DUAL – TEMPERATURE ELECTRON DISTRIBUTION IN A LABORATORY PLASMA

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ABSTRACT

The dual-temperature distribution function is used to investigate theoretically the effect of a perturbation of Maxwell distribution function on density ratios in a laboratory plasma produced solely by collision. By assuming a foreknowledge of collision coefficients and cross-sections and an atomic model which sets at two statistical weights for all states including continuum, calculations show that a collision transition between close neighbouring levels is of more significance than that between widely separated or non-neighbouring levels; also population and electron densities in plasma do not only depend on the energy distribution of free electrons but on both the excitation and binding energies of the atomic levels.

KEYWORDS: Dual – temperature distribution function, Maxwell distribution function, Perturbation, Density ratios, Laboratory plasma, Collision transition.

INTRODUCTION

A perturbation of a Maxwell electron distribution function within a limited energy intervals leads to another function. This new function, called the dual-temperature distribution function, determines the effect of any given perturbation on density ratios. In this work we investigate the theoretical dependence between various densities in laboratory plasma solely produced by collision in relation to the dual-temperature electron distribution, which occurs when there is no thermodynamic equilibrium (Froehlich et al, 2001; Delcroix et al, 2006; Postma, 1969 and Oleka et al, 1989).

MATHEMATICAL DEFINITION

An electron distribution $f(E)$ which corresponds to two electron temperatures T_1 and T_2 ($T_1 > T_2$) in two regions respectively of lower and higher energies of the same Maxwell distribution $f(E)_{Maxw}$ is designated dual-temperature distribution, such that

$$f(E) = c. \, f(E)_{Maxw} . g(E) \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (1)$$

where the perturbation function $g(E)$ is of the form (Goldammer, 1971):

$$g(E) = \left[1 + \frac{\exp(E-E_K)}{kT_F} \right]^{-1}$$

Putting

$$a = \exp(-E_k/KT_F)$$
$$b = (c/\pi^{\frac{1}{2}})(1/KT_1)^{3/2}$$

and

$$\frac{1}{T_2} = \frac{1}{T_1} + \frac{1}{T_F} \quad ,$$

(1) becomes,

$$f(E) = bE^{\frac{1}{2}} \left[\exp(-E/kT_1) + a\exp(E/kT_2) \right]^{-1} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (2)$$

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and can be reduced to two relations:

$$f(E) = bE^{\frac{1}{2}} \cdot \exp(-E/kT_1)/ \left[1 + \exp(E - E_K)/kT_F \right] \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (3)$$

and

$$f(E) = a^{-1}bE^{\frac{1}{2}} \exp(-E/kT_2)/ \left[\exp(E_K - E)/kT_F + 1 \right] \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (4)$$

For low energies the denominator of (3) is approximately unity. The same applies to the denominator of (4) for high energy values. The dual-temperature distribution therefore corresponds to a Maxwell distribution with temperature T_1 in the energy interval $0 \le E \le E_k - kT_F$ and to a lower temperature T_2 for $E > E_k + kT_F$. The transition between two regions is continuous over the complete interval $0 \le E <$ of an approximate width $2\ kT_F$ (Fig. 1). For T_2 T_1, the width of the transition region becomes very large and there is no marked distinction between the dual-temperature distribution and the Maxwell distribution $\overline{T}$ (defined later in text).

SUBSTITUTE DENSITIES

If the energy distribution function of free electrons is an unperturbed Maxwell distribution, population and electron densities are validly represented respectively by the Boltzmann relation (Oleka, 1975).

$$n_k/n_i = \exp(-\chi_{ik}/kT_e)$$

and the Saha equation (Ambarzumjan, 1957): $\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..$ (5)

$$n^2_e/n_i = [2(2\pi m_e kT_e)^{3/2}/h^3] \exp(-\chi_{ik}/kT_e) \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (6)$$

The density ratios at the left hand side of both relations (5) and (6) are designated "equilibrium density ratios" and written as $(n_k/n_i)_E$ and $(n^2_e/n_i)_E$.

Fig.1: Dual Temperature Distribution
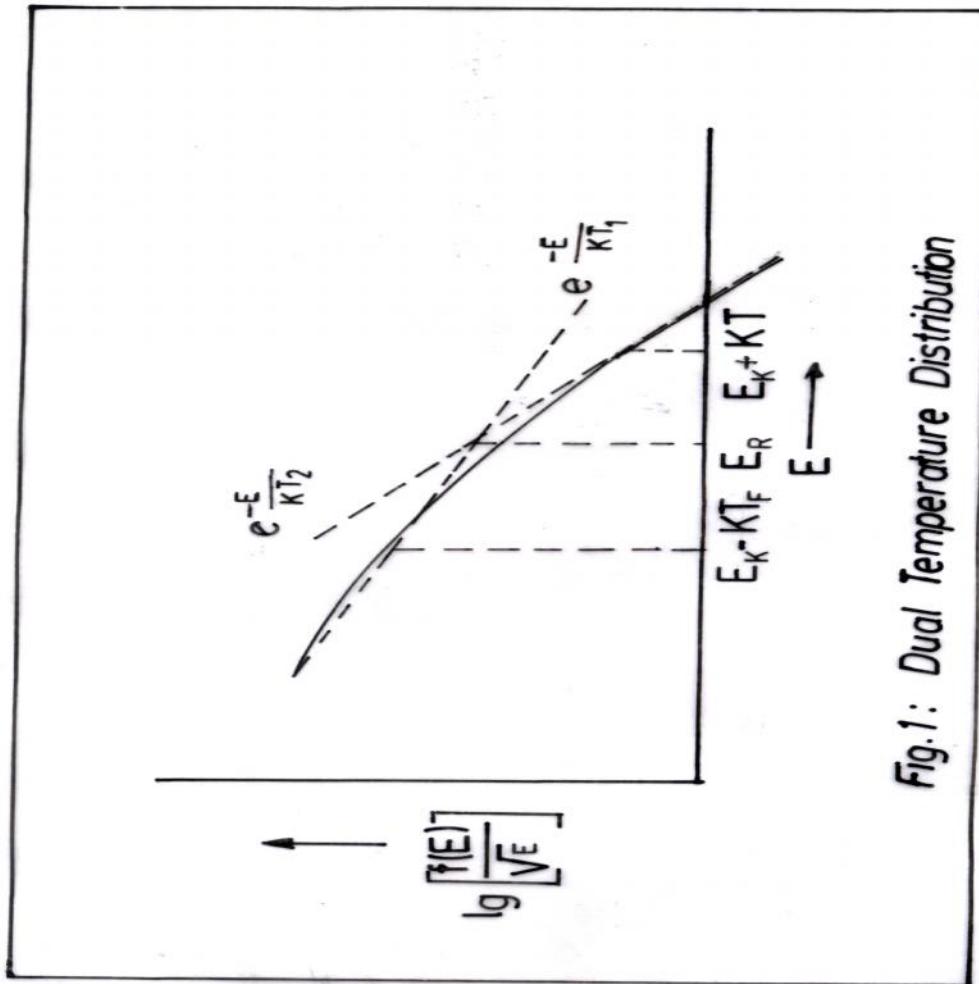
Each ratio is a function of electron temperature, Te. The equivalent temperature with regard to dual-temperature distribution is T_1. An "average electron temperature" T is defined (Goldammer, 1971) as:

$$\overline{T} = 2\overline{E} / 3k,$$

where E is the average energy of free electrons. If $\overline{T}_e = \overline{T}$ is substituted in (5) and (6), "substitute density ratios" written as (n_k/n_i)s and (n^2_e/n_i)s are obtained. They result whenever any given distribution function is replaced by a Maxwell distribution function of equal average energy.

COMPUTATIONAL METHODS

A knowledge of the various collision coefficients – excitation (S_{ik}), collision of second kind ($\overline{S}_{ki}$), ionization ($S_{i\infty}$) and triple collision recombination ($\overline{S}_{\infty i}$) is assumed (Sewing, 1971). S_{ik}, $\overline{S}_{ki}$ and $S_{\infty i}$ contain expressions which involve cross-sections over infinite limits. However, the presence of the factor exp (-E/kT) in the integrand makes finding the upper limit of integration possible. Iteratively, an evaluation is possible since the perturbation function is of the order of one or less. Similarly, the equally complex expression for $\overline{S}_{\infty i}$ is simplified by assuming a perturbation restricted to the high energy region of the Maxwell distribution – a perturbation which has only a negligible influence on $\overline{S}_{\infty i}$.

This assumption leads to a less formidable for $S_{\infty i}$ which contains a factor exp(-E/kT).

RESULTS AND DISCUSSION

It is worthy of note that the average electron temperature T and the already defined substitute densities form a useful basis for the discussion of results. Of interest is that $\overline{T}$ does not necessarily increase monotonically with T_2, should T_1 and E_k be kept constant (see Table I). As T_2 increase T first of all decreases to a minimum value $E_k > 1.5/2T_1$. As T_2 T_1, that is T_F , $\overline{T}$ then increases and tends to T_1. E_k which represents the middle of the transition interval of the distribution function is a determinant factor in the dual-temperature distribution, such that for large values of E_k there is a Maxwell distribution with temperature T_1, while for approximately disappearing values of E_k the distribution function is determined by T_2.

Consequently, population and electron densities in plasma behave similarly. For $E_k = 0$ (Fig. 2), the density ratios lie a little above values which normally would be obtained for a Maxwell distribution of temperature T_2 (See dashed lines drawn on the ordinate axis). The discrepancy results from the fact that the transition from T_1 to T_2 is gradual and not abrupt and implies that the influence of T_1 on the distribution function and electron density is still noticeable even at $E_k = 0$. As E_k increases so also does the electron density in such a manner that as E_k , it approximates asymptotically to the equilibrium density. Most noticeable is that curves which correspond to different values of T_2 as E_k increases do not maintain their position relative to one another. For example, the curve for $T_2 = 0$ rises so fast with increasing E_k such that it intersects all other curves and for $E_k > 1$ eV, the density ratios lie closer to equilibrium values $(n^2_e/n_i)E$ than other density ratios which correspond to higher values of T_2. Table II shows a
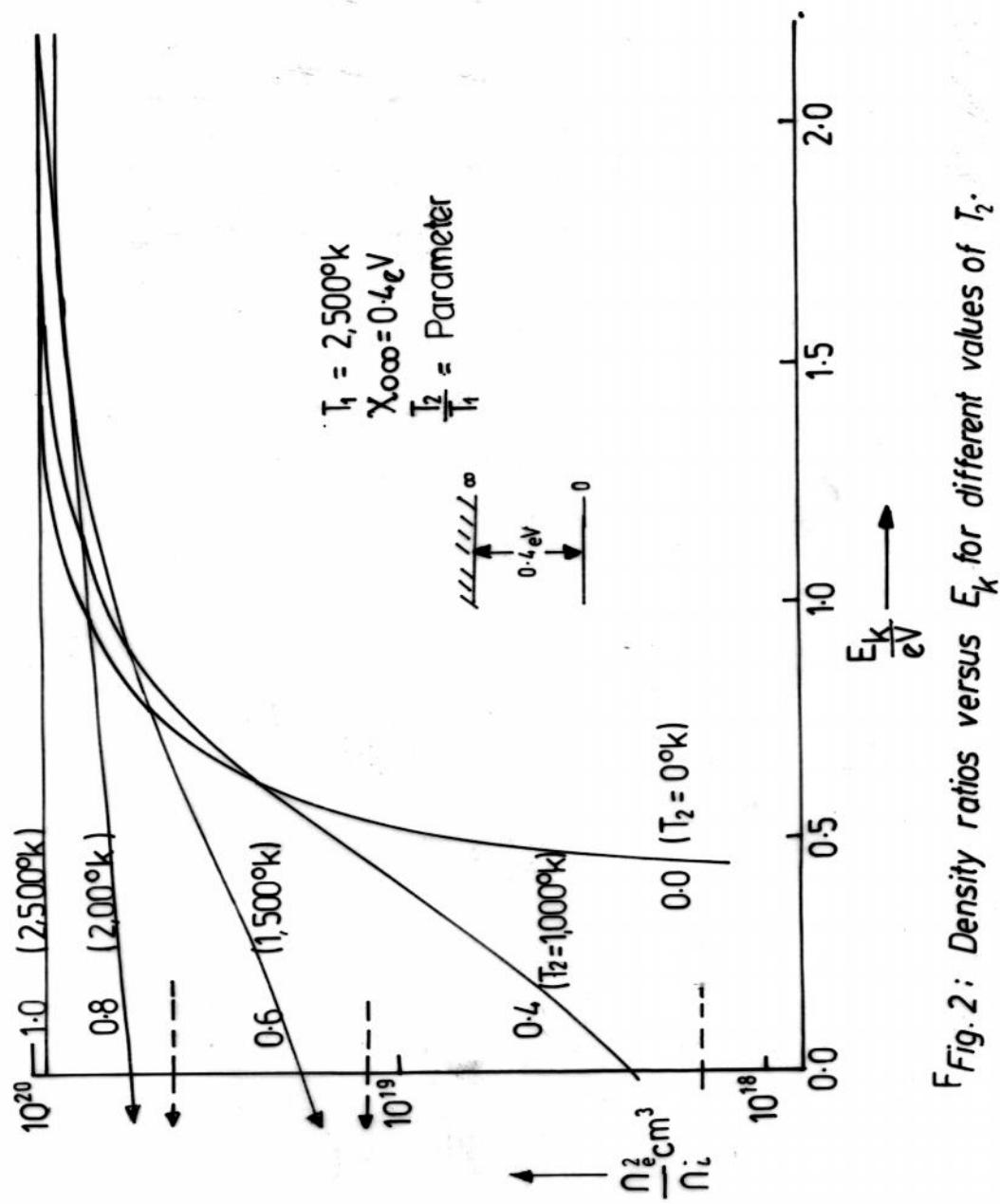
Table I: Dependence of Average Electron Temperature $\overline{T}$ of the Dual-Temperature Distribution on E_k and T_2.

T$_1$ = 2500K; kT$_1$ = 0.2154 eV

T_F E_k T_2	0	625 500	1071 750	1667 1000	2500 1250	3750 1500	10000 2000	22500 2250
0.2				1334	1554	1775	2186	2359
0.4	1447	1468	1517	1608	1735	1887	2213	2365
0.6	1876	1871	1868	1883	1928	2007	2240	2371
0.8	2151	2141	2126	2108	2102	2124	2267	2378
1.0	2314	2306	2292	2268	2241	2226	2294	2385
1.5	2467	2465	2456	2451	2430	2397	2356	2399
2.0	2495	2495	2494	2492	2485	2469	2405	2415
2.5	2499	2499	2499	2499	2497	2492	2441	2429

Table II: Comparison of Electron Densities with Average Electron Temperatures T for various values of E_k and T_2.

$T_1 = 2500$ T_2	E_k	0.6	0.8	1.0	1.5
0	$\overline{T}$ n^2_e/n_o	1876 2.02×10^{19}	2151 5.15×10^{19}	2314 7.26×10^{19}	2467 9.11×10^{19}
1000	$\overline{T}$ n^2_e/n_o	1883 2.34×10^{19}	2108 4.23×10^{19}	2268 6.10×10^{19}	2451 8.71×10^{19}
1500	$\overline{T}$ n^2_e/n_o	2007 3.85×10^{19}	2124 4.85×10^{19}	2226 5.89×10^{19}	2397 7.98×10^{19}
2000	$\overline{T}$ n^2_e/n_o	2240 6.33×10^{19}	2267 6.62×10^{19}	2294 6.91×10^{19}	2356 7.60×10^{19}

Fig. 2 : Density ratios versus E_k for different values of T_2.

comparison of electron densities with the corresponding average electron temperature $\overline{T}$, an doffers an explanation for the behaviour of the curves mentioned above. $\overline{T}$ rises with increasing E_k much faster as T_2 becomes much smaller. Moreover, if E_k is kept constant, electron densities and average electron temperature maintain their least values for the same value of T_2. The nature of the electron density ratio curves, therefore, gives qualitatively the dependence of the average electron temperature on E_k and T_2.

Fig. 3 shows the dependence of electron densities on E_k for three different model atoms. For comparative analysis, the corresponding curve for the substitute densities is also indicated. It is easily seen that the difference between the values of the substitute densities and the densities of an atom with an intermediate level (Curve b) is much smaller than that of an atom without an intermediate level (Curve c) but of the same ionization energy. Also, if there are two intermediate levels, instead of one, between the ground level and continuum (Curve a), the density values of the atom will still be closer to those of the substitute densities.
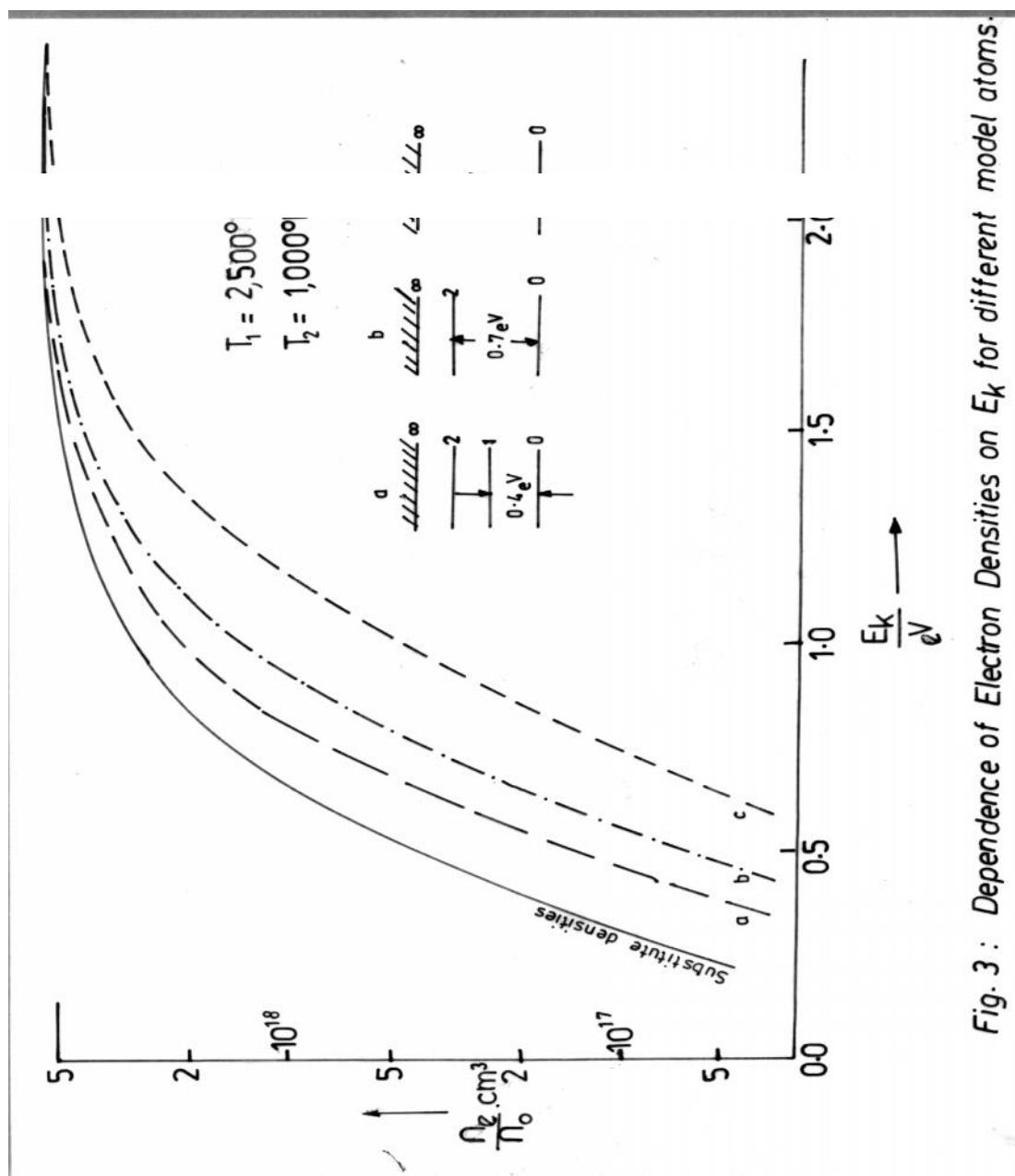

Fig. 3 : Dependence of Electron Densities on E_k for different model atoms.

The behaviour of the ratio of electron density to the population density of an atom in level i in the form n^2_e/n_i with respect to the binding energy when T_1 and E_k are kept constant is shown in Fig. 4. There is hardly any difference between the values of electron densities (Curve 1) and substitute densities (Curve 2) for very low values of $\chi_{i\infty}$. The difference as shown in the diagram is very much exaggerated. A difference only becomes noticeable by about $\chi_{i\infty} \approx 0.3\ E_k$, from where the substitute densities have values relatively higher than those of electron densities. For $\chi_{i\infty} > E_k$, both densities assumes an asymptotic closeness, whereby the asymptote cuts the equilibrium density curve (Curve 3) at $\chi_{i\infty} \approx E_k - 2_k T_F$.

Fig. 5 shows the behaviour of density ratios in relation to the corresponding binding energy for three different model atoms (Curves a, b and c). There is almost a coincidence of the substitute density curve and the curve relating to model atoms from zero value of $\chi_{i\infty}$ to $\chi_{i\infty}$ up to = 0.2 eV. From 0.2 eV, curve (a) becomes discernible as a distinct curve, while curves b and c first show up as separate curves distinct from the substitute density at $\chi_{i\infty} = 0.35$ eV and 0.45 eV respectively. For the various model atoms the values of binding energy referred to above result in the largest energy interval between levels of respectively 0.2 eV, 0.19 eV and 0.18 eV. This points to the fact that the step-by-step coupling of densities from level to level has a greater influence than the direct coupling over and above several intermediate levels.

SUMMARY/CONCLUSION

In plasma arising from collision processes, the following approximate relations for the population densities n_k and n_i of two neighbouring atomic levels k and i hold (Oleka, 1975):

$$(n_k/n_i)\ (n_k/n_i)_s = \exp(-\chi_{i\infty}/k\overline{T}),$$

if the free electrons have a dual – temperature distribution of average temperature $\overline{T}$ and an excitation energy of 0.3 E_k. The energy value at which the temperature T_1 "changes" to T_2 is approximately equal to the excitation energy χ_{oi} of the lowest excited state (Delcroix et al, 2006; Shaw et al, 2007 and Eidmann, 1996). For cesium plasma $\chi_{oi} = 1.38$ eV (Oleka, 1975). This means that the population densities of all atomic levels above 7S can be approximately calculated using the relation above.

Electron and population densities are very close to the values of substitute densities if the number of intermediate levels between ground level and continuum increases. Moreover, the set-by-step coupling of densities from level to level has a greater influence than direct coupling over and above several intermediate levels. The collision cross-section and consequently both the excitation coefficient and collision coefficient of the second kind are inversely proportional to the square of excitation energy. This implies that a collision transition between neighboring levels is more significant than between levels that are widely separated from others. As a result, coupling is more between levels that are direct neighbours.

In general, it is seen that population and electron densities in plasma do not depend only on the energy distribution of the free electrons but also on the excitation and binding energies of the atomic levels.
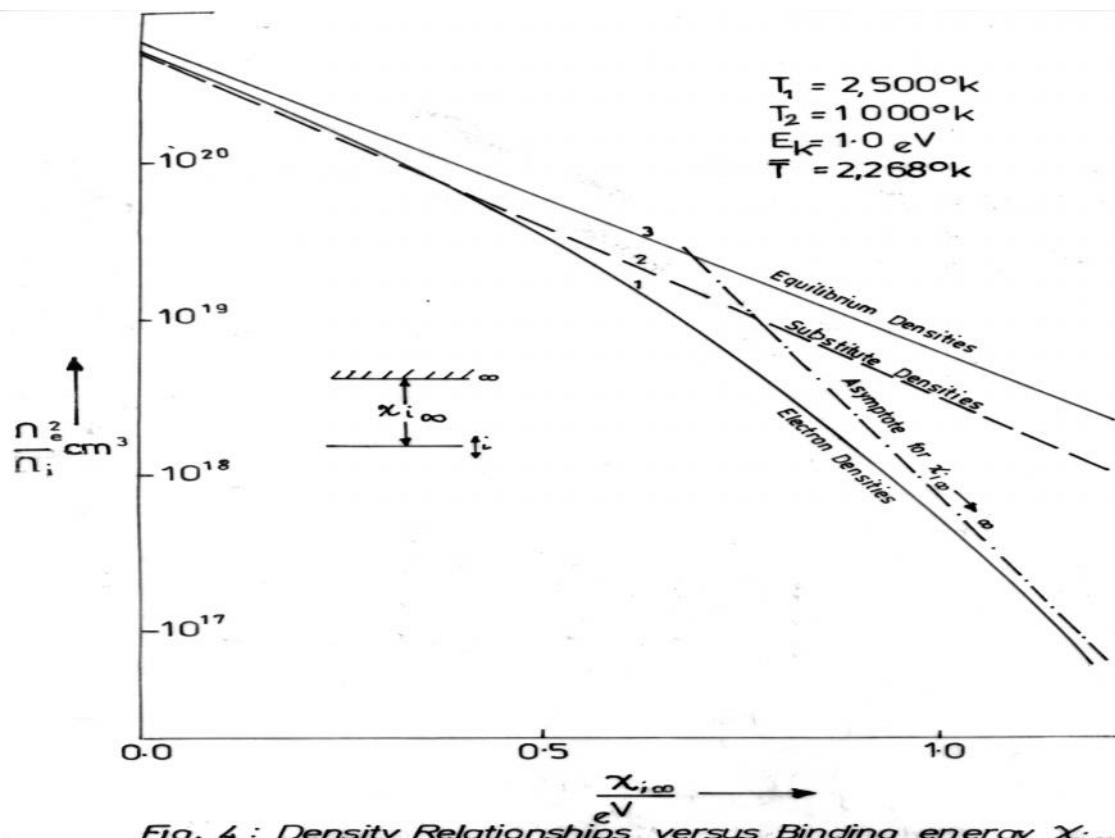

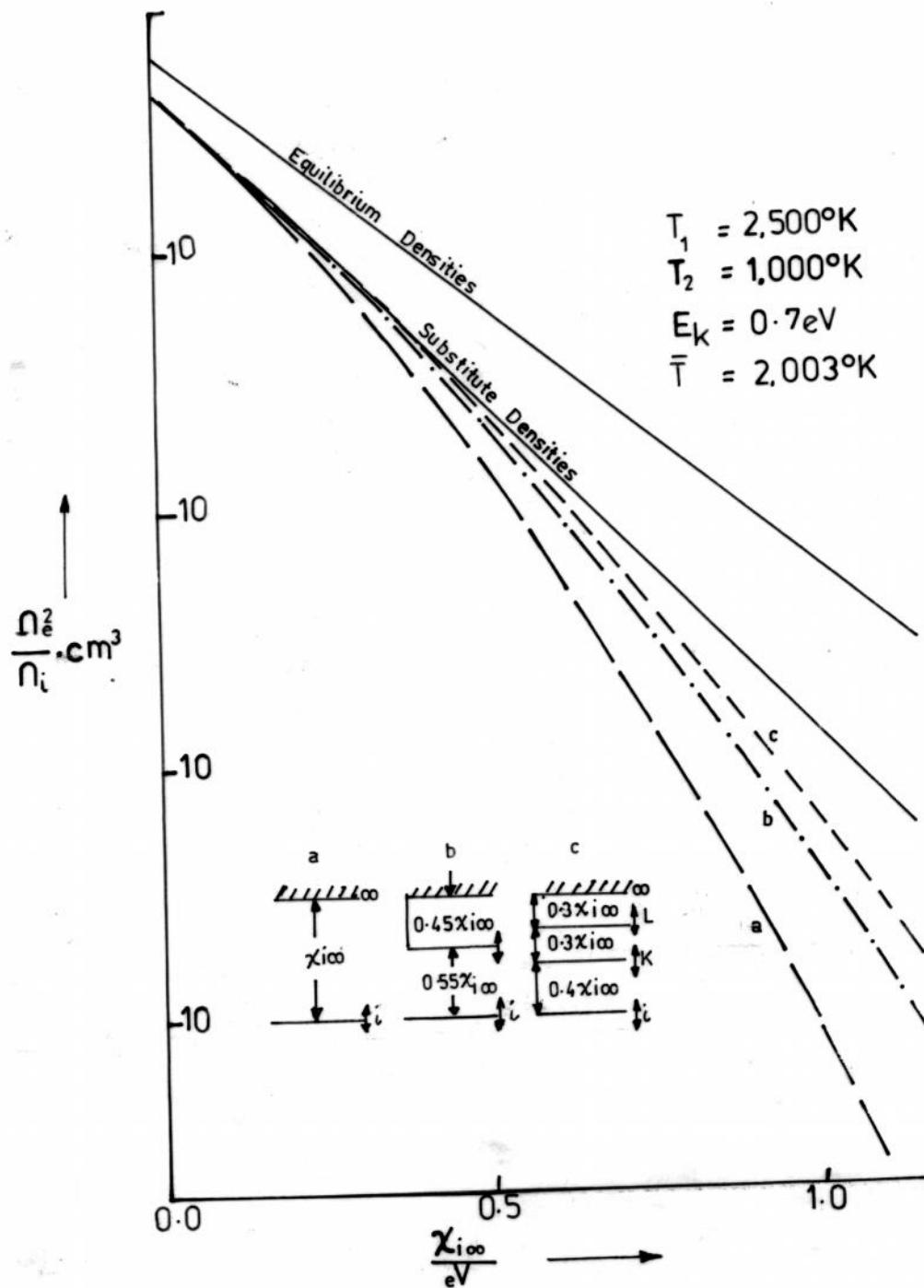Fig. 4 : Density Relationships versus Binding energy $\chi_{i\infty}$

Fig.5 : Density Relationships versus Binding energy $\chi_{i\infty}$ for different Model Atoms.

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