JUMP-DIFFUSION MODEL FOR CRUDE OIL SPOT PRICE PROCESS: PARAMETER ESTIMATION FOR PREDICTING THE MARKET

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Abstract

The jump-diffusion model incorporates a jump component to a diffusion process. It has been very popular in modelling asset prices, foreign exchange and other securities whose dynamics are non-Gaussian. Crude oil spot price is also modelled as a jump-diffusion process. Several methods such as characteristic function CF, maximum likelihood estimation (MLE), method of cummulants, etc. have been used for parameter estimation of the jump-diffusion model. These approaches are inefficient, and estimating parameters of the jump-diffusion model has been challenging. The inability to effectively estimate parameters of the jump-diffusion model makes market prediction and price forecasting challenging. Generation of viable mathematical tool for expressing market perceptions is difficult. The challenges were surmounted, by using the Yuima package in R, for estimating, parameters of the jump-diffusion model. Data for four crude types in the Niger-Delta over the period January 2005 - December 2009 was used. The Yuima package yields accurate estimates, which can help to interpret market behaviour, and possibly make price forecasting and market prediction less challenging.

Introduction

Crude oil is a consumption and investment commodity, with great economic impact on the global market. In the past decades attempts have been made to understand, analyze, track and forecast its price. However, crude oil price fluctuates rapidly. Due to the presence of skewness and kurtosis in the empirical distribution of oil price returns, crude oil price cannot be modeled as a Gaussian process.

Following the shortcomings of the Black-Schole's model and with the realisation that returns on financial assets are not Gaussian, Levy models of the Jump-diffusion type have been used by Jorion (1988) to model asset returns, and by Ball & Torous (1983) to model foreign exchange and other securities. Such Levy models, include models based on the Poisson distribution used by Gencer & Unal (2012) which has slower tail decay than the Gaussian distribution. Using Levy models, higher probabilities were assigned to sudden unexpected events and more realistic models are, therefore, generated.

Again, using Brownian motion in modelling risky asset prices relies on the use of the Gaussian distribution which underestimates the probabilities of sudden unexpected events. Assumption of normality is a poor approximation in the real world because returns have some significant features such as jumps, sem-heavy tails and asymmetry. Also crude oil prices exhibit significant volatility over time. Going by the work of Anatoliy Swishchuk & Kaijie Cui (2013), the distribution of returns on crude oil price shows fat tails, and skewness and barely follow a normal distribution. Attributed to Merton (1976), jump-diffusion models incorporate jumps into a diffusion process through a Poisson process with parameter, l . The nature of the process makes estimation of parameters challenging especially for practical application of the model.

In this paper, the jump-diffusion model was used to describe the dynamics of the crude oil spot price process and the model parameters estimated. The usual methods of parameter estimation such as maximum likelihood estimation, MLE, method of cummulants and characteristic function technique are inefficient; using them have led to incomplete results. Sometimes some parameters are fixed to obtain others, at other times some parameters are assumed to be zero to estimate others. The problem is that parameters obtained this way may not give a good fit for the model, making market analysis inaccurate. It is purposed to estimate the parameters by a more reliable approach obtain more accurate results. Challenges of parameter estimation were encountered by Jorion (1988), Krichene (2006), Ball & Torous (1988), and are discussed in this paper.

Researchers such as Ball & Torous (1983), Jorion (1988) and Krichene (2006) have used the Jump-Diffusion model on some price returns and met with challenges in estimating model parameters. While researchers like Ball & Torous (1985), used(0,1) as (b,d), the parameters of jump size J_t , others like Jorion (1988) estimated (b,d), for their data using maximum likelihood estimation, MLE. Ball & Torous (1985) estimated the parameter,#, using MLE and method of cummulants. They obtained values as low as 0.031 for Dow stock and as high as 22.866 for Zenith stock. However, they restricted some parameters to zero, and others to some constant values, in order to obtain values for some other parameters. These challenges were surmounted by using Yuima for the parameter estimation in this work.

Experimental

The jump - diffusion model for crude oil spot price

A Levy process is defined in order to discuss jump-diffusion model.

Definition

A continuous - time stochastic process $\{X_t : t > 0\}$, defined on a probability space (Ω, F, P) is said to be a Levy process if it possesses the following properties.

(i) P{X0 = 0} = 1.

(ii) Stationary increments: the distribution $X_{t+s} - X_t$ over the interval [t, t+s] does not depend on *t* but on length of interval *s*.

(iii) Independent increments: for every increasing sequence of times $t_0,\ldots,t_n$, the random variables $X_{t_0}, X_{t_1} - X_{t_0}, \ldots, X_{t_n} - X_{t_{n+1}}$ are independent.

(iv) Stochastic continuity: $\forall \varepsilon > 0, \quad Lim_{h\to 0} P(|X_{t+h} - X_t| \geq \varepsilon)$ i.e. discontinuity occurs at random times.

(v) The paths of X_t are P almost surely right continuous with left limits (Cadlag paths).

The Levy model used here is a variant of the Merton's model which has a diffusion component and a jump component. With addition of jumps, a better representation of the real market situation is obtained. By estimating the parameters of the model, empirical data using the model can be computed. Thus a mathematical tool for expressing market perception is obtained and forecasting can be done. Basic features of crude oil price series (like the financial asset price series) include: skewness, excess kurtosis, frequent small and large jumps. The presence of these features have been established in the literature by works of Krichene (2006, 2008). Merton (1976), also recognised the presence of jumps in asset prices. Crude oil has in recent works by Krichene (2005) been studied as a consumption commodity, as well as an asset.

Theorem: *Levy-Ito decomposition* Theorem

If *X* is a Levy process, then there exists $\in R^d, \sigma \in \mathbb{R}$ a Brownian motion W and an independent Poisson random measure *N* on $R^+ \times (R^d - \{0\})$, such that, for each $t \geq 0$

$$X(t) = \mu t + \sigma W(t) + \int_{|x|<1}^{\square} x\tilde{N}(t,dx) + \int_{|x|\geq 1}^{\square} xN(t,dx)$$

$\int_{|x|\geq 1}^{\square} xN(t,dx)$ is the sum of all jumps (finite many) of size bigger than one.

The $\int_{|x|<1}^{\square} x\tilde{N}(t,dx)$ process is the compensated sum of small jumps (of size small than 1

For any given constants R > 0 small jumps and big jumps can be defined as $|x| < R$ and $|x| \geq R$ respectively, such that the corresponding Levy-Ito decomposition is given as

$$X(t) = \mu t + \sigma W(t) + \int_{|x|<R} x\tilde{N}(t,dx) + \int_{|x|\geq R} xN(t,dx)$$

where $\tilde{N}(t,dx) = N(dt,dx) - v(dx)$ is the compensated Poisson random measure, $v(dx)$ is a Levy measure of the Levy process. mand s are drift and volatility of the process respectively.

Tankov & Cont (2004) interpreted the Levy-Ito decomposition theorem, highlighting its important implication, which is that every Levy process is made up of a Brownian motion with drift and possibly an infinite sum of independent compound Poisson processes. The following useful definitions are made:

Definition: *A probability space* consists of the triple $(\Omega, \mathcal{F}, \mathbb{P})$ on the sample space $\mathcal{F}$, where $(\Omega, \mathcal{F})$ is a measurable space, $\mathcal{F}$ is a collection of subsets of Ω, and P is a measure on F For a given set of returns, the rate at which the price of an asset varies (increases or decreases) is called volatility denoted s. Basically, it is the variation from average over a given period.

The *drift* of an asset, denoted μ, is a measure of the average rate of growth of the asset price.

Let J_{t1}, $i = 1,2,...$ be a sequence of independent and identically distributed random variables taking values in $\mathbb{R}$ with common distribution f, (J_t are jump sizes). Let N_t be a Poisson process of intensity λ, independent of all J_{ti}.

The compound Poisson process $J=\{J_t, t \geq 0\}$ is defined by

$$J_t = \sum_{i=1}^{N_t} J_{t_i},$$

The *intensity* λ, of a counting process such as the compound Poisson process is a measure of the rate of change of its predictable path.

Let $(\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t))$ be a filtered probability space. Let $N = (N_t)_{t\geq 0}$ be a counting process, then the process $\lambda=(\lambda_t)_{t\geq 0}$ is called the intensity of the counting process if

$$\int_0^t \lambda_s\, ds < \infty \quad \forall\, t = 0 \text{ and } E\left[\int_0^\infty C_s\, dN_s\right] = E\left[\int_0^\infty C_s \lambda\, ds\right]$$

for all non-negative $(F_t)_{t\geq 0}$ predictable processes $C=(C_t)_{t\geq 0}$

Generally a stochastic differential equation, SDE, with Poissonian jumps for an asset price S_t, is of the form:

$$\frac{dS_t}{S_{t-}} = \mu(t, S_t)dt + \sigma(t, S_t)dW_t + c(t^-, S_{t^-})dN_t, \qquad S(0) = S_0$$

$W = \{W_t, t \geq 0\}$ is a standard Brownian motion, $N = \{N_t, t \geq 0\}$ is a Poisson process with intensity $\lambda > 0$, $t \geq 0$, with $X_0 > 0, c(.,.)$ is the jump coefficient which determines the jump size at an event, i.e. is an impulse function which determines the jump from S_t- to S_t. μ is linear drift and σ is volatility.

In the model for crude oil price, events driven by uncertainty, such as arrival of abnormal information, are incorporated through a Poisson process with intensity λ. The model describes the movement of the crude oil price, by accounting for the jumps which arise from time to time. Thus, event driven uncertainty is expressed by jumps.

Crude oil price process has jumps and is shown by Ogbogbo (2016) to be a Levy process. Let S_t be the crude oil price at time t, S_t is a set of random variables on the probability space, $(\Omega, \mathcal{F}, \mathbb{P})$, Then in line with the work of Ogbogbo for the model of the crude oil spot price, we assume the following jump-diffusion process for the price of crude oil at time t.

$$S_t = S_0 \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t + \Sigma_{t=1}^{N_t} J_t\right\} \quad .. [1]$$

The SDE satisfied by this process, can be obtained using Ito's lemma. S_0 is initial price of crude oil. Standard Brownian motion is associated with the continuous component of the process and distributed as $W_t \sim N(0,1)$. μ is the instantaneous expected return (drift) and σ the volatility of the process. λ is the mean number of arrival of abnormal information; mean number of jumps occurring per unit time.(intensity of the counting process). Jumps of the process are normally distributed with parameters (β,δ) i.e. $J_t \sim N(\beta,\delta)$. β is the mean of jump size J_t and δ is the variance of the jumps.

Crude oil price data and parameter estimation

The empirical data used is on monthly price (in USD) of crude oil for four crude types (fields) in the Niger Delta, over a period of time, January 2005 - December 2009. This price data though specific to the Niger-Delta is in line with crude oil price data in the international oil market available on yahoo finance. The crude types are Bonny Light, (BL), Brass Blend, (BB), Pennington Light, (PL), and Antan (BL, BB, PL and ANTAN). The price data available are monthly data and not daily data as may be quoted on New York Merchantile Exchange, (NYMEX) and International Petroleum Exchange, (IPE). The prices, which are used to collect royalty from oil companies drilling on these fields, are considered as spot prices. The data being used here were provided by the Department of Petroleum Resources (DPR) Victoria Island, Lagos. Different fields produce different types of oil of varying quality, hence, they have different prices.

In order to ascertain viability of the model with respect to empirical data, parameters of the model need to be estimated. Also the size and sign of the parameters help in the analysis of the price trend and explaining the jumps. In this section, there were challenges met in the attempt to use some conventional methods of parameter estimation. By the maximum likelihood estimation and the method of cummulant, exact estimates for all the parameters could not be obtained.

Method of maximum likelihood and quasi maximum likelihood

Challenges faced in estimating the parameters using these methods are discussed here. The method of maximum likelihood chooses the set of values of the model parameters which maximizes the likelihood function. It specifies the joint density function for all n observations $x_1, x_2, \ldots, x_n$, the actual value of the parameter vector is not known. The procedure is aimed at finding an estimator θ which would be as close as possible to the actual value θ_0. The joint density function is given as

$$f(x_1, x_2, \ldots, x_n|\theta) = f(x_1|\theta) \times f(x_2|\theta) \times \ldots \times f(x_n|\theta)$$

The emergent likelihood function is denoted $L(\theta; x_1, x_2, \ldots, x_n)$ and is given as

$$L(\theta; x_1, x_2, \ldots, x_n) = f(x_1, x_2, \ldots, x_n|\theta) = \prod_{i=1}^{n} f(x_i|\theta)$$

The log of the likelihood function, is more convenient to work with, and it is called the log-likelihood

$$\text{In}\, L(\theta; x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} f(x_i|\theta)$$

The likelihood or the log-likelihood function yields the same MLE estimate since log is a strictly monotonically increasing function. Finding the maximum of a function often involves taking the derivative of a function and solving for the parameter being maximized, however, for some complicated problems, difficulties occur; in that maximum-likelihood estimators are not suitable or are non-existent. Other methods such as quasi maximum likelihood estimation, QMLE may be used.

Generally, from equation [1] the parameter vector for the jump model is $\varphi=(\mu, \sigma^2, \lambda, \beta, \delta)$ representing drift, volatility, intensity of counting process, mean and variance of jumps respectively.

The log likelihood function for the jump-diffusion process is given by Ball & Torous (1983) and Krichene (2006) as:

$$L(\varphi, x) = -T\lambda - \frac{T}{2}\ln(2\pi) + \sum_{t=1}^{T} \ln\left[\sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \frac{1}{\sqrt{\sigma^2 + n\delta^2}} \exp\left(\frac{-(x_t - \mu - n\beta)^2}{2(\sigma^2 + n\delta^2)}\right)\right] \ldots [2]$$

In order to numerically optimise the likelihood function, given in [2], the infinite sum is truncated after some value of *N. Jorion* (1988) truncated at N=10 for accuracy while Ball & Torous (1983), truncated at N=15. The upper bond on the summation is replaced by a finite number N. The likelihood function $L_1(\varphi, x)$ is differentiated with respect to each of the parameters on the parameter vector φ, and equated to zero. We differentiate $L_1(\varphi, x)$ in [3] with respect to $\mu, \lambda, \sigma, \beta, \delta$ respectively.

We equated to zero and solved for μ, λ, σ, β, δ respectively. This did not yield explicit expressions for all the parameters.

Method of Cummulant

Press (1967), Beckers (1981), Ball & Torous (1983), used method of cummulants to estimate jump-diffusion parameters. The following discussion on method of cummulants follows from Krichene (2006).

Cumulants of a process X_t denoted k_n, $n = 0,1,2,\ldots$ are obtained from log of the characteristic function of the process. The characteristic function, $\phi_x(u)$ of X_t is given as

$$E\left(e^{iuX_t}\right) = E(\exp iu\,X_t) = \int \exp(iuX_t)\, f(X_t)\, dX_t$$

$f(X_t)$ is the probability density function, pdf, of X_t and u is the transform variable, $i = \sqrt{-1}$. The cumulants k_n of a process X_t are the coefficient in the power series expansion of the logarithm of the characteristics function,CF of X_t and is expressed as

$$\ln\phi(u) = \sum_{n=1}^{\infty} \kappa_n \frac{(iu)^n}{n!} = 1 + \kappa_1\left(\frac{iu}{1!}\right) + \kappa_2\left(\frac{(iu)^2}{2!}\right) + \ldots + \kappa_n \frac{(iu)^n}{n!} +$$

Krichene (2006), gives the characteristic function, for the Jump-Diffusion process as

$$\phi_x(u) = \exp\left[-\frac{\sigma^2 u^2}{2} + iu\mu + \lambda\left(\exp\left(i\beta u - \frac{u^2\delta^2}{2}\right) - 1\right)\right]$$

The first four cummulants of the Jump-Diffusion process are given as follows

$$\kappa_1 = \mu + \lambda\beta, \kappa_2 = \sigma^2 + \lambda\delta^2 + \lambda\beta^2, \kappa_3 = \lambda\beta(3\delta^2 + \beta^2), \kappa_4 = \lambda(3\delta^4 + 6\beta^2\delta^2 + \beta^4)$$

Press (1967), Beckers (1981), Ball & Torous (1983), imposed some restrictions on some of the parameters in order to derive some relations. Press (1967)] put $\mu = 0$, and obtained some relations, but his estimates were wrong-signed and not plausible. Ball & Torous (1983) maintained Becker's restriction, i.e. $\beta = 0$, but used Bernoulli jump process. They derived the same cumulant equations and obtained the estimators as follows:

$$\hat{\mu} = \kappa_1$$

$$\hat{\lambda} = \left(1 \pm \sqrt{\frac{3\kappa^*}{(3\kappa^* + 100)}}\right)\Big/2$$

$$\text{where } \kappa^* = \left(\frac{\kappa_6}{\kappa_4}\right)^2$$

$$\hat{\delta}^2 = \frac{\kappa_6}{\left(\kappa_4\left(5\left(1 - 2\hat{\lambda}\right)\right)\right)}$$

$$\hat{\delta}^2 = \kappa_2 - \hat{\lambda}\hat{\delta}^2.$$

Ball & Torous's modification was applied to the empirical data set used in this work, anomalies were obtained in the parameter values.

TABLE 1

Parameter estimates of the Jump-Diffusion model for BL Crude type using Ball & Torous (1985) approach

Crude Type	Value of λ	Drift μ	Volatility σ	Intensity of CP (λ)	Volatility of jump process (δ)
BL	Negative	0	Nan (not a number)	8.214878e^{-08}	44.88171
	Positive	0	44.8817	0.999999	Nan (not a number)

The YUIMA package: Introduction and design of package

The R package YUIMA is a package from the YUIMA Project which is an open source and collaborative effort that developed a computational framework for simulation and inference of stochastic differential equations, SDE's. The YUIMA package is hosted on R-Forge (2013), and available through the Comprehensive R Archive Network (CRAN) (2014).

The constructor function "setModel" is used to give a mathematical description of the SDE. For example Setmodel gives a description of the SDE with or without jump of the following form:

$$dX_t = a(t, X_t, \theta)dt + b(t, X_t, \theta)dW_t + c(t, X_t, \gamma)dZ_t, \quad X_0 = x_0$$

The functions ( ) ,b( ) , and c ( ) may have a different number of arguments.

In the YUIMA package of Yoshida et al (2014), Quasi Maximum Likelihood Estimation procedure given by Ogihara and Yoshida (2011), is used in estimating parameters of the compound Poisson based SDE model. This is implemented using the "yuima.cp. qmle class" which is a class of the YUIMA package that extends the mle-class of stats4 package of the R-core team (2013).

Results

Description of Levy Process in YUIMA Package

Let Z_t be a compound Poisson process (i.e. jump sizes follow some distribution, like the Gaussian law, and jumps occur at Poisson times). Then, Yoshida et al (2014) in the Yuima -package considered the following SDE which involves jumps

$$dX_t = a(t, X_t, \theta)dt + b(t, X_t, \theta)dW_t + dZ_t,$$

more generally represented as

$$dX_t = a(t, X_t, \theta)dt + b(t, X_t, \theta)dW_t + \int_{|z|>1} c(X_{t-}, z)N(dt, dz) + \int_{0<|z|\le 1} c(X_{t-}, z)[\mu(dt, dz) - \nu(dz)dt], \quad X_0 = x_0 \quad \text{[4]}$$

N is a random measure of the jumps of X. The Levy measure *v* is any measure satisfying

$$v(\{0\}) = 0 \text{ and } \int (1 \wedge |z|^2)v(dz) < \infty$$

A compound Poisson process with intensity ?, with Gaussian jumps can be specified in setModel using the argument measure type = "CP" (for compound Poisson). Quasi Maximum Likelihood Method of Estimation procedure for diffusion model described in the section below is also used by Yoshida et al (2014) in the Yuima package for the jump type model defined in [4].

Estimation of parameter in the YUIMA package: Quasi Maximum Likelihood Estima-tion, QMLE

The estimation method used in the YUIMA package is the Quasi Maximum Likelihood Estimation. In the YUIMA package, a multidimensional diffusion process

$$dX_t = a(t, X_t, \theta_2)dt + b(X_t, \theta_1)dW_t, \quad X_0 = x_0$$

is considered, where W_t is an r-dimensional standard Wiener process independent of the initial variable X_0.

The YUIMA package implements QMLE via the "QMLE function". The interface and the output of the QMLE function are similar to those of the standard MLE function in the stats4 package of the basic R system. Named lists of upper and lower bounds can be specified to identify the search region of the optimizer. The standard optimizer is BFGS when no bounds are specified. If bounds are specified, then L-BFGS-B is used. In the YUIMA package, the likelihood function and the quasi-maximum likelihood estimator for SDE's with jumps are the type by Ogihara and Yoshida (2011).

Parameter estimates for empirical data using YUIMA Package

In this subsection, parameters of the model are estimated using the YUIMA package which uses quasi maximum likelihood estimation method, in its scheme.

The package is programmed to do the following:

- Estimate for the empirical data, the parameters of the model, drift of the diffussion part μ, volatility of the diffusion σ, intensity of the compound Poisson (jump) process λ, (mean of jump size β , variance of jumps δ) .
- Estimate other statistics such as number of jumps
- Randomly generate sample paths for the model, and plot the graphs.
- Plot the empirical data.

The parameters are then estimated, estimates are presented in Table 2. Codes are given in the appendix.

TABLE 2

Parameter estimates price of the crude types

Crude type	*Volatility(σ)*	*Drift(λ)*	*Intensity of CP (β)*	*Mean of J_t (β)*	*Variance of J_t(δ)*	*Number of jumps in 60 months*
BL	0.1154 (SE = 0.0186)	0.0133 (SE = 0.0778)	6.6000 (SE = 1.1489)	0.9405 (1.8282)	10.5025 (1.2928)	33
BB	0.1297 (SE = 0.0213)	0.0497 (SE = 0.0973)	7.6000 (SE = 1.2329)	0.8262 (1.5718)	9.6895 (1.1115)	38
PL	0.1136 (SE = 0.0195)	0.0120 (SE = 0.0797)	7.0000 (SE = 1.1832)	1.0669 (1.6990)	10.0514 (1.2014)	35
ANTAN	0.1272 (SE = 0.0202)	0.0578 (SE = 0.0911)	7.2000 (SE = 1.2000)	0.7964 (1.7207)	10.3242 (1.2167)	36

Discussion

Yuima- Advantage

From the discussions on maximum likelihood estimation (MLE) and cumulant methods, it can be seen that explicit equations cannot be obtained, for all the parameters by these methods. Researchers such as Ball & Torous (1983), Jorion (1988) and Krichene (2006) have used the Jump-Diffusion model on some price returns and met with challenges in estimating model parameters. While, researchers like Ball & Torous (1985), used (0, 1) as (β,δ), the parameters of jump size J_t, others like Jorion (1988) estimated (β,δ) for their data using maximum likelihood estimation, MLE. Ball & Torous (1985) estimated the parameter λ, using MLE and method of cummulants. They obtained values as low as 0.031 for Dow stock and as high as 22.866 for Zenith stock. However, they restricted some parameters to zero, and others to some constant values, in order to obtain values for some other parameters. Ball & Torous (1983) concluded that "difficulties arise with the empirical implementation and verification of this financial process". We surmounted these challenges arising from Maximum likelihood method, method of cummulants and other methods by estimating the parameters using the YUIMA package. This provides a direction in interpreting oil market behaviour and gives a possible guide for predicting the market.

Inference on estimated parameters: implications for oil price behaviour

Going by the structure of the model which has a diffusion part and a jump component, there are periods of normal price distribution (Gaussian) and random intermittent unusual price movements (jumps). This is confirmed by the nature of the estimated parameter values. Statistical conclusions about these parameters are made in this section. For all crude types, the variances of the diffusion and jump components were positive and significant. Volatility or variance values, σ, of 0.1154, 0.1297,0.1136 and 0.1272 for BL, BB, PL and ANTAN respectively, were positive and significant, indicating regular price changes for some periods. While variance values ? of 10.5025, 9.6895, 10.0514, and 10.3242, of the jump component are high and significant, showing that jumps in crude oil price were frequent, with high probability. Also the high values indicate that jumps were quite dominant in the oil price dynamics. The intensity of the compound Poisson, (counting process), which is the probability of the jump is high, because, change in its regular path is altered significantly by occurrence of the jumps. For the drift of the diffusion, estimates were between 0.0133 and 0.0578 were significant showing that oil price had tendency to move up.

Conclusion

Using the Yuima package, we did not have to fix the value of any of the parameters to obtain the value of others. By this method of parameter estimation, more accurate values have been obtained. As mentioned, Ball & Torous (1983) in their paper concluded that "difficulties arise with the empirical implementation and verification of this financial process". These difficulties have been overcome by using the Yuima package. These estimated parameter values can be used via the model equation to simulate the jump-diffusion model and obtain in-sample forecast and out-of-sample forecast for crude oil spot price. Thus implementation and verification of this financial process is now feasible. Inaccurate parameter estimates can yield distorted result for model fit, with misleading information which does not indicate actual market trends.

By this more accurate estimation of parameters, the model becomes a viable mathematical tool for expression of real market situation in terms of oil price. By implication, further steps can be taken to forecast future oil price.

Acknowledgment

The author wishes to appreciate Richard Minkah for his assistance with R-codes.

Appendix

Codes for Parameter Estimation using Yuima

```
### Load the YUIMA package ( you should have installed it earlier)
library(yuima)
## Here you load up the price data: Open excel and and copy the
## four price series using CTRL+C
## 2. Now come to R command and start running this codes
prices<-read.table("clipboard") # Reads price data from to excel R
############################################################
## Dates for data
Date<- c("01/01/05", "01/02/05", "01/03/05", "01/04/05",
  "01/05/05","01/06/05","01/07/05", "01/08/05", "01/09/05",
  "01/10/05", "01/11/05", "01/12/05", "01/01/06", "01/02/06",
  "01/03/06", "01/04/06", "01/05/06", "01/06/06", "01/07/06",
  "01/08/06", "01/09/06", "01/10/06", "01/11/06", "01/12/06",
  "01/01/07", "01/02/07", "01/03/07", "01/04/07", "01/05/07",
  "01/06/07", "01/07/07", "01/08/07", "01/09/07", "01/10/07",
  "01/11/07", "01/12/07", "01/01/08", "01/02/08", "01/03/08",
  "01/04/08", "01/05/08", "01/06/08", "01/07/08", "01/08/08",
  "01/09/08", "01/10/08", "01/11/08", "01/12/08", "01/01/09",
  "01/02/09", "01/03/09", "01/04/09", "01/05/09", "01/06/09",
  "01/07/09", "01/08/09", "01/09/09","01/10/09", "01/11/09",
  "01/12/09")
Function to compute yuima estimates and plot the estimated price
## on the original series
```

```
##################################################################
######
Yum<-function(k,prices,Date)
{
Delta <- 1/12 #
x<-list(BL=prices[,1],BB=prices[,2],PL=prices[,3],prices[,4])
x$Date<-Date
# plotting orriginal data
x$time <- as.Date(x$Date, "%d/%m/%Y")
#tmp <-lapply(1:4,function(k)
tmp <- setYuima(data=setData(zoo(x[[k]],order.by=x$time), delta=Delta))
#plot(tmp, ylab="Price", xlab="Year",main=paste("Crude Price of ", Main[k]))
Main<-c("BL","BB","PL","ANTAN")
# pre-estimates of gBm parameters
a.hat <- var(diff(log(x[[k]])))/Delta
b.hat <- mean(diff(log(x[[k]])))/Delta + 0.5 * a.hat
model <- setModel(drift="mu*x", diffusion="sigma*x",
        jump.coeff="1", measure=list(intensity="lambda",
                df=list("dnorm(z, beta, dels)")),
        measure.type="CP", solve.variable="x")
yuima <- setYuima(model=model, data=tmp@data)
lower <- list(mu=0.01, sigma=0.01,lambda=0.001,beta=0.1,dels=0.1)
upper <- list(mu=100, sigma=100,lambda=25,beta=100,dels=20)
start <- list(mu=b.hat, sigma=a.hat, lambda=5, beta=10, dels=2)
out <- qmle(yuima,start=start,threshold=sqrt(1/Delta),
        upper=upper,lower=lower,method="L-BFGS-B")
```

```
plot(yuima,ylab="Price", xlab="Year", main=paste("Crude Price of ", Main[k]))
#plots data
plot(out,col="red") #what does this plot???
legend("topleft",col=c(1,2),lty=c(1,1),
legend=c("True Value","Forecast"), cex = 0.7)
return(summary(out))
}
#################################################################
#####
## k refers to BL,BB,PL, ANTAN in numbers
k<-1:4
### Here we call the function to compute the estimates and plot it for each
## Price data
sapply(k,function(k)Yum(k,prices,Date))
###################################################################
```

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