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POINCARE SURFACE ANALYSIS OF TWO COUPLED QUINTIC OSCILLATORS IN A SINGLE WELL POTENTIAL

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ABSTRACT

We have investigated the chaotic dynamics of two coupled quintic oscillators in a single well potential as the energy of the oscillator increases, keeping the coupling strength constant. The degree of chaoticity does not increase monotonously with the energy as regular regions reappear within chaotic seas as the energy increases. After the critical energy, however, the motion becomes fully chaotic.

KEYWORDS: Oscillators, Quintic potential, Degree of chaoticity, Coupling, Poincare surface

1.0 INTRODUCTION

The Poincare surface of section, which is one of the commonly used methods in the study of chaotic systems, effectively reduces the number of dimensions needed to analyse such a system. Thus, for a two degree of freedom Hamiltonian system, the Poincare surface is two dimensional, a plane. Traditionally, this could be defined by keeping one of the coordinates constant, and reckoning with only positive values of the corresponding momentum. The latter condition ensures that crossings in only one direction are of relevance.

The Poincare surface of section is the most commonly used method of analyzing chaotic systems. As such, it has found application in chaos literature: the Henon-Heiles system (Henon and Heiles, 1964; Henon, 1981), hydrogen atom in the presence of a uniform magnetic field (Friedrich and Wintgen, 1989; Simonovic, 1997; Inarrea, et al., 2002; Rajan, et al., 2003) and control in the problem of a satellite (Khan, 2010), to mention a few.

As some parameter of the system, energy or coupling increases, the Poincare surface of a chaotic system is usually initially entirely characterised by regular regions, which subsequently degenerate into regions demonstrating regular motion and chaoticity as Kolmogorov-Arnold-Moser (KAM) tori are destroyed. Eventually, after the critical value of the parameter, coupling or energy, the whole surface is covered by chaotic regions. For some systems, the degree of chaoticity increases monotonously with increase in the parameter (Henon, 1964; Friedrich and Wintgen, 1989). In some other chaotic systems, however, the degeneration into chaoticity is not a unidirectional phenomenon as regions of regular motion appear inside chaotic seas as the parameter increases. In some cases there is a regular-partial chaos-regular transition as observed in the case of two linearly coupled double-well oscillators (Paar and Pavin, 2003) or a regular-chaotic-regular transition as reported in the case of a finite chain of damped spins (Jaroszewicz and Sukiennicki, 1998).

The quintic potential is of the form $V(q) = m^2q^2 + gq^4 + hq^6$. The shape of this potential, with the right choice of the parameters m, g and h could be a single well, double well, triple well, single well with double hump, double well with double hump or an inverted single well. This potential has received much attention in recent times. Wang and Yu (2005) explores the bifurcation of limit cycles for planar polynomial systems with even number of degrees. Ganji, et al. (2008) explored the variational approach method for the nonlinear oscillators of the motion of a rigid rod rocking back and cubic-quintic doffing oscillators, Ganji, et al. (2009) applied He's energy balance method to solve strong nonlinear doffing oscillators with cubic-quintic nonlinear restoring force, and Jeyakumari, et al. (2009) analysed the occurrence of vibrational resonance in a damped quintic oscillator.

The above-mentioned authors considered the dynamics of only one quintic oscillator. The present study considers a Hamiltonian system of two coupled quintic oscillators as the coupling parameter increases from a negative value, through zero, and then to positive values. It is known that in some chaotic systems only one range of values of the coupling leads to chaoticity while the other range engenders only regular behaviour as is the case in Friedrich and Wintgen (1989).

2. Theory and Calculations

The Hamiltonian of each of the oscillators is of the form,

$$V = \frac{1}{2}q^2 + \frac{1}{4}q^4 + \frac{1}{6}q^6 \qquad (1)$$

which is a single well potential with a local minimum at $x = 0$ (Fig. 1). The line E = 1.0 in the figure represents the energy of the oscillator chosen for this work.

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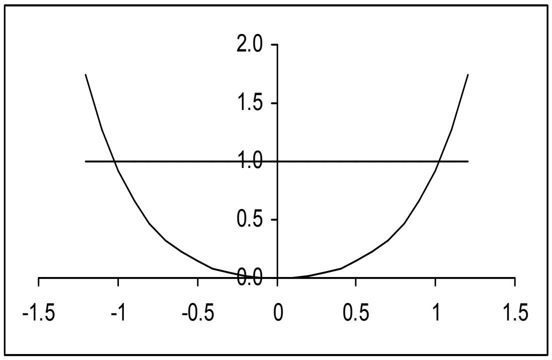


Fig. 1: The potential *V*(*q*) and energy E = 1.0

With the energy fixed, the displacement of the oscillator is limited by the extents of the potential.

When two such oscillators are coupled, the resulting potential is,

$$V=\frac{1}{2}(q_1^2+q_2^2)+\frac{1}{4}(q_1^4+q_2^4)+\frac{1}{6}(q_1^6+q_2^6)+\frac{s}{2}(q_1^2q_2^2) \quad (2)$$

where the coupling parameter is s. When s is zero, the two oscillators are effectively decoupled. The coupling term, $s(q_1^2q_2^2)/2$ has been chosen to ensure that the potential is symmetric for q_1 and q_2.

The corresponding Hamiltonian is,

$$H=\frac{1}{2}(p_1^2+p_2^2)+\frac{1}{2}(q_1^2+q_2^2)+\frac{1}{4}(q_1^4+q_2^4)+\frac{1}{6}(q_1^6+q_2^6)+\frac{s}{2}(q_1^2q_2^2) \quad (3)$$

The resulting equations of motion are:

$$\frac{dq_1}{dt}=p_1 \quad (4)$$

$$\frac{dq_2}{dt}=p_2 \quad (5)$$

$$\frac{dp_1}{dt}=-(q_1+q_1^3+q_1^5+s\,q_1q_2^2) \quad (6)$$

$$\frac{dp_2}{dt}=-(q_2+q_2^3+q_2^5+s\,q_1^2q_2) \quad (7)$$

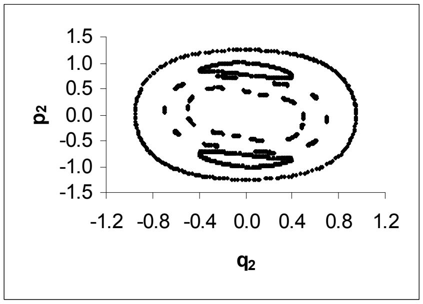
For a given coupling strength, 0.5, the system of equations (4) to (7) was solved by the fourth-order Runge-Kutta method. The phase space is four dimensional, (q_1,q_2,p_1,p_2), representing the two coordinates and the corresponding momenta. A subspace of the phase space is defined by the Poincare surface as ($q_1=0$, $p_1>0$). Setting $q_1=0$ fixes one of the coordinates; $p_1>0$ ensures that the Poincare surface is crossed in only one direction. Thus, the Poincare surface is characterised by the coordinate and momentum, (q_2,p_2). With various initial conditions, the intersections of the orbits with the two-dimensional surface, (q_2,p_2) were noted and plotted (Figs. 2 and 3).

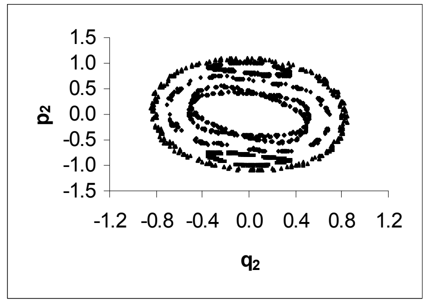
3. RESULTS AND DISCUSSION

Up to energy 0.8, the Poincare surface consists of only regular orbits (Fig. 2a). Fig. 2b shows that by energy 0.85, homoclinic points are evident. In Fig. 2c, the energy is 0.9, and by this energy, chaotic orbits have appeared. The innermost regular region shown in Fig. 2c has degenerated into a homoclinic orbit at energy 0.95 (Fig. 2d). The Poincare surface of section varies little between energy 0.95 and 1.0 as is evident in Fig. 2e (compared to 2d). However, the chaotic sea grows at the expense of the three regular regions between energies 1.0 and 1.2 (Fig. 2f). With a further increase in the energy to 1.4, the middle regular region narrows and divides into two, albeit with the upper and lower regular regions slightly increasing (Fig. 2g). Fig. 2h shows that at energy 1.6, the central regular regions have disappeared. By energy 1.7, the upper and the lower regular regions have increased in extent. In addition, two new areas of regular motion are evident (Fig. 3a). With the energy increasing to 1.75, all the regular regions hitherto visible have shrunken and are barely perceptible (Fig. 3b). Thereafter, the upper and lower regular regions disappear altogether, but the ones in the middle of the figure widen until the energy is 1.9 (Fig. 3c). These remain practically unchanged until the energy is 2.0 (Fig. 3d). From energy 2.0 to 2.1, the remaining regular regions shrink (Fig. 3e). If Fig. 3f, these regular regions have become two localised chaotic seas. In Fig. 3g, the two localised seas have degenerated further as the energy increases to 2.18, giving rise to the outer fringe surrounding the entire Poincare surface. Beyond this energy, the Poincare surface shows that the system is fully chaotic (Fig. 3h).

4. CONCLUSION

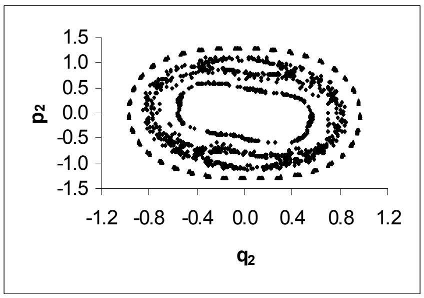
The motion of a coupled quintic oscillator in a single well with a fixed coupling strength 0.5 demonstrates regular and chaotic behaviour as the energy increases. Only regular motion is observed before energy 0.85. Chaotic motion has sets in at energy 0.9. The degree of chaoticity increases until the energy is 1.6. Between 1.6 and 1.7, the regular regions grow in extent. The regular regions decay yet again until the energy is 1.75. Another set of regular regions open up after energy 1.75, increasing in extent until 1.9 value of energy. From energy 2.0, the system becomes progressively more chaotic as the regular regions shrink. Beyond the threshold energy 2.18, the system becomes fully chaotic, and remains so as the energy increases further.

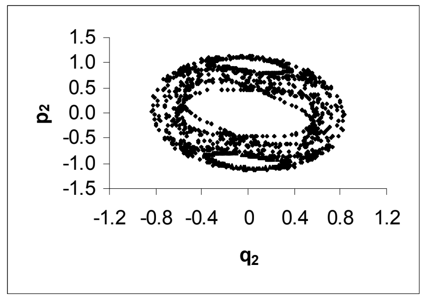




(a) $E = 0.8$

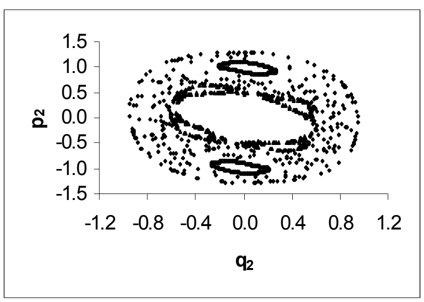
(b) $E = 0.85$

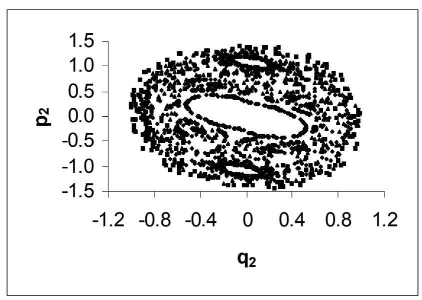




(c) $E = 0.9$

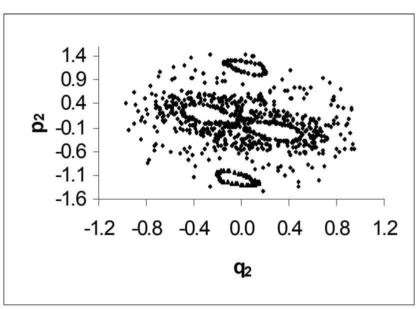
(d) $E = 0.95$

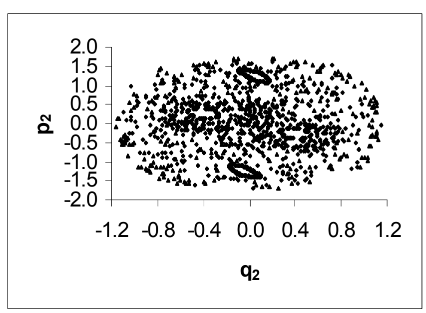




(e) $E = 1.0$

(f) $E = 1.2$

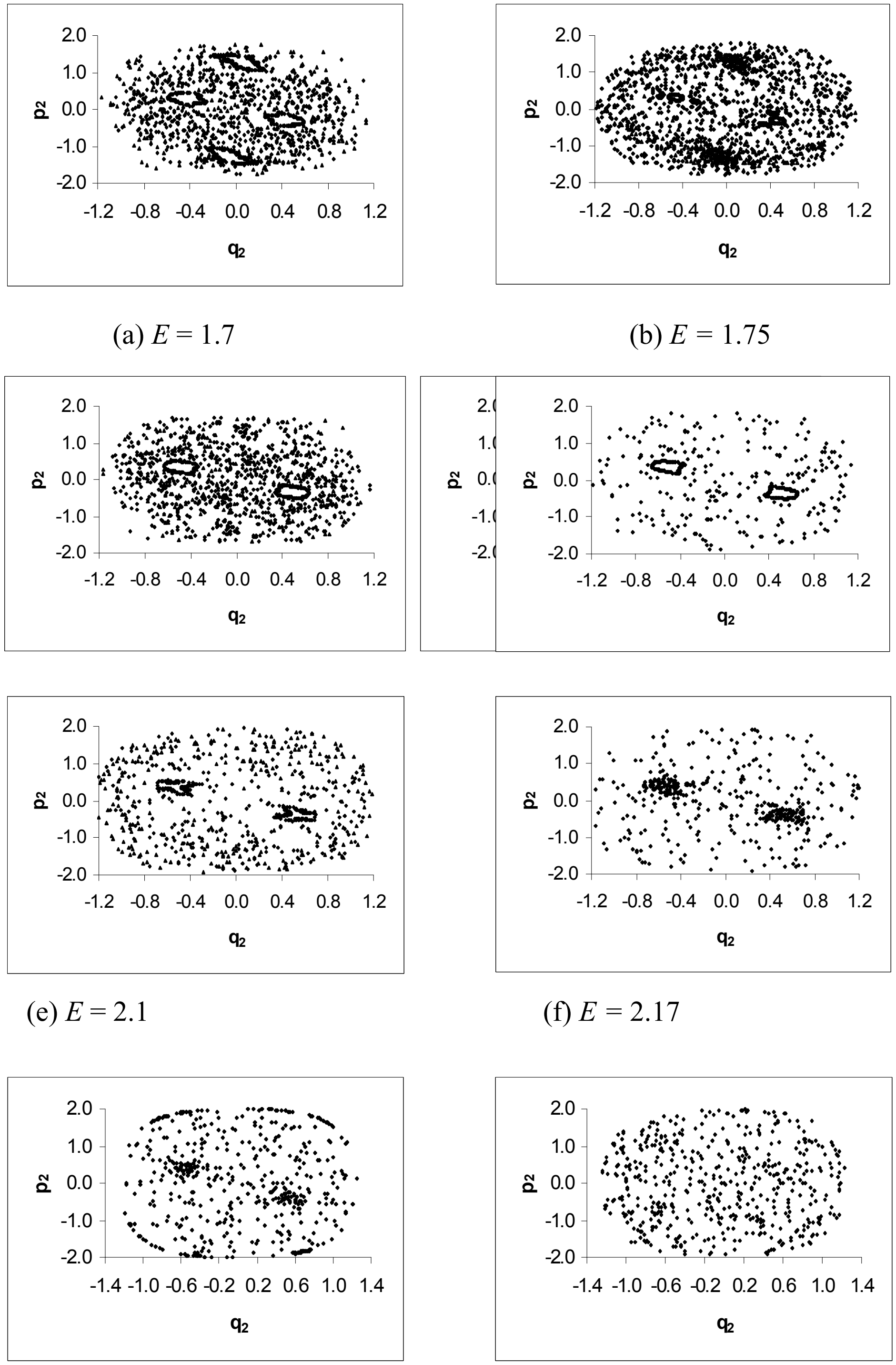




(g) $E = 1.4$

(h) $E = 1.6$

Fig. 2: Poincare surface of section for $s = 0.5$, with energy from 0.8 to 1.6



(a) $E = 1.7$

(b) $E = 1.75$

(e) $E = 2.1$

(f) $E = 2.17$

(g) $E = 2.18$

(h) $E = 2.19$

Fig. 3: Poincare surface of section for *s* = 0.5, with energy from 1.7 to 2.19

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