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SPATIAL VARIATION OF MAGNETOTELLURIC FIELD COMPONENTS IN SIMPLE 2D AND 3D ENVIRONMENTS

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ABSTRACT

The spatial variation of magnetotelluric field components over a buried three-dimensional conductive inhomogeneity is investigated numerically. The E-polarization mode electromagnetic field components were computed for different aspect ratios of the inhomogeneity and for different frequencies of the incident waves. The results show that as aspect ratio of the inhomogeneity is reduced the spatial variation of the electric field component E_x is reduced and that of the magnetic field component H_y is increased. Furthermore, the variation of H_y is strongly influenced by frequency as aspect ratio is reduced. Familiarity with these variations is important in understanding the behaviour of theoretically computed apparent resistivities which are useful in the interpretation of magnetotelluric field measurements for investigating geologic structures.

KEYSWORDS: Spatial variation, E- polarization mode, Magnetotelluric field components, Electromagnetic field components, Three – dimensional, Two – dimensional, Aspect ratio.

INTRODUCTION

The electromagnetic (EM) field components measured at the Earth's surface during a magnetotelluric (MT) survey are normally considered as raw data and are usually manipulated to yield such quantities as impedance, apparent resistivity, phase and other quantities which allow for meaningful physical interpretation (Linde and Pederson, 2004). However an understanding of the behaviour of these field components in space is essential to understanding the behaviour of theoretically computed apparent resistivities, especially when approximating a three-dimensional (3D) body by a two-dimensional (2D) body because of computational difficulties associated with 3D algorithms (Mackie et al, 1993). A 3D MT response calculated using the integral equation technique required about 20 minutes – CPU time – per frequency on a SDSC cray Y-MP while the corresponding 2D response calculated on the same machine required about 3 minutes – CPU time (Wannamaker, 1991). Furthermore, approximate solutions to EM problems are often as useful as the more complete solutions (Chave and Booker, 1987).

Stodt et al (1981) obtained horizontal MT field components over a buried conductive prismatic 3D body (aspect ratio = ½) and over the corresponding 2D body at three frequencies. The aspect ratio of a body is the width to length ratio (Sheriff, 2002). The results of their investigation show that for a 2D case the transverse electric (TE) mode horizontal magnetic field may vary by more than a factor of three; but this variation is not as great over 3D inhomogeneities.

In this work I have carried out further investigation of these variations as the 3D body is made to approach a 2D body. E-polarization mode MT field components are computed for three different frequencies using 2D and 3D forward modeling computer programs written by Daniyan (1983). The programs are based on the Rayleigh – FFT numerical technique, which is valid for curved separating regions of different electrical conductivity.

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THEORY

Maxwell's equations govern large scale EM phenomena. These equations, in the frequency domain, for a plane wave with exp(iωt) time dependence and normally incident on the air-earth interface are

$$\text{Curl } \overline{E} = - i\omega\mu\overline{H} \tag{1a}$$

$$\text{Curl } \overline{H} = (\sigma + i\omega\varepsilon)\, \overline{E} \tag{2a}$$

Rearranging the above equations we obtain

$$\nabla^2 \overline{E} - \nabla(\nabla.\overline{E}) = k^2 \overline{E} \tag{1b}$$

$$\nabla^2 \overline{H} - \nabla(\nabla.\overline{H}) = k^2 \overline{H} \tag{2b}$$

where $k^2 = i\omega\mu\sigma - \omega^2\varepsilon\mu$. (3)

Let R represent either E or H, then

$$\nabla^2 \overline{R} - \nabla(\nabla.\overline{R}) = k^2 \overline{R}$$

Notations:

$\overline{E}$ is electric field vector
H is magnetic field vector
ω is radian frequency
μ is magnetic permeability
σ is electric conductivity
ε is electric permittivity
i is equal to $\sqrt{-1}$

In equations (1a) and (2a) it is assumed that the values of μ and ε are those of free space.

Consider the general subsurface structure and the Cartesian coordinate system shown in Figure 1. The x-direction is into the paper. The air-earth interface is flat, the first subsurface interface varies in one direction (2D case) or in two directions (3D case) and the second subsurface interface is planar. The air layer has zero conductivity σ_o and the other layers have conductivities σ_1, σ_2 and σ_3 respectively. Subscripts are indices of the different regions as follows:

Index of region (*l*)	region	
0	$z < 0$	
1	$A + F(y) > z > 0$	(2 – D case)
	$A + F(x, y) > z > 0$	(3 – D case)
2	$D > z > A + F(y)$	(2 – D case)
	$D > z > A + F(x,y)$	(3 – D case)
3	$z > D$	

where A and D represent depths. To model a conductive inhomogeneity in a more resistive homogeneous half-space, D is made equal to A, the value of σ_3 equal to σ_1, and σ_2 is greater than σ_1 (Figure 2).

In a 2-D geometry the inhomoegeneity is infinite in one direction (x-direction here) which is the strike direction and the scalar conductivity varies with one horizontal coordinate (y) and with depth (z). Maxwell's equations for a 2-D structure excited by a plane-wave source generally decouple into two independent sets of equations. In one set the electric field is parallel to strike direction; this situation is termed E-polarization mode. In the other set the magnetic field is parallel to strike direction and this is known as the H-polarization mode.

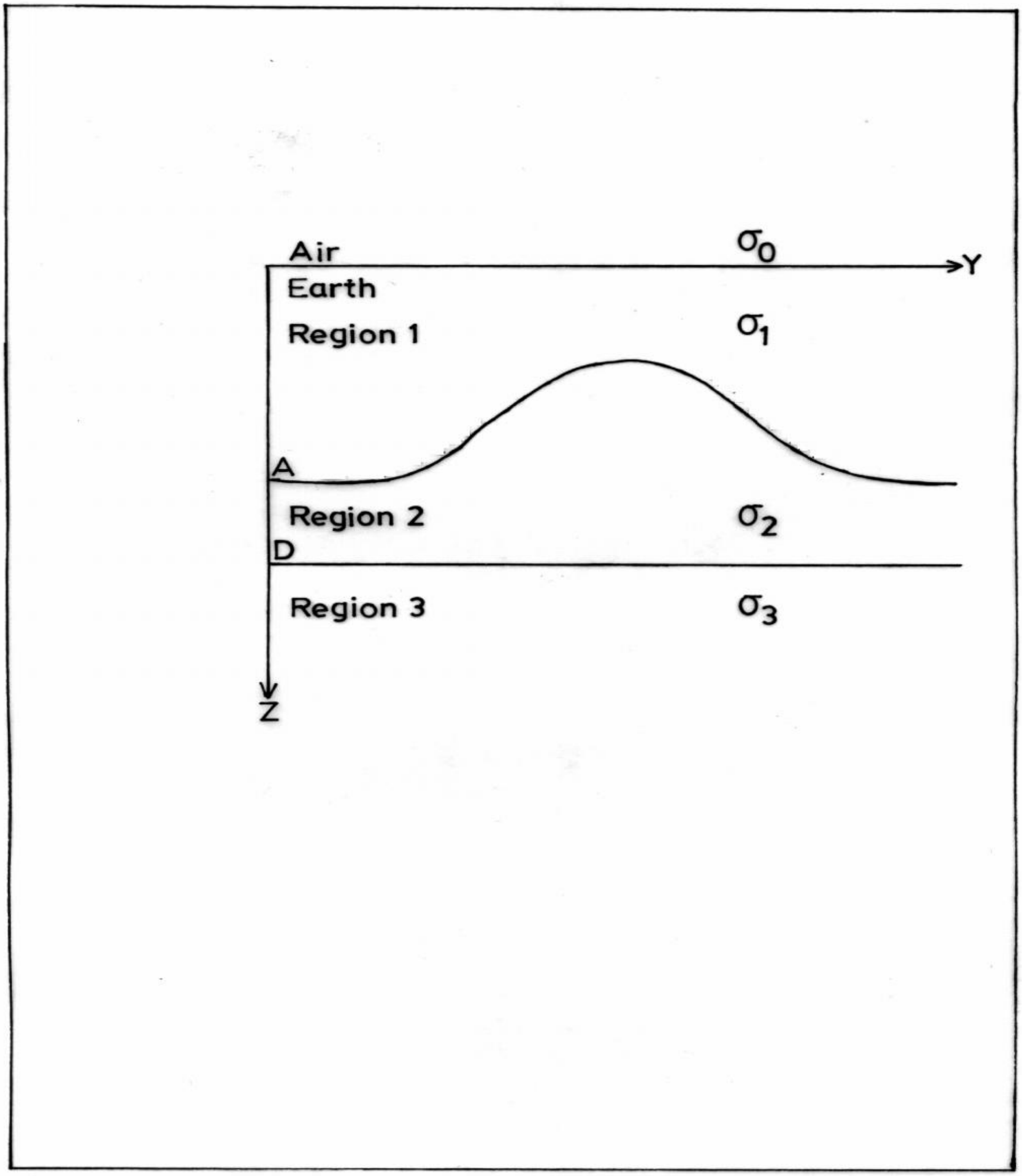

Fig. 1. General subsurface structure.

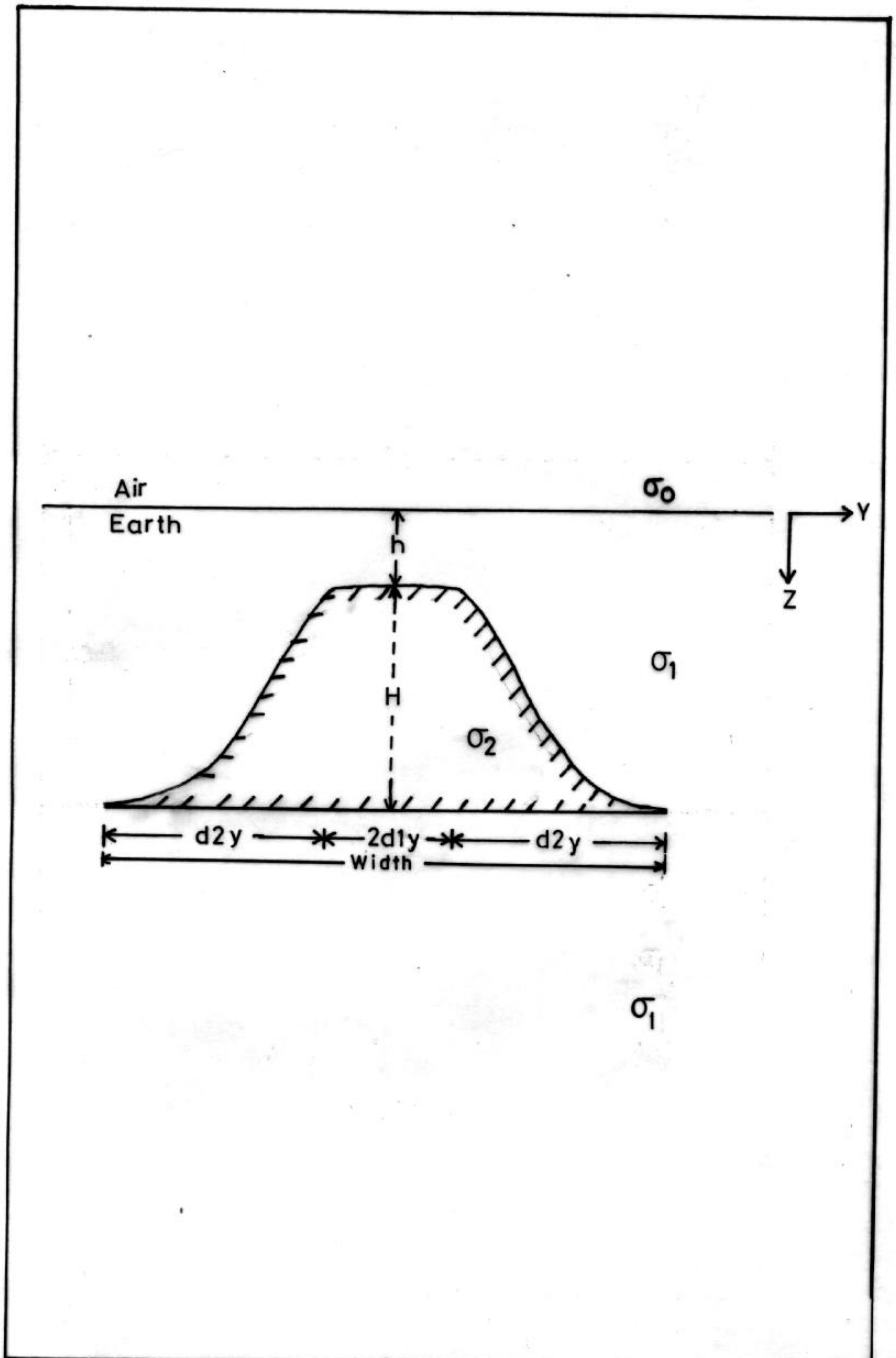

FIG. 2. Cross-section of two- and three-dimensional models.

Maxwell's equations (1(a) and 2(a) for these modes can be written as:
For E-polarization mode:

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = (\sigma + i\omega\varepsilon)\, E_x \qquad (4a)$$

$$\frac{\partial E_x}{\partial z} = -\, i\omega\mu H_y \qquad (4b)$$

$$\frac{\partial E_x}{\partial y} = -\, i\omega\mu H_z \qquad (4c)$$

For H-polarization:

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial_z} = -\, i\omega\mu H_x \qquad (5a)$$

$$\frac{\partial H_x}{\partial z} = (\sigma + i\omega\varepsilon)\, E_y \qquad (5b)$$

$$\frac{\partial H_x}{\partial y} = (\sigma + i\omega\varepsilon)\, E_z \qquad (5c)$$

The solutions of the above equations evaluated at the surface of the earth model yield the MT field components.

In the case of 3 – D geometry the inhomogeneity is of finite extent in all directions; there is usually no defined strike and the scalar conductivity varies with both horizontal coordinates and with depth. Maxwell's equations for a 3-D structure excited by a plane-wave source do not generally decouple as in 2–D case. All the EM field components are always present when the incident electric field is polarized in any particular direction. When the incident electric field is parallel to the long axis of the body the mode of polarization is termed E-polarization and when the electric field is perpendicular to the long axis the mode is designated H- polarization. The vector equation (3) may be written in Cartesian coordinates as:

$$\frac{\partial^2 R_x}{\partial_y^{\,2}} + \frac{\partial^2 R_x}{\partial_z^{\,2}} - \frac{\partial}{\partial_x}\left(\frac{\partial R_y}{\partial_y} + \frac{\partial R_z}{\partial_z}\right) = k^2 R_X \qquad (6a)$$

$$\frac{\partial^2 R_y}{\partial_x^{\,2}} + \frac{\partial^2 R_y}{\partial_z^{\,2}} - \frac{\partial}{\partial_y}\left(\frac{\partial R_z}{\partial_z} + \frac{\partial R_x}{\partial_x}\right) = k^2 R_y \qquad (6b)$$

$$\frac{\partial^2 R_z}{\partial_x^{\,2}} + \frac{\partial^2 R_z}{\partial_y^{\,2}} - \frac{\partial}{\partial_z}\left(\frac{\partial R_x}{\partial_x} + \frac{\partial R_y}{\partial_y}\right) = k^2 R_X \qquad (6c)$$

Again the solutions of the above simultaneous equations evaluated at the model's surface are the MT field components due to 3D anomalies.

The solutions to equations (4), (5), and (6) are obtained by first reducing them to ordinary differential equations using the Fourier Integral Transform method and then solving them in a normal way (see Daniyan, 1983). The results are

For 2 – D models:

$$R_{Lx}\,(y, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [A_L\,(\eta)\ \exp\ (\alpha_L z) + B_L(\eta)\ \exp\,(-\alpha_L z)]$$
$$\times\ \exp\,(i\eta y)\, d\eta \qquad (7a)$$

$$R_{Lx}\,(x, y, z) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [C_L\ (\xi, \eta)\ \exp\ (\sim_L' z) + D_L\ (\xi, \eta)]$$
$$\times\ \exp\ (-\sim_L' z)\,\exp\,(i\xi x + i\eta y)\, dy\, d\eta \qquad (7b)$$

where R_{Lx} represents either the electric or magnetic field component along the x-direction in region *l* (see Figure 1) which can be calculated for any coordinate position (y, x) or (x, y, z), α_L and $\sim_L'$ are propagation constants, A_L, B_L etc are spectral amplitudes, and ξ and η are wave numbers in the horizontal coordinate directions.

In order to evaluate the field components the unknown quatities in equations (7a) and (7b), which are the spectral amplitudes, must be determined. To find them the usual EM boundary conditions are applied. The application of the boundary conditions at the interfaces yield coupled integral equations (Aki and Larner, 1970), which are solved numerically using the Rayleigh – FFT technique (Aki and Larner, 1970; Daniyan, 1983) to obtain the spectral coefficients. Once the R_{Lx}'s are evaluated the complete EM field components can be found by using the appropriate Maxwells' equations as the case may be.

MODEL STUDIES

The models used in this investigation consist of an inhomogeneity of conductivity σ_2 buried in a more resistive homogeneous half-space of conductivity σ_1 (see Figure 2). For the 2D case the anomaly is infinite in the x-direction and for the 3D case it is of finite extent in all directions. The non-planar interface in the diagram is described as follows:

For 2D case:

$$F(y) = H \qquad \text{for } |y| < |d1y|$$

$$F(y) = \frac{H}{2}\left(1 + \cos\pi \frac{(y-d1y)}{d2y}\right) \qquad \text{for } d1y < |y| < d1y + d2y$$

$$F(y) = 0 \qquad \text{for } |y| > d1y + d2y$$

For 3 – D case:

$$F(x, y) = H \qquad \text{for } |x| < d1y \text{ and } |y| < d1y$$

$$F(x, y) = \frac{H}{4}\left(1 + \cos\pi \frac{(x-d1x)}{d2y}\right)\left(1 + \cos\pi \frac{(y-d1y)}{d2y}\right) \text{ for}$$

$$d1x < |x| < d1x + d2x \text{ and}$$
$$d1y < |y| < d1y + d2y$$

$$F(x, y) = 0 \qquad \text{for } |x| > d1x + d2x \text{ and } |y| > d1y + d2y$$

H is the height of the anomaly and h is its depth of burial. The width of the anomaly is taken as its dimension at the base in the y-direction, which is 2(d1y + d2y).

Note that the long axis of the 3-D body is in the x-direction and that the models are symmetric. The essential parameters of the model are H = 300 m, 2(d1y + d2y) = 2 km, d1y = 200 m, ρ_1 = 50 ohm – m, and $^\circ{}^2_2$ = 5 ohm-m. (dlx = 1200m, d2x = d2y = 800 m, h = 10 m and repetition distance = 20 km in both horizontal directions).

RESULTS

Horizontal transverse electric field components were computed for three aspect ratios of the inhomogeneity, namely ½, ¼ and ~0 (= 2D) and for three frequencies 0.556, 1.000 and 1.500 Hz using 2D and 3D computer programs of Daniyan (1983). This choice of aspect ratios is based on the finding that an inhomogeneity loses most of its three-dimensionality when its aspect ratio is less than $^1/_3$ (Onwuegbuche, 1984).

The normalized amplitudes of the horizontal electric and magnetic fields calculated along the profile in the symmetry plane through the centre and perpendicular to the long axis of the bodies are presented in Figures 3, 4 and 5. The normalization factor is the distant field, which allows for direct assessment of the relative spatial variations in the magnitudes of the fields. Only one half of each profile is plotted out because of symmetry. It is apparent that the effect of the reduced aspect ratio of the 3D body is to reduce the spatial variation in the electric field, E_x and to increase that in the magnetic field H_y.

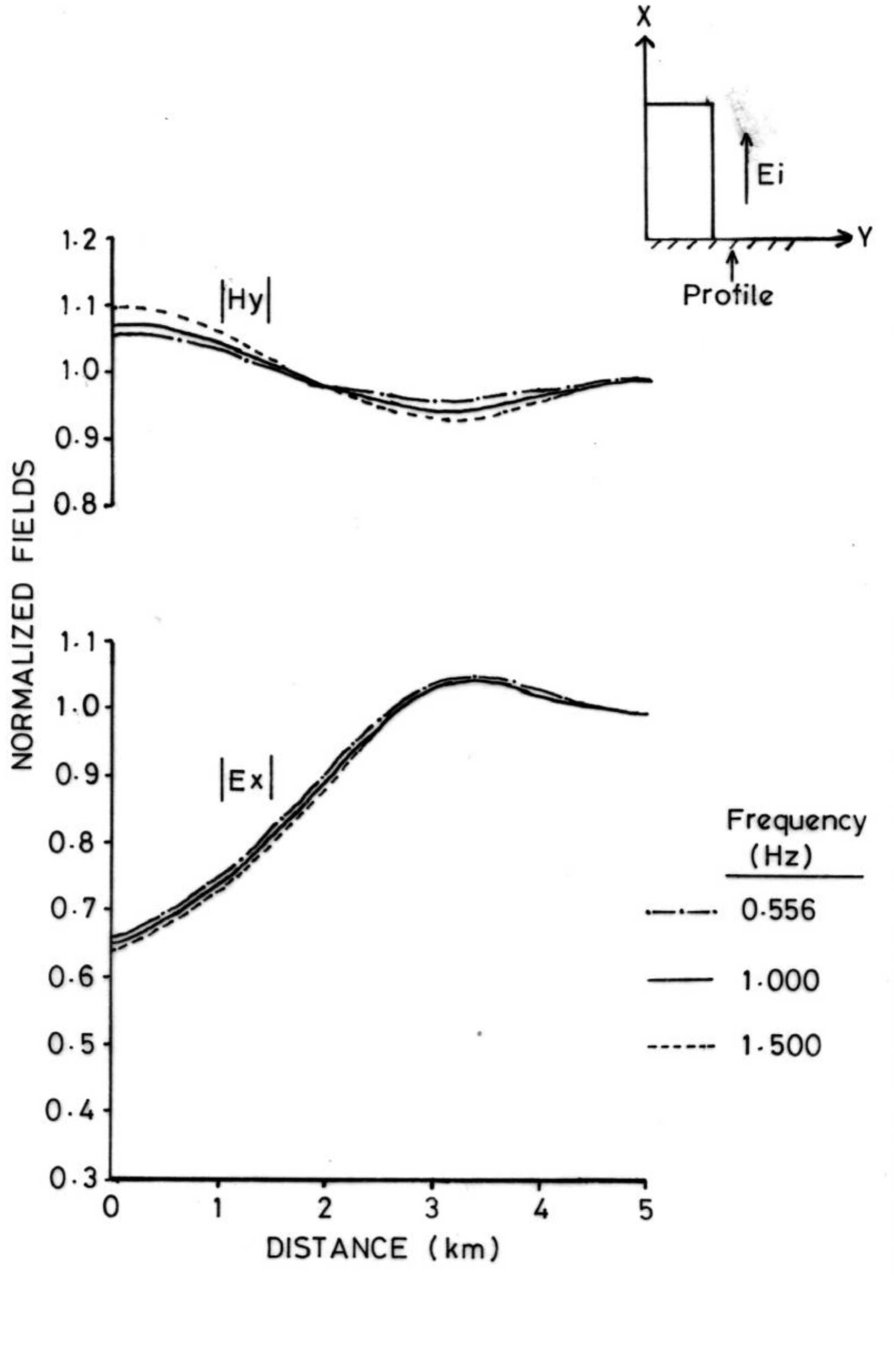

Fig. 3. Spatial variation of E – parallel mode horizontal field components of a 3 – D body (aspect ratio = ½) Ei is the incident electric field.

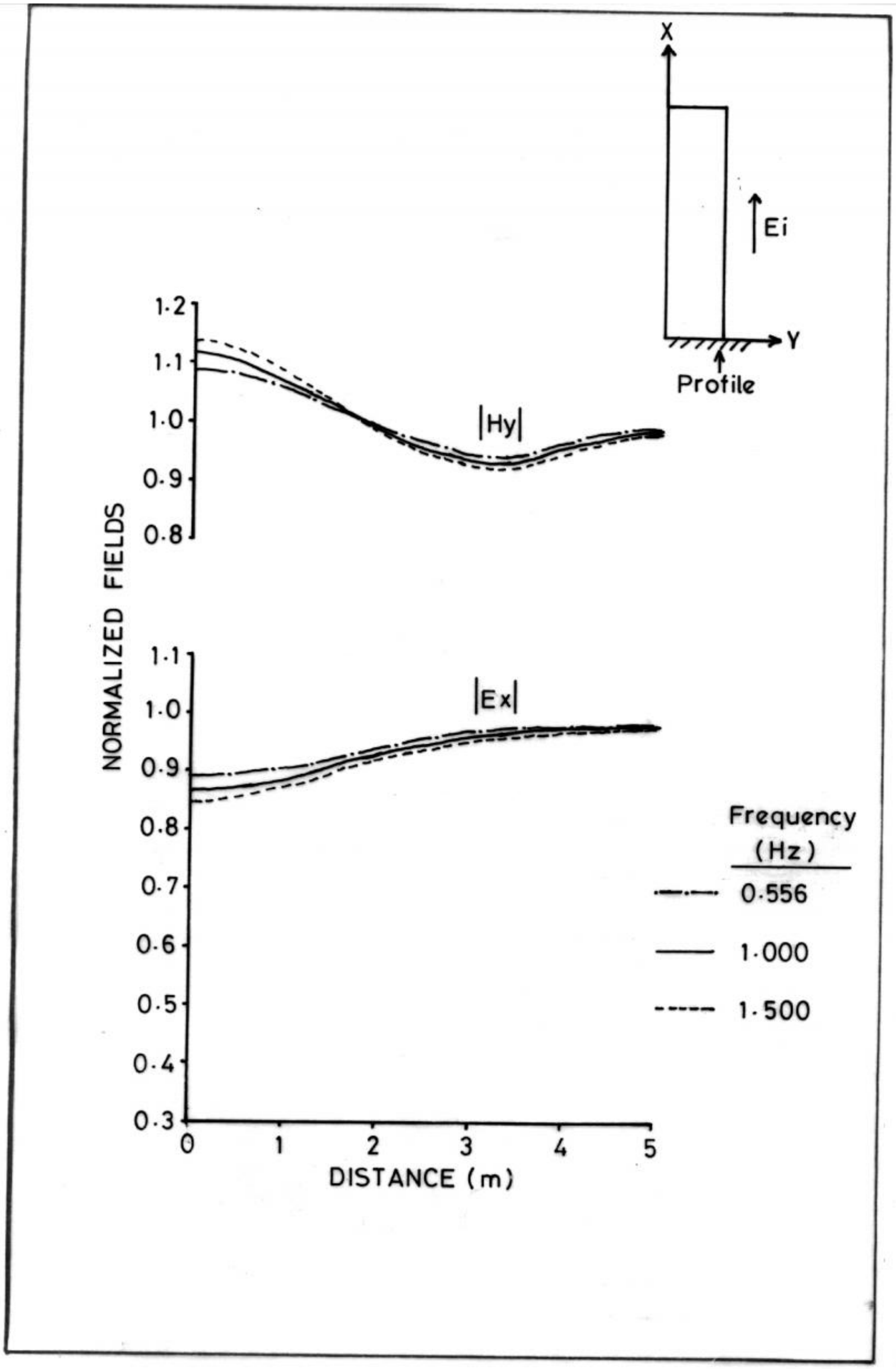

Fig. 4. Spatial variation of E – parallel mode horizontal field components of a 3 – D body (aspect ratio = ¼) Ei is the incident electric field.

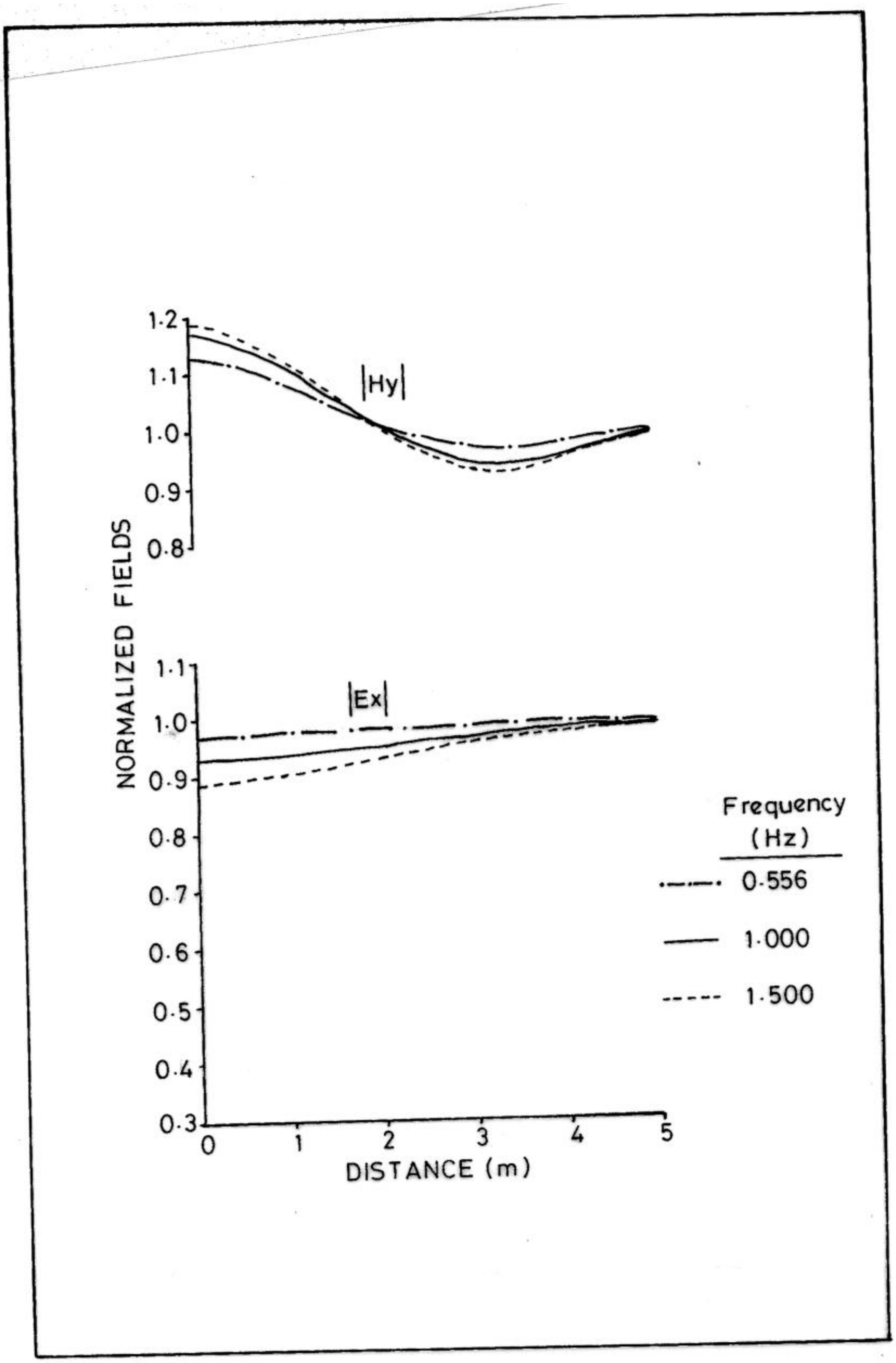

Fig. 5: Spatial variation in E-polarization mode horizontal field components Ex and Hy over a 2-D body.

DISCUSSION AND CONCLUSION

Large scale EM induction is responsible for regional fields and currents. These fields and currents are perturbed by lateral and vertical conductivity discontinuities in a homogeneous half-space. The spatial behaviour of the EM field components computed for 2D and 3D earth-models in this work may be explained by a consideration of the mechanisms responsible for the perturbations in these environments. These mechanisms are local induction of current loops and current gathering.

When the aspect ratio of the inhomogeneity is equal to ½ the fields are perturbed mainly by current gathering. This is demonstrated in Figure 3 where the spatial variation in E_x for the three frequencies are almost the same. The spatial variation in E_x is greater than that in H_y. Thus the behaviour of apparent resisitivities [proportional to $(E_x/H_y)^2$] will be strongly influenced by the spatial variation in E_x. When the aspect ratio is changed to ¼ (Figure 4) local induction of current loops becomes more important than current gathering. Induction effects appear as different spatial variation in E_x for different frequencies and an increase in the spatial variations in H_y. The behaviour of apparent resistivities will thus be controlled more by the variations in H_y than by variations in E_x.

In 2D environment (Figure 5) induction is the dominant mechanism responsible for field perturbations. E_x in this case can be characterized as spatially uniform. The spatial variation of H_y is greater than that for aspect ratio of ½ and ¼ (cf Stodt et al., 1981). Thus the influence of H_y on apparent resistivities will be greater than for the case where the aspect ratio is greater than or equal to ¼ particularly at low frequencies.

The results obtained in this study lead to the conclusion that the effect of decreased aspect ratio of a 3D body (aspect ratio of ½) on EM field components is to reduce the spatial variation in the electric field component E_x and to increase the spatial variation in the magnetic field component H_y. Furthermore, the variation in the magnetic field component is strongly influenced by frequency as aspect ratio is decreased.

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