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A SET OF RULES FOR CONSTRUCTING AN ADMISSIBLE SET OF D-OPTIMAL EXACT DESIGNS

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ABSTRACT

In the search for a D-optimal exact design using the combinatorial iterative technique introduced by Onukogu and Iwundu, 2008, all the support points that make up the experimental region are grouped into H concentric balls according to their distances from the centre. Any selection of N support points from the balls defines a class of designs ξ_N. The technique requires the identification of best D-optimal exact design(s) within a class and moves progressively from one class to another with a higher determinant value until it reaches an optimum.

Quite often, each class contains several N-point designs and a hundred per cent search is required to identify best design(s) within a class. A set of rules for selecting best design(s) within a class, alternative to one hundred per cent search is developed. The effectiveness of the rules is measured by the reduction in the number of determinant evaluations relative to a hundred per cent search

KEY WORDS: Concentric Ball; D-optimal Exact Designs; Class of Designs; Admissible Designs; Support Point.

1. INTRODUCTION OF THE PROBLEM

Given a response function $f(\underline{x})$ $\underline{x} \in \tilde{X}$ ($\tilde{X}$ is the experimental region), the problem of optimal experimental design is to select N points out of the $\tilde{N}$ support points contained in a given experimental region $\tilde{X}$ so as to optimize the value of a criterion function $\phi(.)$. Over the years, a good number of methods (both analytical and iterative) have been developed for achieving this aim. See for instance Fedorov (1972), Atkinson and Donev (1992) and Onukogu (1997). These methods are developed according to the optimality criterion of interest such as D-, A-, E- and G-optimality criteria for both exact and continuous designs.

In the technique developed by Onukogu and Iwundu (2008), all the $\tilde{N}$ distinct support points that make up the experimental region are grouped into H concentric balls g_1, g_2, …, g_H according to their distances from the centre of the experimental region such that g_1 contains the support points farthest away from the centre, g_2 contains the support points second farthest away from the centre, …, and g_H contains the support points nearest to the centre of the experimental region. A ball g_h (h = 1, 2, …, H) defined on a Euclidean space R^m is an m-directional sphere with radius d_h, Chidume (1989) and balls defined on an experimental region are said to be concentric if they are centered at the same point in the region with different radii, such that the ball with the smallest radius takes the innermost position in the region, while the ball with the highest radius takes the outermost position in the region. The radius of the hth ball is $d_h = \left(\underline{x}'_{ih}\underline{x}_{ih}\right)^{1/2}$ $i = 1, 2, ..., n_h$ where n_h is the number of support points in ball h and $d_h > d_{h+1}$, h = 1, 2, …, H-1.

The $\tilde{N}$ distinct support points in the experimental region are grouped such that n_1 support points are contained in ball one, n_2 support points are contained in ball two, …, and n_H support points are contained in ball H, where $n_1 + n_2 + \ldots + n_H = \tilde{N}$ class of N. Then, a class of N-point designs is defined by the set $\{N_1, N_2, ..., N_H\}$, where N_1 support points are selected from the n_1 support points in g_1, N_2 support points are selected from the n_2 support points in g_2 and so on such that $N_1 + N_2 + \ldots + N_H = N$ and the size of a class is defined as the number of N-point designs in the class; such that size of the kth class is given by: $Z_k = \binom{n_1}{N_1}\binom{n_2}{N_2}\cdots\binom{n_H}{N_H} = \prod_{h=1}^{H}\binom{n_h}{N_h}$, k = 1, 2, ….

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This varies from class to class and in some cases, it is always large.

To obtain the best D-optimal exact design within a class from the several N-point designs contained in the class, one hundred per cent search is required. In this case, all the N-point designs are to be considered. For instance, consider a 5-point design defined in the experimental region $\tilde{X} = \{x_1, x_2 : x_1, x_2 = -1, -1/2, 0, 1/2, 1\}$ for a first order linear bivariate response function: $f(x_1, x_2) = a_{00} + a_{10}x_1 + a_{20}x_2 + a_{12}x_1x_2 + e$. The experimental region is given geometrically by:

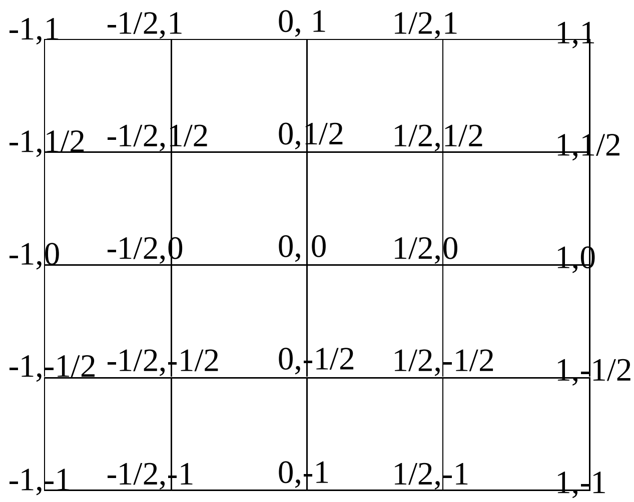

Figure 1.1 A 25-design point squared experimental region.

The twenty-five (25) distinct points in the region are grouped into six (6) concentric balls with their corresponding ball sizes as given below:

$$g_1 = \begin{pmatrix} -1 & 1 \\ 1 & -1 \\ -1 & -1 \\ 1 & 1 \end{pmatrix}, g_2 = \begin{pmatrix} -1/2 & 1 \\ 1/2 & -1 \\ -1/2 & -1 \\ 1/2 & 1 \\ -1 & 1/2 \\ 1 & -1/2 \\ -1 & -1/2 \\ 1 & 1/2 \end{pmatrix}, g_3 = \begin{pmatrix} 0 & -1 \\ 0 & 1 \\ -1 & 0 \\ 1 & 0 \end{pmatrix}, g_4 = \begin{pmatrix} -1/2 & 1/2 \\ 1/2 & -1/2 \\ -1/2 & -1/2 \\ 1/2 & 1/2 \end{pmatrix}, g_5 = \begin{pmatrix} -1/2 & 0 \\ 1/2 & 0 \\ 0 & -1/2 \\ 0 & 1/2 \end{pmatrix}, g_6 = \begin{pmatrix} 0 & 0 \end{pmatrix};$$

$$n_1 = n_3 = n_4 = n_5 = 4, n_2 = 8, n_6 = 1.$$

Define a class of 5-point designs as one with $N_1 = 1$, $N_2 = 2$, $N_3 = 1$, $N_4 = 1$, $N_5 = 0$ and $N_6 = 0$ such that the size of the class is $\binom{4}{1}\binom{8}{2}\binom{4}{1}\binom{4}{1}\binom{4}{0}\binom{1}{0} = 1792$. This simple illustrative example requires one thousand seven hundred and ninety two determinant evaluations in order to identify a local D-optimal design using the one hundred per cent search. This sequential procedure finds it difficult to reach a global optimum.

The purpose of this paper therefore is to provide an alternative to a hundred per cent search by introducing a set of rules which reduces the number of determinant evaluations within each class. The application of the rules leads to an admissible set of designs within each class. A set of design measures is said to be an admissible set if there is no design outside the set which is better than any one within the set, Onukogu (1997).

2 FUNDAMENTAL CONCEPTS

2.1 The Information Matrix of a Design

Consider an N-point design measure for an n-variate response surface:

$$y(\underline{x}) = f(x_1, x_2, \ldots, x_n) + e,$$

defined in an experimental region $\tilde{X}$.

$$\xi_N = \begin{bmatrix} \underline{x}'_1 & \omega_1 \\ \underline{x}'_2 & \omega_2 \\ \dots & \dots \\ \underline{x}'_N & \omega_N \end{bmatrix}; \qquad \underline{x}'_i = (x_{i1}, x_{i2}, \dots, \underline{x}'_{in}) \quad \omega_i \ge 0; \ \sum_{i=1}^{N} \omega_i = 1$$

where n is the number of variables (factors) in the experiment and ω_i is the probability of selecting the design point $\underline{x}_i$, that is the weight of design point $\underline{x}_i$. This design measure is said to be exact if the number of trials is specified. If on the other hand the distribution of trials over the experimental space is only specified by a measure ξ regardless of the number of support points N, the design is called a continuous or an approximate design, Atkinson and Donev (1992). The N x p coefficient (design) matrix for the design measure ξ_N is defined by:

$$\underset{N\times P}{X}(\xi_N) = (x_{ki}) = \begin{pmatrix} \underline{x}'_1 \\ \underline{x}'_2 \\ \dots \\ \underline{x}'_N \end{pmatrix} = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1P} \\ x_{21} & x_{22} & \dots & x_{2P} \\ \dots & \dots & \dots & \dots \\ x_{N1} & x_{N2} & \dots & x_{NP} \end{pmatrix}; k = 1, 2, \dots, N, \ and \ i = 1, 2, \dots, p$$

$$\text{Then, } M(\xi_N) = \mathrm{X}(\xi_{\mathrm{N}})' X(\xi_N) = (\mathrm{m_{ij}}) = \begin{bmatrix} \mathrm{m}_{11} & \mathrm{m}_{12} & \dots & \mathrm{m}_{1\mathrm{p}} \\ \mathrm{m}_{21} & \mathrm{m}_{22} & \dots & \mathrm{m}_{2\mathrm{p}} \\ \dots & \dots & \dots & \dots \\ \mathrm{m}_{\mathrm{p}1} & \mathrm{m}_{\mathrm{p}2} & \dots & \mathrm{m}_{\mathrm{pp}} \end{bmatrix}; i, j = 1, 2, \dots, p$$

$$where \ m_{ii} = \sum_{k=1}^{N} x_{ki}^2 \ \forall \ i = i \ and \ m_{ij} = \sum_{k=1}^{N} x_{ki} x_{kj} \ \forall \ i < j$$

If the model $f(\underline{x})$ defining an experimental design is unbiased, then the adequacy of the design is determined by its information matrix. Therefore, the admissibility (or in more strict terms optimality) of a design is determined by comparing its information matrix (or some operators of its information matrix) with those of others.

2.2 D-Optimal Designs within a Class

Onukogu (1997) defines D-optimality criterion as a function of the information matrix of a design:

$$\phi\left[M(\xi_N)\right] = \det\left[M(\xi_N)\right] = -\log \ \det\left[M(\xi_N)\right]$$

A design ξ_N is therefore said to be D-optimal if

$$\det\left[M^{-1}(\xi_N^*)\right] = \min\left\{\det\left[M^{-1}(\xi_N)\right]\right\} \quad \forall \ M \ (\xi_N) \in S^{P\times P}$$

or

$$\det\left[M(\xi_N^*)\right] = \max\left\{\det\left[M(\xi_N)\right]\right\} \quad \forall \ M \ (\xi_N) \in S^{P\times P}$$

where $S^{P\times P}$ is a set of all the possible p×p non-singular information matrices.

In a class with class size of Z_k, let $S_Z^{P\times P}$ be a set of all the p×p non-singular information matrices in the class, $S_Z^{P\times P} \le Z_k$.

Then a design ξ_N^* is said to be D-optimal within the class if

$$\det\left[M(\xi_N^*)\right] = \max\left\{\det\left[M(\xi_N)\right]\right\} \quad \forall M \ (\xi_N) \in S_Z^{P\times P}.$$

Statement 2.2.1

Given two non-singular pxp symmetric matrices $A = (a_{ij})$ and $B = (b_{ij})$, then $\det(A) > \det(B)$ if

(1). $\|a_{ij}\| \le \|b_{ij}\| \ \forall \ i, j; \ i \ne j; i, j = 1, 2, \dots, p$ and (2). $a_{ii} > b_{ii} \ \forall \ i, i = 1, 2, \dots, p$.

Proof

Theorems by Searle (1982) and Arua et al (1999) state that addition of any multiple of a row (column) of a matrix to another row (column) of the same matrix does not affect the value of the determinant. Based on that, Rao (1973), Searle (1982) and Kreyszig (1999) state that the determinant of any matrix can be obtained by taking the product of the diagonal elements of the reduced triangular matrix obtained from it.

Applying the above statements, let $M = (m_{ij})$ be a non-singular p×p information matrix .

$$M = \left(m_{ij}\right) = \begin{pmatrix} m_{11} & m_{12} & m_{13} & \dots & m_{1p} \\ m_{21} & m_{22} & m_{23} & \dots & m_{2p} \\ m_{31} & m_{32} & m_{33} & \dots & m_{3p} \\ \dots & \dots & \dots & \dots & \dots \\ m_{p1} & m_{p2} & m_{p3} & \dots & m_{pp} \end{pmatrix}$$

Let $M^{(P-1)}$ be the reduced triangular form of the matrix M. Using Gaussian elimination technique, we obtain $M^{(P-1)}$ as follows:

To obtain $M^{(1)}$ which is the reduced matrix of M after the first iteration, m_{11} is made the pivotal element. Row one is multiplied by $-\frac{m_{i1}}{m_{11}}$ and added to row i (i = 2, 3,…, p) to have

$$M^{(1)} = \left(m_{ij}^{(1)}\right) = \begin{pmatrix} m_{11} & m_{12} & m_{13} & \dots & m_{1P} \\ 0 & m_{22}^{(1)} & m_{23}^{(1)} & \dots & m_{2P}^{(1)} \\ 0 & m_{32}^{(1)} & m_{33}^{(1)} & \dots & m_{3P}^{(1)} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & m_{P2}^{(1)} & m_{P3}^{(1)} & \dots & m_{PP}^{(1)} \end{pmatrix},$$

where $m_{ij}^{(1)} = m_{ij} - \left\|\frac{m_{i1}m_{1j}}{m_{11}}\right\|; \quad i > 1; i = j \quad \text{and} \quad \left\|m_{ij}^{(1)}\right\| = \left\|m_{ij}\right\| + \left\|\frac{m_{i1}m_{1j}}{m_{11}}\right\|; i > 1, i \neq j$.

To obtain $M^{(2)}$ which is the reduced matrix of M after the second iteration, $m_{22}^{(1)}$ is made the pivotal element. Row two of $M^{(1)}$ is multiplied by $-\frac{m_{i2}^{(1)}}{m_{22}^{(1)}}$ and added to row i (i = 3,4,…,p) to have

$$M^{(2)} = \left(m_{ij}^{(2)}\right) = \begin{pmatrix} m_{11} & m_{12} & m_{13} & \dots & m_{1P} \\ 0 & m_{22}^{(1)} & m_{23}^{(1)} & \dots & m_{2P}^{(1)} \\ 0 & 0 & m_{33}^{(2)} & \dots & m_{3P}^{(2)} \\ 0 & 0 & m_{43}^{(2)} & \dots & m_{4P}^{(2)} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & m_{P3}^{(1)} & \dots & m_{PP}^{(1)} \end{pmatrix}$$

where $m_{ij}^{(2)} = m_{ij}^{(1)} - \left\|\frac{m_{i2}^{(1)}m_{2j}^{(1)}}{m_{22}^{(1)}}\right\|; \quad i > 2, \ i = j \quad \text{and} \ \left\|m_{ij}^{(2)}\right\| = \left\|m_{ij}^{(1)}\right\| + \left\|\frac{m_{i2}^{(1)}m_{2j}^{(1)}}{m_{22}^{(1)}}\right\|; \quad i > 2, \ i \neq j$.

The iteration continues until we obtain $M^{(P-1)}$, an upper triangular matrix of M as:

$$M^{(p-1)} = \left(\mathrm{m}_{\mathrm{ij}}^{(\mathrm{p-1})}\right) = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & \dots & m_{1p} \\ 0 & m_{22}^{(1)} & m_{23}^{(1)} & m_{24}^{(1)} & \dots & m_{2p}^{(1)} \\ 0 & 0 & m_{33}^{(2)} & m_{34}^{(2)} & \dots & m_{3p}^{(2)} \\ 0 & 0 & 0 & m_{44}^{(3)} & \dots & m_{4p}^{(3)} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & m_{pp}^{(p-1)} \end{bmatrix}$$

At the end of kth iteration,

$$m_{ij}^{(k)} = m_{ij}^{(k-1)} - \left\| \frac{m_{ik}^{(k-i)} m_{kj}^{(k-1)}}{m_{kk}^{(k-1)}} \right\|; \ i > k; i = j, \text{ and } \left\| m_{ij}^{(k-1)} \right\| = \left\| m_{ij}^{(k-1)} \right\| + \left\| \frac{m_{ik}^{(k-1)} m_{kj}^{(k-1)}}{m_{kk}^{(k-1)}} \right\|; \ i > k, \ i \neq j$$

Hence the rate of change of both $m_{ij}^{(k)}; i = j$ and $\left\| m_{ij}^{(k)} \right\|; i \neq j$ for all j at each kth iteration is $\left\| \frac{m_{ik}^{(k-1)}}{m_{kk}^{(k-1)}} \right\|$. Now comparing the matrices A and B at the end of their kth iterations, $\left\| \frac{a_{ik}^{(k-1)}}{a_{kk}^{(k-1)}} \right\| < \left\| \frac{b_{ik}^{(k-1)}}{b_{kk}^{(k-1)}} \right\| \forall i, k$ .So $a_{ik}^{(k)} > b_{ik}^{(k)}; \ i = j, \ \forall \ i > k$ and $\left\| a_{ik}^{(k)} \right\| \leq \left\| b_{ik}^{(k)} \right\|; i \neq j \forall \ i > k$.

Define $\det[M] = \det\left[M^{(p-1)}\right] = \prod_{i=1}^{p}\left(m_{ii}^{i-1}\right)$; then $\prod_{i=1}^{p} a_{ii}^{(i-1)} > \prod_{i=1}^{p} b_{ii}^{(i-1)}$; therefore det(*A*) > det(*B*).

It is evident from the above statement that to obtain a D-optimal design within a class, the information matrix of all the Z_k designs within the class will be investigated. Then ξ_N^* is D-optimal within the class if it has the least pair-wise absolute off-diagonal elements and the highest pair-wise diagonal elements among all the designs in the class.

Corollary to statement 2.2.1

Define a 1/2p(p-1) dimensional vector of absolute ratios of the off-diagonal elements to the diagonal element appearing in the same column of the information matrix of a design in a class of designs by: $\underline{v} = \left(v_1, \ v_2, \ v_3, \ \ldots, \ v_{1/2\,p(p-1)}\right)$

$Then, \ \det\left[M\left(\xi_N^*\right)\right] = \max\left\{\det\left[M\left(\xi_N\right)\right]\right\} \text{ over all } M\left(\xi_N\right) \in S_z^{pxp}$ if

1). $\left\| \sum_{k=1}^{N} x_{kj} x_{kj'} \right\|$ is minimum

2). $\sum_{k=1}^{N} x_{kj}^2$ is maximum, such that

3). $\theta^* = \max\left\{\theta_i; \ i = 1, 2, \ldots, z_k\right\}$, and

4). $\underline{v}^* = \min\left\{\underline{v}_i; \ i = 1, 2, \ldots, z_k\right\}$,

where θ_i is the product of the diagonal elements in the information matrix of the ith design and $\underline{v}_i$ is the vector of ratios as defined above for the ith design in the kth class.

Proof:

From statement 2.2.1 above, at each kth iteration the ith diagonal element is reduced by $\left\| \frac{m_{ik}^{(k-1)} m_{kj}^{(k-1)}}{m_{kk}^{(k-1)}} \right\|$; i = j. The absolute ratio of the kth off-diagonal element to its corresponding diagonal element increases (decreases) by $\left\| \frac{m_{ik}^{(k-1)}}{m_{kk}^{(k-1)}} \right\|$. Consequently, the reduction in the ith diagonal element leads to a reduction in the product of the p diagonal elements which by definition is the determinant of the information matrix when all the necessary reductions have been made.

Based on the corollary above, any design that satisfies the above conditions is said to belong to an admissible set of D-optimal exact designs.

3. NUMERICAL DEMONSTRATION

To demonstrate the following rules, we shall use a bivariate first order linear interactive model defined in a regular experimental region $\tilde{X}$ = {x_1, x_2: x_1 = -1, 0, 1; x_2 = -2, -1, 0, 1, 2} without blocking. The 15 support points in the experimental region are grouped into five balls as shown below:

with n_1 = 4, n_2 = 2, n_3 = 4, n_4 = 4 and n_5 1.

(1). Consider a six-point design for a bivariate linear response function: $f\left(x_1 x_2\right) = a_{00} + a_{10}x_1 + a_{20}x_2 + a_{12}x_1x_2 + e$.

$$g_1 = \begin{pmatrix} -1 & 2 \\ 1 & -2 \\ -1 & -2 \\ 1 & 2 \end{pmatrix};\ g_2 = \begin{pmatrix} 0 & -2 \\ 0 & 2 \end{pmatrix};\ g_3 = \begin{pmatrix} -1 & 1 \\ 1 & -1 \\ -1 & -1 \\ 1 & 1 \end{pmatrix};\ g_4 = \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix};\ g_5 = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

Selected Class: $N_1 = 5$, $N_2 = 0$, $N_3 = 0$, $N_4 = 1$, $N_5 = 0$, and size, $Z_k = \binom{4}{1}\binom{2}{0}\binom{4}{0}\binom{4}{1}\binom{1}{0} = 16$.

Table 3.1: Table of Results

S/N of Designs	Set of $\sum_{k=1}^{N} x_{kj}^2$	θ_i	$\underline{v}_i'$	Determinant
1	{6,5,21,20}	12600	-1, 1, -2, -2, 2, -4	10688
2	{6,5,21,20}	12600	1, -3, -2, -2, -2, 4	10176
3	{6,5,21,20}	12600	-1, -3, 2,2, -2, -4	10176
4	{6,5,21,20}	12600	1, 1, 2, 2, 2, 4	10688
5	{6,5,21,20}	12600	-1, 3, -2, -2, 2, -4	10176
6	{6,5,21,20}	12600	1, -1, -2, -2, -2, 4	10688
7	{6,5,21,20}	12600	-1, -1, 2, 2, -2, -4	10688
8	{6,5,21,20}	12600	1, 3, 2, 2, 2, 4	10176
9	{6,6,20,20}	14400	-2, 2, -2, -2, 2, -4	11264
10	{6,6,20,20}	14400	0, -2, -2, -2, -2, 4	12288*
11	{6,6,20,20}	14400	-2, -2, 2, 2, -2, -4	11264
12	{6,6,20,20}	14400	0, 2, 2, 2, 2, 4	12288*
13	{6,6,20,20}	14400	2, 2, 2, 2, 2, 4	11264
14	{6,6,20,20}	14400	0, 2, -2, -2, 2, -4	12288*
15	{6,6,20,20}	14400	2, -2, -2, -2, -2, 4	11264
16	{6,6,20,20}	14400	0, -2, 2, 2, -2, -4	12288*

(2). Consider a seven-point design for the same bivariate linear response function: $f(x_1x_2) = a_{00} + a_{10}x_1 + a_{20}x_2 + a_{12}x_1x_2 + e$

Selected Class: N_1=4, N_2=1, N_3=2, N_4=0 and N_5=0, with size, $Z_k = \binom{4}{4}\binom{2}{1}\binom{4}{2}\binom{4}{0}\binom{1}{0} = 12$.

Table 3.2: Table of Results.

S/N of Designs	Set of $\sum_{k=1}^{N} x_{kj}^2$	θ_i	$\underline{v}_i'$	Determinant
1	{7, 6, 22, 18}	16632	0, -2, -2, -2, 0, 0	15184
2	{7, 6, 22, 18}	16632	-2, -2, 0, 0, 0, -2	14464
3	{7, 6, 22, 18}	16632	0, 0, 0, 0, 2, 0	16016*
4	{7, 6, 22, 18}	16632	0, -4, 0, 0, -2, 0	14352
5	{7, 6, 22, 18}	16632	2, -2, 0, 0, 0, 2	14464
6	{7, 6, 22, 18}	16632	0, -2, 2, 2, 0, 0	15184
7	{7, 6, 22, 18}	16632	0, 2, -2, -2, 0, 0	15184
8	{7, 6, 22, 18}	16632	-2, 2, 0, 0, 0, -2	14464
9	{7, 6, 22, 18}	16632	0, 4, 0, 0, 2, 0	14352
10	{7, 6, 22, 18}	16632	0, 0, 0, 0, -2, 0	16016*
11	{7, 6, 22, 18}	16632	2, 2, 0, 0, 0, 2	14464
12	{7, 6, 22, 18}	16632	0, 2, 2, 2, 0, 0	15184

Note that the classes were chosen arbitrarily. (*) shows D-optimal exact designs in the classes.

4. CONCLUSION

In this paper, D-optimal exact designs within a class of designs are examined and it is established that such designs must satisfy the conditions given in corollary 2.2.1 in section two of this paper. The set of rules were applied in the two examples in section three above in the regular experimental region: $\tilde{X}$ = {x_1, x_2: x_1 = -1, 0, 1; x_2 = -2, -1, 0, 1, 2} for a bivariate response function: $f\left(x_1x_2\right) = a_{00} + a_{10}x_1 + a_{20}x_2 + a_{12}x_1x_2 + e$. At the end, four best D-optimal exact designs with equal determinant value of 12288 were identified out of the sixteen possible 6-point designs in the class. Also, two D-optimal exact designs were identified in the second example with equal determinant of 16016. Finally, a computer-based software for determining the best D-optimal exact design within a class will be computationally easier to be developed in respect of the rules than applying a hundred per cent search.

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