Comparative Study on Linear and Non-Linear Geostatistical Methods: A Case Study on Kalsaka Hill Gold Deposit, Burkina Faso*

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Abstract

Selecting an appropriate method to evaluate an ore deposit is imperative in resource estimation since it becomes the basis for reliable planning and development of a mine. Even though linear geostatistical methods such as Ordinary Kriging (OK) give reasonable estimates, there may be instances where recoverable resource estimates are difficult to obtain, particularly when the deposit is characterised by a positively skewed grade distribution with some outliers. Multiple Indicator Kriging (MIK), which is a non-linear estimation technique, is not based on any assumption about the distribution underlying the data and offers realistic solutions to problems associated with skewness and outliers. This paper looks at the use of MIK technique as an alternative method of recoverable resource estimation to OK by comparing the resource estimates obtained from MIK and OK models. OK model showed more smoothing effect on its estimates than the MIK model as evidenced in swath plots. Underestimated grades and tonnages were observed when OK was used to estimate a gold deposit at Kalsaka, according to the grade and tonnage reconciliation. MIK model yields estimates which are higher and closer to the actual than the OK model estimates.

Keywords: Gold, Multiple Indicator Kriging, Ordinary Kriging, Outlier, Variography

1 Introduction

Reliable estimation of a mineral resource is the first and key phase in evaluating its economic worth and this becomes the basis for reliable planning and development of a mine. In recent times, geostatistical estimation methods, such as ordinary kriging, have been put to effective use, especially in gold mining industry. Ordinary kriging, which is a linear geostatistical estimation method, assumes a normal distribution for sample data. However, in reality, grade data are mostly not normally distributed but skewed (Annels, 1991). Besides, the presence of outliers makes semi-variogram modelling very difficult and this affects grade estimates. Thus, using an ordinary kriging method to interpolate grade into blocks may have its own problems. Ordinary kriging was used to estimate the Kalsaka Hill gold deposit, in Burkina Faso, and there was recognition of underestimated resource estimates from the grade and tonnage reconciliations during mining. Skewness and the presence of outliers influence the grade distribution and this requires an appropriate method of mean grade estimation if grade overestimation or underestimation is to be minimised (Glacken and Blackney, 2003). In order to deal with the problem of underestimated grades and tonnages, a non-linear geostatistical estimation approach which addresses skewed distribution of data and resistant to the influence of outliers should be considered (Lipton *et al*., 2003). At present, multiple indicator kriging, which makes no explicit assumption about the distribution underlying the sample data, is widely used in the mining industry because it offers realistic solutions to problems associated with skewness and outliers (Jones, 2003). In view of this, the study verifies the propriety of multiple indicator kriging, which transforms grade data into zeros and ones at various cut-offs, as an alternative method of recoverable resource estimation to ordinary kriging.

2 Resources and Methods Used

2.1 Study Area

Kalsaka Gold Mine is located in the Yatenga Province, about 150 km north-west of Ouagadougou, Burkina Faso. It lies approximately on latitude 13° 11' 12" N and longitude 1° 59' 28" West (Fig. 1). The mine is accessed through 100 km sealed road and a further 80 km gravel roads. The Kalsaka area is 350 m above sea level in relatively flat undulating terrain. Locally, hills are about 50 m above the surrounding plains. The climate of the area is dry Sudanese-Saharan type with sharply contrasting wet and dry seasons. The average rainfall is 619 mm and the wettest months are July, August and September.

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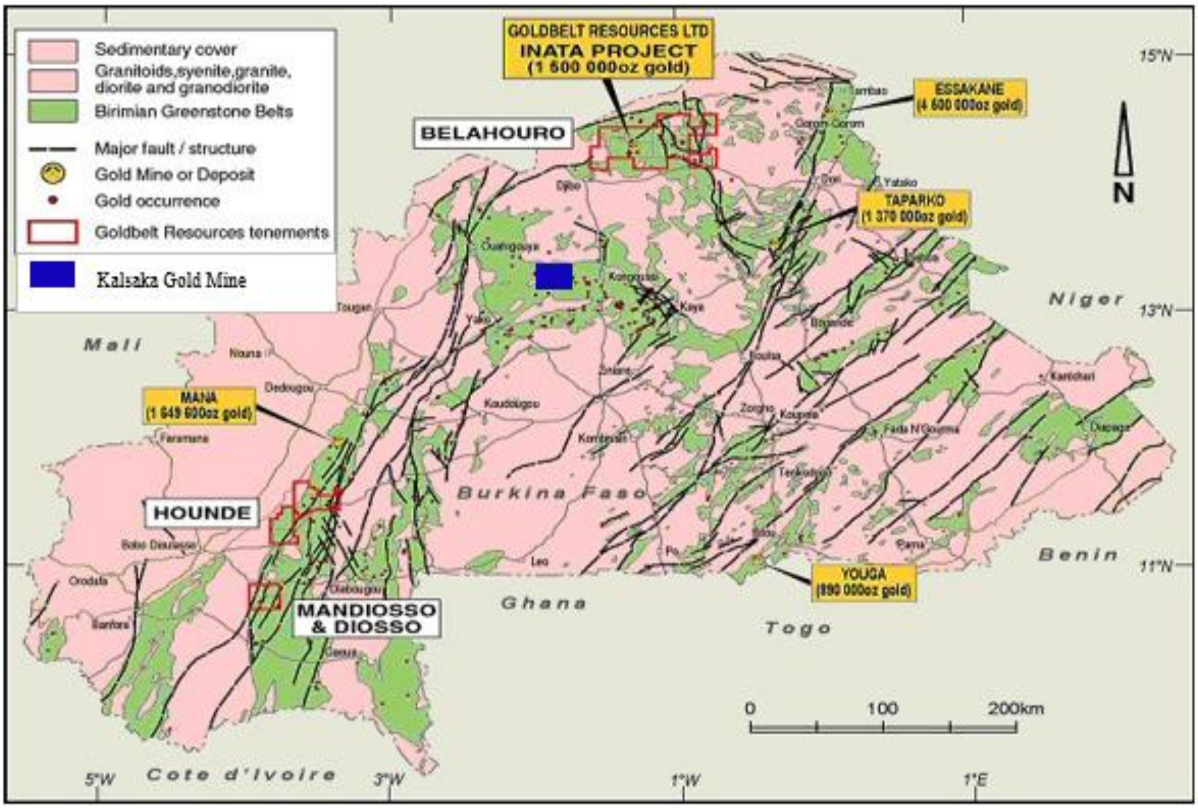

Fig. 1 A Geological Map of Burkina Faso showing the Study Area (Anon., 2007)

The monthly average temperatures range from 22 °C to 35.8 °C (Anon. 2003).

2.2 Geology of Kalsaka

Basalt and andesite, which form part of the central greenstone belt of Burkina Faso, host the steeply dipping east-west striking shear zones. Each shear zone is typically 3-10 m wide and dips at 50 – 60° to the south in the east of the area, becoming sub-vertical to the west. Gold mineralisation occurs in narrow shear zones and contain thick quartz veins whose brecciated margins host the highest grades.

2.3 Data Used

The data used for this study were obtained from the available Reverse Circulation (RC) exploration drill hole database compiled by the Kalsaka Gold Mine, Burkina Faso. The data comprised collar coordinates, downhole survey, lithology codes, assay values, topography and densities. Universal Transverse Mercator (UTM) projection was used for all data collection and surveying. A total number of 7303 drill hole samples, covering a strike length of 862 m, were used. The samples were taken at an interval of 1 m. Holes were drilled at angles ranging from 30° to 80° on 50 m drill spacing along strike and 20 m perpendicular to the general strike of the mineralisation. Zones of high grade were infilled to 25 m by 10 m spacing.

2.4 Data Validation

The drill hole data were imported into Datamine Studio 3 and validated to check for repeat assay values, overlapping sample intervals, gaps in the data and other errors. The validation only discovered some sample intervals with no assay values and such samples were removed.

2.5 Domain Interpretation

The validated data were used to digitise the boundaries of the mineralisation in sections at a cut-off grade of 0.25 g/t. Fig. 2 shows a section of the drill holes along 21200 mE. The resulting digitised boundaries were used to create wireframe outlining the three-dimensional geometry of the mineralisation (Fig. 3). The boundaries were interpreted based on grade information. The wireframe defined the spatial limits of mineralisation.

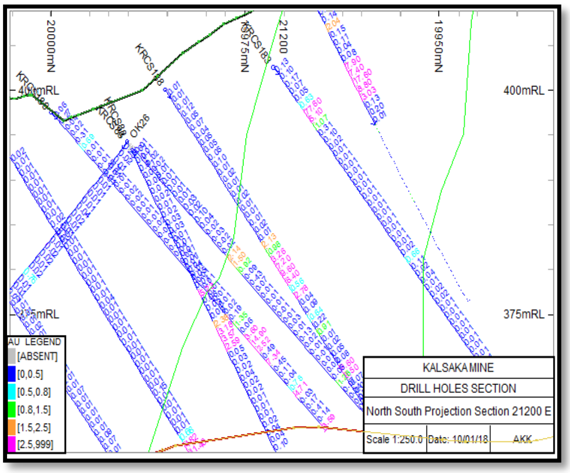

Fig. 2 Cross-section of Drill Holes along 21200 mE

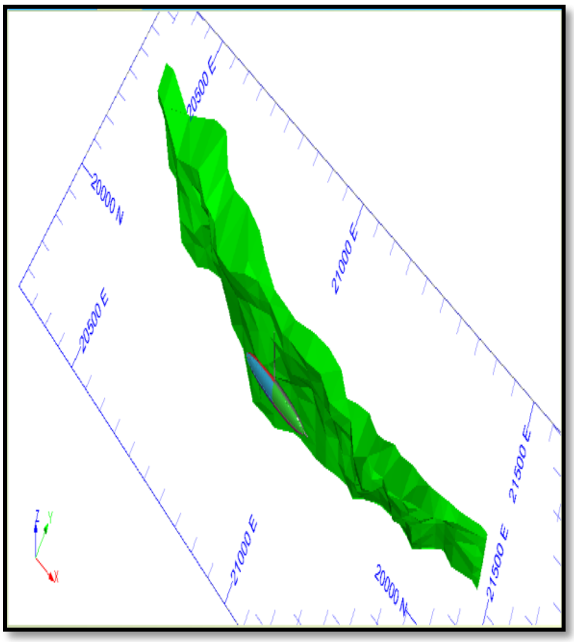

Fig. 3 An Isometric View of 3-D Solid

2.6 Statistical Analysis

The downhole composites were computed from the samples within the wireframe using a constant length interval of 1 m. The reason was to equalise the lengths of the samples for meaningful statistical and geostatistical analyses. The summary statistics of the 1 m Au composites are presented in Table 1. From the summary, the coefficient of variation, 3.49, indicates a significant variation of the mineralisation (Rossi and Deutsch, 2014) which implies that outliers may exist. The high grade values contribute significantly to the mean grade. The mineralisation is also characterised by a highly positive skewed distribution as shown in Fig. 4.

Table 1 Summary Statistics of 1 m Au Composites

Count	7303
Minimum value	0.0001
Maximum value	132
Mean	0.93
Median	0.03
Variance	9.88
Standard deviation	3.14
Coefficient of variation	3.49
Skewness	14.88
Kurtosis	478.34

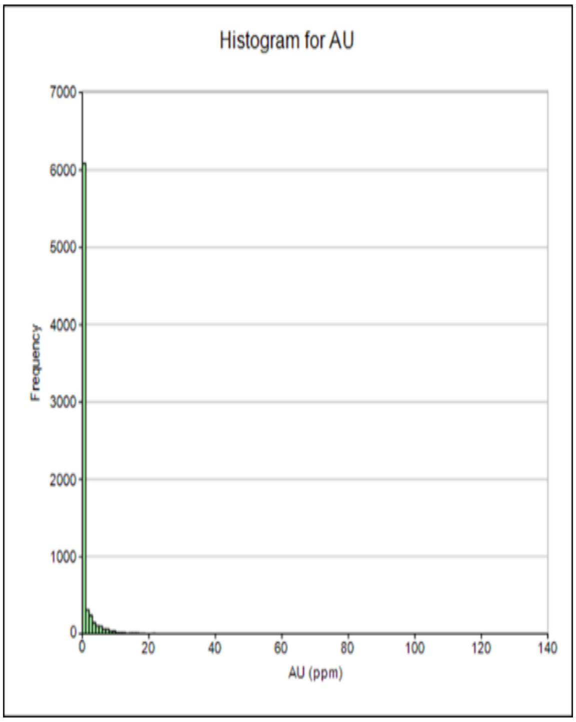

Fig. 4 Histogram of 1 m Au Composites

2.7 Outlier Analysis

The normal probability plot (Fig. 5) shows some scattered high grade values at the tail of grade distribution. The plot shows a kink at 15 ppm and this was used as a top-cut. The Kalsaka Hill deposit displays about fifty-six high-grade values, which represent 0.77% of the entire dataset, above 15 ppm that may result in overestimation of the blocks. The risk associated with estimating the high grade values into blocks may be high since the metal content may not be achieved when the blocks are mined. Hence, all the fifty-six high-grade values were reduced to 15 ppm prior to estimation.

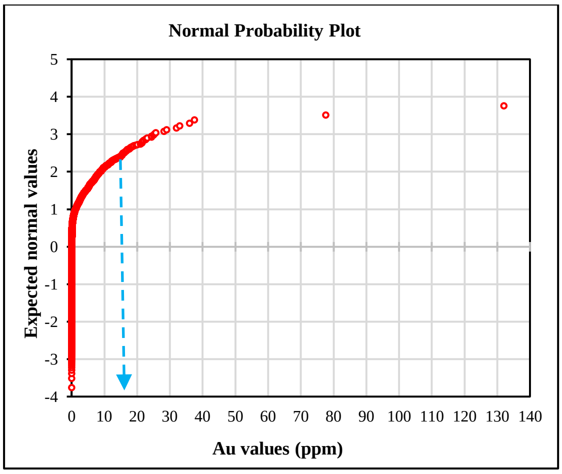

Fig. 5 Probability Plot of 1 m Au Composites

2.8 Grade Variography

Downhole experimental semi-variograms were computed and modelled. A two-structure spherical scheme was found to be appropriate for modelling the semi-variograms. The downhole semi-variogram was to provide an estimate of the nugget variance since it was calculated from a small distance along the paths of drill holes. The downhole semi-variogram is illustrated in Fig. 6. Horizontal experimental semi-variograms were also computed and modelled with a two- structure spherical model along strike and across strike as shown in Figs. 7 and 8 respectively. Lag spacing and angular tolerance were selected appropriately to capture more samples. From the semi-variogram models, the continuity of mineralisation was not the same in all the three orientations which means that the mineralisation is anisotropic. The direction of maximum continuity is along strike. Table 2 shows the directional semi-variogram model parameters where C_0, C_1, C_2, a_1 and a_2 represent the nugget variance, first sill, second sill, first range and second range respectively.

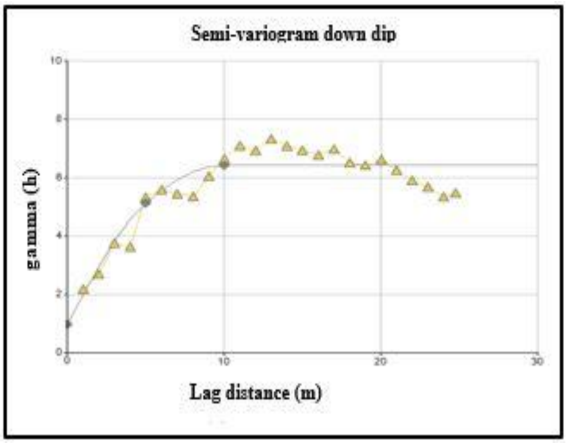

Fig. 6 Semi-variogram Down Dip

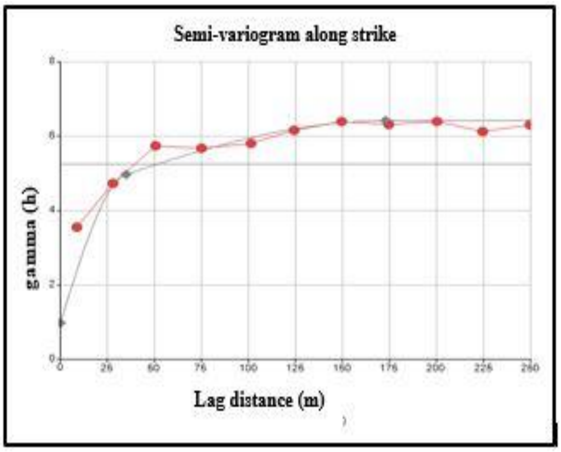

Fig. 7 Directional Semi-variogram along Strike

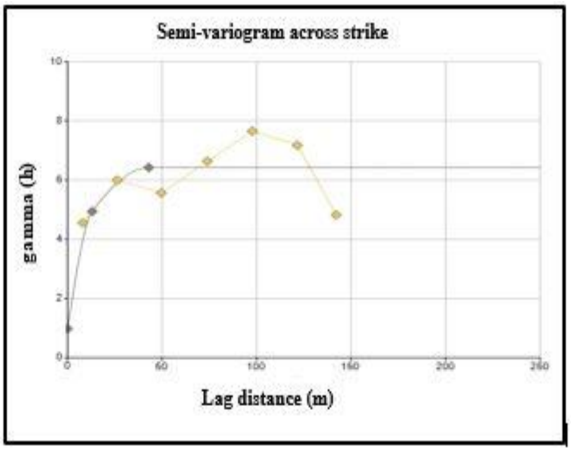

Fig. 8 Directional Semi-variogram across Strike

Table 2 Directional Semi-variogram Parameters

Direction	Spherical model parameters				
	C_0	C_1	C_2	a_1	a_2
Downhole	0.982	1.308	4.134	5	10
Across strike	0.982	2.791	2.651	13	43
Along strike	0.982	3.369	2.069	35	173

2.9 Indicator Variography

A series of four cut-offs, 0.25 g/t, 0.5g/t, 0.75 g/t and 1 g/t, were used for the indicator variography with a view to quantify the spatial continuity at these cut-offs. The basic statistics tool in Datamine Studio 3 software was used to generate the interval statistics of the dataset based on a bin size of 0.25 g/t. These cut-offs are the reported mining cut-offs of the mine based on waste, low, medium and high grade ores. Table 3 shows the summary of the interval statistics of the Kalsaka Hill deposit. The grade samples within the class were transformed into zeros and ones. The transform indicators were used to compute experimental semi-variograms and modelled. A two-structure spherical scheme was found to be appropriate for modelling the semi-variograms at each cut-off in the three different directions. Lag spacing and angular tolerance were selected appropriately to capture enough samples. The binary transform of the grades was defined at each cut-off as (Rossi and Deutsch, 2014):

$$\mathrm{I}(u; Z_k) = \begin{cases} 1, & if\ Z(u) \le Z_k \\ 0, & if\ Z(u) > Z_k \end{cases} \qquad (1)$$

where Z(u) is a sample grade at u location and I(u; Z_k) is the threshold grade defined at a location of u for a cut-off of Z_k. Samples of such model for 0.25 g/t cut-off are shown in Figs. 9, 10, and 11, and the parameters are displayed in Table 4.

Table 3 Interval Statistics of the Dataset

Class interval	Class count	Class mean	Class median	Class frequency
0.0-0.25	5407	0.035	0.010	0.740
0.25-0.5	326	0.358	0.350	0.045
0.5-0.75	196	0.606	0.600	0.027
0.75-1	154	0.870	0.860	0.021
≥ 1	1220	5.084	3.360	0.167
Total	7303			1

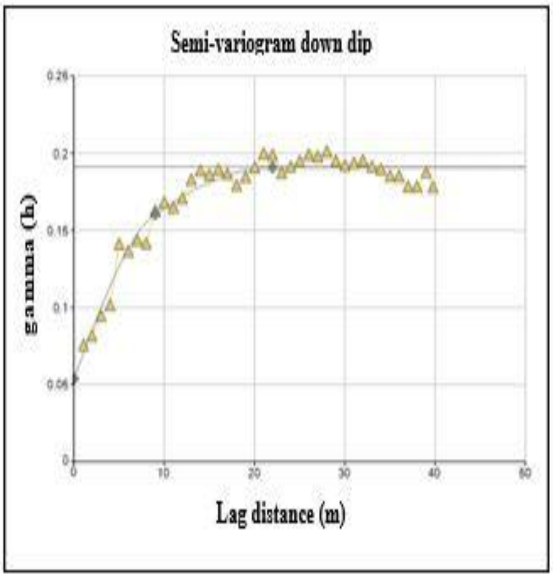

Fig. 9 Indicator Semi-variogram Downhole for 0.25 g/t

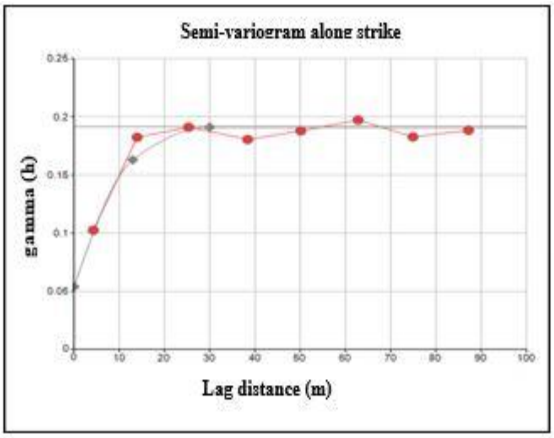

Fig. 10 Indicator Semi-variogram along Strike for 0.25 g/t

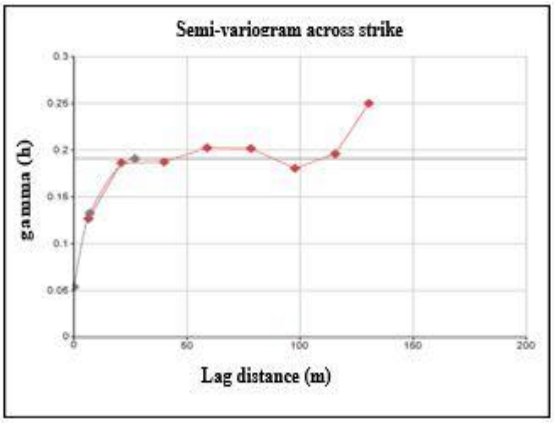

Fig. 11 Indicator Semi-variogram across Strike for 0.25 g/t

Table 4 Indicator Semi-variogram Parameters for 0.25 g/t Cut-off

Direction	Spherical model parameters				
	C_0	C_1	C_2	a_1	a_2
Downhole	0.054	0.065	0.072	9	22
Across strike	0.054	0.043	0.094	7	27
Along strike	0.054	0.065	0.072	13	30

2.10 Block Modelling

A 3-D block model was obtained using Datamine Studio 3 software. It covered the interpreted mineralisation domain. The model was constrained at the top by the surface topography at Kalsaka. A parent block size of 20 mE x 10 mN x 5 mRL was selected based on the drill hole spacing along x and y axes. The z dimension was chosen based on the bench height used at the mine.

Sub-blocking was done to ensure adequate volume representation. Table 5 shows the dimensions of the sub-blocks as well as the number of parent blocks. Attributes coded into the block models included the densities of the sundry weathering zones.

Table 5. Block model limits

Orientation	Easting	Northing	Elevation
Max. coordinates	21500	20060	460
Min. coordinates	20600	19900	230
Cell size (Parent)	20	10	5
Sub-block size	5	2.5	2.5

2.11 Resource Estimation (OK)

Ordinary kriging was used to interpolate gold grade directly into the block model using parameters derived from the grade semi-variogram model (Table 2) and sample search ellipsoid (Table 6). The search parameters used for the estimation were defined based on the variography and data spacing.

Table 6 Search Parameters for Ordinary Kriging Estimate

Search distance (m)	Major	173
	Semi-major	43
	Minor	10
Rotation angles (°)	Azimuth	90
	Dip	75

2.11.1 Validation of OK Model Results

The grade estimates were validated visually by moving through cross sections and superimposing colour coded drill hole data on analogous colour coded block models. Fig. 12 shows one such sections along N-S section 21200 mE. The colour codes of drill hole data matched well with those of the block models and therefore the local estimates were deemed satisfactorily accurate. Additional validation check was done on the block estimates by comparing them with average borehole composites in a swath plot, one of which is shown in Fig. 13.

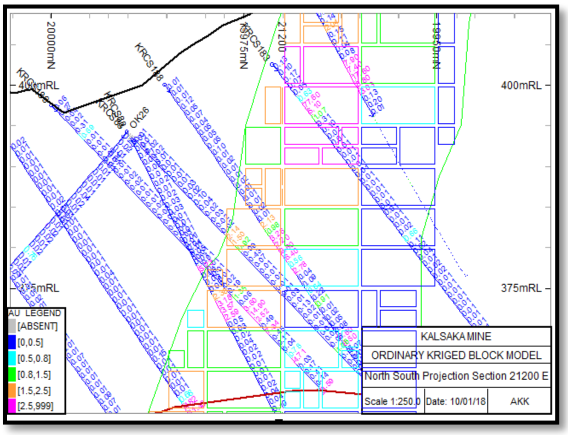

Fig. 12 Cross-sectional View of Validated OK Model along N-S Section 21200 mE

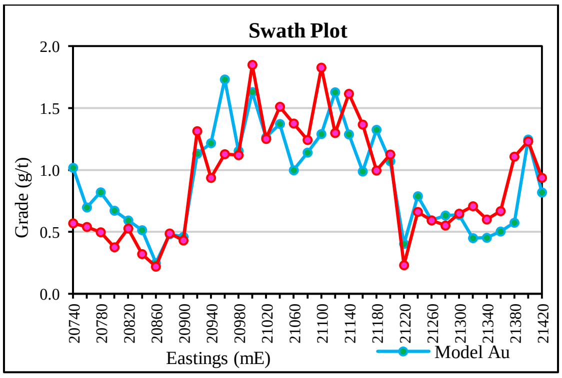

Fig. 13 Swath Plot of Drill Hole Grades and OK Model at 20 mE

2.12 Resource Estimation (MIK)

Multiple indicator kriging estimates were computed using ordinary kriging with parameters derived from the indicator semi-variogram models and sample search ellipsoid which were defined based on the variography and data spacing. The approach used to model ordinary kriging estimates was used for multiple indicator kriging estimates. The resource estimates obtained at the various cut-off grades are shown in Table 7.

2.12.1 Validation of MIK Model Results

The grade estimates were also validated visually by moving through cross sections and superimposing colour coded drill hole data on analogous colour coded block models. Fig. 14 shows one such case along 21200 mE. The colour codes of drill hole data matched well with those of the block models hence the local estimates were deemed satisfactorily accurate. Additional validation check was done on the block estimates by comparing them with average borehole composites in a swath plot, one of which is shown in Fig. 15.

Table 7 Summary of OK and MIK Global Resource Estimates

Cut-off (g/t)	Estimation Method	Tonnage (Mt)	Grade (g/t)	Ounces (millions)
0.25	OK	6.10	1.35	0.266
	MIK	6.91	1.92	0.427
0.50	OK	4.51	1.56	0.227
	MIK	5.88	2.15	0.407
0.75	OK	3.74	1.97	0.237
	MIK	5.03	2.52	0.407
1	OK	3.14	2.11	0.213
	MIK	4.16	2.60	0.348

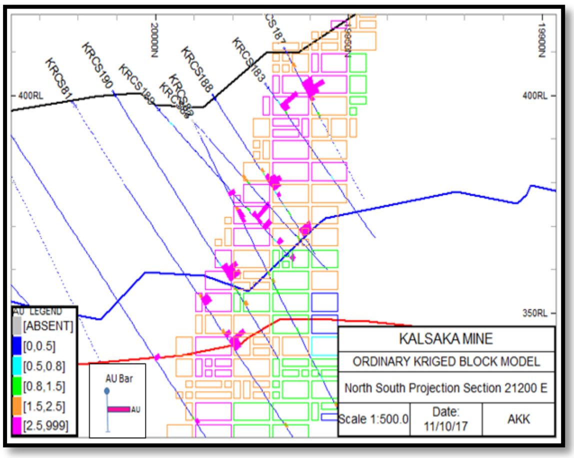

Fig. 14 Cross-sectional View of Validated MIK Model along N-S 21200 mE

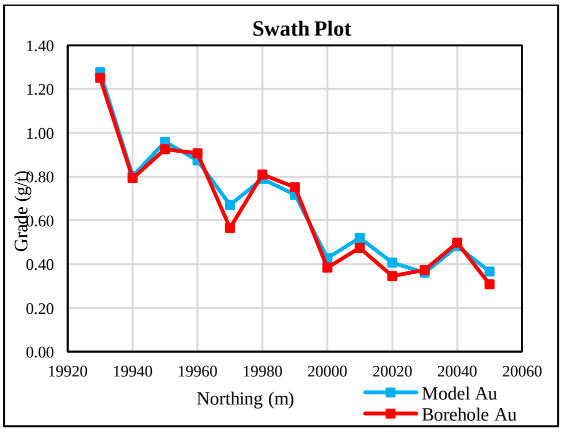

Fig. 15 Swath Plot of Drill Hole Grades and MIK Model at 20 m

3 Results and Discussion

Cross-sectional views of the model estimates in Figs. 12 and 14 show that the grade estimates of the multiple indicator kriging blocks are closer to the drill hole grades than that of the ordinary kriging. The swath plots compare the general trend of both the drill hole grades and the estimated block grades and these confirm that the trends are reasonably similar. The plots show the average of the drill hole grades as well as the average of block estimates from sliced block models and drill holes at a defined interval. In Fig. 13, the plot indicates that although analogous trends are occurring, there is a significant bias with the block model grades being higher than the drill hole grades in regions of low Au values. Regions of high Au values are somewhat underestimated and these variations are as a result of smoothing effect on the ordinary kriging block estimates.

Fig. 16 compares the block model grades for OK and MIK at an elevation of 5 m. The ordinary kriging model grades are consistently lower than the drill hole grades except at 320 m, 335 m and 395 m where the block model grades are higher than the drill hole grades. At these locations, there is overestimation of grades by the ordinary kriging model. Underestimation of high Au grades is also common but it is so much at the extreme right end of the plots where high Au values are observed. Overestimation of low Au values and underestimation of high Au values indicate smoothing effect on the ordinary kriging grade estimates. Even though, the multiple indicator kriging grade estimates are also lower than the drill hole grades at some points, they are more close to the drill hole grades than the ordinary kriging grade estimates. It means that the multiple indicator kriging model gives estimates which are close to the actual grades. Comparatively, the multiple indicator kriging model is seen to give more accurate recoverable resource estimates than the ordinary kriging model.

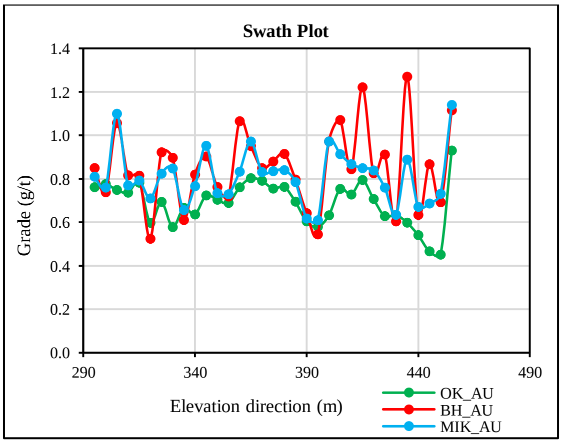

Fig. 16 OK and MIK Grades with Drill Hole Grade at Elevation of 5 m

In Table 7, detailed analysis shows that at cut-off grades of 0.25 g/t and 0.50 g/t, multiple indicator kriging yields about 13% and 30% more tonnes of material with 42% and 38% more grades than ordinary kriging respectively. At cut-off grades of 0.75 g/t and 1.0 g/t, multiple indicator kriging produces about 34% and 32% more tonnes of material with 28% and 24% more grade than the ordinary kriging model. The low tonnage and low grade produced by the ordinary kriging model may have resulted in the underestimated resource estimates according to the tonnage and grade reconciliation during the production period of the mine. In terms of metal content, multiple indicator kriging showed consistent high ounces of gold at all the cut-off grades.

The study also established a grade-tonnage relationship, which relates the tonnes of material to its grade above cut-off grades. In Fig. 17, the multiple indicator kriging model produces higher tonnage of material with higher grade than the ordinary kriging model. The grade curve for multiple indicator kriging is somewhat above the grade curve for ordinary kriging except at cut-offs of 2.0 g/t and 2.5 g/t where the average grades are very close. Considering the four cut-offs used by the mine, the multiple indicator kriging model yields high ounces of gold more than the ordinary kriging model. Using the swath plots together with the grade tonnage curve, it can be said that the multiple indicator kriging model gives recoverable resource estimates of the deposit, where the distribution of grade is highly skewed.

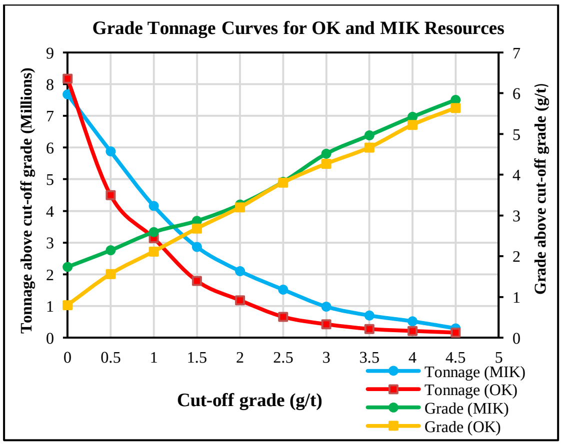

Fig. 17 Grade Tonnage Curves for OK and MIK

4 Conclusions

(i) Smoothing effect is more in the ordinary kriging estimates than in the multiple indicator kriging estimates as evidenced in the swath plots.

(ii) Multiple indicator kriging model yields grade estimates which are higher and closer to the actual grades than the ordinary kriging model. Thus grade and tonnage reconciliation could be improved using MIK for resource estimation at the Kalsaka Gold Mine.

Acknowledgements

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