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RAINFALL MODELLING WITH A TRANSECT VIEW IN GHANA

F. F. TORGBOR*, D. A. STERN, B. K. NKANSAH AND R. D. STERN
(F. F. T.: African Institute for Mathematical Sciences, Ghana (AIMS Ghana); D. A. S.: IDEMS International, UK; B. K. N.: Department of Mathematics and Statistics, University of Cape Coast, Cape Coast Ghana; R. D. S.: University of Reading, UK)
**Corresponding author's email: f.torgbor@aims.edu.gh*

Abstract

Rainfall variability is an inherent part of African climate. This variability has important implications for food production and general livelihoods in countries such as Ghana. 53 years of rainfall data for 15 stations were obtained from the Ghana Meteorological Agency and used to describe the variability in the pattern of rainfall in Ghana. The direct method was used to analyse the annual rainfall totals and the total number of rainy days. A Markov chain modelling approach, which involves the fitting of harmonic regression curves to model the probability of rain within the year was also used. The effects of the previous rainy day was obtained throughout the year. The first order Markov model for each station was significant with the probability of rain given dry being higher than the probability of rain given rain in the north, while the probability of rain given rain was higher than the probability of rain given dry in the south.

Introduction

Climate change and variability are essential phenomena because of the great impact they have on livelihoods. In the temperate zones, temperature is the most important climatic parameter because of its variable nature within the region. In tropical regions such as West Africa, however, rainfall is the climatic parameter that raises enormous concerns because it is subject to high variability both temporarily and spatially. West African climatology is mainly influenced by two major wind or air mass systems; the south-western maritime air and the north-eastern continental air. Both systems and their influence on the local climates are controlled by the position of the Inter Tropical Convergence Zone (ITCZ) (Hayward & Oguntoyinbo, 1987).

Ghana is in the tropical region and is located within the Guinea Coast of West Africa. The rainfall regime within the country differs from north to south. This is because the north and the south falls within the Sahel and Guinea Coast regions, respectively. A warm South Atlantic usually results in above average rainfall in the Guinea region while below average rainfall is recorded in the Sahel region (Gu & Adler, 2003), and this implies different rainfall patterns in the south and north of Ghana. The rainfall season of Ghana is controlled by the movement of the Inter Tropical Convergence Zone (ITCZ), which oscillates between northern and southern Ghana within the year. The dominant wind direction in regions south, of the ITCZ is south westerly, blowing warm moist air from the Atlantic over the continent. However, the prevailing winds over the north of the ITCZ originate from the north east, bringing hot and dusty air from the Sahara desert, also known as the Harmattan.

As the ITCZ migrates between its north and south positions within the year, the regions between these northern and southern positions of the ITCZ experience a shift between the two opposing prevailing wind directions. This pattern is referred to as the West African Monsoon. The southern part has a bi-modal rainfall regime with a major season from March to June and a minor season from September to

October, however, the northern part experiences a uni-modal rainfall regime from June to September (Sultan *et al.*, 2004). The responses of these regions differ with respect to major external causes of climate variability such as the Southern Oscillation (ENSO) and the South Atlantic sea surface temperatures (SST).

In spite of the general pattern in rainfall, it is still highly variable and unpredictable in most cases with climate change being attributed to this variability. The interest of stakeholders especially farmers in this rain-fed agriculture regions is to know when they should expect rain and how soon the rains will continue but this has not been clearly communicated to them. In this study, an attempt was made to investigate the behaviour of the rainfall spatially across the country using some modelling techniques. Overall, we generated suitable models to describe the rainfall pattern in Ghana and discussed whether there are visible trends in the rainfall amounts and rainy days from north to south of the country.

Concepts of rainfall analysis

There are two main types of analysis in this paper. A very common and simple way of analyzing rainfall data has been the *direct or simple* methods. In this analysis, we define events of particular interest which are extracted from the daily data. This analysis is in two stages, the extraction of events and then the summary of the extracted events. The other approach which we call the *modeling approach* in this paper, produces a model that summarizes all possible events of interest. This model is on a continuous daily basis within the year. It applies the Markov chains which is useful in spell length phenomena and also uses harmonics regression which produces smooth curves that model the seasonal nature of rainfall. This approach produces two models; one for the chance of rain, and the other for the amount of rain on rainy days.

The simple or direct method (Stern *et al.*, 1982a) of analysis is less complicated and faster as compared to the modelling approach. However, the direct method has some inadequacies due to short records of data available for many analyses. These short records result in large standard errors of estimates. Furthermore, it becomes difficult to compare data from different sites, as the years for which data are available may vary between the sites. The modelling approach (Stern *et al.*, 1982b) helps to resolve these inadequacies in addition to it being used to simulate data for many hundred years. The direct approach can then be used to analyse the simulated data.

Adequate information on rainfall is necessary and important in various aspects of this region. Getting the right rainfall information requires exploration of the above concepts because both complement each other.

Experimental

The Ghana Meteorological Agency has been in existence since the first weather observations were made in the 1830s, when Aburi Gardens were established, though the records from these times have not survived (Web, 2014, Accessed May 2014b). Data received by the Head Office of the Ghana Meteorological Agency from its local weather stations are transferred into a database system known as CLIDATA (Web, 2014, Accessed May 2014a) for data management.

Data used in this study were obtained from 15 stations out of about 310 nationwide weather stations of the Ghana Meteorological Agency (GMet). The data comprises amounts of rainfall per day throughout each year 53 years. Since the study of the transect is a key part of this work, the stations were selected to be on a five-tier basis from the north to the south of the country and in a three-tier basis from west to east. The selection criteria for the 15 specific stations chosen were mainly on their geographical positions, availability of quality controlled and long records of data with low number of missing values. These stations are shown in Fig. 1.

The statistical analyses were carried out in Instat+ (Web, 2014, Accessed May 2014d) and the R statistical package (Web, 2014, Accessed May 2014e).

TABLE 1

Data from transect stations

No.	*Station*	*Lat.*	*Long.*	*Alt.* (m)	*Available data*	*Years with Complete* data (%)
1	Manga	11.02	-0.27	250.0	1979 - 2012	29(85)
2	Navrongo	10.90	-1.10	201.3	1960 - 2012	48(91)
3	Babile	10.52	-2.82	304.7	1960 - 2012	41(77)
4	Yendi	9.45	-0.02	195.2	1960 - 2012	52(98)
5	Tamale	9.42	-0.85	183.3	1960 - 2012	51(96)
6	Bole	9.03	-2.48	299.5	1960 - 2012	47(89)
7	Kete-Krachi	7.81	-0.03	122.0	1960 - 2012	53(100)
8	Atebubu	7.75	-0.98	121.9	1960 - 2012	45(85)
9	Wenchi	7.75	-2.10	338.9	1960 - 2012	48(91)
10	Goaso	6.80	-2.52	213.3	1960 - 2012	44(83)
11	Kumasi	6.72	-1.62	286.3	1960 - 2012	50(96)
12	Ho	6.60	0.47	157.6	1960 - 2012	49(94)
13	Ada	5.8	0.03	5.20	1960 - 2012	48(91)
14	Accra	5.6	-0.17	67.7	1960 - 2012	48(91)
15	Axim	4.87	-2.23	37.8	1960 -2012	46(87)

*A summary of meta data of the sixteen stations used in the transect analysis. Data from all stations start from 1960 and end in 2012, except for Manga, which starts in 1979. It also contains information on years with complete data.

Fig. 1. Stations for transect

Rainfall model

Below are notations used in this rainfall model;

$Z \in \mathbb{R} \geq 0$	General rainfall model
$X \in \{0, 1\}$	Model for raining or not raining on a day of the year
$Y \in \mathbb{R} > 0$	Model for the amount of rain on a rainy day of the year
$p \in [0, 1]$	Probability of success (rain) on a day of the year.
$n \in \{1, \ldots, 366\}$	Day number in the year that is, January, 1 = 1 and December, 31 = 366

$\mathbf{t} \in \left\{\frac{2\pi}{366}, \frac{2\times 2\pi}{366}, \ldots, 2\pi\right\}$	Radians equivalent to day number in the year for fitting curves
w	Represents rainy day (wet day) in the year
d	Represents dry day in the year

The rainfall model (Z) formulated follows that of a two part model, that is, $Z = XY$ where X models the probability of rain on a given day of the year, and Y models the amount of rain that is likely to fall given it rains on a day in year. Most of the ideas here follow from (Garbutt *et al.*, 1981).

It is obvious in real life what is meant to be a rainy or dry day. However, for analytic purposes and in comparing stations, one often finds that different observers are not equally conscientious in the recording of small rainfalls, or in the extent to which data measurements are rounded. In this paper a threshold value of 0.85 mm is set, which avoids rounding problems in both millimetres and inches (it is between 0.03 and 0.04 inches) (Web 2014, Accessed May 2014c). This means that days with rainfall amounts below 0.85 mm are deemed as dry days (**d**) otherwise they are deemed as wet days (**w**). The set threshold value ensures that rainfall stations considered in the transect analysis are easily and accurately comparable. Also in general less than 1 mm may account for 20 percent of rainy days in Ghana (West Africa), although they contribute less than 2 percent of the yearly rainfall (Garbutt *et al.*, 1981). When these small amounts of rainfall are ignored by the effect of a threshold, it decreases the chance of rain but increases the mean amount of rainfall on a rainy day.

Each part of the rainfall model entails a Markov chain component and a harmonic regression component which are shown in the following sections.

Chance of rain

The threshold classifies our data into two main states as described before, wet (**w**) and dry (**d**). Thus, our data follows the Binomial Distribution, $B \sim (1, p)$ possible outcomes ($\mathbf{w} = \mathbf{1}$ or $\mathbf{d} = \mathbf{0}$) and with a probability of occurrence (p). If days were independent of each other with constant probability of success (rain) of p then our data on the occurrence of rain would follow a Bernoulli distribution ($B \sim (1, p)$) and the total number of rainy days in any period of n days would have a binomial distribution ($B \sim (n, p)$). We therefore define the variable,

$$X = \begin{cases} 1 & \text{with probability p} \\ 0 & \textit{with probability } 1 - p \end{cases}$$

These probabilities are estimated from proportions based on the historical data which are then smoothed throughout the year by fitting a curve. The harmonic regression is chosen for this fitting because of its Fourier series component that has the ability to easily fitting both unimodal and multimodal seasonal patterns within the year. Furthermore the harmonic is best suited for the periodic nature of rainfall within the year, where days (x) of the year are transformed into radians, $t = 2\pi \times \text{day}(x)/366$, for all $x = 1, 2\ldots366$. In fitting the curve using the harmonic regression, the observed probability (p) of rain on any day is transformed using the logit transformation. That is,

$$y = log\left(\frac{p}{1-p}\right) = a_0 + \sum_{j=1}^{n}\left(a_j \cos jt + b_j \sin jt\right) + \varepsilon \qquad [1]$$

a_0, a_j, b_j are the coefficient parameters. Now $p \in [0, 1]$, however, the Fourier series component of the equation above allows values within $(-\infty, \infty)$.

The fitted probability (p) for the occurrence of rain now becomes;

$$p = \frac{exp(y)}{(1+exp(y))}$$

Markov Chain Orders (The Conditioned chance of rain)

The probability of rain on a given day in the year may vary based on the influence of the state of previous days. This is where the Markov chain component of our model is made evident. If the probabilities depend on only the previous day then this a *first order Markov chain* as explained in previous sections whilst a zero order Markov chain is one which does not depend on its previous state.

A typical example of modelling the first order Markov chain can be investigated by examining conditional probabilities. For instance in Babile, it is noticed that out of the 53 years of historical data 15th May was a rainy day 12 times. 16th May was a rainy day 11 times, 2 of them following a rainy 15th May. From this information, it can be seen that the proportion of rainy days which estimates the probability (p) of rain on the 16th May given a rainy day (15th May) is 2/12 = 0.1667. The chance of a rainy day on 16th May following a dry day was 9/41 = 0.2195. Again these proportions on days within the year can then be smoothed using the curve fitting methods above for each type of conditioned chance of rain. The conditioned chance of rain following a dry day also represents the probability that a dry spell will end, while the conditioned chance that a rainy day follows a rainy day is the probability that a rainy spell continues.

Rainfall amounts

The second part of the model (Y) deals with the amount of rainfall on a rainy day. This is assumed to follow a gamma model. This is consistent with the historical data exhibiting a distribution with a large proportion of small values and a few large values of rainfall amounts with a bound at 0.85 mm of rainfall. It shows the positive skewness of the distribution and hence the gamma model. The gamma model is often shown in one of three different forms, the first ($G\sim(k,\theta)$) with a shape parameter k and a scale parameter θ, the second ($G\sim(k,\theta)$) which has a shape parameter k and an inverse scale parameter $\beta=\frac{1}{\theta}$, ($G\sim(k,\mu)$) has a shape parameter k and a mean $\mu=\frac{k}{\beta}=k\theta$ as its parameters.

The third form of the gamma model $G\sim(k,\mu)$, is adopted which gives a frequency distribution,

$$f(x)=\left(\frac{k}{\mu}\right)^{k}\frac{(x-a)^{k-1}e^{-k(x-a)/\mu}}{\Gamma(k)} \qquad [3]$$

with the rainfall x shifted by the threshold value, a = 0.85 whilst having all values less than a ignored. Therefore the mean rainfall per rainy day is estimated as the sum of the mean rain and the threshold (m + a). The shape parameter, k affects the proportion of extreme observations and therefore the variability and skewness of the distribution. In modelling the amounts of rainfall, only data from wet days are considered. As with the chance of rain, the distribution of rainfall amounts may be influenced by the states of preceding days. Again the harmonic regression is used to fit the curves of the mean amount of rain (m) on rainy days but with the log link function.

$$y=\log(\mu)=a_0+\sum_{j=1}^{n}\left(a_j\cos jt+b_j\sin jt\right)+\varepsilon \qquad [4]$$

Therefore, with a_0, a_j, b_j as the coefficient parameters. Again μ is an expected value that falls within $(0,\infty)$, however, the fourier series component of the equation above allows values within ((-∞ , ∞), The estimated mean amount of rain μ on a rainy day now becomes;

$$Y=\mu=\exp(y)$$

Transect View of Rainfall Models

As observed in earlier sections, the main source of rainfall in this tropical region originates from the Inter Tropical Convergence Zone (ITCZ) which appears as a band of clouds, that circle the earth near the equator. The transect in this section looks at the variability in the pattern of rainfall and hence gives consideration to the geographical location of stations particularly on their latitudes, since it matches up with the ITCZ movement. The transect analysis in this paper is in two parts, the preliminary transect analysis (Annual Rainfall Totals and Annual Rainy Day Totals) and the model transect analysis (Chance of Rain and Amount of Rain).

TABLE 2

Statistics on total annual rainfall amounts and annual rainy days

Tier (Av. latitude)	*Station*	*Min.*	*Rain Mean*	*Amts Max.*	*(mm) Stdv.*	*Min.*	*Rain Mean*	*Days Max.*	*Stdv.*
First (10.8)	Manga	686	963	1563	191.1	43	62	76	7
	Navrongo	671	991	1365	152.2	51	65	81	7
	Babile	564	1030	1417	166.0	41	69	89	10
Second (9.2)	Yendi	833	1248	1712	207.3	54	81	100	8
	Tamale	695	1098	1666	185.3	50	72	93	9
	Bole	762	1104	1820	198.1	60	77	113	10
Third (7.8)	Kete-Krachi	916	1349	2432	278.0	59	84	108	11
	Atebubu	714	1325	1984	271.2	36	82	128	16
	Wenchi	850	1254	1758	199.0	68	92	127	12
Fourth (6.6)	Goaso	830	1345	2037	222.9	71	98	157	16
	Kumasi	892	1383	2345	276.4	73	101	144	13
	Ho	747	1303.	2056	220.0	58	95	135	15
Fifth (5.2)	Ada	359	846	1696	271.5	28	54	91	11
	Accra	333	804	1413	241.6	30	56	101	12
	Axim	1065	2027	3332	489.2	79	117	164	16

Rainfall amounts and rainy days are both highest in Axim for both minimum and maximum values. This is the most southerly station and also on the west. Accra which has the lowest minimum rainfall amount also has the minimum rainy days. However, its maximum rainfall is not the least among the maximum rainfall amounts and is the same as the case for the maximum number of rainy days.

Annual rainfall totals

There is large variability in the annual rainfall totals from the north to south of Ghana as is shown in Table 2. Fig.2 expresses this variability in time series plots for the various stations across their tiers.

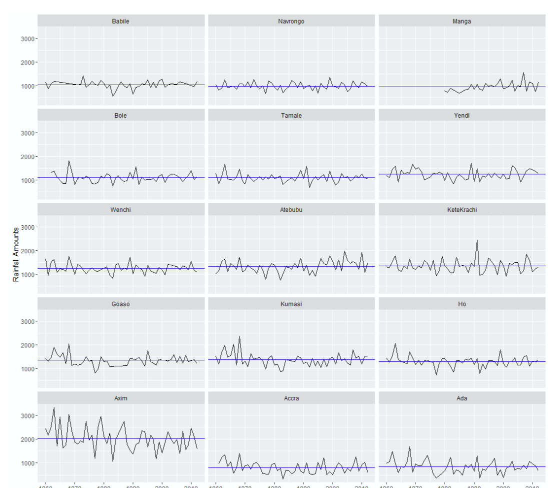

Fig. 2. Transect of annual rainfall totals

As shown in Fig.2, the first tier which constitutes the time series plots of stations Babile, Navrongo and Manga, altogether, do not show any form of visible trend. A general view of the transect of the annual rainfall totals does not indicate trends at the individual stations. This was confirmed with a formal regression analysis. Nevertheless, a high variability of annual rainfall totals is shown across the entire transect with an increasing average annual rainfall totals exhibited from north-east to south-west of the country. Ada and Accra are also noted to be the areas with the lowest average annual rainfall totals.

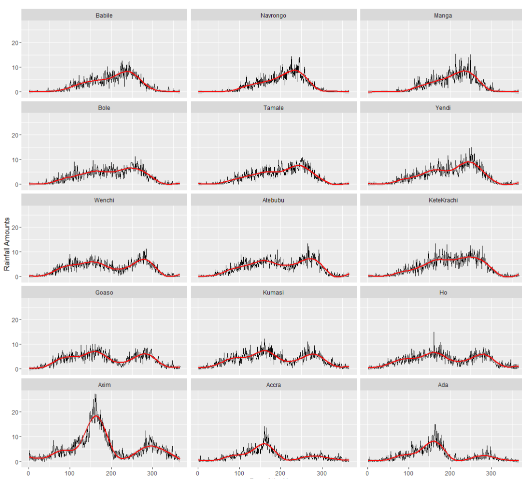

Fig. 3. Mean per rainy day

Fig. 3 shows a plot of the mean amount of rain per rainy day, together with smoothed lines for the stations within the transect. The first tier of this plot shows a gradual rise of about 2.50 mm of rain per rainy day from day 122 (01 May) which peaks between day 212 (30 July) and day 225 (12 August) at about 12.5 - 15 mm of rain. The second tier follows a more gradual rise and peaks between day 252 (8 September) and day 257 (13 September) at about 11 - 15 mm of rain. These two tiers discussed above show clear unimodality with regards to the mean amount of rain per rainy day. The third tier shows a subtle bimodality in the mean amount of rains on a rainy day with an early rise from day 80 (20 March), with a first peak between day 150 (29 May) and day 160 (08 June) at about 8 mm of rain. A gradual shallow drop follows this on day 225 (12 August) and then peaks for the second time between day 269 (25 September) and day 279 (5 October) at about 10 - 12 mm.

Tiers four and five show a more distinct bimodality as stations within these tiers are more southerly. Their plots also show earlier rises in early March and generally get to their first peaks between late May and early June at about 10 - 15 mm, however, with Axim peaking at about 27 mm. Their plots show a steep drop in the mean amount of rain per rainy day in August and then rise again to peak in late September to early October within a range of 7.5 – 12.5 mm.

Annual rainy day totals

Analogous to the chance of rain in the modelling approach, the annual rainy day totals shows the number of rainy days observed within a year considering the span of years of data available at individual stations across the transect. Summary statistics of the annual rainy day totals is shown in Table and with time series plots of individual stations shown in Fig. 4.

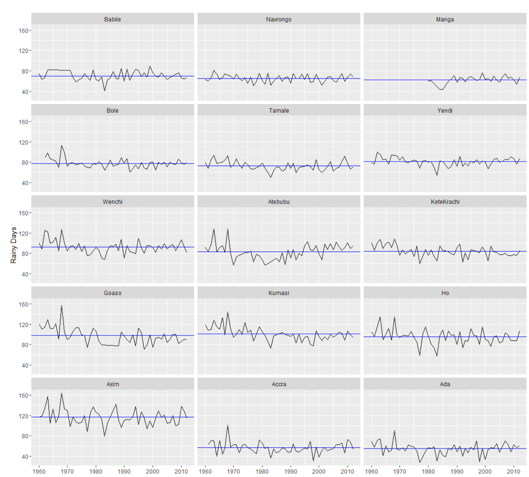

Fig. 4. Transect of annual rainy day totals

The number of rainy days experienced annually in the transect indicates an increase south-westwards from the north eastern direction of the country. There is also increased variability in the same direction. Again, the annual rainy day totals do not indicate trends at the individual stations. Ada and Accra continue to be the stations with low averages of annual rainy day totals. Considering the phenomenon exhibited by the annual rainy day totals and the annual amount of rainfall totals, it could be seen along the transect that the higher the rainy days the higher the rainfall amounts.

Transect of Models
In this section of the transect analyses, comparison are mainly done on the models of the chance of rain and the mean rain amount per day. The northern part of Ghana is observed to have a different rainfall pattern from that of the southern part.

Chance of rain.
The probability of rain on a day within the year is the first part of analysis across the transect. Curves were fitted for the chance of rain in all stations within the tiers of the transect as shown in Fig.5 .

Table 3 consists of the Markov order, the model type with harmonic orders, the number of parameters in the model, the residual vs. null devaince and the respective degrees of freedom for the residual vs. null deviance, the Akaike information criterion (AIC) values and the Bayesian information criterion (BIC) values respectively.

TABLE 3

Models comparison (Chance of rain) - Babile

Markov	*Type*	*Par*	*Deviance*	*D.F*	*AIC*	*BIC*
0	with four harmonics	9	13040/16449	16790/16798	13058	13127
0	with three harmonics	7	13041/16449	16792/16798	13055	13109
0	with two harmonics	5	13044/16449	16794/16798	13054	13093
1	Parallel (2 harmonics)	6	13024/16442	16783/16788	13036	13082
1	*Interacting (2 harmonics)*	*10*	*12935/16442*	*16779/16788*	*12955*	*13033*
2	Parallel (2 harmonics)	8	13015/16432	16771/16778	13031	13092
2	Interacting (2 harmonics)	20	12911/16432	16759/16778	12951	13106

For each station just as it is with Babile station (shown in Table 3), was compared to zero, first and second order Markov chains. We also compared seasonal pattern using two, three and four harmonics. Finally, it was considered if parallel curves would be adequate for the first or second order chains, or whether a more complicated model was needed. Table 3 shows that the interacting models of the first order and the second order expresses a better deviance than the other models considered. However, it was observed that there doesn't seem to be a need for the second order, since it expresses only a little less of the AIC value than the first order. This could mean the presence of over-fitting with the second order and also an increase in model complexity. Hence, using the BIC proves that the first order interacting Markov model with two harmonics suffices for Babile in terms of the chance of rain as shown in Fig. 5.

Following this argument, models for the other stations were obtained as shown in the Appendix.

Fig. 5 shows that a first order Markov interacting model sufficed for all individual stations across the transect. Fitted curves for stations in the first tier up north of the country evidently shows to be unimodal with the probability of rain being explained by two curves. The turquoise colour representing the probability of rain given rain whilst the red colour represents the probability of rain given dry. All stations within this tier expressed an overall parameter number of 10, each in explaining the probability of rain with two harmonic orders. It was observed that the first tier shows some form of steepness in the unimodality of the fitted curves as one moves from from Babile to Manga.

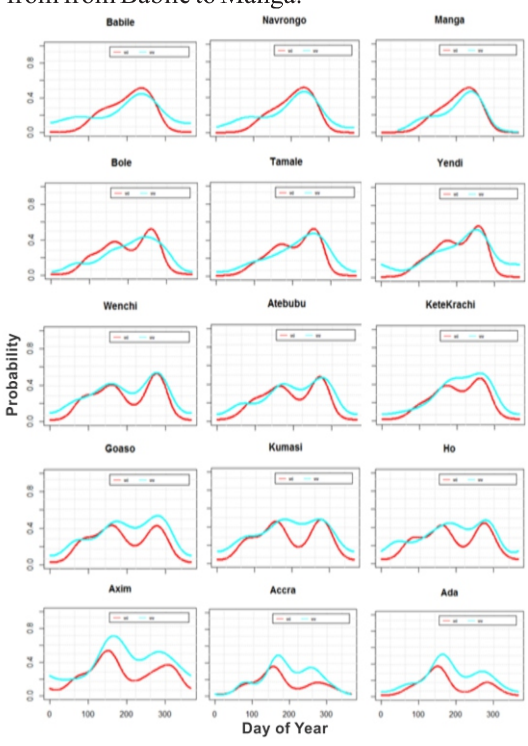

Fig.5. Transect of probability of rain models

Just into the season (in April), Fig. 5 shows an intersection of the different probabilities of rain at about 0.2 probability. This implies that both the probability of rain given rain and that of rain given dry has about a one in five days chance of occurring. This is extended by some days but still occurs in April as one looks from Babile to Manga. Immediately after this intersection the probability of rain given dry day takes a steady rise over the probability of rain given a rainy day till it reaches a peak of one in two days chance of rain occurrence around mid-August when the ITCZ is high up in the north. Both curves for the probability of rain then falls till they intersect again in September to about a probability of 0.3 and then finally the probability of rain given a rainy gets a bit higher than that of rain given a dry day moving towards the end of the season.

The second tier in the above plot also shows fitted curves for the probability of rain given a rainy day and that of rain given a dry day. Each station in this tier constituted curves with 18 parameters which included four harmonic orders. Generally across the tier it can be seen that the curve for the probability of rain given a dry day (red) is higher than the probability of rain given a rainy day (turquoise) in the 2nd and 3rd quarters of the year. The probability of rain given a dry day peaks in September with a two in three days chance of occurrence specifically at Yendi whilst both Tamale and Bole express a one in two days chance of occurrence at their peak. Unlike the former tier where both probability curves had a similar shape, in this tier there is an overall unimodality expressed by the probability of rain given a rainy day curve (turquoise) whereas the probability of rain given a dry day indicates an element of bimodality.

In the third tier the curves are made up of 18 parameters per station. They follow a different pattern as compared to the curves from the stations of the previous tiers. Here, there are less intersections between the probability models with the probability of rain given a rainy day dominating throughout the season. There is a more visible form of bimodality expressed by the probability of rain given a dry day created around the mid-July to August period and this could be the effect of the ITCZ being further away from this tier zone. This bimodality is more visible at Wenchi with an indication of bimodality in the probability of rain given a rainy day but fades out gradually as one moves towards the eastern direction with probability of rain given a rainy day turning almost unimodal at Kete-krachi. The highest peak of rainfall in this tier is expressed by the probability of rain given a rainy day with a one in two days chance of occurrence in September. Atebubu, however, shows an intersection between curves at its peak and, therefore, the probability of it raining given it was a dry or wet on the day before is equivalent to a simple toss of a coin.

Tier four also consists stations with interacting first order Markov chain which has four harmonics (18-parameter curves). Again in this tier, the probability of rain given a rainy day dominates the whole season. There is a pronounced bimodality expressed by the probability of rain given a dry day and this is below the unimodality expressed by the probability of rain given a rainy day curve. The peak of the first part of the season for the probability of rain given a dry day is reached at 0.4 chance during June and intersects with the gradual rising unimodality part expressed by the probability of rain given a rainy day across all stations in this tier. The probability of rain given a rainy day reaches its peak in September is about one in two days chance. This phenomenon is much pronounced in Goaso than in the other stations within this tier.

Clear bimodality is finally shown among the two fitted curves at the south-most part of the transect in the fifth tier. As in the three previous tiers an 18-parameter model is sufficient to explain the chance of rain. It can be seen that there is an early start (early April even in March for Axim) in the season. This implies that at the start of the season the probability of having rain after a dry day or wet day is the same. After this period which ends in April for Axim, and in early May for Ada and Accra, there is a steep rise in the probability of rain given a rainy day over that of rain given a dry day. This rise hits a peak of about 0.7 probability at Axim and a probability of a one in two days chance at both Ada and Accra in June. It also establishes the fact that the peak of the rainy days in this tier is in June. After this peak, both curves fall gradually till August (when the ITCZ is in the north) before rising to a peak in mid-September to early October and finally drops down towards the end of the year.

Mean amount of rain

The models for the mean amount of rain on a rainy day is explored across the transect in this section.

A first order Markov model is fitted to each station in the transect as shown in Figure.6.

TABLE 4

Shape parameter for mean amount of rain for rainy days

Tier (Av. latitude)	*Station*	*Shape (k)*	*Std error*	*Coefficient of variation (C.V) %*
First (10.8)	Manga	1.081	0.030	96.18
	Navrongo	1.069	0.023	96.71
	Babile	1.081	0.024	96.18
Second (9.2)	Yendi	1.021	0.019	98.97
	Tamale	1.033	0.021	98.39
	Bole	1.039	0.021	98.11
Third (7.8)	Kete-Krachi	0.903	0.017	105.23
	Atebubu	1.108	0.021	95
	Wenchi	1.049	0.019	97.64
Fourth (6.6)	Goaso	1.058	0.019	97.22
	Kumasi	0.999	0.017	100.01
	Ho	0.978	0.017	101.12
Fifth (5.2)	Ada	0.899	0.021	105.47
	Accra	0.887	0.020	106.17
	Axim	0.841	0.013	109.04

Table 4 estimates the shape parameter of the gamma distribution of the fitted models across the transect with their corresponding coefficient of variation. Table contains the different models looked at in choosing the appropriate model for the mean amount of rain on a rainy day in Babile.

TABLE 5

Models Comparison (Mean Amount of Rain) - Babile

Markov	*Type*	*Par*	*Deviance*	*D.F*	*AIC*	*BIC*
0	with 4 harmonics	9	3440/3493	3222/3230	23966	24027
0	with 3 harmonics	7	3445/3493	3224/3230	23967	24016
0	with 2 harmonics	5	3460/3493	3226/3230	23979	24016
1	*Parallel(3 harmonics)*	*8*	*3419/3493*	*3222/3229*	*23933*	*23988*
1	Interacting(3 harmonics)	14	3414/3493	3216/3229	23939	24030

The Babile station model in terms of the amount of rain with reference to Table 5 shows that a parallel first order Markov model with three harmonics is the most suitable model. It shows enough deviance explanation with correspon-ding lowest AIC and BIC values among the other models considered. The interacting model of the first order is seen to just display over-fitting with higher AIC and BIC values. The chosen model (parallel first order Markov with three harmonics) which has eight parameters is shown in Fig. 6 Following a similar argument, models for the other stations are obtained as shown in the Appendix.

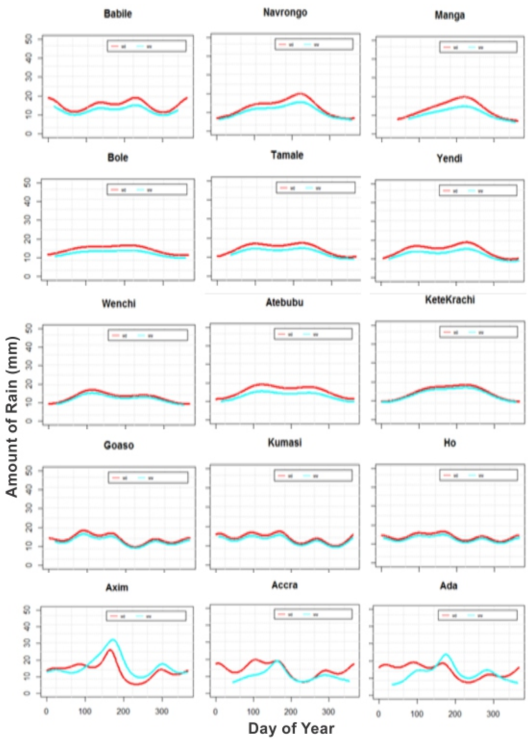

Fig. 6. Transect of mean rain amount models

Other than Babile, which has fitted parallel curves of the first order with three harmonics, both Navrongo and Manga in the first tier exhibit parallel curves with two harmonics. The model for Babile constitutes eight parameters whilst those of Navrongo and Manga is made up of six-parameter model. The mean amount of rain on a rainy day after a dry day (red) is a little higher than that of a rainy day following a rainy day (turquoise). In this tier the mean rain amount expressed by a rainy day following a dry day hits a peak of about 20 mm in August. This is the period when the probability of rain given a dry day is also highest while that of the rainy day following a rainy day hits its peak with about 15 mm of rain. An average estimate of k in Table 4 for this tier is from 1.077, which indicates a high level of variability. Table 4 shows that all the stations here have a value of the shape parameter, k that is close to one. The gamma model with k = 1 is known as the simple exponential distribution.

For stations in tier two, parallel curves of the first Markov order with a two-harmonic order are fitted. With these curves, there is a general constant mean rain amount of about 17.5 mm per rainy day for a rainy day which follows a dry day while it is about 15 mm for a rainy day that follows a rainy day. Higher variability is generally still emphasized in this tier with an estimated k of the amounts distribution ranging between 1.021 to 1.039.

The third tier also shows six parameter models for the curves fitted in each station, with the mean amount of rain following a dry day's curve just higher than that following a rainy day. There is roughly a constant mean rainfall amount for a major part of the season at about 20 mm, declining towards the end of the season in September. In this tier, there appears to be a slightly reduced variability as compared to the previous tiers in Table 4.

Except for Goaso, tier four expresses higher variability than previous tiers in the transect of the mean amount of rainfall models. Models in tier four constitute a parallel first order Markov but with four harmonics unlike the previous tiers where two harmonics proved sufficient for their fitting, therefore, 10 parameters were used in fitting models in this tier. The curves lie close to each other with the mean rain amount following a dry day still just a bit higher than that following a rainy day. The curves generally show an almost constant mean rain amount of about 17.5 mm for the first part of the rainy season which then varies slightly towards the end of the year.

An interacting first order Markov model with four harmonics is needed for the stations in the fifth tier. These models have 18 parameters per station. Table 4 shows that the highest variability of rainfall amount on a day in the country is expressed in this tier with estimates, $k < 1$. It is also only in this tier that bimodality in the rain amounts is pronounced, especially at Axim. Axim has its first peak in the first part of rainy season with a mean of just over 30 mm of rain for rain given a rainy day and about 25 mm for rain given a dry day. The second peak is in late September to early October at about 20 mm and

15 mm respectively for the two curves. This shows why Axim has the highest annual amount of rainfall. Accra and Ada both show peaks of less than 25 mm of rain in the first season and less than 16.5 mm in the second season.

A general view of the transect mean rain amount models expresses the subtle unimodal change from the north eastern direction (Manga) to a clear bimodality in the south western direction (Axim) of the country. It also shows relatively constant models of mean rainfall amounts in the middle belt of the transect. The change from a parallel model with two harmonics to an interacting model with four harmonics (last tier) is also observed in the transect. The parameter estimates of the various models are seen in the Appendix. The study shows to an extent that the higher the probability of rain (annual rainy day totals), the higher the mean rain amounts (annual rainfall amount totals).

Discussion and conclusion

Using 15 rainfall stations across the country, the paper has described the rainfall phenomenon in Ghana. It has established the variable nature of rainfall and emphasized the need for effective techniques for modelling rainfall. The analyses of the stations within the transect has provided detailed description of the rainfall patterns within individual tiers and also that of the entire transect. The influence of the location of stations within the transect region on the rainfall patterns has also been discussed. The use of both the direct and the modelling approaches of modelling rainfall as shown by (*Stern et al.,* 1982a) and (Stern *et al.,* 1982b), respectively, used in this paper has proved adequate. The direct approach showed simple methods of extracting information such as the annual rainfall total and annual rainy day total from available rainfall data. Other aspects such as the start of the rains, seasonal rainfall totals and the length of the season can also be considered using this approach. The modelling approach which separates the chance of rain from the mean amount of rain on a rainy, has allowed clear descriptions and comparisons of the various regions in this study. These two methods together are very much complemen-ting, with the direct method being simple but less detailed whereas the modelling approach is quite complex but more detailed.

The time series plots showed no visible trend with the annual rainfall totals and the annual rainy day totals of the stations within the transect region. This emphasized the absence of evidence for rainfall change within the period of study for the transect. It, however, expresses the high variability of rainfall within the region. This finding is consistent with findings from studies in Uganda (Osbahr *et al.*, 2011) and southern Zambia (Stern & Cooper, 2011). Together with these other studies it is evident that climate change has to date not affected rainfall as many perceive, however, the high variability of rainfall cannot be overlooked. Separating the chance of rain from the amount of rain as it is in the modelling approach has clearly permitted comparisons between climates of the different locations within the transect. The results of the modelling approach analyses showed that the first order Markov process is sufficient in modelling rainfall within Ghana. For the topmost tier of the transect, a first order Markov process with two harmonics is found sufficient for the chance of rain whilst the other stations within the region exhibit a first order Markov with four harmonics. Modelling the amount of rain on a rainy day showed that a first order interacting model was sufficient for the last tier in the transect while all the other stations expressed first order parallel models.

The study has shown that in terms of the chance of rain within the year, stations in the north of the transect exhibit unimodal patterns whilst those in the south exhibit bimodal patterns due to the movement of the ITCZ. The results of having a relative peak in August to September in the northern part of the transect and peaks of June and October in the south is also seen. The higher chance of getting a rainy day given a previous dry day in the north and a greater chance of getting a rainy day after a rainy day in the south during the season is noted. The relative constant models exhibited by the models for the mean amount of rain on a rainy day with high variability expressed by the gamma distribution parameter, k ranging from 0.841 to 1.108 are also noted. These results are consistent with a study by (Garbutt *et al*., 1981) in West Africa.

The Markov chain modelling used in this paper has shown that further investigations could be done to improve the models generated. As mentioned earlier, the models produced have all years considered together, however, it has been shown that the modelling approach can be used with shorter records.

This permits for example, a split of the years, depending on the ENSO condition (El Nino, Normal, La Nina) and the difference in the pattern of rainfall in these "types" of years can then be quantified. This follows the approach used by (Stern & Cooper, 2011). Furthermore, a look at models on the tier (latitude) level shows that models look similar and are quite consistent, hence, an approach to attain a combined model with additional stations within the tier regions could be an interesting study. Also a keener look at figures show that the fifth tier is the only part of the transect with a pronounced east to west difference. Again, this warrants a more detailed study. Fortunately, there are more stations with data available at GMet on this tier level, hence, this is feasible to explore in a further study.

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Appendix

TABLE A1

Chance of Rain Parameter Estimates

Station	Markov Order	dc_1	ds_1	dc_2	ds_2	dc_3	ds_3	dc_4	ds_4	Int.
Manga	wd	-2.95292	-0.97145	-0.83213	-0.03356					-2.7361
	ww	1.09551	0.43694	0.03469	0.29657					0.56336
Navrongo	wd	-2.80926	-0.80611	-0.84299	0.04671					-2.5795
	ww	1.91528	0.42721	0.51004	0.37855					1.09595
Babile	wd	-2.52826	-0.73583	-0.84865	-0.01609					-2.2807
	ww	1.97372	0.29175	0.62549	0.36933					0.97696
Yendi	wd	-2.21668	-0.58049	-0.84280	0.08348	-0.00661	0.25998	0.19410	-0.06920	-1.9579
	ww	1.44214	-0.17344	0.73342	-0.07823	0.23237	0.23237	-0.12759	0.06215	0.75233
Tamale	wd	-2.15969	-0.53100	-0.82396	0.11630	0.01861	0.24796	0.15393	-0.10399	-2.1094
	ww	1.18565	-0.17087	0.37508	-0.06579	-0.08092	-0.21768	-0.14566	0.08945	0.66341
Bole	wd	-1.87406	-0.44406	-0.87032	-0.09576	-0.09576	0.18160	0.17684	-0.14962	-1.8523
	ww	0.89766	-0.18181	0.47285	0.18845	-0.18483	-0.08246	-0.28565	0.11257	0.40739
Kete-Krachi	wd	-1.66270	-0.51356	-0.63833	-0.09087	-0.09632	0.16920	0.13462	-0.06436	-1.7486
	ww	0.59878	-0.24413	0.40183	0.12764	-0.01910	-0.01328	-0.09875	0.08858	0.57197
Atebubu	wd	-1.42938	-0.26904	-0.81058	-0.14473	-0.20583	-0.20583	0.14587	-0.04052	-1.6260
	ww	0.66324	-0.18299	0.47222	0.11952	-0.06168	-0.08019	-0.14584	0.02604	0.51153
Wenchi	wd	-1.22932	-0.24855	-0.95762	-0.36655	-0.39036	0.26886	0.10887	-0.07524	-1.477
	ww	0.53938	-0.07762	0.48103	0.20629	0.19684	-0.02181	-0.09236	0.07746	0.55366
Goaso	wd	-1.12270	-0.11742	-0.73351	-0.34565	-0.36221	0.16961	0.05389	-0.09318	-1.3805
	ww	0.39323	-0.30278	0.36428	0.22815	0.06350	-0.04053	-0.06828	0.06181	0.60655
Kumasi	wd	-0.90166	-0.14019	-0.68994	-0.38815	-0.35320	0.27516	0.05344	-0.12901	-1.2527
	ww	0.20594	-0.16760	0.42840	0.35361	0.14240	-0.19499	-0.04015	0.11491	0.51958
Ho	wd	-0.89429	-0.096449	-0.633692	-0.266573	-0.31006	0.243264	0.119335	-0.11703	-1.3023
	ww	0.278398	-0.177291	0.429443	0.429443	0.111857	0.003071	-0.11361	0.225510	0.47138
Ada	wd	-1.00223	0.227067	0.227067	-0.690910	-0.69091	0.266497	0.030045	-0.11799	-2.2054
	ww	-0.00881	-0.393804	0.392498	0.425499	0.036356	-0.028822	-0.01364	0.001164	-0.118
Accra	wd	-0.92039	0.10955	-0.36935	-0.51922	-0.28053	0.13182	0.01348	-0.21085	-2.0436
	ww	-0.48608	-0.48746	-0.01117	0.32689	-0.09524	-0.08983	0.03736	-0.06691	0.40240
Axim	wd	-0.59576	-0.01416	-0.27988	-0.67897	-0.23868	0.08404	-0.09325	-0.18932	-1.1013
	ww	-0.16593	-0.38549	0.43802	0.24067	0.08092	0.11541	0.11863	0.13112	0.70096

TABLE A2
Amount of Rain Parameter Estimates

Station	Markov Order	dc_1	ds_1	dc_2	ds_2	dc_3	ds_3	dc_4	ds_4	Int.
Manga	wd	0.042896	0.009606	0.008880	-0.00247					0.090640
	ww									0.017547
Navrongo	wd	0.043537	-0.00411	0.012912	-0.01138					0.089995
	ww									0.014324
Babile	wd	0.005343	0.001295	-0.00970	-0.00323	-0.01101	-0.0002			0.068024
	ww									0.013579
Yendi	wd	0.018946	-0.00424	0.007950	-0.00824					0.071784
	ww									0.012268
Tamale	wd	0.017567	-0.00631	0.006803	-0.00645					0.072082
	ww									0.010727
Bole	wd	0.012606	-0.00331	0.003241	-0.00333					0.070693
	ww									0.011654
Kete-Krachi	wd	0.024061	0.002241	0.007045	-0.00026					0.073325
	ww									0.004321
Atebubu	wd	0.015271	-0.00107	0.007751	0.001234					0.065693
	ww									0.012360
Wenchi	wd	0.017368	-0.00598	0.010987	0.000550					0.079561
	ww									0.006569
Goaso	wd	-0.00067	-0.01290	0.002805	0.007582	0.001375	-0.0041	-0.00788	0.002740	0.073831 9
	ww									0.006308
Kumasi	wd	0.003986	-0.01314	-0.00210	-0.00110	-0.00302	-0.0060	-0.00902	0.001096	0.073579
	ww									0.006757
Ho	wd	0.004312	-0.00723	-0.00046	0.003142	-0.00266	-0.0033	-0.00615	0.001891	0.073686
	ww									0.006602
Ada	wd	-0.00694	-0.02713	-0.00437	0.009543	0.007087	-0.0084	-0.01027	-0.00212	0.076945
	ww	0.045625	0.031807	0.014274	0.011427	0.002471	0.01105	-0.00042	0.002673	0.016185
Accra	wd	-0.00854	-0.02500	-0.00424	0.021662	0.004665	-0.0094	-0.01341	-0.00353	0.079577
	ww	0.036652	0.018880	0.010719	0.007571	0.007870	0.00458	0.006012	0.008574	0.032493
Axim	wd	-0.01190	-0.03471	-0.00630	0.026952	0.020309	-0.0149	-0.01307	-0.00269	0.084230
	ww	0.023948	0.025421	-0.00262	-0.01131	-0.01016	0.00923	0.010453	-0.00439	-0.01693

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