# Computer-Aided Cut-off Grade Optimisation for Open Pit Mines*

[1]J. W. Muriuki and [2]V. A. Temeng
[1]Jomo Kenyatta University of Agriculture and Technology, Nairobi, Kenya
[2]University of Mines and Technology, Tarkwa, Ghana

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## Abstract

A mine planning team is tasked among other duties with designing a feasible mine plan which in turn maps out the daily running of the mining project. A mine plan revolves around a cut-off grade which is thoughtfully and uniquely selected while considering various aspects such as grade tonnage distribution, economic and operational parameters specific to a mine. Selection of a cut-off grade can be a daunting task often involving iterative and lengthy mathematical formulas which take huge amounts of time to execute, often leaving room for error. In the occurrence of such errors, a mining project can be faced with sequential outcomes that could even lead to premature closure. The cut-off grade is therefore a strategic variable that determines the economic viability of a mine, and hence return on investment. It is critical that the cut-off grade is optimal so as to maximise the net present value. Lane's approach is a model that utilises several steps to yield one cut-off grade value. This algorithm is flexible and can be adjusted to include other factors specific to a mine. Regrettably, many mining companies continue to operate using inaccurate cut-off grades wrongly calculated or assumed. This has continuously led to frustrations due to losses and prematurely abandoned mines. This study focused on the development and implementation of an easy to use computer application based on Lane's approach that runs on Windows platform, and hence targeting a larger user base for choosing an optimum cut-off grade for open pit mines.

**Keywords:** Cut-Off Optimisation, Cut-Off Optimiser, Optimum Cut-Off Grades, Whittle

## 1 Introduction

Mining processes are faced with certain intricacies during their operations and decision making processes. To achieve the objective of resource utilisation, a value designating the amount of metal per tonne of ore that can be economically mined and processed (cut-off grade) must be agreed upon. Certain factors must be taken into consideration to decide the best cut-off grade to use in the objective of maximising the economic benefit from the mining operation (Rendu, 2008).

An optimum cut-off grade gives the advantage of considering discount rates and process capacities, and these are subject to change. Over the life of a mine, the optimum cut-off grade changes in response to these variations (Ataei and Osanloo, 2003).

The principles of Lane's approach are derived from the future project cash flows summarised in present value of internal rate of return. The cash flow model is based on three main components of a mining operation: the mining component, the concentrator or processing component and the refining component (Dagdelen and Asad, 1997).

This study focused on developing and implementing an easy to use computer application (Cut-off Optimiser ver.1.0) using Lane's approach that runs on Windows platform, and hence targeting a larger user base as most PCs come installed with Microsoft Windows operating system. In this paper, data from a gold mine is used to illustrate the working of the application. Cut-off grade optimisation using the same data was done on the Cut-off Grade Optimisation Node (Type 2) in Whittle 4X software, a commercially available package, which uses a proprietary approach to optimise the cut-off grade. The results obtained are discussed and compared.

### 1.1 Lane's Approach

Ken Lane first presented a model for optimising cut-off grade in 1964 and modified it in 1988. The approach identifies a mining operation as having three distinct stages (mining, concentrating and refining) and it involves selection of a cut-off grade between intermediate grades to provide an optimum cut-off grade that maximises present value.

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1.1.1 Parameters

The parameters used in Lane's cut-off grade policy are shown in Table 1.

**Table 1 Notations of Lane's Algorithm**

| Notation | Description | Unit |
|---|---|---|
| M | Mining capacity | tonne/year |
| C | Concentrator capacity | tonne/year |
| R | Refining capacity | grams/year |
| m | Mining costs | $/tonne |
| c | Concentrating costs | $/tonne |
| r | Refining costs | $/g |
| s | Sale price | $/g |
| Qm | Mine output | tonne/year |
| Qc | Concentrator feed | tonne/year |
| Qr | Refinery product | grams/year |
| y | Recovery | % |
| V | Present value | $ |
| T | Production period | year |
| f | Fixed cost | $/year |

1.1.2 Selection of the Optimum Cut-Off Grade

Economic limiting cut-off grades gm, gc, and gr represent the mining, concentrating and refining cut-off grades respectively, and balancing cut-off grades gmr, gmc, and grc that relate to the mining, concentrating and refining cut-off grades are used in choosing the grade that maximises the present value of the mine. The mathematical formulae of these grades are presented in Table 2.

**Table 2 Mathematical Formulae of Grades**

| Cut-off formula |
|---|
| $g_m = \dfrac{c}{y(s-r)}$ |
| $g_c = \dfrac{c + \dfrac{f+Vd}{C}}{y(s-r)}$ |
| $g_r = \dfrac{c}{\left(s - r - \dfrac{f+dV}{R}\right)y}$ |
| gmc = f (Qm, Qc, grade distribution) |
| gmr = f (Qm, Qr, grade distribution) |
| grc = f (Qr, Qc, grade distribution) |

The optimum cut-off grade is selected using the six cut-off grades shown in Table 2 as follows:

$$G_{mr} = \begin{cases} g_m & \text{if } g_{mr} \le g_m \\ g_r & \text{if } g_{mr} \ge g_r \\ g_{mr} & \text{otherwise} \end{cases}$$

$$G_{rc} = \begin{cases} g_r & \text{if } g_{rc} \le g_r \\ g_c & \text{if } g_{rc} \ge g_c \\ g_{rc} & \text{otherwise} \end{cases}$$

$$G_{mc} = \begin{cases} g_m & \text{if } g_{mc} \le g_m \\ g_c & \text{if } g_{mc} \ge g_c \\ g_{mc} & \text{otherwise} \end{cases}$$

G=middle value ($G_{rc}$, $G_{mr}$, $G_{mc}$)

## 1.2 Cut-Off Optimiser Ver.1.0 Development and Implementation

The general steps outlined by Lane (1964 and 1988) and modified by Dagdelen (1992) for determining Lane's cut-off grade policy:

(i) Read the input files:
   a. Grade-tonnage distribution in each pushback for the whole deposit.
   b. Economic and operational parameters to be used in the cut-off grade policy are shown in Table 1.

(ii) Determine limiting and balancing cut-off grades and then determine the optimum cut-off grade, G.

(iii) From the grade-tonnage curve of the deposit compute:
   a. The ore tonnage (To) above the optimum cut-off grade (G) obtained.
   b. The waste tonnage (Tw) that is below the optimum cut-off grade (G) obtained.
   c. Average grade of ore.

(iv) Determine the values of Qm, Qc and Qr.

(v) Compute the life of deposit N based on the limiting capacity among mine, mill, and refinery and then calculate the annual profit for the life of mine.

(vi) Adjust the grade tonnage distribution by subtracting Qc from the grade distribution intervals above optimum cut-off grade (G) and the waste tonnes (Qm-Qc) from the intervals below the optimum cut-off grade (G) in proportionate amount such that the distribution is not changed.

(vii) Check if Qc is less than the milling capacity C, then set mine life N= i and go to step (x); otherwise set the year indicator i=i+1 and go to step (iv).

(viii) Compute the accumulated future NPVs based on the profits P calculated in step (vii) for each year from i to N.

(ix) This NPV value becomes the second approximation of V (the first was V=0) for use in the formulas to calculate the optimum cut-off grade.

(x) If it is the first iteration then knowing the profits obtained in each year, find the net present value year by year by discounting back those profits.

(xi) Use the net present values obtained in step (xi) as initial NPV's for each corresponding year for the second iteration.

(xii) Repeat the computation from step (iv) until the value V converges.

(xiii) If the computed NPV converges, the Application stops.

The steps outlined were implemented using C++/CLI programming in Visual Studio 2008 Integrated Development Environment (IDE) to create the Cut-Off Optimiser Ver.1.0. application. The logic flow diagram of the application is shown in Fig.3.

Cut-Off Optimiser Ver.1.0 is an end-user productivity program that enables the user to choose the optimum cut-off grade for a single deposit in an open pit mine. The application runs on the windows platform. Double clicking the application icon will launch Cut-Off Optimiser Ver.1.0. Once started, the application displays graphical user interface equipped with icons, buttons, windows, and dialog boxes.

The application requires the user to input the grade, tonnage and the parameters illustrated in Table 1. The result tabulation showing the production schedule, yearly profits, NPV and number of iterations per year is displayed on the output window and subsequently displayed on a Microsoft Excel spreadsheet that is automatically launched by the program.

## 2 Resources and Methods Used

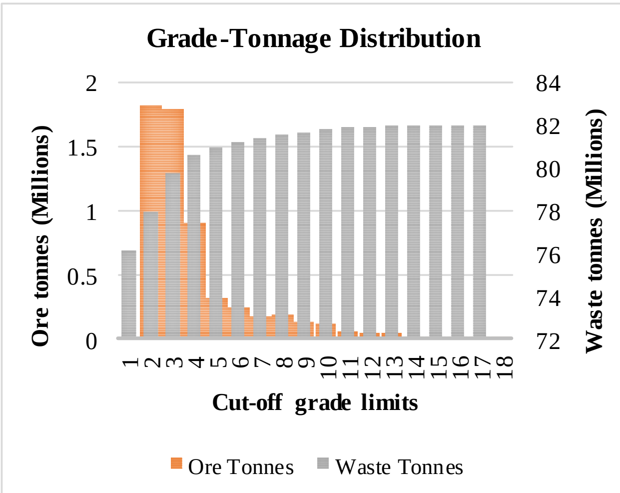
Data from an optimised block model from a gold mine whose name is withheld for privacy purposes is used to demonstrate the working of the application. The tonnage-grade distribution and mine design parameters are shown in Fig. 1 and Table 3 respectively. The values in Table 4 give implicit capacities and accepted costs to mine the deposit.

**Table 3 Economic Parameters and Operational Capacities**

| Parameters | Values |
|---|---|
| Price (P) | $38.58 /g |
| Sale price | $0.16 /g |
| Concentrating cost | $29.43 / tonne |
| Mining cost | $1.20 / tonne |
| Fixed cost | $ 10 950 000/ year |
| Mining capacity | 29 000 000 tonnes |
| Milling capacity | 1 050 000 tonnes |
| Refining capacity | 5 400 000 g |
| Capital costs (CC) | $ 90 000 000 |
| Processing recovery | 90% |
| Discount rate (d) | 10% |
| Mining dilution (k) | 5% |

### 2.2 Running Data on Cut-Off Optimiser Ver. 1.0

The process flow diagram shown in Fig. 4 describes the working of the Cut-Off Optimiser ver.1.0.




## 3 Results and Discussion

Tables 4, 5 and 6 show the results of the cut-off grade policy as reported from Cut-Off Optimiser Ver.1.0, Whittle 4X's cut-off optimisation Type (2) Node and the grade tonnage output as reported in both scenarios respectively. Fig. 2 shows the variable cut-off grades results obtained in Whittle 4X and Cut-Off Optimiser Ver.1.0. The grades are higher in earlier years and they reduce over the life of the mine.

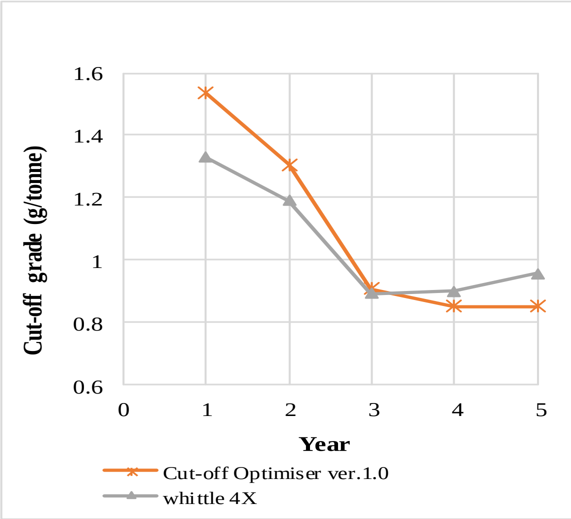



**Fig. 2 Cut-off Grade Results**

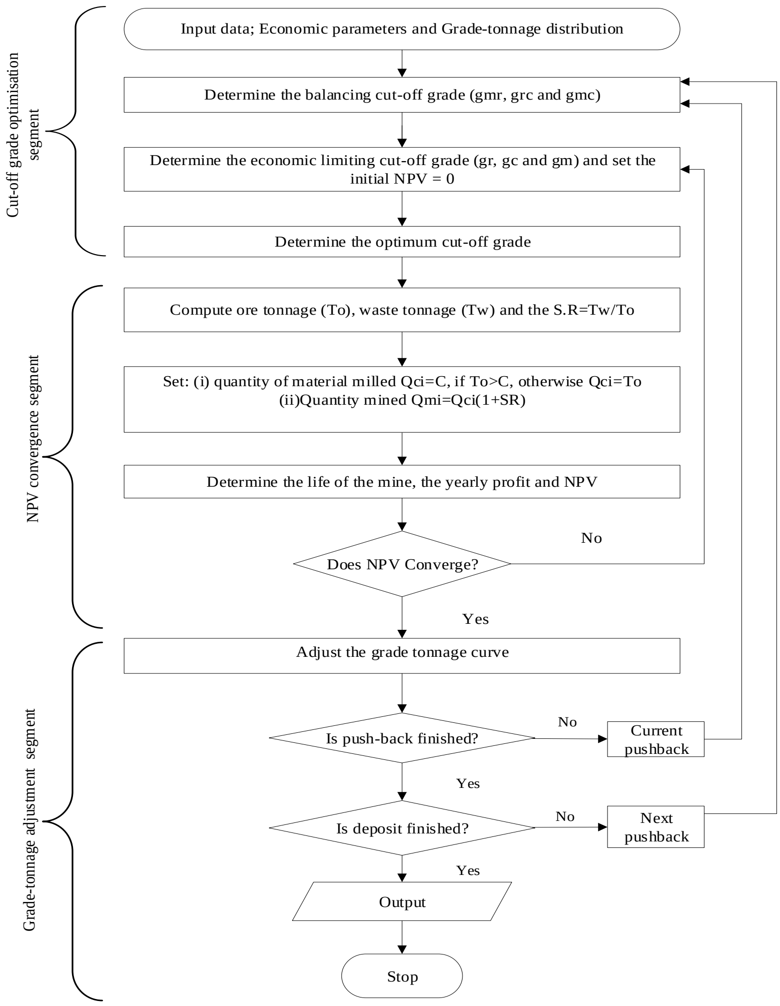



**Fig. 3 Logic Flow Diagram of the Lane's Approach (After Bascetin and Nieto, 2007)**

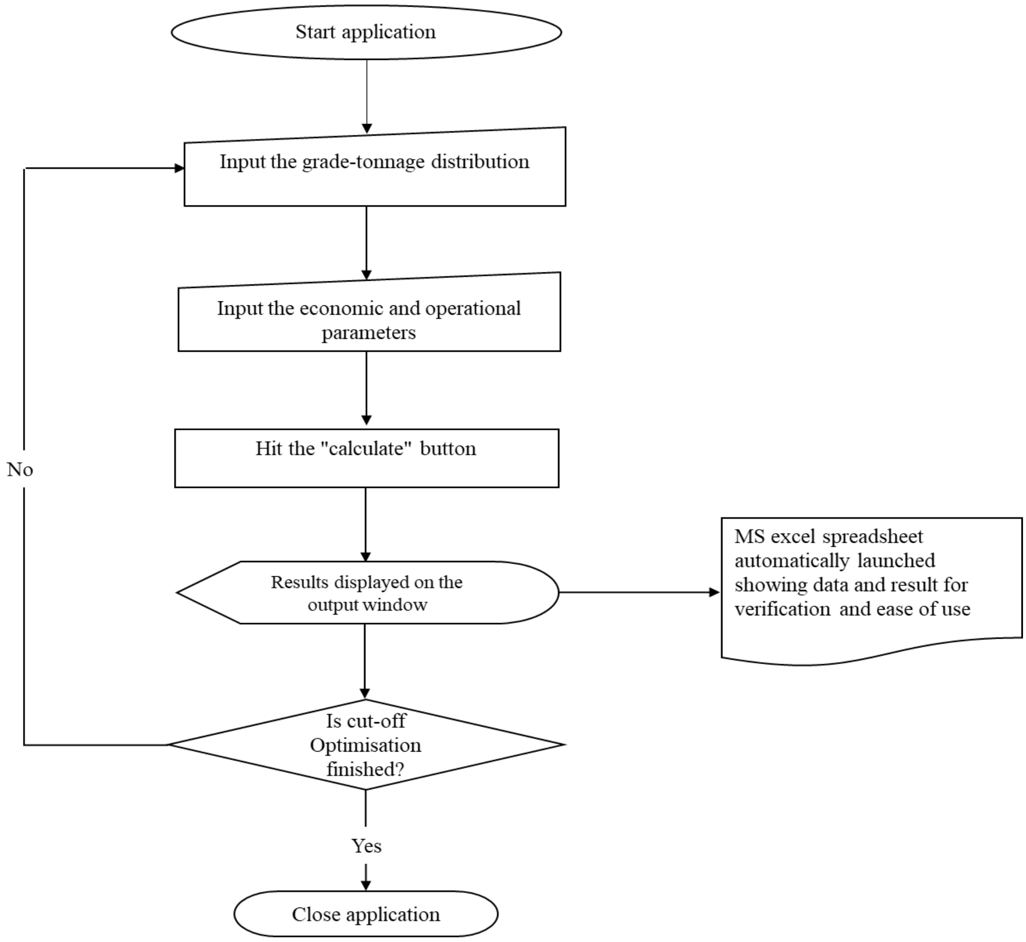



**Fig. 4 Process Flowchart of the Working of Cut-Off Optimiser Ver.1.0**

Cut-off grades obtained from Cut-Off Optimiser Ver.1.0 ranged between 1.54 g/t and 0.85 g/t while in Whittle's 4X Cut-Off Optimisation (Type 2) Node, the grades ranged between 1.328 g/t and 0.893g/t.

Fig. 5 shows the net present value results. The total NPV value over five years obtained in Cut-Off Optimiser Ver.1.0 is $231 673 816 while in Whittle 4X the NPV obtained is $216 112 946. The net present values also follow a declining trend, and this ensures quicker payback of the capital invested in the earlier years. Using this example, the net present value improves by 7% in Whittle 4X compared to Cut-Off Optimiser Ver.1.0

Menabde *et al.* (2004) noted that variable cut-off grades maximise the net present value and they have been known to the mining community for a long time. Such cut-off grade values yield substantial improvement in terms of expected NPV as was demonstrated by Lane (1964 and 1988).

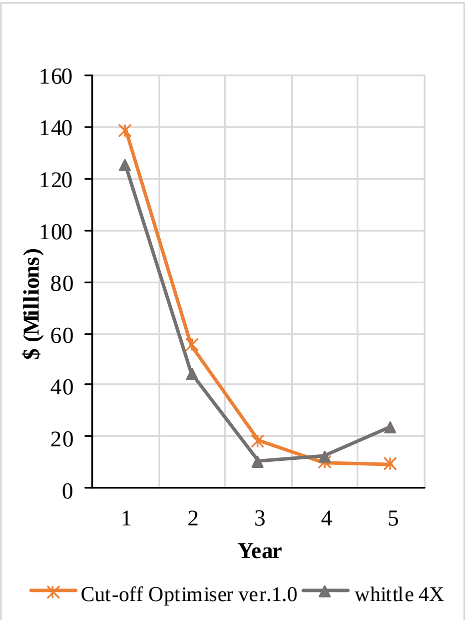



**Fig. 5 Net Present Value Results**

It is important to note that Whittle's 4X Cut-Off Optimisation (Type 2) Node is not used in this study to subvert Whittle 4X but to illustrate the importance of variable cut-off grades over the life of a mine to achieve optimum economic returns.

Cut-Off Optimiser Ver. 1.0 utilises an iterative approach based on Lane's approach which uses the grade-tonnage distribution, prices, costs, recoveries, and capacities. Whittle 4X on the other hand applies a proprietary search algorithm combined with powerful Linear Programming, and it involves multiple mines, multiple mineral elements, multiple processing paths, alternative products, blending, changing prices, costs, recoveries, and capacities. The distinctiveness of the approaches used in these two applications is the likely result of the variance in the results obtained.

To test if the net present value could be improved further, mining dilution is introduced in the formulae equations. A mining dilution (k) of 5% is incorporated in the adjusted mathematical formulae shown in Appendix 2. As a result, the net present value improves by $1 633 319, which is a 1% increase from the initial value of $216 112 946.

It is estimated that only about 20% of mines in the world apply Lane's approach of optimising cut-off grade and this is because Mining engineers look for constant mining rates which seem attractive but are, in the long run, not optimal (Whittle, 2007 and Anon, 2011). Mining higher grader ore in earlier years means one can process higher grade material and make more money. Lower grade ore can be stockpiled for later processing when it is economic to process at that grade (Asad, 2007 and Wooler, 1999).

Analysis between the average grades is shown in Table 6. Average grades are included in the calculations of the balancing cut-off grades. Fig. 6 shows a parity of the average grades in the two scenarios used

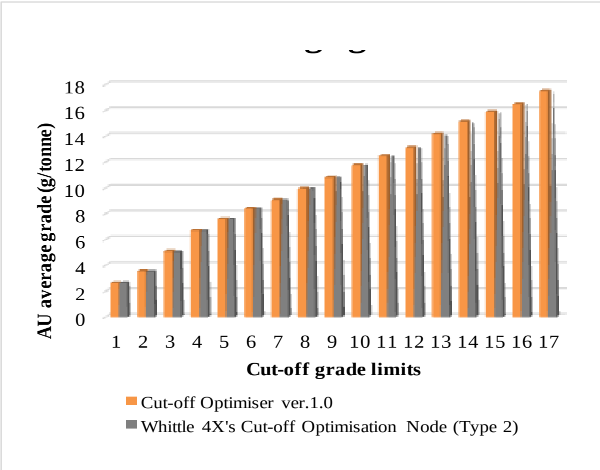



**Fig. 6 Grade Tonnage Output Report in the Two Scenarios**

**Table 4 Results of Cut-Off Grade Policy as Reported from Cut-off Optimiser ver.1.0**

| Year | Cut-off grade (grams/tonne) | Qm (tonnes) | Qc (tonnes) | Qr (grams) | NPV ($) |
|---|---|---|---|---|---|
| 1 | 1.535 | 26 281 763 | 1 050 000 | 3 363 255 | 138 832 595 |
| 2 | 1.305 | 22 180 888 | 1 050 000 | 12 362 304 | 55 434 814 |
| 3 | 0.907 | 14 674 405 | 1 050 000 | 21 821 960 | 18 290 682 |
| 4 | 0.851 | 16 013 837 | 1 050 000 | 16 738 800 | 9 885 399 |
| 5 | 0.851 | 1 817 411 | 242 281 | 15 720 390 | 9 230 324 |
| | | | | **Total NPV** | **231 673 816** |

**Table 5 Results of Cut-Off Grade Policy as Reported from Whittle 4X per Period**

| Yr. | Cut-off Grade g/tonne | Qm (tonnes) | Qc (tonnes) | Qr (grams) | NPV ($) |
|---|---|---|---|---|---|
| 1 | 1.328 | 13 903 385 | 1050 000 | 4 938 104 | 125 092 167 |
| 2 | 1.189 | 17 687 887 | 1 050 000 | 13 141 487 | 44 465 862 |
| 3 | 0.893 | 16 290 119 | 1 050 000 | 17 180 420 | 10 415 063 |
| 4 | 0.900 | 28 866 934 | 1 050 000 | 18 075 774 | 12 498 293 |
| 5 | 0.956 | 5 256 682 | 233 633 | 16 459 378 | 23 641 561 |
| | | | | **Total NPV** | **216 112 946** |

**Table 6 Grade Tonnage Output as Reported in Whittle 4X's Cut-Off Optimisation Node (Type 2) and Cut-Off Optimiser Ver.1.0**

| AU cut-off | Cumulative Ore tonne | Au Avrg. (g/tonne) | |
|---|---|---|---|
| | | Cut-Off Optimiser ver.1.0 | Whittle 4X's Cut-off Optimisation Node (Type 2) |
| 0 | 5 899 475 | 2.625 | 2.631 |
| 1.063 | 4 080 124 | 3.559 | 3.511 |
| 2.125 | 2 291 368 | 5.094 | 5.031 |
| 3.188 | 1 382 223 | 6.697 | 6.687 |
| 4.250 | 1 066 170 | 7.580 | 7.565 |
| 5.313 | 823 682 | 8.404 | 8.388 |
| 6.375 | 652 129 | 9.077 | 9.051 |
| 7.438 | 461 395 | 9.975 | 9.952 |
| 8.500 | 326 357 | 10.805 | 10.779 |
| 9.563 | 212 685 | 11.753 | 11.703 |
| 10.625 | 148 763 | 12.465 | 12.399 |
| 11.688 | 99 756 | 13.108 | 13.037 |
| 12.750 | 45 623 | 14.164 | 14.027 |
| 13.813 | 21 696 | 15.138 | 14.979 |
| 14.875 | 11 171 | 15.886 | 15.717 |
| 15.938 | 4 928 | 16.493 | 16.417 |
| 17.000 | 116 | 17.500 | 17.952 |

## 4 Conclusions

Lane's cut-off grade optimisation approach formed the foundation of this study, and is used to show the role of variable cut-off grades over the life of a mine to improve the value of a mine.

Lane (1988) notes that an optimum cut-off policy largely depicts a variable cut-off grade over the life of the mine. Uneconomic grades in early years may become economic in the latter years. This reality presents the possibility of a stockpiling strategy with accompanying problems in logistics, additional cost and the possibility that the stockpiled material may deteriorate over time.

Further improvements on Cut-off Optimiser Ver.1.0 are necessary and will be made over time to integrate other factors into the model such as multiple mines, multiple mineral elements and processing paths and stockpiling culminating into new versions of the application.

C++/CLI was chosen as the programming language to use as it is modular, and the source code is editable, making it highly customisable. This research is directed at mining companies looking to reduce their costs by using familiar software and average hardware on an ordinary PC. A mining company seeking to find an optimum cut-off grade in the earlier years of a mine can use Cut-Off Optimiser Ver.1.0 for cut-off grade optimisation.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## Authors

**Juliah Wangari Muriuki** is a Tutorial Fellow in the Mining, Material and Petroleum Engineering (MMPE) Department at the Jomo Kenyatta University of Agriculture and Technology (JKUAT), Kenya. She holds a BSc (Hons) degree in Mining and Mineral Processing from JKUAT and a Master of Science degree in Mining Engineering from the University of Mines and Technology (UMaT), Tarkwa, Ghana. She is currently a PhD student at the Okayama University, Japan. Areas of her research interests include mineral economics, mine economic and financial evaluation, mining geology, mine design and planning and operations research.

**Assoc Prof Victor Amoako Temeng** is an Associate Professor of Mining Engineering at the University of Mines and Technology (UMaT), Tarkwa, Ghana. He holds the degrees of BSc (Hons), PgD in mining engineering from the Kwame Nkrumah University of Science and Technology School of Mines, Tarkwa, Ghana, MSc in mining engineering from the University of Zambia, and PhD in mining engineering from the Michigan Technological University. He is a member of the Honour Society of Phi Kappa, Michigan Technological University Chapter and a member of the Ghana Institution of Engineers (GIHE). His research areas cover simulation and animation of mine production and machinery systems, materials handling, operations research, mine economic evaluation, and mine design and planning.