 2 |
| UTM and GNG | 2 |

**Table 18 Summary HPE Transformation Results between LMG and GNG for the 2D Conformal Model (unit: metres)**

| Statistical Indicators | New Parameters | Existing Parameters |
|---|---|---|
| $RMSE_{HPE}$ | 0.3666 | 1.1181 |
| $SD_{HPE}$ | 0.0921 | 0.2868 |
| $Max_{HPE}$ | 0.3666 | 1.6666 |
| $Min_{HPE}$ | 0.0339 | 0.6515 |

**Table 19 Summary HPE Transformation Results between LMG and UTM for the 2D Conformal Model (unit: metres)**

| Statistical Indicators | New Parameters | Existing Parameters |
|---|---|---|
| $RMSE_{HPE}$ | 0.1031 | 0.7538 |
| $SD_{HPE}$ | 0.0406 | 0.0402 |
| $Max_{HPE}$ | 0.1503 | 0.8099 |
| $Min_{HPE}$ | 0.0523 | 0.7061 |

**Table 20 Summary HPE Transformation Results between GNG and UTM for the 2D Conformal Model (unit: metres)**

| Statistical Indicators | New Parameters | Existing Parameters |
|---|---|---|
| $RMSE_{HPE}$ | 0.4036 | 26.8274 |
| $SD_{HPE}$ | 0.1231 | 6.4450 |
| $Max_{HPE}$ | 0.4036 | 34.0197 |
| $Min_{HPE}$ | 0.0644 | 14.4812 |

## 4 Conclusions and Recommendation

This study applied, evaluated, and compared the capability of 2D conformal and 2D affine transformation models for coordinate transformation between LMG, GNG and UTM grid systems. The results indicated that the two transformation models do not have any absolute superiority between them. Hence, both models are suitable to be used to perform coordinate transformation in Mine X. To this end, it is concluded that both the 2D conformal and 2D affine models could produce transformation results with sufficient accuracy and can be used by the exploration geologist, mine geologist and surveyors for ore block transformation between LMG, GNG and UTM. In the future work, the authors plan to use artificial intelligence to refine the results achieved by the 2D conformal and 2D affine transformation models.

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