

DERIVATION OF EPSILON (E) AND PI (π) FROM THE KINETIC MOLECULAR THEORY OF MATTER

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ABSTRACT

This paper considers the possible physical origins of the important natural constants epsilon ($e = 2.7182\dots$) and pi ($\pi = 3.1415\dots$). They are suggested to originate from the kinetic molecular nature of matter. Epsilon (e) is suggested to be the ratio of the driving force on a randomly moving particle accelerated with a quantum of energy, for instance thermal action (kT), in a specified direction to the restoring force acting in the opposite direction. Pi (π), on the other hand, is suggested to be the ratio of the sum of all the other forces pulling the particle in different directions, towards random motion, to the driving force in the direction of acceleration. The paper proposes new methods for the estimation of these parameters from physical first principles.

KEYWORDS: kinetic molecular, epsilon, pi, random motion, natural constants, quantum of energy.

INTRODUCTION

The natural numbers epsilon (e) and Pi (π) are two of the most important parameters in nature (Uvarov, 1979; Krasnov, 1987; Dickerson, 1969; Dence, 1975). They are very widely applied in mathematical and natural systems (Moore, 1963; Botlyansky, 1977; Zachmann, 1972). Base of the napieran logarithm, epsilon, should not be confused with the molar absorptivity discussed in Beer's law (Joseph, 2005; Strong, 1952). But the actual physical origin of both epsilon and pi is yet to be fully understood (<http://en.wikipedia.org/wiki/pi>). These constants tend to occur in natural processes or systems in which dynamism is of the essence. Epsilon is the well known base of natural logarithm; Pi has been noted to be the base the powers of which yield a sequence as the dimensions of the erythrocyte (Osuagwu, 2006). But "Pi also appears as the average ratio of the actual length and the direct distance between source and mouth in a meandering river" (Strolum, 1996;

www.mathworld.wolfram.com/pi.html, 2007). So these numbers are not just mathematical conventions, as at times assumed (<http://en.wikipedia.org/wiki/Pi>, 2007), but have important physical manifestations.

An earlier paper reported that epsilon (e), the base of natural logarithm, possibly, originated from the kinetic molecular nature of matter (Osuagwu, 1999). A "particle-in-a-pipe" model, employed in the derivation of e , depicted a randomly moving particle, with equal probability ($1/6 = 0.1667$) of moving in any of the six directions of three-dimensional space, trapped in a "heuristic" one-dimensional pipe. The model had the 'particle-in-the-pipe' accelerated in one direction by the injection of energy. On exiting from the theoretical pipe, it turned out that there had been an apparent redistribution of motional probabilities among the six coordinates around the particle in three-dimensional space, in favour of the direction of acceleration, as illustrated in the scheme below (see fig. 1):

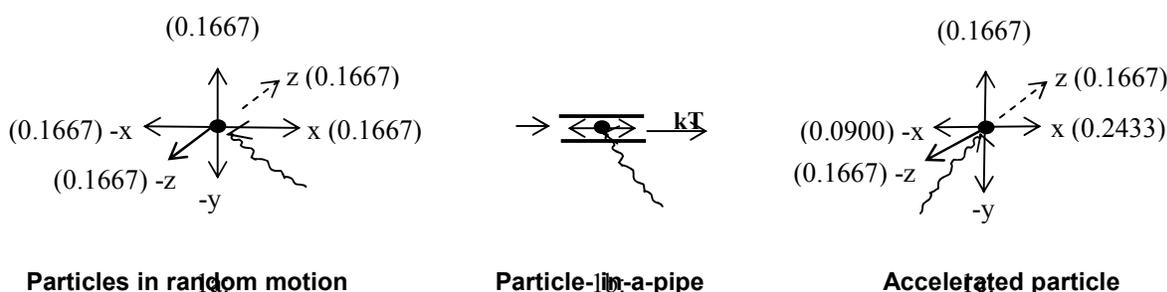


Fig 1: (a) Particle in random motion (b) Particle in a pipe; (c) Accelerated particle.

kT = quantum of thermodynamic action, where k is Boltzmann's constant ($k = R/6.02 \times 10^{23}$; R is gas constant) and T is Kelvin temperature.

It was observed that one quantum of thermodynamic action (RT), if a mole of particles is assumed, redistributed the motional probabilities along the x-axis from ratio of 0.1667: 0.1667 or 1.00: 1.00 to a new ratio of 0.0900: 0.2433 or 1.0: 2.7. Epsilon, e, is

$$e = 1 + \left\{ \sum_{n=1}^{\infty} 2 \left(\frac{1-n}{n} \right)^{0.7707} \right\}^{0.7707} = 2.7182... \quad \text{Eq. 1.}$$

This paper proposes an explanation for the origin of epsilon (e) in terms of the mass of particles (mole) instead of that of a single particle. The paper also suggests that pi (π) is one of the probability ratios that emerge, necessarily, from the particle-in-a-pipe model at the same time as epsilon (e). This suggests its origin, also, from the kinetic molecular nature of matter.

EPSILON (e) SUGGESTED AS THERMODYNAMIC MASS-PARTITION COEFFICIENT

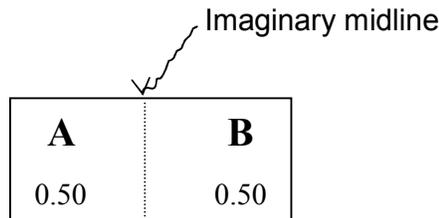
For the purpose of argument, consider a closed system, a 'heuristic' container, containing a mole of

thus the coefficient of energy-driven quantum partition in a system in equilibrium (Osugwu, 1999).

Estimation of the value of epsilon (e) from model

A systematic analysis had suggested an alternative method and an equation for the estimation of the value of epsilon (e) as:

particles (see the illustration of scheme 1). Kinetic theory suggests that all the molecules in the container will be in pure random motion (Dickerson, 1969). If, for the purpose of argument, an imaginary line is drawn through the center of the container it would, theoretically, partition the particles into two equal amounts (0.50 moles each) in two (virtual) compartments, which we designate as compartments A and B (scheme 1). The ratio of particles in each of the two compartments, A: B is 0.50: 0.50 or 1: 1.



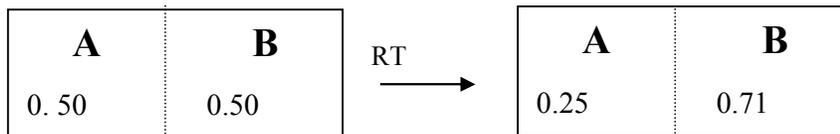
Scheme1: A closed system containing a mole of particles in two imaginary compartments (A and B).

Each virtual compartment has equal probability (0.50) of containing the particles. If it is supposed that a virtual pump is employed, to pump particles from the compartment A to the B compartment of the system, then the numerical value of particles on the A compartment will be decreasing, becoming less than 0.50 mole, while the numerical value in the B will, complementarily, be increasing, becoming more than 0.50 mole.

Derivation of Epsilon (e)

Mathematically, one magnitude of compression of the amount in A can be considered equivalent to one magnitude compression of the numerical value 0.50, which is a fraction: $(0.50)^2 = 0.25$. The inverse event will take place to expand the amount in the B compartment: $(0.50)^{1/2} = 0.71$. (The result of squaring a fraction is decreased numerical value, the converse of which is also true; this is treated, in this paper, as numerical equivalents of physical compression and expansion, respectively).

Scheme 2 illustrates, below, what would, numerically, approximate the new distribution:



Scheme 2: Distribution of mole of particles after one magnitude redistribution by directional supply of one quantum of energy (RT).

We note that the mathematical process of redistribution accounted for only, approximately, 0.96 moles ($0.25 + 0.71 = 0.96$ moles) of the 1.00 mole originally put in the system, before partition by energy input. The first law of thermodynamics, which requires that the mass in a closed system remain constant, also requires an accounting for the shortfall of 0.04 moles,

after the partition, for system integrity (note that $1.00 - 0.96 = 0.04$).

We note the necessary emergence of a boundary when an actual partition occurs between compartments A and B. It seems reasonable to suggest that the, apparently, missing amount (0.04 moles) has to be located in this shared boundary. If not it would

have to be located in either of the two compartments, to ensure the conservation of mass. The final distribution of particles in each compartment (A and B) will, therefore, have to include half this, theoretically, equally shared boundary amount for each compartment. This consideration should result in the following, suggested new distribution, between the two compartments:
 A=0.25 + 0.50 (0.04) = 0.25 + 0.02 = 0.27 moles
 B=0.71 + 0.50 (0.04) = 0.71 + 0.02 = 0.73 moles
 New ratio is 0.27: 0.73 moles = 0.27: 0.73 = 1.00: 2.70
 This implies that the kinetic molecular nature of matter is the probable origin of the parameter epsilon (e ≈ 2.71

...), the base of natural logarithm. This agrees with the finding of the earlier paper on this subject (Osuagwu, 1999).

PROBABLE PHYSICAL ORIGIN OF PI (π)

The finding that epsilon (e) is a ratio of the opposed forces on the x-axis (x and -x) of a particle that has been accelerated in the x-direction, suggested the possible existence of other ratios of probabilistic forces among the co-ordinates of accelerated particle with directional orientation in space (see fig. 2).

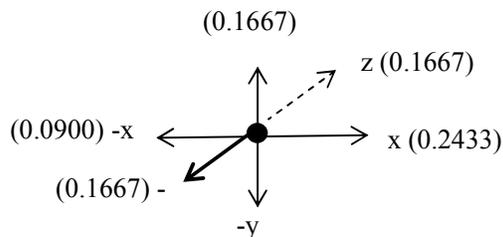


Fig. 2: Distribution of motional Probabilities on a Particle Accelerated by one quantum of Energy, kT (k = Boltzmann’s constant; T = Kelvin temperature).

On inspection, it seemed a number of other ratios of interest could be constructed from the consideration of the relationships between the different forces acting on the accelerated particle. One of these is the ratio of all the other forces to the force acting in the direction of

acceleration, such that (x, as earlier defined represents the force of acceleration along the x-axis or driving force on the particle; 1.00 represents the total sum of, probabilistic, forces acting on the particle):

$$\frac{1.00 - x}{x} = \frac{1.00 - 0.2433}{0.2433} = \frac{0.7567}{0.2433} = 3.1101... \quad \text{Eq. 2.}$$

This number (3.1101...) is a reasonable approximation of the natural constant pi (π), which is, conventionally, approximated to 3.1415...

Estimation of the value of pi (π) from model

It can be recalled, from the earlier paper (Osuagwu, 1999), that an equation for the, first approximation, estimation of the value of epsilon (e) is:

$$e = 1 + \sum_{n=1}^{\infty} 2 \left(\frac{1-n^2}{n} \right) \quad \text{Eq. 3.}$$

Noting the presence of epsilon and pi relationships as ratios on the same accelerated particle, reason suggests that the equations for estimating them are related. We also note that the relationships that yield pi (π) are more, generally, distributed in the three dimensions of space. It turns out that when the term “

” is operated upon, exponentially, with “n” to give the term a higher geometric character, in reflection of the multidimensional origin of pi (π), the following transformation results:

$$2 \left(\frac{1-n^2}{n} \right)^n = 2 \left(\frac{n-n^3}{n} \right) = 2 \left(1-n^2 \right) \quad \dots \text{Eq. 4.}$$

Now, the sum of the series described by the term “ $2 \left(1-n^2 \right)$ ” yields:

2(

This is an even closer approximation of pi (π), to the standard value ($3.1289/3.1416 = 0.9960$) and strengthens the possibility of common link to the kinetic

molecular nature of matter. Equations for the derivations of epsilon (ϵ) and pi (π) appear, mutually, interconvertible.

$$\sum_{n=0} 2^{(1-n^2)} = 2 + 1 + 0.125 \dots \text{etc} = 3.1289 \quad \dots \text{Eq 5}$$

Normalization, by inspection and the employment of a computational modulator, 0.91363, as outlined in the original paper on epsilon (Osuagwu, 1999), yields a much closer approximation of p (π) to the standard value: This result strengthens the suggestion that pi (π) originates, like epsilon (ϵ), from the kinetic molecular nature of matter.

$$\pi = 2 + \left\{ \sum_{n=1}^{\infty} 2^{\left(\frac{1-n^2}{n}\right)^{0.91363}} \right\}^{0.91353} = 3.14159\dots \quad \text{Eq. 6.}$$

CONCLUSION

This paper suggests that the parameters pi ($\pi = 3.14159\dots$), and epsilon ($\epsilon = 2.7182\dots$), which are important natural constants have their origin in the kinetic molecular nature of matter. Epsilon (ϵ) is associated with the redistribution of motion along the axis of particle acceleration in space in favour of direction of acceleration. Pi (π) describes the ratio of forces, on an accelerated particle, which tends to restore pure random motion in space to force of acceleration in a given direction. The manifestations of these natural constants in systems, therefore, reflect the dynamics of the processes at work within the systems. They are, therefore, approximately, derivable from the first physical principles of the kinetic molecular theory of matter. The kinetic molecular origin of these constants would explain the random behaviour of numbers, normality, observed in the estimations of pi (Bailey and Crandall, 2002; Lagarias, 2001).

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